# New Constraints On Lepton Nonconserving R-parity Violating Couplings 

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#### Abstract

Strong upper bounds are derived on certain product combinations of lepton nonconserving couplings in the minimal supersymmetric standard model with explicit $R$-parity violation. The input is information from rare leptonic decays of the long-lived neutral kaon, the muon and the tau as well as from the mixings of neutral $K$ - and $B$-mesons. One of these bounds is comparable and another superior to corresponding ones obtained recently from neutrinoless double beta decay.


[^0]The minimal supersymmetric standard model MSSM [1] is now a leading candidate for physics beyond the standard model. A natural question to ask is whether global conservation laws (such as those of baryon number $B$ and lepton number $L$ ), valid in the standard model, hold as one goes to MSSM. This is related to the question of $R$-parity $\left(R_{p}=(-1)^{3 B+L+2 S}\right.$, with $S$ the intrinsic spin $[2]$ ) conservation for which no credible theoretical argument is in existence. It is therefore important to consider the phenomenology of possible $R_{p}$-violating terms [3, [] in the Lagrangian. We direct our attention to scenarios [3, 4] of explicit $R_{p}$ violation instead of those with a spontaneous one since the former can be accommodated within the particle/sparticle spectrum of MSSM while the latter needs at least one additional singlet superfield.

It is difficult to accommodate both $L$-violating and $B$-violating $\mathbb{R}_{p}$ terms within the constraints of proton decay [5]. Assuming that the baryon-number violating terms are identically zero [6] helps evade this constraint in a natural way, apart from rendering simpler the cosmological requirement of the survival of GUT baryogenesis through to the present day. This survival can then be assured if at least one of the lepton numbers $L_{i}$ is conserved over cosmological time scales [7].

The $R_{p}$-violating and lepton-nonconserving part of the superpotential, with the MSSM superfields in usual notation, involves two kinds of couplings $\lambda_{i j k}$ and $\lambda_{i j k}^{\prime}(i, j, k$ being family indices), and can be written as

$$
\begin{equation*}
W=\frac{1}{2} \lambda_{i j k} L_{i} L_{j} \overline{E_{k}}+\lambda_{i j k}^{\prime} L_{i} Q_{j} \overline{D_{k}}, \tag{1}
\end{equation*}
$$

where $\lambda_{i j k}=-\lambda_{j i k}$. We have here omitted possible bilinear terms as they are not relevant to the discussion. Most phenomenological studies of these couplings have been aimed at deriving upper bounds [8] on the magnitudes of individual $\lambda$ - or $\lambda^{\prime}$-coupling in terms of observed or unobserved processes. However, recently there has been some interest in deriving bounds on the products [9, 5, 10, [1] of two such couplings which are stronger than the products of known bounds on the respective individual couplings. This will be the approach taken here.

Before such an analysis is attempted, one needs to consider the possible effects of fermion mixing on the $\not R_{p}$ sector [5, 11], if only because this can itself be a cause for the simultaneous presence of more than one such coupling. These effects are twofold. For one, it is, in some sense, more natural to consider eq.(1) to be defined in the gauge basis rather than in the mass basis. In that case, even if there were only one particular nonzero $\lambda^{\prime}$-coupling, quark mixing would generate a plethora of such couplings, albeit related to one another. In this Letter, we do not confine ourselves to this particular theoretical motivation, but rather consider the most general lepton number violating $\mathbb{R}_{p}$ sector. The second effect is more subtle. Since the Cabibbo-Kobayashi-Maskawa matrix $K$ is different from identity, the $S U(2)_{L}$ symmetry of the $\lambda^{\prime}$ terms is no longer manifest when rexpressed in terms of the mass eigenstates. To wit,

$$
\begin{equation*}
W_{\lambda^{\prime}}=\lambda_{i j k}^{\prime}\left(N_{i} D_{j}-\sum_{p} K_{j p}^{\dagger} E_{i} U_{p}\right) \overline{D_{k}}, \tag{2}
\end{equation*}
$$

where the $\lambda_{i j k}^{\prime}$ have already been redefined to absorb some field rotation effects. In eq.(2) we have chosen to suppress the effects of possible non-alignment of fermion and sfermion mass
matrices. However, since any such non-alignment is already constrained severely [12], it is reasonable to neglect such interplay of different effects.

