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NEW LIMITS TO BIAS AND THE AMOUNT OF DARK MATTER IN THE UNIVERSE

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ABSTRACT

The observed thermodynamic distribution function $f(N)$ for galaxies places significant constraints on the amount of structured dark matter in the universe. The simplest models of cold dark matter require the cosmological density parameter $\Omega_0 \lesssim 0.4$. Biased galaxy formation in more complicated models must have specific forms which depend on the amount and structure of dark matter in the model.

Subject headings: cosmology — galaxies: clustering — galaxies: formation

I. INTRODUCTION

The value of Ω_0, the ratio of the average density in the universe to the density needed to just close the present Einstein-Friedmann models, continues to tantalize cosmologists. Luminous matter in the universe contributes a value of $\Omega_0 \approx 0.03$ (Faber and Gallagher 1979). Dark matter within galaxies, whose presence may be indicated by extended flat rotation curves, may increase this value by a factor ~ 3 (Rubin *et al.* 1982). Additional dark matter between galaxies suggested by the virial masses of clusters and groups of galaxies could increase Ω_0 by a further factor ~ 2 (Faber and Gallagher 1979), although there is now evidence (Davies 1985) that earlier virial mass estimates for many groups may have been too high. The resulting value of $\Omega_0 \approx 0.1$–0.2 is also the maximum baryon density consistent with the observed deuterium/hydrogen ratio in standard models of nucleosynthesis (Yang *et al.* 1984).

If the additional matter needed to bring the value of Ω_0 up to unity exists, it is therefore likely to be nonbaryonic and to be distributed more smoothly than galaxies so as not to conflict with the virial cluster estimates. Recent motivation for $\Omega_0 = 1$ has come from the requirements of inflationary cosmologies, and many hypothetical particles have been suggested as major constituents of the dark matter (see Blumenthal *et al.* 1984 for a review). The smoother distribution of dark matter has given rise to the notion of "biased galaxy formation" in which luminous galaxies may form preferentially in regions of greater large-scale density enhancement (e.g., Kaiser 1984; Schaeffer and Silk 1985; Rees 1985*a*, *b*; Silk 1985; Umemura and Ikeuchi 1985).

In this *Letter*, we show that there are significant constraints on the amount of dark matter and on the physical processes which may produce bias. These constraints are independent of the detailed nature of the dark matter as long as it gravitates. Our results follow from the observed large-scale distribution of galaxies.

II. CONSTRAINTS ON THE AMOUNT OF DARK MATTER AND THE FORMS OF BIAS

The $f(N)$ statistic is a useful way to characterize the large-scale distribution of galaxies. This statistic is just the probability of finding N galaxies in a volume V whose average number density is $n = \bar{N}/V$. For a homogeneous isotropic distribution, $f(N)$ is essentially independent of the shape of the volume V. Therefore, it is especially useful to describe the two-dimensional area projection of truncated cones on the celestial sphere. Although $f(N)$ contains much more information about the galaxy distribution than the low-order correlation functions which are often used, it does not describe the distribution completely (see Saslaw 1985*a*).

Recent work (Crane and Saslaw 1986) shows that the observed $f(N)$ distribution for the Zwicky catalog of galaxies is described very well by the function

$$f(N) = \frac{\bar{N}(1-b)}{N!}\left[\bar{N}(1-b) + Nb\right]^{N-1} e^{-\bar{N}(1-b)-Nb}. \quad (1)$$

This distribution was predicted (Saslaw and Hamilton 1984) for the asymptotic state of gravitational clustering of point masses in an expanding universe. Here $b \equiv -W/2K$ is essentially the ratio of gravitational correlation energy to peculiar energy. For an asymptotically relaxed system, both N-body simulations (Miyoshi and Kihara 1975; Saslaw and Hamilton 1984; Saslaw 1985*b*) and simple theory (Saslaw 1986) based on the cosmic energy equation and the BBGKY hierarchy predict $0.65 \lesssim b \lesssim 0.75$. The value of b which best fits the observations is $b = 0.70 \pm 0.05$ (Crane and Saslaw 1986).

