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A General Treatment of Oblique Parameters

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Abstract

A reexamination is made of one-loop oblique electroweak corrections. General definitions are given of the oblique parameters without reference to any q^2-expansion scheme. The old oblique parameters S,T and U are defined as differences of gauge boson vacuum polarization Π-functions and suffice to describe certain observable ratios on the Z-peak and the ρ parameter at $q^2 = 0$. Regarding the new oblique parameters V,W and X, the first two are defined in terms of differences of Π-functions as well as the wavefunction renormalization of the corresponding weak boson, and the third in terms of the difference of differences of two Π-functions for $\gamma - Z$ mixing. Explicit expressions for measurable quantities involving all six oblique parameters are given and experimental bounds are obtained on the latter, some for the first time. A review of these constraints suggests that the linear approximation of Peskin and Takeuchi is robust.

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I. **Introduction**

A lot of interest in recent years has centred around electroweak precision parameters [1]. These are very useful in describing the possible effects of interactions *beyond the Standard Model* (BSM). Hints for such interactions have been sought ardently in many precision experiments. To this end, one has had to first carefully compute various quantities in the Standard Model (SM). These calculations take into account electroweak one-loop radiative corrections [2] as well as higher loop corrections in QCD and those involving the top quark. Given these computed results, tiny deviations between predicted and measured values are looked for. In their absence, bounds are put on the precision parameters.

Consider processes without any external top quark. The coupling between the Z boson and a bottom quark-antiquark pair is special in that the specific top-mediated large vertex correction needs to be explicitly included. Otherwise, to a reasonable accuracy, one-loop corrections to all remaining SM quantities, measured to date, can be approximated by gauge vector-boson self-energy graphs [3], known as *oblique corrections.* (For more precise estimates [4], of course, general vertex corrections and box graphs at the one-loop level as well as top-induced two-loop terms need to be taken into account.) The aforementioned parameters can now be defined as linear combinations [5, 6] of differences between various gauge boson vacuum polarization Π-functions at the same scale or at two separate scales and henceforth will be called old oblique parameters. These are the three constants S,T,U or equivalently ϵ_1, ϵ_2,ϵ_3 [1] — with $\epsilon_1 = \alpha T$, $\epsilon_2 = -\alpha(4s^2)^{-1}U$,$\epsilon_3 = \alpha(4s^2)^{-1}S$, α and s respectively being the fine structure constant and the sine of the Weinberg angle at the tree level. LEP and lower energy data have been used to obtain interesting bounds on them [3, 4, 5, 6].

One can classify the nongauge SM fields into light (all fermions except the top) and heavy (top and Higgs) sectors. The contributions of the light fields to perturbatively calculable quantities are known quite precisely, those from the heavy sector less so. Moreover, BSM contributions — if any — are likely to get mixed up with the latter. The effects of such heavy physics can be added on to the light physics SM calculation by using the Π-formalism. We define $\Pi_{AB}(q^2)$ in terms of the real part of the current propagator

$$\mathrm{Re} \int d^4x e^{iq.x} \langle\Omega| T J_\mu^A(x) J_\nu^B(0)|\Omega\rangle = -\Pi_{AB}(q^2)\eta_{\mu\nu} + q_\mu q_\nu \text{ terms}, \quad (1)$$

$\eta_{\mu\nu}$ being the flat Minkowski metric and A,B being gauge group indices. The $q_\mu q_\nu$ terms drop out when contracted with the external fermion current (as we are considering light fermions only in the external legs), so that $\Pi_{\mu\nu}$, and all quantities derived from it, are gauge invariant. Physics enters the LHS of (1) through the insertion of a complete set of states $\sum_n |n\rangle\langle n|$ between the current operators. The sum can be decomposed neatly as $\sum_{n_l} |n_l\rangle\langle n_l| + \sum_{n_h} |n_h\rangle\langle n_h|$ where $|n_l\rangle$ refers to the light SM sector and $|n_h\rangle$ to the heavy sector including both heavy SM and any possible BSM fields. Thus the heavy and the light sector contributions add linearly to the Π-functions and hence to the oblique parameters.