In terms of the component fields, the interaction term in the Lagrangian density can then be expressed as

$$
\begin{align*}
\mathcal{L}_{I}= & \lambda_{i j k}\left(\tilde{\nu}_{i L} \overline{e_{k R}} e_{j L}+\tilde{e}_{j L} \overline{e_{k R}} \nu_{i L}+\tilde{e}_{k R}^{\star} \overline{\left(\nu_{i L}\right)^{C}} e_{j L}\right) \\
+ & \lambda_{i j k}^{\prime}\left[\tilde{\nu}_{i L} \overline{d_{k R}} d_{j L}+\tilde{d}_{j L} \overline{d_{k R}} \nu_{i L}+\tilde{d}_{k R}^{\star} \overline{\left(\nu_{i L}\right)^{C}} d_{j L}\right.  \tag{3}\\
& \left.\quad-\sum_{p} K_{j p}^{\dagger}\left(\tilde{e}_{i L} \overline{d_{k R}} u_{p L}+\tilde{u}_{p L} \overline{d_{k R}} e_{i L}+\tilde{d}_{k R}^{\star} \overline{\left(e_{i L}\right)^{C}} u_{p L}\right)\right]+ \text { h.c. }
\end{align*}
$$

Here $e(\tilde{e})$ stands for a charged lepton (slepton) and the fields for the other particles have been designated by the corresponding name letters.

| Quantity | Experimental <br> value (bound) | Quantity | Experimental <br> value (bound) |
| :--- | :--- | :--- | :--- |
| $\delta m_{K}$ | $3.5 \times 10^{-12} \mathrm{MeV}$ | $\delta m_{B}$ | $3.4 \times 10^{-10} \mathrm{MeV}$ |
| $\operatorname{Br}\left(K_{L} \rightarrow \bar{e} e\right)$ | $<4.1 \times 10^{-11}$ | $\operatorname{Br}\left(K_{S} \rightarrow \bar{e} e\right)$ | $<1.0 \times 10^{-5}$ |
| $\operatorname{Br}\left(K_{L} \rightarrow \bar{\mu} \mu\right)$ |  |  |  |
| $\operatorname{Br}\left(K_{L} \rightarrow \bar{\mu} e+\mu \bar{e}\right)$ | $(7.4 \pm 0.4) \times 10^{-11}$ <br> $<3.3 \times 10^{-11}$ | $\operatorname{Br}\left(K_{S} \rightarrow \bar{\mu} \mu\right)$ <br> $\operatorname{Br}\left(K^{+} \rightarrow \pi \nu \bar{\nu}\right)$ | $<3.2 \times 10^{-7}$ <br> $<5.2 \times 10^{-9}$ |
| $\operatorname{Br}(\mu \rightarrow 3 e)$ | $<10^{-12}$ | $\operatorname{Br}(\tau \rightarrow 3 e)$ | $<1.3 \times 10^{-5}$ |
| $\operatorname{Br}(\tau \rightarrow \bar{\mu} e e)$ | $<1.4 \times 10^{-5}$ | $\operatorname{Br}(\tau \rightarrow \bar{e} e \mu)$ | $<1.4 \times 10^{-5}$ |
| $\operatorname{Br}(\tau \rightarrow \bar{\mu} \mu e)$ | $<1.9 \times 10^{-5}$ <br> $\operatorname{Br}(\tau \rightarrow 3 \mu)$ | $\operatorname{Br}(\tau \rightarrow \bar{e} \mu \mu)$ | $<1.6 \times 10^{-5}$ |

Table 1: Experimental numbers [13] used as upper bounds on the contribution of the $\not \mathbb{R}_{p}$ terms to the various observables.

