SOLAR FIVE-MINUTE OSCILLATIONS

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ABSTRACT

The stability of acoustic modes trapped in the solar envelope is studied in the framework of linearized perturbation theory, by incorporating the mechanical and thermal effects of turbulence on the mean flow through the eddy transport coefficients. Many of these acoustic modes are found to be overstable, with the most rapidly growing modes occupying a region centred around 3.2 mHz spread over a wide range of length-scales. The numerical results turn out to be in reasonable agreement with the observed power-spectrum of the five-minute oscillations. It is demonstrated that these oscillations are most likely to be driven by a simultaneous operation of the k-mechanism and the turbulent conduction mechanism, the dominant contribution to the generation of self-excited acoustic waves arising from the turbulent mechanism.

The discovery of solar five-minute oscillations by Leighton, Noyes and Simon (1962) has provided a very valuable tool to probe the interior of the sun. The science of solar seismology originated with the observations of Deubner (1975) who resolved the spatial and temporal structure of these oscillations. A detailed power-spectrum of the five-minute oscillations of high degree (spherical harmonic degree $\ell > 150$) was provided by the work of Rhodes, Ulrich and Simon (1977) and Deubner and Rhodes (1979). The low degree oscillations ($\ell < 3$) were detected by Claverie et al (1979) and Grec et al (1980) using the integrated sunlight and the gap between the observations of high degree and those of low degree was bridged by the recent observations of solar oscillations of intermediate degree ($1 < \ell < 150$). These observations have confirmed the suggestion of Ulrich (1970) and Leibacher and Stein (1971) that the five-minute oscillations represent non-radial acoustic modes trapped in the solar envelope.

The important question of the excitation mechanism responsible for these oscillations was addressed by Ando and Osaki (1975) and Ulrich and Rhodes (1977). The stability of non-radial oscillations in a realistic solar envelope model was investigated by these authors with full effects of radiative exchange included, although the interaction between turbulent convection and oscillation was neglected. This situation was remedied by Goldreich and Keeley (1977) by incorporating the influence of turbulent convection on the stability of acoustic modes which were shown to be stabilized by the presence of turbulent viscosity.

It is well known that, except for the top few tens of kilometers, the major fraction of the total flux in the solar envelope is transported by convection. Furthermore, the turbulent conductivity is much larger than the radiative conductivity for the most part of the convection zone. Therefore, the turbulence is expected to play a major role in modulating the heat flux and oscillations. This prompted Antia, Chitre and Narasimha (1982) to embark on a study of the over-stability of acoustic modes in the solar envelope by approximately including the mechanical and thermal effects of turbulence through the eddy transport coefficients.

The governing equations for this purpose are the usual hydrodynamical equations:

(mass) $$\frac{\partial \rho}{\partial t} + \nabla . (\rho \underline{v}) = 0;$$

(momentum) $$\rho \frac{\partial \underline{v}}{\partial t} + \rho (\underline{v} . \nabla) \underline{v} = \rho \underline{I} - \nabla P - \nabla P_t$$

$$- \frac{2}{3} \mu_t \nabla (\nabla . \underline{v}) - \frac{2}{3} (\nabla . \underline{v}) \nabla \mu_t + \nabla . \left[\mu_t (\nabla \underline{v} + \underline{v} \nabla) \right];$$

(energy) $$\rho T (\frac{\partial s}{\partial t} + \underline{v} . \nabla s) + \rho (\frac{\partial E_t}{\partial t} + \underline{v} . \nabla E_t) + P_t \nabla . \underline{v} = - \nabla . \underline{F}.$$

Here μ_t is the coefficient of turbulent viscosity, $\underline{P}$ the thermodynamic pressure including gas and radiation pressure, $\underline{P}_t$ the turbulent pressure, E_t the turbulent energy density, $\underline{F}$ is the total flux made up of the radiative flux, $\underline{F}^R$ and convective flux, $\underline{F}^C$. The radiative flux calculated in the Eddington approximation is given by

$$\underline{F}^R = - \frac{4}{3k\rho} \nabla J,$$

where

$$J = \sigma T^4 - \frac{\nabla . \underline{F}^R}{4k\rho} .$$

The convective flux is computed adopting the standard mixing length formalism. Thus,