Computer N-body experiments also show (Saslaw 1985*b*) that a fairly wide range of homogeneous, initial conditions,

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both Poisson and non-Poisson, relax to the distribution of equation (1). This does not depend significantly on the mass spectrum of the galaxies, since their later clustering is dominated by the attraction of large density inhomogeneities containing many galaxies. Very inhomogeneous initial states, some cold, dark matter scenarios, and artificial lattice states, however, may not be able to relax to equation (1) in the available Hubble time. All biased models of galaxy clustering should be checked for their agreement with the observed $f(N)$ distribution. So far, this seems to have been done only in a preliminary way for $N = 0$ in an explosion scenario dominated by subsequent gravitational clustering (Saarinen, Dekel, and Carr 1986). However, it is important to show agreement for all values of N (insofar as practical) since each $f(N)$ emphasizes different aspects of clustering. There are many distributions which fit equation (1) either for small values of N, or for large values of N, but not for both.

The agreement between the theoretical result for simple gravitational clustering predicted by equation (1) and the distribution observed for luminous galaxies would be destroyed by any significant biasing. There are various models of bias. In some, the dark matter is collected into massive non-luminous galactic structures (e.g., Rees 1985*a*, *b*); in others the dark matter is more diffusely distributed. If all galactic structures, luminous as well as dark, share the same $f(N)$ distribution, then luminous galaxies will be biased by the *selection process* which makes them light up. If dark and luminous structure does not have the same $f(N)$ distribution, there will be an additional bias to the $f(N)$ distribution of the luminous galaxies caused by the *different distributions* which evolve gravitationally.

We illustrate some limits which $f(N)$ imposes on bias by considering a fiducial model based on random selection. Although this may not be a "realistic" model in itself, we emphasize it here for two reasons. First, it is the simplest model; it has only one adjustable parameter so its behavior is clear. Second, it is the least biased model, so that its departures from observations show clearly where more bias is needed to obtain better agreement. We prefer this point of view which starts close to the observations, because the physical processes which may produce bias are very uncertain at present.

The fiducial model assumes that the dark matter is collected into dark galaxies (cf. Rees 1985*a*, *b*). Let the distribution of matter in the universe be characterized by $f(N)$, the probability that there are a total of N galactic structures (luminous or dark) in a volume of size V. Let $f_L(N)$ and $f_D(N)$ denote the respective probabilities of finding N luminous and N dark galaxies in V. It is easy to apply this description to different levels of luminosity by supposing that $f_L(N)$ galaxies can be detected to a given apparent magnitude and $f_D(N)$ are not detectable. Then we may write

$$f_L(N) = \sum_{M=N}^{\infty} \mathscr{P}(N; M) f(M), \tag{2}$$

where $\mathscr{P}(N; M)$ is the probability that N luminous galaxies are found in a volume containing a total of M galaxies.

We next suppose that luminous galaxies occur independently at random throughout the whole distribution and the average number of luminous galaxies in a volume is $\bar{N}_L$. The only bias in the fiducial model is random selection of luminous galaxies. This could occur if the reason for a galaxy becoming luminous were purely internal, unaffected by its environment. Then the average probability for finding a luminous galaxy is

$$p = \frac{\bar{N}_L}{\bar{N}} \tag{3}$$

and $\mathscr{P}(N; M)$ is the binomial distribution for N successes in M trials:

$$\mathscr{P}(N; M) = \binom{M}{N} p^N q^{M-N}, \tag{4}$$

where $q = 1 - p$ and the binomial coefficient $\binom{M}{N} = M!/[N!(M-N)!]$. Therefore,

$$f_L(N) = \sum_{M=N}^{\infty} \binom{M}{N} p^N q^{M-N} f(M). \tag{5}$$

For computations it is useful to have the generating function of $f_L(N)$ which is

$$\sum_{N=0}^{\infty} f_L(N) z^N = \sum_{N=0}^{\infty} f(N)(q + pz)^N. \tag{6}$$

Equating the coefficients of powers of z on both sides of equation (6) gives $f_L(N)$ for all N as a function of p.