One can also consider the difference of two Π-functions at different q^2-values by defining a function $\Pi'_{AB}(q^2)$:

$$\Pi_{AB}(q^2) \equiv \Pi_{AB}(0) + q^2\Pi'_{AB}(q^2). \tag{2}$$

At $q^2 = 0$, $\Pi'_{AB}(q^2)$ and $d\Pi_{AB}(q^2)/dq^2$ are equal; otherwise they are different. This procedure can be extended one step further by introducing $\Pi''_{AB}(q^2)$. This is defined as proportional to the difference of two Π'_{AB} functions, *i.e.*, to the difference of differences of Π_{AB} functions. Thus define

$$\Pi''_{AB}(q^2) \equiv 2q^{-2}[\Pi'_{AB}(q^2) - \Pi'_{AB}(0)]. \tag{3}$$

Again, this equals $d^2\Pi_{AB}(q^2)/d(q^2)^2$ only at $q^2 = 0$. It follows from (2) and (3) that

$$\Pi_{AB}(q^2) = \Pi_{AB}(0) + q^2\Pi'_{AB}(0) + \frac{1}{2}q^4\Pi''_{AB}(q^2). \tag{4}$$

The extraction of S,T,U from the data, obtained in experiments performed both on the Z- and the W-masses as well as at low energies, has so far been made with some kind of an approximation on the q^2-dependence of the concerned Π-functions. Peskin and Takeuchi proposed [3] a linear expansion approximation which we briefly recount. This approximation amounts to replacing $\Pi'(q^2)$ by $\Pi'(0)$. Thus (2) becomes

$$\Pi_{AB}(q^2) = \Pi_{AB}(0) + q^2\Pi'_{AB}(0). \tag{5}$$

A quadratic expansion approximation, which is a quadratic extension of (5), *i.e.*,

$$\Pi_{AB}(q^2) = \Pi_{AB}(0) + q^2\Pi'_{AB}(0) + \frac{q^4}{2}\Pi''_{AB}(0), \tag{6}$$

has also been suggested in [7]. Our aim in this paper is to avoid — to the extent possible — this kind of approximation in extracting the oblique parameters from experimental data. Thus, we need to identify those segments of the data which make this feasible for S,T and U. We will see that the utilization of the full set of electroweak data in determining the oblique parameters requires the introduction of three more parameters [8] V,W and X. These vanish in the linear approximation and will be called the new oblique parameters.

Before proceeding further, it is useful to review the motivation behind the linear approximation employed in [3] and related subsequent works [5, 6]. In the SM the difference between (2) and (5) for $0 \ll q^2 \ll M_{W,Z}^2$ can be computed and is known to be negligibly tiny. Coming to BSM contributions, Peskin and Takeuchi [3] were inspired by scenarios of electroweak symmetry breakdown, in which the BSM mass scale M controlling the dimensional denominators in $\Pi'(0)$, $\Pi''(0)$ etc. was expected to be $\gtrsim 1$ TeV. In such a situation, since q^2 varies between zero and $M_{W,Z}^2$ only, it would not make much sense to retain the $\mathcal{O}(q^4/M^4)$ terms, specially when one is neglecting both the non-oblique corections to one-loop terms and the two-loop contributions. In this picture the linear approximation could a priori be deemed as accurate. There are, however, other scenarios of electroweak symmetry breaking in which some BSM scales could have a significantly lower value — such as 100-200 GeV. An example would be a low-lying techniparticle or a low electroweak gaugino mass in supersymmetry [9]. Now the accuracy of the linear approximation could be called into question. It would be desirable to give more general definitions of the oblique parameters, independent of any q^2-expansion, and hence without any commitment to the magnitude of the scale controlling electroweak symmetry breaking dynamics.

In this paper we formulate these general definitions. The old oblique parameters S,T,U are expressed as differences [6] of Π-functions while the new ones V,W and X are given in terms of corresponding differences of differences and their derivatives. A similar approach was taken earlier [8], but we have made several improvements. We give definitions that preserve the correct symmetry properties of S and U unlike in [8]. Furthermore, not only do we describe all one-loop oblique electroweak corrections in terms of the full set of six parameters S,T,U,V,W and X, we are able to put bounds on *all* of them using the data. Moreover, we clarify the issue of q^2-expansion and discuss the robustness of the linear approximation.

Focussing on the q^2-expansion procedure, we examine the question why it was used by previous authors [3, 5, 7] at all. This expansion entered the picture only in relating the Z- and W-wavefunction renormalization constants which involve derivatives of Π-functions to S,T,U which involve only differences. That step was necessitated in the calculation of certain physical observables in terms of those parameters since the said renormalization constants occur in their expressions. Ratios, which do not depend upon the wavefunction renormalization constants, are directly computable in terms of S,T and U without any use of the expansion approximation. Turning then to quantities which do, we show precisely how V,W and X can be defined so that all such quantities are expressible in terms of six oblique parameters. Using the first set of ratios and experimental data, one can obtain numerical bounds on S,T and U *independently of any expansion approximation.* These can then be utilized with respect to experimental data on the second set of quantities to derive the best bounds on V,W and X. The bounds on V and X are again obtained without any expansion; however, that on W is derived within the quadratic approximation. SM contributions to all the oblique parameters (with a reference point of top and Higgs mass values) can then be subtracted to yield bounds on the corresponding BSM contributions.

