OVERSTABLE SOLAR OSCILLATIONS OF INTERMEDIATE AND HIGH DEGREE

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ABSTRACT

The stability of linear acoustic modes trapped in the solar envelope is investigated with the thermal and mechanical effects of turbulence on the mean flow incorporated through turbulent conductivity, viscosity and turbulent pressure. A number of these modes turn out to be overstable, and the most rapidly growing acoustic modes are found to occupy a region centred around 3.2mHz and spread over a wide range of length scales. These numerical results are in reasonable accord with the observed power-spectrum of the five-minute oscillations.

A considerable amount of work has been done to study the oscillatory velocity field in the solar atmosphere. The work of Rhodes et al.(1977) and Deubner et al.(1979) has yielded a detailed power spectrum of the five-minute oscillations for high degree($\ell \gtrsim 150$) and opened up a new field of solar seismology to probe the solar interior. The observations have undoubtedly established that the five-minute oscillations represent non-radial acoustic modes in the solar envelope, thus confirming the suggestion of Ulrich(1970) and Leibacher and Stein(1971) regarding the nature of these oscillations. Later observations using integral sunlight (Claverie et al., 1979) revealed the existence of five-minute oscillations of low degree($\ell \lesssim 3$). Finally the recent observations by Duvall and Harvey(1983) have yielded the power spectrum of the five-minute oscillations in the intermediate range($1 \leqslant \ell \lesssim 150$) thus bridging the gap between the observations for high degree and those for low degree.

The excitation mechanisms for these modes have been studied by various authors. Ando and Osaki(1975) and Ulrich and Rhodes(1977) investigated the stability of nonradial oscillations for a solar envelope model with full effects of radiative dissipation included in the calculation and found many overstable acoustic modes with their periods centred around 300s and with a wide range of horizontal wave numbers. However the interaction between turbulent convection and oscillation was completely neglected in their work. Goldreich and Keeley(1977a) and Berthomieu et al.(1979) studied the influence of turbulent convection on the stability of acoustic modes to find that the turbulent viscosity stabilizes all these modes. Goldreich and Keeley(1977b) studied the stochastic excitation of these modes.

Antia et al.(1982, hereinafter referred to as paper I) studied the overstability of acoustic modes in the solar envelope model with mechanical and thermal effects of turbulence included in an approximate manner through the eddy transport coefficients. It was concluded that the simultaneous operation of the κ-mechanism and the turbulent conduction (convective Cowling) mechanism (Unno, 1976) is responsible for exciting the five-minute oscillations, the dominant contribution to the generation of self excited acoustic waves arising from the convective Cowling mechanism. The turbulent pressure however was neglected in this work. Later Antia et al.(1983) found that the turbulent pressure has a significant influence on the convective modes yielding a double peak in the curve of growth rate against horizontal wavelength. These two peaks can be readily identified with the two distinct scales of solar convection corresponding to granulation and supergranulation. In the present work we choose a solar envelope model which yields a reasonable agreement with observations concerning granulation and supergranulation and study the overstability of acoustic modes in such a model. In view of the recent observations of these modes at intermediate values of ℓ we have extended our solar model inwards to about 1/4 $R_\odot$ so that we can study the modes upto $\ell \approx 10$. The model yields reasonable agreement between the calculated and the observed frequencies.

The equilibrium solar envelope model was constructed using the mixing length formalism with mixing length $L = z + 459$ km where z is the depth measured from the level with optical depth $\tau = 1$. At the lower boundary of our envelope model the temperature is about 8.2×10^6 K and the energy generation in layers above this level is expected to be only about two percent of the total solar luminosity and hence the nuclear energy generation is not included. The upper boundary of the envelope model is placed at the temperature minimum and the atmosphere is assumed to be in radiative equilibrium. We study only those modes which are completely trapped in the envelope model, since the characteristics of untrapped modes may depend critically on the presence of adjoining layers. It turns out that only the modes for $\ell \gtrsim 10$ are trapped in the envelope model. In fact, even for $\ell = 10$ the higher frequency modes are not trapped and hence their frequencies will tend to be overestimated.