$\underline{F}^C = -K_t \frac{T}{C_p} \underset{\sim}{\nabla} s$, where the turbulent heat conductivity $K_t = \alpha \rho C_p W L$. In this expression, α is an efficiency factor of order unity, L is the mixing length and W the mean convective velocity. The foregoing equations are linearized and the generalized eigenvalue problem is solved numerically with realistic boundary conditions to obtain the complex eigenvalues.

Many of the acoustic modes trapped in the solar envelope turn out to be overstable with the most unstable modes occupying a region centred around a period of 300 s. The calculations in fact show that turbulent heat exchange plays a significant role in destabilizing the acoustic modes. In an earlier analysis Ando and Osaki (1975) had concluded that the acoustic modes are largely overstabilized by the k-mechanism operating in the hydrogen ionization zone, while our analysis indicated that a simultaneous operation of the k-mechanism and the turbulent conduction mechanism is responsible for the excitation of the five-minute oscillations, the latter making the dominant contribution to the generation of self-excited acoustic waves. It should be stressed that both the radiative and turbulent conduction mechanisms have their origin in the strongly superadiabatic region near the surface, but the efficiency of the turbulent mechanism is larger by a factor (cf Unno, 1976)

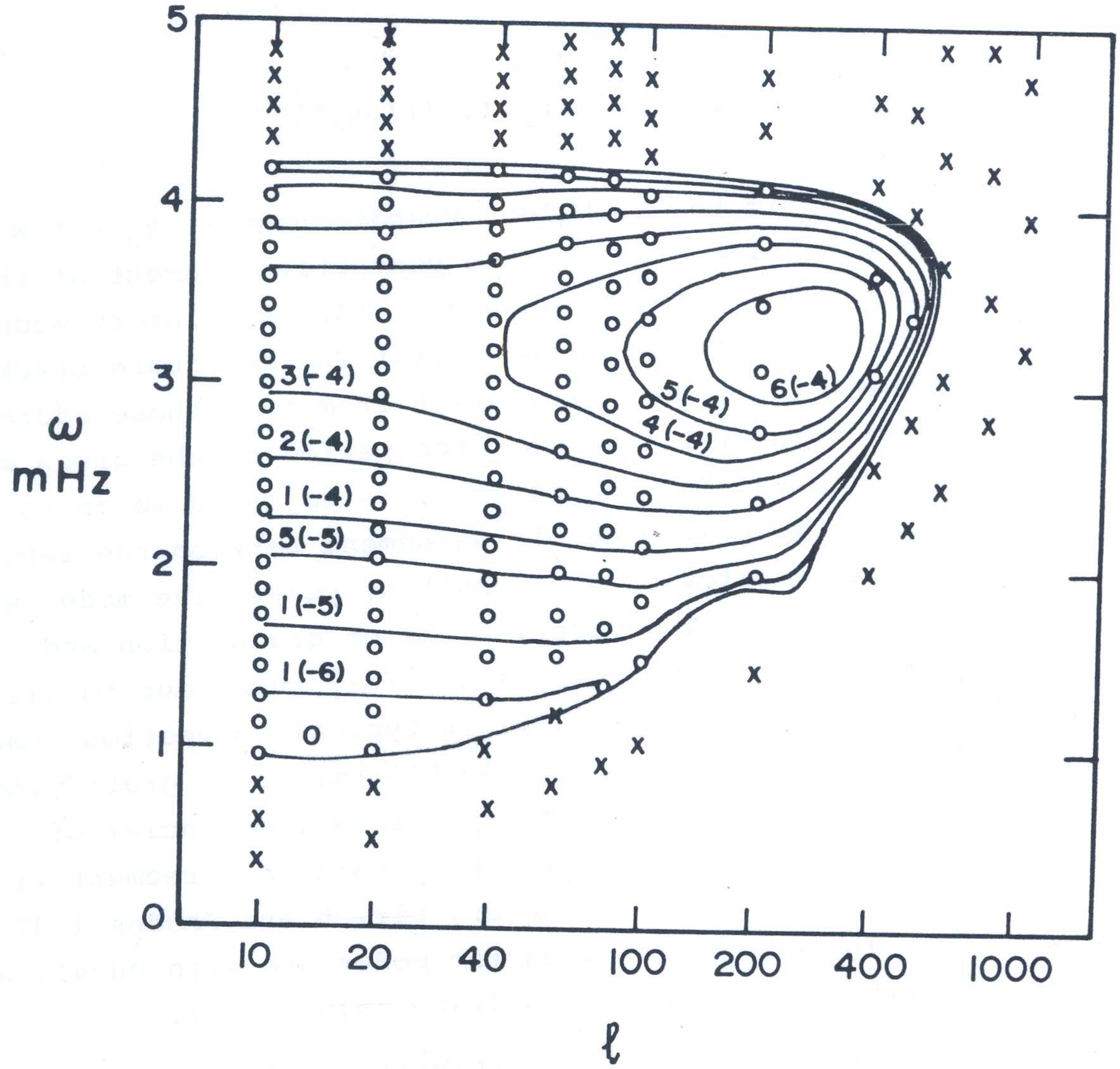
$$\frac{F^C}{F^R} \quad \frac{\nabla}{\nabla - \nabla_{ad}} .$$