We first consider the constraints on products of $\lambda^{\prime}$-couplings given by our knowledge of the neutral $B$ - and $K$-meson mass levels. The mass difference $\delta m_{B}$ between $B_{1}$ and $B_{2}$ (and similarly $\delta m_{K}$ between $K_{1}$ and $K_{2}$ ) arises out of the mixing between the $B_{d}^{0}, \overline{B_{d}^{0}}$ (and $K^{0}, \overline{K^{0}}$ ) mesons. The $\lambda^{\prime}$-couplings of eq.(3) make such mixing amplitudes possible at the tree-level through the exchange of a sneutrino $\tilde{\nu}_{i}$ both in the $s$ - and $t$-channels. In fact, the effective Lagrangian terms for $B_{d}-\overline{B_{d}}$ and $K-\bar{K}$ mixings are

$$
\begin{align*}
& \mathcal{L}_{e f f}(B \rightarrow \bar{B})=-\sum_{n} \frac{\lambda_{n 31}^{\prime} \lambda_{n 13}^{\prime \star}}{m_{\tilde{\nu}_{n}}^{2}} \overline{d_{R}} b_{L} \overline{d_{L}} b_{R}  \tag{4a}\\
& \mathcal{L}_{e f f}(K \rightarrow \bar{K})=-\sum_{n} \frac{\lambda_{n 21}^{\prime} \lambda_{n 12}^{\prime \star}}{m_{\tilde{\nu}_{n}}^{2}} \overline{d_{R}} s_{L} \overline{d_{L}} s_{R} \tag{4b}
\end{align*}
$$

We calculate the contributions of (4a) and (4b) to $\delta m_{B}, \delta m_{K}$ respectively and require them not to exceed the corresponding experimental numbers (see Table (1). Though it is tempting to improve the bounds by first subtracting the SM contributions, we choose not do so as the latter involve considerable uncertainties, both experimental and theoretical. Before we present our numbers, it is useful (for notational purposes) to define the following "normalized" quantities :

$$
\begin{align*}
n_{n} & \equiv\left(\frac{100 \mathrm{GeV}}{m_{\tilde{\nu}_{n L}}}\right)^{2} & u_{n} & \equiv\left(\frac{100 \mathrm{GeV}}{m_{\tilde{u}_{n L}}}\right)^{2}  \tag{5}\\
d_{n}^{L} & \equiv\left(\frac{100 \mathrm{GeV}}{m_{\tilde{d}_{n L}}}\right)^{2} & & d_{n}^{R}
\end{align*}
$$

Finally, our numbers from $\delta m_{B}$ and $\delta m_{K}$ respectively are $\ddagger:$

$$
\begin{align*}
& \sum_{i} \lambda_{i 31}^{\prime *} \lambda_{i 13}^{\prime} n_{i} \lesssim 3.3 \times 10^{-8}  \tag{6a}\\
& \sum_{i} \lambda_{i 21}^{\prime *} \lambda_{i 12}^{\prime} n_{i} \lesssim 4.5 \times 10^{-9} \tag{6b}
\end{align*}
$$

The upper bound in eq. (6a) is only marginally weaker than that $\left(3 \times 10^{-8}\right)$ obtained by Babu and Mohapatra 10 very recently from the lack of observation of neutrinoless double-beta decay. In contrast, the bound of eq.(6B) is three orders of magnitude stronger than the $10^{-6}$ obtained by those authors.

Next, we focus on rare leptonic decay modes of the neutral $K$-mesons: $K_{L, S} \rightarrow e \bar{e}$ or $\mu \bar{\mu}$, $e \bar{\mu}, \mu \bar{e}$ as well as the semileptonic decay $K^{+} \rightarrow \pi^{+} e_{i} \bar{e}_{j}$. At the partonic level, the generic subprocess is one in which a down-type quark-antiquark pair ( $d_{k}$ and $\bar{d}_{\ell}$, say) tranform into a charged lepton-antilepton pair (assumed to be $e_{i}$ and $\bar{e}_{j}$ ): $d_{k}+\overline{d_{\ell}} \rightarrow e_{i}+\bar{e}_{j}$. Here $i, j, k, \ell$ are generation indices. The reaction can proceed via the exchange of a $u$-squark $\tilde{u}_{n L}$ in the $t$-channel as well as via an $s$-channel exchange of a sneutrino $\tilde{\nu}_{n}$. The effective Lagrangian terms can be obtained in the same manner as above. We have then