We have studied the stability of acoustic modes in the envelope model by using the usual linearization procedure (cf. paper I). We include the convective flux given by usual mixing length expression,

$$F^c = \alpha\rho W L C_p T(\nabla - \nabla')/H_p \ ,$$

where all symbols have their usual meaning, α is a parameter of order unity which in the present model is assumed to be 0.25. We have included the perturbation in F^c by assuming the same expression to be valid in the time dependent situation also. It is found that this perturbation plays a very significant role in destabilizing the acoustic modes. Further we have also included the turbulent viscosity given by

$$\nu_t = \sigma_t \alpha W L \min[\, 1, \, (\frac{\pi}{\tau_c \omega})^2 \,] \, ,$$

where σ_t can be considered as the turbulent Prandtl number, ω is frequency of the given mode and $\tau_c = L/W$ is the turnover time for the average convective eddies at that depth. The factor in the square brackets is supposed to ensure that contribution from only those eddies whose turnover time is less than half the period of given mode is included. We have adjusted the value of σ_t so as to get the best possible agreement between the length and time scale of calculated convective modes and observed values for granulation and supergranulation. This value is found to be 0.2 for the present model. Let us compare the typical magnitude of our turbulent viscosity with that used by other workers, say Goldreich and Keeley(1977 a). Their expression corresponds to $\sigma_t = 1$, $\alpha = 1$ and 1 in place of π. Thus we have a factor of 0.05 coming from $\alpha\sigma_t$ and over most of the convection zone the quantity $\pi/\tau_c\omega < 1$, giving an extra factor of order 10 coming from π^2. This makes our value of the viscosity roughly half that of Goldreich and Keeley.

The linearized equations including perturbation of convective flux, the turbulent viscosity and turbulent pressure are solved numerically to obtain the frequencies and the associated growth rates for a specified value of the horizontal harmonic number ℓ. It may be noted that the main emphasis of our calculations is on the stability of p-modes and we have made no effort to adjust the envelope model in order to match the calculated frequencies with observations. We choose a model which gives theoretically calculated convective modes in agreement with the observed features of granulation and supergranulation. It is encouraging to find that the same model yields frequencies which are in reasonable accord with observations of Deubner et al.(1979) for high ℓ and Duvall and Harvey(1983) for intermediate ℓ, as can be readily seen from Figure 1. It should be noted, however, that there is marked disagreement at low ℓ for higher harmonics. This could be due to our neglect of the solar interior region or the energy generation terms, which may affect the results somewhat. As noted earlier for $\ell = 10$ the acoustic cut off frequency at the lower boundary is about 4 mHz, but even for modes with frequencies somewhat lower than this, there should be some effect on account of penetration into the underlying layer which is likely to reduce the frequency of these modes. If we compare the present results with those of Ando and Osaki(1975) or with paper I, the computed frequencies are consistently lower, although the difference is small for $\ell \geqslant 40$; at lower ℓ the difference is rather large as these modes are not trapped in the truncated envelope model considered there. Finally it should be noted that the difference in our present frequencies and those in paper I is almost entirely due to small difference in model, while the turbulent pressure makes no noticeable difference as far as frequencies are concerned.

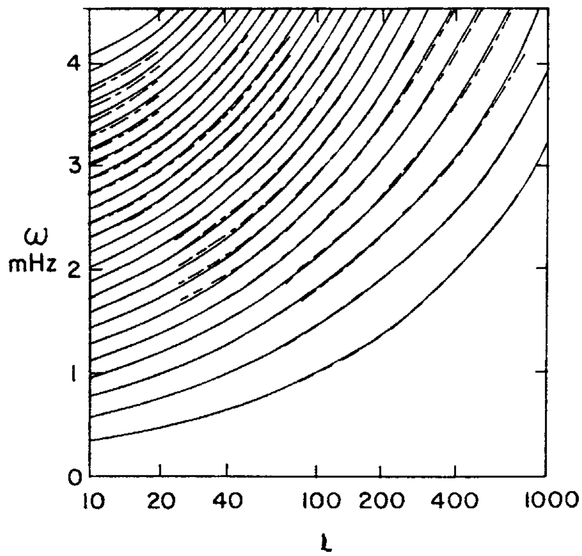

Fig.1: The frequencies ω as a function of ℓ for the first 25 p-modes(F-P24). The solid lines represent the frequencies calculated from the envelope model while the dashed lines show the observed frequencies.

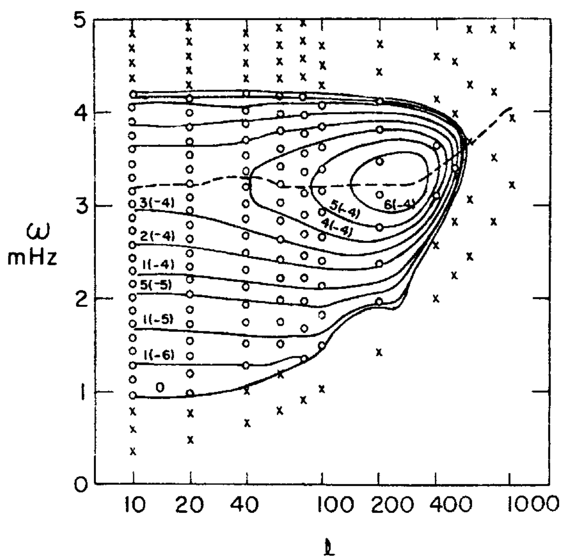

Fig.2: The diagnostic (ω - ℓ) diagram showing contours of equal stability coefficient η by which the contours are labelled. The stable modes are indicated by crosses(X) and unstable ones by open circles(O). The dashed line represents the locus of frequencies where η is maximum for a given ℓ.