In adopting an expression for the turbulent viscosity we follow essentially the treatment of Goldreich and Keeley (1977) and write

$$\nu_t = \sigma_t \; \alpha \; WL\left[\min\left\{1, \; (1/\omega t_c)^2\right\}\right],$$

where σ_t is the turbulent Prandtl number, $t_c = L/W$ is the turn-over time for the convective element at that depth and ω the frequency of the oscillatory mode under consideration. The factor in the square brackets ensures that the contribution from only those eddies whose turn-over time is $\lesssim$ the period of the given mode is included. The parameter σ_t is adjusted so as to obtain the best possible agreement between the length and time-scales of the resultant convective modes with the corresponding observations of granulation and supergranulation. The value of σ_t turns out to lie in the range of 0.2 - 0.3 for a typical convection zone having a thickness of ~ 200,000 km. It is gratifying to find that the same model yields frequencies of acoustic modes which are in reasonable agreement with the observations of Deubner, Ulrich and Rhodes (1979) for high degree (ℓ) acoustic modes and with Duvall and Harvey (1983) for intermediate values of ℓ.

The results of the stability calculations are displayed in the accompanying figure which shows the contours of constant stability coefficient η (= growth rate/frequency) of a given acoustic mode in the horizontal wave number, k_h-frequency, ω diagram. The outermost contour corresponds to the marginally stable case (η = 0), within which all the modes are unstable, while the modes outside this region are stable. We make the plausible assumption that only those modes with significant growth rates will have substantial observed power. It is interesting to note that the region in the figure where $\eta > 10^{-4}$ approximately coincides with the region where substantial amount of power has been observed. In particular, the high frequency cut-off yielded by our analysis around 4-5 mHz, more or less



The contours of equal stability coefficient η are shown in the ($\omega - \ell$) diagram where the crosses (X) denote the stable modes and open circles (0) the unstable modes.

independent of ℓ, is in rough agreement with the observations of Duvall and Harvey (1983). Furthermore, for low ℓ, the lower harmonics are either stable or have an extremely small growth rate - this is consistent with the low observed power in these harmonics.

Our analysis indicates, in agreement with the earlier results of Ando and Osaki (1975) that the most unstable acoustic modes are spread over a region centred mainly around 3.2 mHz with a wide range of horizontal length scales. However, there is one significant difference, namely, we get closed contours of the stability coefficient η with a distinct peak, while Ando and Osaki have open contours with η increasing with ℓ. This is clearly the influence of turbulent viscosity included in our work which because of its effectiveness at short length scale end decreases the growth rates at high ℓ.

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