For any form of $f(M)$, we can use equation (5) or (6) to determine the range of p giving $f_L(N)$ consistent with observations. This gives an upper limit to the amount of dark matter in the model. For a numerical example we take $f(M)$ to be given by equation (1), as would be expected if the total distribution is gravitationally relaxed. Figure 1 shows the result for $f_L(N)$ using values of $p = 2/3$, $1/2$, $1/3$, $1/4$, and $1/6$. The solid line fits the observed $f_L(N)$ histogram from Figure 1 of Crane and Saslaw (1986) for $2° \times 5°$ areas of the Zwicky catalog at galactic latitudes $> 30°$, a typical distribution. All curves are normalized to the observed value of the peak.

The observed spread $\Delta b = 0.05$ in the value of b is equivalent to a percentage uncertainty in $f(N)$ of approximately $\Delta f/f = f^{-1}(\partial f/\partial b)\Delta b$ for $\Delta b \ll 1$. This percentage uncertainty in f will depend on N. For a value $\bar{N} = 20.53$, appropriate to Figure 1, we find that for $N = 5$, 10, and 15, the respective values of $|\Delta f/f|$ are 0.3, $\lesssim 0.0002$, and 0.12. Note that values of f in the neighborhood of $N = 10$ are not affected much by the spread in b because all the curves of $f(N)$ for this range pass through nearly the same value at $N = 10$. [See Saslaw 1985*b* for examples of the effect of Δb on $f(N)$.] The important point, however, is that the observed uncertainties in $f(N)$ are not strongly systematic.

Although the value of $p = 2/3$ may be marginally consistent with uncertainties in the data, the distributions for

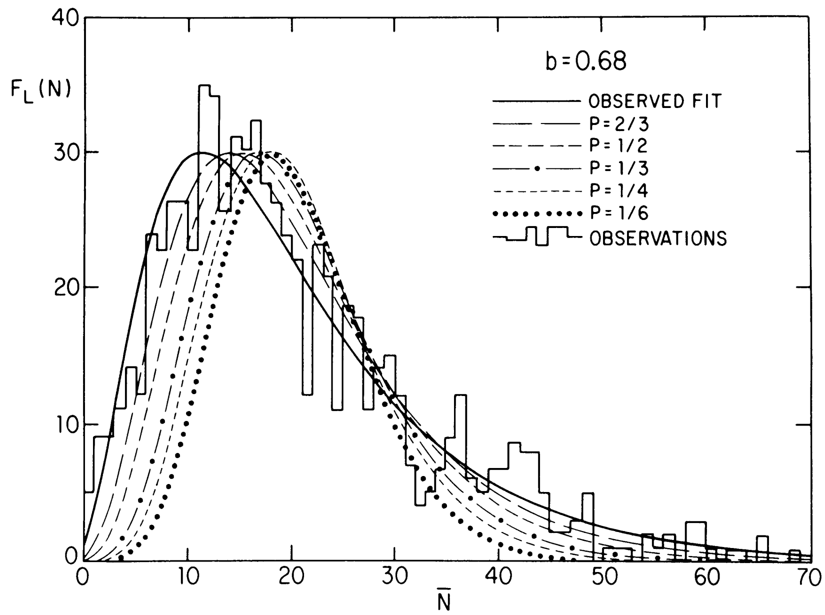

FIG. 1.—The distribution $f_L(N)$ as a function of N for a luminous subset which is a randomly selected fraction p of all galactic structures. The solid line taken from Crane and Saslaw (1986) is the best fit to the observed distribution for $2° \times 5°$ areas of the Zwicky catalog above galactic latitude 30°.

$p \lesssim 1/2$ do not agree with observations. The disagreement is systematic: the distributions for $p \lesssim 1/2$ have too low probabilities at small N (because a random selection reduces small-scale correlations), their most probable value of N is too high, and their tail is too low. These results also hold relative to the other observed distributions in Crane and Saslaw (1986).