The rest of the paper is organized as follows. Section II contains a review of the definitions of the old oblique parameters S,T and U, emphasizing their symmetry content. Section III discusses the issue of wavefunction renormalization of the weak vector bosons and the two new oblique parameters V,W as well as the need for the sixth parameter X; general definitions of these three are also given. In Section IV explicit one-loop corrected formulae (listed in the Appendix) for various observables are derived in the $\star$ scheme in terms of all six oblique parameters distinguishing between certain ratios which involve only S,T and U, and quantities which also involve the new parameters. Numerical constraints on the old and new oblique parameters are discussed in Section V. The final Section VI summarizes our conclusions and contains a discussion of the robustness of the linear approximation.

II. **Old Oblique Parameters**

Though a sizable literature exists on S,T and U, it would be useful to review their definitions here before introducing the new oblique parameters. In the generic Π-function $\Pi_{AB}(q^2)$ of (1), the combination (AB) is allowed to take the values (11), (22), (33), (3Q) and (QQ). Here 1,2,3 refer to the generators of the $SU(2)_L$ gauge group and Q to the electromagnetic current. On account of the unbroken part of the weak isospin symmetry, Π_{11} equals Π_{22}; hence there are four independent Π-functions. These are generally written in the gauge boson basis as follows:

$$\Pi_{\gamma\gamma} = e^2\Pi_{QQ}, \tag{7}$$

$$\Pi_{\gamma Z} = \frac{e^2}{sc}(\Pi_{3Q} - s^2\Pi_{QQ}), \tag{8}$$

$$\Pi_{ZZ} = \frac{e^2}{s^2c^2}(\Pi_{33} - 2s^2\Pi_{3Q} + s^4\Pi_{QQ}), \tag{9}$$

$$\Pi_{WW} = \frac{e^2}{s^2}\Pi_{11}. \tag{10}$$

In our notation the electromagnetic, the weak charged and the weak neutral current interaction terms in the Lagrangian density are given respectively by

$$\mathcal{L}_{em} \equiv eJ^Q_\mu A^\mu, \tag{11}$$

$$\mathcal{L}_{W;cc} \equiv \frac{e}{\sqrt{2}s}(J^{1-i2}_\mu W^{\mu+} + J^{1+i2}_\mu W^{\mu-}), \tag{12}$$

$$\mathcal{L}_{W;nc} \equiv \frac{e}{sc}(J^3_\mu - s^2 J^Q_\mu)Z^\mu, \tag{13}$$

with A^μ, $W^{\mu\pm}$ and Z^μ as the corresponding electromagnetic, charged weak and neutral weak gauge boson fields respectively. The tree-level quantities s^2 and $e \equiv \sqrt{4\pi\alpha}$ are defined from (i) $s^2 = 1 - {M_W^0}^2/{M_Z^0}^2$ where M_W^0 and M_Z^0 are the tree-level masses of the W and the Z respectively (without the finite self-energy corrections) and (ii) the Thompson cross-section $\sigma_T = 8\pi(\alpha/m_e)^{2/3}$ [10]. The tree-level four-Fermi coupling G_F^0 is given by $\sqrt{2}e^2/(8s^2c^2{M_Z^0}^2)$.

Let us restrict ourselves only to on-shell or near-peak Z,W data plus low energy measurements which define the electroweak couplings. Let us also first consider dimensionless ratios of measurable quantities from which the W or Z wavefunction renormalizations cancel out. Then it suffices to take $\Pi_{\gamma\gamma}$, $\Pi_{\gamma Z}$ and Π_{ZZ} at $q^2 = M_Z^2, 0$ and Π_{WW} at $q^2 = M_W^2, 0$. That would give us eight one-loop corrected quantities, but the QED Ward identities imply

$$\Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0, \tag{14}$$

leaving six such independent quantities. We have already related tree-level parameters directly to experimental data. For one-loop corrected quantities, one would like to utilize three accurate numbers obtained from three precision measurements [10]: (i) the inverse of the fine structure constant, measured [11] from the Quantum Hall Effect, to be 137.036, (ii) the mass of the Z determined from LEP — namely (91.1888$\pm$ 0.0022) GeV, and (iii) the Fermi constant, inferred from the muon lifetime calculated to one-loop, as $1.16639(2) \times 10^{-5}$ GeV^{-2}. These can be used to eliminate three of the Πs. Finally, three more one-loop corrected quantities remain and the system is completely described by the choice [6]:

$$S \equiv -\frac{8\pi}{M_Z^2}\Big[\Pi_{3Y}(M_Z^2) - \Pi_{3Y}(0)\Big], \tag{15}$$

$$T \equiv \frac{4\pi}{c^2 s^2 M_Z^2}\Big[\Pi_{11}(0) - \Pi_{33}(0)\Big], \tag{16}$$

$$U \equiv \frac{16\pi}{M_W^2}\Big[\Pi_{11}(M_W^2) - \Pi_{11}(0)\Big] - \frac{16\pi}{M_Z^2}\Big[\Pi_{33}(M_Z^2) - \Pi_{33}(0)\Big]. \tag{17}$$

In (15) $\Pi_{3Y} = 2(\Pi_{3Q} - \Pi_{33})$ since $J_\mu^Q = J_\mu^3 + \frac{1}{2}J_\mu^Y$.

What needs to be stressed in the definitions (15)-(17) is the physical meaning behind the symmetry content of each of the three oblique parameters. S quantifies the difference in mixing between the hypercharge and the third weak isospin currents at $q^2 = M_Z^2$ and $q^2 = 0$ (the mixing itself between two operators in the two factor groups is absent classically but arises at the quantum one-loop level owing to spontaneous symmetry breakdown). In contrast, T and U are directly concerned with weak isospin rather than hypercharge. T describes the amount of weak isospin breaking at $q^2 = 0$ and is linearly proportional to the deviation from unity of the ρ-parameter, measured at low energies. On the other hand, U measures the contribution of the W,Z mass nondegeneracy to weak isospin breaking. The additional oblique parameters V,W and X get introduced when one goes beyond measurable quantities which do not depend on the wavefunction renormalization and starts to consider those which do. Nevertheless, the definitions (15)-(17) stand and do not suffer any modification since the latter are independent of the q^2-expansion procedure.

At this point, it would be useful to contrast our definitions (15)-(17) with those of ref. [8]. We have extended the S,T,U definitions of Peskin and Takeuchi (PT) [3] beyond the linear approximation to a q^2-independent form. Burgess *et al* [8] aimed to do the same with the Marciano-Rosner (MR) [5] definitions of the corresponding quantities. With only linear terms retained, the PT and the MR definitions agree while ours agree with those of ref. [8] only for T, but not for S and U. The difference concerns the broken and custodial symmetry contents of the Π-function combinations relevant to these oblique parameters. Specifically, we can show that while our definitions preserve the desired symmetry properties, theirs do not. One could, of course, argue that the choice of definitions is a matter of convenience since it is possible to go from phenomenological fits for one set to those for the other by appropriate substitutions. However, different definitions mean different physical quantities and this important point needs to be kept in mind.

Let us reconsider the definitions of S and U. Recasting (15) and (17) in the gauge boson basis [cf. (7)-(10)], we have

$$\frac{\alpha}{4s^2c^2}S = \frac{\Pi_{ZZ}(M_Z^2)-\Pi_{ZZ}(0)}{M_Z^2} - \frac{c^2-s^2}{cs}\frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2} - \frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2}, \tag{18}$$

$$\frac{\alpha}{4s^2}U = \frac{\Pi_{WW}(M_W^2)-\Pi_{WW}(0)}{M_W^2} - c^2\frac{\Pi_{ZZ}(M_Z^2)-\Pi_{ZZ}(0)}{M_Z^2} - s^2\frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2} - 2sc\frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2}. \tag{19}$$

In place of the ratios $\Pi_{\gamma Z}(M_Z^2)/M_Z^2$ and $\Pi_{\gamma\gamma}(M_Z^2)/M_Z^2$, the authors of [8] had used $\Pi'_{\gamma Z}(0)$ and $\Pi'_{\gamma\gamma}(0)$ respectively. With such a choice, S could not be written as a difference of Π_{3Y} functions between the scales $q^2 = M_Z^2$ and $q^2 = 0$ except only in the linear approximation. Thus the physical meaning of S as a measure of the quantum mixing between J_μ^3 and J_μ^Y currents would be lost. Similar would be the fate of U as a measure of the contribution of the W,Z mass nondegeneracy to weak isospin breaking. The "S" and "U", defined in [8], are therefore quantities that are physically different from what have been considered so far. The two sets are related, via the X-parameter (see Section III), by "S"=S$-4(c^2-s^2)$X, "U"=U$-8s^2$X. In consequence, renormalization independent observable ratios, which depend only on S,T and U, become functions of "S",T,"U" and X.