$$
\begin{align*}
\mathcal{L}_{e f f}\left(d_{k}+\bar{d}_{\ell} \rightarrow e_{i}+\bar{e}_{j}\right) & =\sum_{n}\left[\frac{\lambda_{n i j}^{\star} \lambda_{n k \ell}^{\prime}}{m_{\tilde{\nu}_{n}}^{2}} \overline{d_{\ell R}} d_{k L} \overline{e_{i L}} e_{j R}+\frac{\lambda_{n j i}^{\star} \lambda_{n l k}^{\prime}}{m_{\tilde{\nu}_{n}}^{2}} \overline{d_{\ell L}} d_{k R} \overline{e_{i R}} e_{j L}\right]  \tag{7}\\
& -\sum_{n, p} \frac{K_{n p} \lambda_{i p k}^{\star} \lambda_{j n l}^{\prime}}{2 m_{\tilde{u}_{n L}}^{2}} \overline{e_{i L}} \gamma_{\mu} e_{j L} \overline{d_{l R}} \gamma^{\mu} d_{k R} .
\end{align*}
$$

The last term on the RHS comes from the $t$-channel exchange of a $u$-squark while the first two arise from the two $s$-channel diagrams (the first term with the sneutrino entering the hadronic vertex and the second with the sneutrino leaving it). On calculating the relevant matrix element of the effective Hamiltonian from (7), we find that the new contributions to the decay $K_{L} \rightarrow e_{i} \bar{e}_{j}$ can be parametrized in terms of the combinations

$$
\begin{align*}
\mathcal{A}_{i j} & \equiv \sum_{n, p} u_{n} K_{n p}\left(\lambda_{i p 1}^{\prime \star} \lambda_{j n 2}^{\prime}-\lambda_{i p 2}^{\prime \star} \lambda_{j n 1}^{\prime}\right), \\
\mathcal{B}_{i j} & \equiv \sum_{n} n_{n} \lambda_{n i j}^{\star}\left(\lambda_{n 12}^{\prime}-\lambda_{n 21}^{\prime}\right) . \tag{8}
\end{align*}
$$

[^1]| Decay Mode | Combinations constrained | Upper bound |
| :---: | :---: | :---: |
| $K_{L} \rightarrow \mu \bar{\mu}$ | $\lambda_{122} \lambda_{112}^{\prime} n_{1}, \lambda_{122} \lambda_{121}^{\prime} n_{1}, \lambda_{232} \lambda_{312}^{\prime} n_{3}, \lambda_{232} \lambda_{321}^{\prime} n_{3}$ | $3.8 \times 10^{-7}$ |
|  | (*) $\lambda_{211}^{\prime} \lambda_{222}^{\prime}\left(u_{1}+u_{2}\right), \lambda_{212}^{\prime} \lambda_{221}^{\prime}\left(u_{1}+u_{2}\right)$ | $3.3 \times 10^{-5}$ |
| $K_{L} \rightarrow e \bar{e}$ | $\lambda_{121} \lambda_{212}^{\prime} n_{2}, \lambda_{121} \lambda_{221}^{\prime} n_{2}, \lambda_{131} \lambda_{312}^{\prime} n_{3}, \lambda_{131} \lambda_{321}^{\prime} n_{3}$ | $2.5 \times 10^{-8}$ |
| $K_{L} \rightarrow e \bar{\mu}+\bar{e} \mu$ | $\begin{array}{lll} \lambda_{122} \lambda_{212}^{\prime} n_{2}, & \lambda_{122} \lambda_{221}^{\prime} n_{2}, & \lambda_{132} \lambda_{312}^{\prime} n_{3}, \\ \lambda_{132} \lambda_{321}^{\prime} n_{3} \\ \lambda_{121} \lambda_{112}^{\prime} n_{1}, & \lambda_{121} \lambda_{121}^{\prime} n_{1}, & \lambda_{231} \lambda_{312}^{\prime} n_{3}, \end{array} \lambda_{231} \lambda_{321}^{\prime} n_{3} .$ | $2.3 \times 10^{-8}$ |
|  | $\begin{array}{lll} \hline \lambda_{111}^{\prime} \lambda_{212}^{\prime} u_{1}, & \lambda_{112}^{\prime} \lambda_{211}^{\prime} u_{2}, & \lambda_{121}^{\prime} \lambda_{222}^{\prime} u_{2} \\ \lambda_{122}^{\prime} \lambda_{221}^{\prime} u_{2}, & \lambda_{131}^{\prime} \lambda_{232}^{\prime} u_{3}, & \lambda_{132}^{\prime} \lambda_{231}^{\prime} u_{3} \end{array}$ | $3.5 \times 10^{-7}$ |
| $K^{+} \rightarrow \pi \nu \bar{\nu}$ | $\lambda_{i 1 n}^{\prime} \lambda_{j 2 n}^{\prime} d_{n}^{R}, \lambda_{i n 2}^{\prime} \lambda_{j n 1}^{\prime} d_{n}^{L} \quad($ For all $i, j, n)$ | $4.8 \times 10^{-5}$ |