There is a possible caveat to this result. Random selection produces a less correlated sample than the original distribution. Thus it may be possible to reproduce the observed $f(N)$ distribution by random selection from a more clustered population with a larger value of b. We have checked this and found that it is indeed the case. For example, random selection with $p = 1/3$ from a total thermodynamic $f(N)$ distribution having $b = 0.81$ and $\bar{N} = 3\bar{N}_L$ also agrees very well with the observed $f_L(N)$ distribution. However, this interpretation would have two basic problems. First, the Zwicky catalog which is the basis of the observational analysis is reasonably complete and uniform to an apparent $m_{\rm Zw} \approx 14.5$ and there is no reason why the $f(N)$ statistic should be modified substantially by a fairly bright *apparent* magnitude limit. In other words, galaxies of $m_{\rm Zw} < 14.5$ should, on average, have the same gravitational correlations and trace the mass distribution just as well as apparently fainter luminous galaxies. Moreover, an apparent magnitude limit does not give a random selection of galaxies. The second basic problem with fits having $p < 1$ and $b \gtrsim 0.75$ is that the computer N-body simulations mentioned previously predicted an asymptotic value $0.65 \lesssim b \lesssim 0.75$ for galaxy clustering under a wide range of initial conditions and expansion rates. Furthermore, kinetic theory also gives $b \approx 0.70$. Therefore, it seems that $b \approx 0.70$ with $p \gtrsim 1/2$ is the most likely interpretation of Figure 1.

Examination of the $f_L(N)$ distribution as a function of scale for particular small values of N, including the void distribution $f(0)$, confirms that values of $p \lesssim 1/2$ fail to fit the observations.

Therefore, we conclude that if luminous and dark structures have an $f(N)$ distribution which characterizes simple gravitational clustering, and luminosity occurs at random, the number of dark structures is less than the number of luminous structures. The total mass in dark structures will be less than in luminous structures if the average dark structure has less mass, as models for their darkness suggest.

The fiducial model provides some insight into the properties of more complicated biased models. For example, luminous galaxies might not be selected randomly from the total distribution. Figure 1 [and corresponding figures for $f(0)$, $f(1)$, etc.] would then give the quantitative bias function —the difference between the observed distribution and the random selection distributions for $p < 1$ which any physical theory of bias must satisfy. For values of $N \gg \bar{N}$, Figure 1 shows that if $P \lesssim 1/2$ the luminous distribution $f_L(N)$ must be increased to agree with the observed $f(N)$, i.e., there must be a greater than random probability for finding luminous galaxies in regions of large overdensity. This agrees with current prejudices about bias. The surprise, however, is that Figure 1 shows that underdense regions must also have a greater than random probability of containing luminous galaxies, and somewhat overdense regions must have a less than random probability of containing luminous galaxies. Therefore the constraints that $f(N)$ places on biased galaxy theories are quite significant. More complicated theories of bias could be treated by modifying $\mathscr{P}(N; M)$ in equation (4).

In these more complicated theories of bias, the $f(N)$ distributions of dark and luminous structures could have different forms, not only because of selection, but because they have different physical origins. For example the dark structure distribution might be gravitationally relaxed, but luminous galaxies might result from explosions (Ostriker and Cowie 1981; Ikeuchi 1981). Or, luminous galaxies may result from the fragmentation of unrelaxed large-scale filaments (Zel'dovich, Einasto, and Shandarin 1982). Or dark matter may be much more uniformly distributed and not clumped into galactic structures.

In these more general cases, the initial distributions of luminous and dark matter, their inevitable redistribution caused by gravity, and their biased selection would have to combine (some might say conspire) to yield the same observed $f_L(N)$ distribution which is predicted as the asymptotic state of simple gravitational clustering.

III. CONCLUSION

The agreement between the predictions of nonlinear gravitational clustering theory, *N*-body experiments, and the observed probability distribution $f(N)$ for finding N galaxies in an area of given size on the sky provides significant constraints on the amount and distribution of structured, nonuniform dark matter in the universe. Theories in which luminous galaxies are selected at random from a gravitationally clustered distribution of dark and luminous galaxies fit the observed $f(N)$ distribution only if the amount of dark matter is less than about half the total matter content, or if the actual value of b is considerably greater than the observed value. Theories in which luminous galaxies are selected in a biased way from a gravitationally clustered distribution of dark and luminous galaxies must have a positive selection bias at both low and high densities, and a negative bias at intermediate densities. Theories in which luminous galaxies are selected from a non–gravitationally clustered distribution must carefully adjust both the selection bias and the total population distribution to obtain the observed $f(N)$ distributions.

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