An alternative approach might be to replace, in the above definitions, the differences $[\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0)]/M_Z^2$ and $[\Pi_{WW}(M_W^2) - \Pi_{WW}(0)]/M_W^2$ by $\Pi'_{ZZ}(0)$ and $\Pi'_{WW}(0)$ respectively, alongwith the replacements done in ref. [8]. Now, 'S'$=-8\pi\Pi'_{3Y}(0)$ and 'U'$=16\pi[\Pi'_{11}(0)-\Pi'_{33}(0)]$, so that 'S' still quantifies the 3Y mixing albeit at $q^2 = 0$, while 'U' measures weak isospin breaking for the $\Pi'(0)$s. With these definitions, however, no physical observable involving 'S' and 'U' at the one-loop level would be computable without invoking the q^2-expansion procedure. Our definitions (15)-(17) have been chosen so as to preserve the symmetry properties of the oblique parameters *as well as* to enable the determination of S,T,U without reference to that expansion approximation.

III. **Renormalization and the New Oblique Parameters**

Once again, it would be useful to start with a brief review – this time of the wavefunction renormalizations of the Z and of the W and of their role in computing one-loop observables – along the lines of Peskin and Takeuchi [3]. The tree-level masses of the gauge bosons M_W^0, M_Z^0 get modified by vacuum polarization diagrams. Now the wavefunction renormalization constants Z_V (V=W,Z) — defined consistently to one-loop as the coefficients of the poles of the corresponding propagators $Z_V \equiv (1 - d/dq^2\ \Pi_{VV}|_{q^2=M_V^2})^{-1}$ $\approx 1 + d/dq^2 \Pi_{VV}|_{q^2=M_V^2}$ — develop nontrivial dependences on the oblique parameters. By using (4), one can then write

$$Z_Z = 1 + \Pi'_{ZZ}(0) + M_Z^2 \Pi''_{ZZ}(M_Z^2) + \frac{1}{2} M_Z^4 \frac{d}{dq^2} \Pi''(q^2)|_{q^2=M_Z^2}, \tag{20}$$

$$Z_W = 1 + \Pi'_{WW}(0) + M_W^2 \Pi''_{WW}(M_W^2) + M_W^4 \frac{d}{dq^2} \Pi''(q^2)|_{q^2=M_W^2}. \tag{21}$$

If the concerned Π-functions were only linearly dependent on q^2 [3], $d\,/dq^2\ \Pi|_{q^2=M_V^2}$ would be expressible as proportional to the difference $\Pi(M_V^2) - \Pi(0)$ and would not be an independent quantity. However, any general nonlinear q^2-dependence in $\Pi_{VV}(q^2)$ would make the derivative independent of $\Pi(M_V^2)$ and $\Pi(0)$.

We now define two new oblique parameters V and W as follows:

$$\alpha V \equiv \frac{1}{2}\Big[\frac{d}{dq^2}[q^4 \Pi''_{ZZ}(q^2)]_{q^2=M_Z^2} - M_Z^2 \Pi''_{ZZ}(q^2)\Big], \tag{22}$$

$$\alpha W \equiv \frac{1}{2}\Big[\frac{d}{dq^2}[q^4 \Pi''_{WW}(q^2)]_{q^2=M_W^2} - M_W^2 \Pi''_{WW}(q^2)\Big]. \tag{23}$$

These are new in the sense that they do not contribute in the linear approximation [3] under which $\Pi''_{AB}(q^2) \to 0$. Further, under the quadratic approximation [7], namely $\Pi''_{AB}(q^2) \approx \Pi''_{AB}(0)$, they become $\alpha V \approx \Pi'_{ZZ}(M_Z^2) -$

$\Pi'_{ZZ}(0)$, $\alpha W \approx \Pi'_{WW}(M_W^2) - \Pi'_{WW}(0)$. It is to be noted that (20)-(23) yield the Z, W wavefunction renormalization constants to be

$$Z_Z = 1 + M_Z^{-2}[\Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0)] + \alpha V, \tag{24}$$

$$Z_W = 1 + M_W^{-2}[\Pi_{WW}(M_W^2) - \Pi_{WW}(0)] + \alpha W. \tag{25}$$

When we extend our consideration of measurable quantities on each vector boson pole from only dimensionless ratios (described by α,G_F,M_Z and S,T,U) to also include dimensional quantities, two more parameters, namely, Z_Z and Z_W (equivalently two new oblique parameters V and W), enter their formulae calculated to one-loop.