Table 2: Upper bounds on the magnitudes of coupling products derived from rare $K$ decays, under the assumption that only one such product is nonzero. In the entry marked with ( $\star$ ), we have assumed that $K_{12}=-K_{21}$. For definitions, see eq.(5).

Two points need to be emphasised here :

- On account of the particular Lorentz structure of the operators, the contributions of the $\mathcal{A}_{i j}$ terms are chirality-suppressed. Hence, in the event of vanishing $\mathcal{B}_{i j}$, the constraints on $\mathcal{A}_{i j}$ would be weaker than those operative in the reverse case.
- In the limit of a trivial CKM matrix $\left(K_{n p}=\delta_{n p}\right), \mathcal{A}_{i j}^{*}=-\mathcal{A}_{j i}$, and thus $\mathcal{A}_{11}$ and $\mathcal{A}_{22}$ are purely imaginary. Though large imaginary parts in $\lambda_{i j k}^{\prime}$ cannot be ruled out per se, these are liable to generate unacceptably large CP violating effects.

Nonetheless, we retain, for the time being, the possibility of complex $\not R_{p}$ couplings. Ignoring again the SM contributions, we demand that the $\not_{p}$ contribution by itself does not exceed the experimental upper bounds. Considering [13] $K_{L} \rightarrow e \bar{e}, \mu \bar{\mu}$ and $e \bar{\mu}+\mu \bar{e}$ in turn, we have

$$
\begin{align*}
2.5 \times 10^{-8} & \gtrsim\left|\mathcal{B}_{11}\right| \\
1.3 \times 10^{-13} & \gtrsim\left[\left|\mathcal{B}_{22}\right|^{2}-0.099 \operatorname{Re}\left(\mathcal{B}_{22}^{2}\right)\right]+0.0024\left|\mathcal{A}_{22}\right|^{2}+0.10 \operatorname{Im}\left(\mathcal{A}_{22}\right) \operatorname{Im}\left(\mathcal{B}_{22}\right)  \tag{9}\\
5.4 \times 10^{-16} & \gtrsim\left(\left|\mathcal{B}_{12}\right|^{2}+\left|\mathcal{B}_{21}\right|^{2}\right)+0.0022\left(\left|\mathcal{A}_{12}\right|^{2}+\left|\mathcal{A}_{21}\right|^{2}\right) \\
& =0.047 \operatorname{Re}\left(\left\{\mathcal{A}_{12}^{*}-\mathcal{A}_{21}\right\} \mathcal{B}_{12}\right)
\end{align*}
$$