The results of our stability analysis are displayed in Figure 2 which shows the contours of constant stability coefficient η(which is the ratio of growth rate to frequency) of given mode in the ω - ℓ plane. The outermost contour corresponds to the marginally stable case $\eta = 0$ within which all modes are unstable while the modes outside the region are all stable. The nearly horizontal dashed line in the middle marks the locus of frequency where the stability coefficient is maximum for a given ℓ. It can be seen that there is a distinct peak at $\ell \approx 300$ and $\omega \approx 3.2$ mHz, while the growth rates drop rather abruptly at higher ℓ but at lower ℓ the peak extends in the form of a rather broad ridge roughly parallel to ℓ-axis centred around a frequency of 3.2 mHz.

The most marked difference between our results and those of Ando and Osaki(1975) is that while we get closed contours yielding a distinct peak they have open contours with stability coefficient increasing with ℓ upto $\ell = 1500$ which is the largest value they considered. This is clearly due to the effect of turbulent viscosity which is more effective at high ℓ. We also find a high frequency cut off where $\eta = 0$ around 4.2 mHz more or less independent of ℓ, while they find the $\eta = 0$ contour going to higher frequency at high ℓ. Finally in our results the growth rates decrease with decreasing ℓ, while they find an increase in growth rate as ℓ decreases below about 30. This feature is also present in paper I and we believe it is due to the fact that for this range of values of ℓ the modes are not trapped in the envelope model truncated at $r \approx 0.5\ R_\odot$ and hence the growth rates tend to be overestimated.

We should try to compare our results with observations. Evidently, from our linear stability analysis it is not possible to find the relative amplitudes of individual modes. In fact it is not clear why the amplitudes of these modes are limited to such small observed values. Nevertheless it should be noted that here we are dealing with a very large number of modes simultaneously excited in the solar envelope model and it is not clear how they will interact with each other and with the general field of turbulence in the convection zone. Under these circumstances the best we can do is to try some sort of qualitative comparison between theory and observations based on the assumption that only those modes which have significant growth rate will have substantial power in observations. With this assumption it can be seen that the region of significant growth rate(say $\eta > 5\times10^{-5}$) in Figure 2 roughly coincides with region in which significant power has been observed. In particular it may be noted that the high frequency cut off in our results at around 4.2 mHz roughly agrees with observations of Duvall and Harvey(1983). Further as we go to low values of ℓ, the lower harmonics are either stable or have a small growth rate which is consistent with low observed power in these harmonics at low ℓ. For example, for $\ell = 10$ F, P1, P2 modes are stable while P3 to P10 modes have η less than 5×10^{-5}. On the other hand, we find the F-mode to be always stable, although Deubner et al.(1979) have detected some power in this mode and also we find all the modes to be stable for $\ell \gtrsim 600$ where again some power is observed. This could be the result of an overestimate of turbulent viscosity in the atmosphere.

Finally, regarding the excitation mechanism for these modes, in addition to the usual κ-mechanism and the Cowling-Spiegel mechanism, we have also included the perturbation in convective flux or the convective Cowling mechanism, to find that this makes the most significant contribution in destabilizing these modes(cf. paper I). In particular the convective Cowling mechanism is found to be sufficiently strong to overstabilize the acoustic modes by itself. This is probably the result of the turbulent conductivity being much larger than the radiative conductivity in the convection zone. Christensen-Dalsgaard and Frandsen (1982) have pointed out that the κ-mechanism tends to stabilize the acoustic modes if the departure from radiative equilibrium in the atmosphere is taken into account. Our calculations do support their conclusion but even in that situation the convective Cowling mechanism is found to overstabilize the p-modes provided the viscous damping is not too strong. In the absence of turbulent viscosity almost all the modes in the range of Figure 2 turn out to unstable, but the turbulent viscosity ofcourse tends to stabilize these modes, the effect being more pronounced for higher ℓ or higher harmonics. Furthermore, we also find that the turbulent pressure tends to destabilize these modes. It will be most interesting to examine the role of convective Cowling mechanism in driving the oscillations in other stars.

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