In order to avoid the q^2-expansion and analytically use the new oblique parameters, we find it convenient to use the $\star$ renormalization scheme of Kennedy and Lynn [12]. This is for illustrative purposes only. The $\star$ scheme has deficiencies with respect to the level of precision now achievable in electroweak testing. It does not incorporate the one-loop vertex corrections and box graphs which contribute nonnegligibly to low-energy phenomena. With the leading log approximation, it also cannot fully cover significant top-induced two-loop terms which are part and parcel of the present day high precision calculations. However, the focus in this paper is not on the numerical accuracy of radiative corrections, but rather on a formal discussion of Π-functions with analytic expressions. It is for that purpose that we employ the $\star$ scheme.

In the $\star$ scheme, the key step is to replace all the bare parameters at the tree-level with their starred counterparts *evaluated at appropriate momenta.* Thus in this scheme the running wavefunction renormalizations $Z_{Z^\star}(q^2)$ and $Z_{W^\star}(q^2)$ are given by [3] the relations

$$\frac{e_\star^2}{s_\star^2 c_\star^2} Z_{Z^\star} = \frac{e^2}{s^2 c^2} Z_Z, \tag{26}$$

$$\frac{e_\star^2}{s_\star^2} Z_{W^\star} = \frac{e^2}{s^2} Z_W. \tag{27}$$

The quantities $e_\star^2(q^2)$, $s_\star^2(q^2)$ and $c_\star^2(q^2)$, which appear in (26)-(27), are related at the one-loop level to e^2, s^2 and c^2 respectively through appropriate Π'-functions as follows:

$$e_\star^2(q^2) = e^2[1 + \Pi'_{\gamma\gamma}(q^2)], \tag{28}$$

$$s_\star^2(q^2) = s^2 \left[1 - \frac{c}{s}\Pi'_{\gamma Z}(q^2)\right], \tag{29}$$

$$c_\star^2(q^2) = c^2 \left[1 + \frac{s}{c}\Pi'_{\gamma Z}(q^2)\right], \tag{30}$$

and we maintain $s_*^2(q^2) + c_*^2(q^2) = 1$. In (28)-(30) we have once again retained only the terms that are linear in the Π's as a consistent one-loop approximation. It now follows that

$$Z_{Z^*}(M_Z^2) = Z_Z \frac{e^2}{s^2c^2}\left(\frac{s_*^2c_*^2}{e_*^2}\right)_{M_Z^2} \tag{31}$$

$$= 1 + \frac{\alpha}{4s^2c^2}S + \alpha V, \tag{32}$$

where we have used (18), (24), (26) and (28)-(30). Similarly,

$$Z_{W^*}(M_W^2) = 1 + \frac{\alpha}{4s^2}(S+U) + \alpha W. \tag{33}$$

When we extend our set of calculable quantities to include measurables at $q^2 = 0$, two additional quantities enter, namely, $e_*^2(0)$ and $s_*^2(0)$. Of these, $e_*^2(0)$ equals $4\pi\alpha_*(0)$ where $\alpha_*(0)$ is the fine structure constant measured [11] from the Quantum Hall Effect. This has already been taken as an input parameter (see Section II). In contrast, $s_*^2(0)$ is an independent quantity. While $s_*^2(M_Z^2)$, a dimensionless measurable on the Z-pole, is given in terms of old oblique parameters, $s_*^2(0)$ can only be related to $s_*^2(M_Z^2)$ through the introduction of another new oblique parameter X to describe the running:

$$\alpha X \equiv -sc[\Pi'_{\gamma Z}(M_Z^2) - \Pi'_{\gamma Z}(0)]. \tag{34}$$

$\Pi'_{\gamma Z}(M_Z^2)$ has already appeared in S (vide eq. (18)), but $\Pi'_{\gamma Z}(0)$ is an independent quantity, not necessarily small. For example, in the SM $\Pi'_{\gamma Z}(0)$ is $\mathcal{O}(10^{-4})$. This underscores the need for the new parameter X. By comparison, $\Pi'_{\gamma\gamma}(0) \equiv 1 - \alpha\alpha_*^{-1}(0)$; taking α from Thompson scattering [10] and $\alpha_*(0)$ from the Quantum Hall effect [11], we find $\Pi'_{\gamma\gamma}(0) \approx 2 \times 10^{-6}$, which can be safely neglected. Henceforth we shall therefore replace $\alpha_*(0)$ by α everywhere. Returning to X, we see that it vanishes in the linear approximation and that its definition (34) agrees with that given in ref. [8]. An additional point to note is that neither $e_*^2(M_W^2)$ nor $e_*^2(M_Z^2)$ is a measurable quantity on the W- or Z-pole. In order to make contact with experimentally measured dimensional quantities on each vector boson pole, one needs to theoretically evolve e_*^2 from $q^2 = 0$ to $q^2 = M_W^2, M_Z^2$, via the renormalization group, calculating all the light SM contributions to it (*i.e.*, excluding those of the top and the Higgs). This quantity will be called [3] $e_{*,0}^2(M_V^2)$, and we shall use $\alpha_{*,0}(M_V^2) \equiv e_{*,0}^2(M_V^2)/(4\pi)$, (V=W,Z).