The $\not R_{p}$ contribution to the rare $K_{S}$ decays, on the other hand, are parametrized by the
combinations

$$
\begin{align*}
\mathcal{C}_{i j} & \equiv \sum_{n, p} u_{n} K_{n p}\left(\lambda_{i n 1}^{\prime \star} \lambda_{j n 2}^{\prime}+\lambda_{i n 2}^{\prime \star} \lambda_{j n 1}^{\prime}\right),  \tag{10}\\
\mathcal{D}_{i j} & \equiv \sum_{n} n_{n} \lambda_{n i j}^{\star}\left(\lambda_{n 12}^{\prime}+\lambda_{n 21}^{\prime}\right) .
\end{align*}
$$

Since all of the relevant $\mathcal{C}_{i j} \mathrm{~s}$ are real even in the limit of a trivial CKM matrix, we cannot use the arguments proffered in the case of the $K_{L}$ to disentangle the contributions (the mass suppression continues nonetheless). The resulting expressions are quite analogous to those for $K_{L}$ decays and, in the case of [13] $K_{S} \rightarrow \mu^{-} \mu^{+}$, looks like

$$
\begin{equation*}
\left[\left|\mathcal{D}_{22}\right|^{2}+0.099 \operatorname{Re}\left(\mathcal{D}_{22}^{2}\right)\right]+0.1 \operatorname{Re}\left(\mathcal{C}_{22}^{\star} \mathcal{D}_{22}\right)+0.0025\left|\mathcal{C}_{22}\right|^{2} \lesssim 3.1 \times 10^{-9} \tag{11}
\end{equation*}
$$

The additional contribution to the decay $K^{+} \rightarrow \pi^{+} \nu_{i} \bar{\nu}_{j}$ due to the $\not R_{p}$ terms is in the form of two sets of $t$-channel diagrams (for each $i, j$ ), one each with $\tilde{d}_{L n}$ and $\tilde{d}_{R n}$. The effective Lagrangian can be parametrized as

$$
\begin{equation*}
\mathcal{L}_{e f f}\left(K^{+} \rightarrow \pi^{+} \nu_{i} \bar{\nu}_{j}\right)=\frac{1}{2} \overline{\nu_{i L}} \gamma_{\mu} \nu_{j L} \sum_{n}\left[\frac{\lambda_{i 2 n}^{\prime \star} \lambda_{j 1 n}^{\prime}}{m_{\tilde{d}_{n R}}^{2}} \overline{s_{L}} \gamma^{\mu} d_{L}-\frac{\lambda_{i n 1}^{\prime \star} \lambda_{j n 2}^{\prime}}{m_{\tilde{d}_{n L}}^{2}} \overline{s_{R}} \gamma^{\mu} d_{R}\right] \tag{12}
\end{equation*}
$$

Since the nine possible combinations ( $i j$ ) cannot be distinguished experimentally, the bound [13] from the non-observation of such a mode leads to an upper bound on the incoherent sum (and hence also on the individual quantities):

$$
\begin{equation*}
\sum_{i, j}\left|\mathcal{E}_{i j}\right|^{2} \lesssim 2.3 \times 10^{-9} \tag{13}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathcal{E}_{i j} \equiv \sum_{n}\left(d_{n}^{R} \lambda_{i 2 n}^{\prime \star} \lambda_{j 1 n}^{\prime}-d_{n}^{L} \lambda_{i n 1}^{\prime \star} \lambda_{j n 2}^{\prime}\right) \tag{14}
\end{equation*}
$$

Finally, we come to products of $\lambda$-couplings which are probed by information on rare three-body leptonic decays of $\mu$ and $\tau$. All these processes can be characterized by the generic transition $e_{a} \rightarrow e_{b}+e_{c}+\bar{e}_{d}$ where $a, b, c, d$ are generation indices. The reaction proceeds by the exchange of a sneutrino $\tilde{\nu}_{n}$ in the $t$-channel as well as in the $u$-channel. Analogous to the neutral $K$-decays, here too the sneutrino propagator may have both directions. The effective Lagrangian term now is