IV. **Observables in Terms of Oblique Parameters**

We now present the calculations of several observables and check their dependences on the oblique parameters, old and new. First, let us consider the ρ-parameter which involves the ratio of the neutral current neutrino scattering amplitude to the charged current one at $q^2 = 0$. The $q^2 = 0$ condition can be used to one's advantage by writing [3]

$$\rho_\star(0) = 1 + \frac{4\pi\alpha}{s^2c^2M_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)] = 1 + \alpha T. \tag{35}$$

It may be noted here that in the low-energy limit the one-loop corrected four fermion weak interaction term is

$$\frac{G_F}{\sqrt{2}}\left[J_\mu^+ J^{\mu^-} + \rho_\star(0) J_\mu^{NC} J^{\mu NC}\right], \tag{36}$$

where $J_\mu^+ \equiv J_\mu^1 - iJ_\mu^2$ and $J_\mu^{NC} \equiv J_\mu^3 - s_\star^2(0)J_\mu^Q$. We ought to reemphasize here that in terms of numerical accuracy that is now attainable [4] in precision testing at $q^2 = 0$, the obliqueness approximation and hence the $\star$ formalism is rather inaccurate since contributions from non-oblique terms are not entirely negligible.

We come next to s_*^2. At the tree-level, we have

$$s^2c^2 = \frac{\pi\alpha}{\sqrt{2}G_F^0 M_Z^{0^2}}. \tag{37}$$

Our Z-standard [3] is to identify $\sin^2\theta_W|_Z$, defined by

$$\sin^2\theta_W|_Z \equiv \frac{1}{2}\left[1 - \left\{1 - \frac{4\pi\alpha_{*,0}(M_Z^2)}{\sqrt{2}G_F M_Z^2}\right\}^{1/2}\right], \tag{38}$$

as the renormalized value of s^2, *i.e.*, equal to

$$s^2 + \delta s^2 = s^2 + \frac{s^2c^2}{c^2 - s^2}\delta\ln(s^2c^2). \tag{39}$$

with α evolved to $\alpha_{0,\star}(M_Z^2)$. (37) then leads to

$$\sin^2\theta_W|_Z - s^2 = \frac{s^2c^2}{c^2 - s^2}\left(\frac{\delta\alpha_{0,\star}}{\alpha_{0,\star}} - \frac{\delta G_F}{G_F} - \frac{\delta M_Z^2}{M_Z^2}\right). \tag{40}$$

Following Lynn, Peskin and Stuart [13], we can write

$$\frac{\delta\alpha_{0,\star}}{\alpha_{0,\star}} = \Pi'_{\gamma\gamma}(M_Z^2), \tag{41}$$

$$\frac{\delta G_F}{G_F} = -\frac{\Pi_{WW}(0)}{M_W^2}, \tag{42}$$

$$\frac{\delta M_Z^2}{M_Z^2} = \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2}. \tag{43}$$

(40)-(43) yield

$$\sin^2\theta_W|_Z - s^2 = e^2\frac{s^2c^2}{c^2-s^2}\Big[\Pi'_{QQ}(M_Z^2) + \frac{1}{s^2c^2M_Z^2}\{\Pi_{11}(0) - \Pi_{33}(M_Z^2) + 2s^2\Pi_{3Q}(M_Z^2) - s^4\Pi_{QQ}(M_Z^2)\}\Big]. \quad (44)$$

On the other hand, from (29), we can write

$$s_*^2(q^2) - s^2 = -e^2[\Pi'_{3Q}(q^2) - s^2\Pi'_{QQ}(q^2)]. \quad (45)$$

(44) and (45) lead to the results

$$s_*^2(M_Z^2) = \sin^2\theta_W|_Z + \frac{\alpha}{c^2-s^2}(\frac{1}{4}S - s^2c^2T), \quad (46)$$

$$s_*^2(0) = \sin^2\theta_W|_Z + \frac{\alpha}{c^2-s^2}(\frac{1}{4}S - s^2c^2T) - \alpha X. \quad (47)$$

We note that X does not appear in the observables measured on the Z-pole, while it would appear in the low-energy observables that contain $s_*^2(0)$. It is also noteworthy that we can write s_*^2 exactly only at $q^2=0$ and $q^2=M_Z^2$ and at no other value of q^2, as is evident from the expression for X. Thus, for example, in order to evaluate $s_*^2(M_W^2)$, one has to use the quadratic expansion approximation [7]. This is why the latter will be needed in obtaining bounds on W. It is also interesting that X appears through $s_*^2(0)$ in the expression [14] for the left-right parity-violating Moller asymmetry.