$$
\begin{align*}
(100 \mathrm{GeV})^{2} \mathcal{L}_{\mathrm{eff}}\left(\mathrm{e}_{\mathrm{a}} \rightarrow \mathrm{e}_{\mathrm{b}}+\mathrm{e}_{\mathrm{c}}+\overline{\mathrm{e}}_{\mathrm{d}}\right) & =\mathcal{F}_{a b c d} \overline{e_{b R}} e_{a L} \overline{e_{c L}} e_{d R}+\mathcal{F}_{d c b a} \overline{e_{b L}} e_{a R} \overline{e_{c R}} e_{d L}  \tag{15}\\
& +\mathcal{F}_{a c b d} \overline{e_{c R}} e_{a L} \overline{e_{b L}} e_{d R}+\mathcal{F}_{d b c a} \overline{e_{c L}} e_{a R} \overline{e_{b R}} e_{d L},
\end{align*}
$$

where

$$
\begin{equation*}
\mathcal{F}_{a b c d} \equiv \sum_{n} n_{n} \lambda_{n a b} \lambda_{n c d}^{\star} . \tag{16}
\end{equation*}
$$

In eq.(15) the first (last) two terms on the RHS correspond to the $t$ - ( $u$-) channel exchange diagrams. Utilizing the known experimental upper limits 13 on the relevant partial widths,

| Decay Mode | Combinations constrained | Upper bound |
| :---: | :---: | :---: |
| $\mu \rightarrow 3 e$ | $\lambda_{121} \lambda_{122} n_{2}, \lambda_{131} \lambda_{132} n_{3}, \lambda_{231} \lambda_{131} n_{3}$ | $6.6 \times 10^{-7}$ |
| $\tau^{-} \rightarrow 3 e$ | $\lambda_{121} \lambda_{123} n_{2}, \lambda_{131} \lambda_{133} n_{3}, \lambda_{231} \lambda_{121} n_{2}$ | $5.6 \times 10^{-3}$ |
| $\tau^{-} \rightarrow e^{-} e^{-} \mu^{+}$ | $\lambda_{231} \lambda_{112} n_{2}, \lambda_{231} \lambda_{133} n_{3}$ | $5.7 \times 10^{-3}$ |
| $\tau^{-} \rightarrow \mu^{-} e^{+} e^{-}$ | $\lambda_{131} \lambda_{121} n_{1}, \lambda_{122} \lambda_{123} n_{2}, \lambda_{132} \lambda_{133} n_{3}, \mid$ <br> $\lambda_{232} \lambda_{121} n_{2}, \lambda_{131} \lambda_{233} n_{3}$ | $5.7 \times 10^{-3}$ |
| $\tau^{-} \rightarrow e^{+} \mu^{-} \mu^{-}$ | $\lambda_{131} \lambda_{121} n_{1}, \lambda_{132} \lambda_{233} n_{3}$ | $6.2 \times 10^{-3}$ |
| $\tau^{-} \rightarrow e^{-} \mu^{+} \mu^{-}$ | $\lambda_{131} \lambda_{122} n_{1}, \lambda_{232} \lambda_{133} n_{3}, \lambda_{232} \lambda_{122} n_{2}$, <br> $\lambda_{121} \lambda_{123} n_{1}, \lambda_{231} \lambda_{233} n_{3}$ | $6.7 \times 10^{-3}$ |
| $\tau \rightarrow 3 \mu$ | $\lambda_{132} \lambda_{122} n_{1}, \lambda_{122} \lambda_{123} n_{1}, \lambda_{232} \lambda_{233} n_{3}$ | $6.4 \times 10^{-3}$ |

Table 3: Upper bounds on the magnitudes of coupling products derived from flavour changing lepton decays, under the assumption that only one such product is nonzero. For definitions, see eq.(5).
we have,

$$
\begin{array}{lll}
\mu \rightarrow 3 e & : & \left|\mathcal{F}_{1112}\right|^{2}+\left|\mathcal{F}_{2111}\right|^{2} \lesssim 4.3 \times 10^{-13}, \\
\tau \rightarrow 3 e & : & \left|\mathcal{F}_{1113}\right|^{2}+\left|\mathcal{F}_{3111}\right|^{2} \lesssim 3.1 \times 10^{-5}, \\
\tau \rightarrow 3 \mu & : & \left|\mathcal{F}_{2223}\right|^{2}+\left|\mathcal{F}_{3222}\right|^{2} \lesssim 4.1 \times 10^{-5}, \\
\tau^{-} \rightarrow \mu^{+} e^{-} e^{-} & : & \left|\mathcal{F}_{3112}\right|^{2}+\left|\mathcal{F}_{2113}\right|^{2} \lesssim 3.3 \times 10^{-5},  \tag{17}\\
\tau^{-} \rightarrow e^{+} \mu^{-} \mu^{-} & : & \left|\mathcal{F}_{3221}\right|^{2}+\left|\mathcal{F}_{1223}\right|^{2} \lesssim 3.8 \times 10^{-5}, \\
\tau^{-} \rightarrow e^{+} e^{-} \mu^{-} & : & \left|\mathcal{F}_{1123}\right|^{2}+\left|\mathcal{F}_{3211}\right|^{2}+\left|\mathcal{F}_{1213}\right|^{2}+\left|\mathcal{F}_{3121}\right|^{2} \lesssim 3.3 \times 10^{-5}, \\
\tau^{-} \rightarrow e^{-} \mu^{+} \mu^{-} & : & \left|\mathcal{F}_{3122}\right|^{2}+\left|\mathcal{F}_{2213}\right|^{2}+\left|\mathcal{F}_{3212}\right|^{2}+\left|\mathcal{F}_{2123}\right|^{2} \lesssim 4.5 \times 10^{-5}
\end{array}
$$

It is thus clear that the rare decays considered here lead to myriad bounds on combinations of $R_{p}$ couplings. Since particular products do occur more than once (albeit with different scalar masses), it is instructive to ask what the bounds would be if only one product were non-zero. In Table 2, we give the constraints, under such assumptions, for the products relevant to $K$ decays. The corresponding $\lambda \lambda$ products (which are constrained from flavour violating lepton
decays) are listed in Table 3. We should mention here that weaker constraints on $n_{2}\left|\lambda_{121} \lambda_{122}\right|$ and $n_{3}\left|\lambda_{131} \lambda_{132}\right|$ were earlier estimated from the non-observation of $\mu \rightarrow 3 e$ decay [14].

To conclude, we have derived quite strong upper bounds on certain product combinations of $\lambda$ and $\lambda^{\prime}$ couplings. The reason that the bounds are so restrictive is that they come from processes which are permitted by such couplings at the tree-level but are disallowed or have to proceed via loops both in the SM and in the MSSM. If we consider transitions such as $\mu \rightarrow e \gamma$ or $\tau \rightarrow e \gamma$, which are loop-induced even with such couplings, the corresponding bounds would not be anything like as strong. Of course, our list is not fully exhaustive all possible coupling product combinations have not been covered. It is interesting to note that, barring the constraints from $\mu \rightarrow 3 e$, the bounds on the $\lambda \lambda$ products are, in general, weaker than those on the $\lambda \lambda^{\prime}$ combinations ${ }^{2}$. On the other hand, every single $\lambda$-coupling appears in Tables 1 and 2. The same is not the case for the $\lambda^{\prime}$-couplings. For instance, $\lambda_{322}^{\prime}$ and $\lambda_{323}^{\prime}$ are unconstrained while couplings like $\lambda_{22 k}^{\prime}, \lambda_{13 k}^{\prime}$ (except $\lambda_{133}^{\prime}$ ) are only weakly constrained [B]. Thus one may still harbor good hope of detecting a nonzero signal, say at LEP2, from some of these couplings [15]. A large top quark event sample, accumulated at Fermilab, will also provide [16] a good opportunity to probe certain $\not R_{p}$ couplings from possible bounds on nonstandard top decays.

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[^2]
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[^1]:    ${ }^{1}$ Here, and in the rest of the discussion, bounds on any (complex) quantity will apply to its magnitude.

[^2]:    ${ }^{2}$ Since $\lambda$ and $\lambda^{\prime}$ couplings mimic a class of dilepton and leptoquark phenomenology respectively, it is likely that a similar conclusion may be reached in the generic case as well 17.