The ratio of the gauge boson masses can be expressed in terms of the old oblique parameters. Starting with

$$\begin{aligned} M_W^2 &= {M_W^0}^2 + \frac{e^2}{s^2}\Pi_{11}(M_W^2) \\ &= (M_Z^2 - \delta M_Z^2)(\cos^2\theta_W|_Z - \delta\cos^2\theta_W|_Z) + \frac{e^2}{s^2}\Pi_{11}(M_W^2), \end{aligned} \quad (48)$$

and using (40), we derive

$$M_W^2/M_Z^2 = \cos^2\theta_W|_Z + \frac{\alpha c^2}{c^2-s^2}\Bigg[-\frac{1}{2}S + c^2T + \frac{c^2-s^2}{4s^2}U\Bigg]. \quad (49)$$

(The derivation proceeds much in the same manner as that given in the longer article by Peskin and Takeuchi [3].)

The partial widths measured at the Z-pole contain S and T among the old obique parameters, and V among the new ones. This is evident from the following expressions in the $\star$ scheme.

$$\Gamma(Z\to\nu\bar\nu) = Z_{Z^\star}(M_Z^2)\frac{\alpha_{\star,0}(M_Z^2)M_Z}{24s_\star^2(M_Z^2)c_\star^2(M_Z^2)}, \quad (50)$$

$$\Gamma(Z \to \ell\bar{\ell}) = Z_{Z^\star}(M_Z^2)\frac{\alpha_{\star,0}(M_Z^2)M_Z}{6s_\star^2(M_Z^2)c_\star^2(M_Z^2)}(1+\delta_{\rm QED}) \left[\left\{-\frac{1}{2}+s_\star^2(M_Z^2)\right\}^2+\left\{s_\star^2(M_Z^2)\right\}^2\right], \quad (51)$$

$$\Gamma(Z \to u\bar{u}) = \Gamma(Z \to c\bar{c}) = Z_{Z^\star}(M_Z^2)\frac{\alpha_{\star,0}(M_Z^2)M_Z}{2s_\star^2(M_Z^2)c_\star^2(M_Z^2)}(1+\delta_{\rm QED}+\delta_{\rm QCD}) \left[\left\{\frac{1}{2}-\frac{2}{3}s_\star^2(M_Z^2)\right\}^2+\left\{-\frac{2}{3}s_\star^2(M_Z^2)\right\}^2\right], \quad (52)$$

$$\Gamma(Z \to d\bar{d}) = \Gamma(Z \to s\bar{s}) = Z_{Z^\star}(M_Z^2)\frac{\alpha_{\star,0}(M_Z^2)M_Z}{2s_\star^2(M_Z^2)c_\star^2(M_Z^2)}(1+\delta_{\rm QED}+\delta_{\rm QCD}) \left[\left\{-\frac{1}{2}+\frac{1}{3}s_\star^2(M_Z^2)\right\}^2+\left\{\frac{1}{3}s_\star^2(M_Z^2)\right\}^2\right], \quad (53)$$

where

$$\delta_{\rm QED} = 3\alpha Q_f^2/4\pi, \quad \delta_{\rm QCD} = \frac{\alpha_S}{\pi}+1.409\left(\frac{\alpha_S}{\pi}\right)^2-12.77\left(\frac{\alpha_S}{\pi}\right)^3, \quad (54)$$

α_S being the QCD fine structure constant and a colour factor of 3 has been included in the $Z \to q\bar{q}$ cases. The decay of Z into a $b\bar{b}$ pair is on a slightly different footing, since the vertex correction mediated by a virtual top quark has to be taken care of. This correction can be accounted for by writing $\Delta\rho_t = 3m_t^2 G_F/(8\pi\sqrt{2})$ and replacing s_*^2 in (53) by $s_*^2(1+\frac{2}{3}\Delta\rho_t)$, assuming that the BSM physics does not contribute significantly to the non-oblique part.

Employing the numerical values of α, $\alpha_{*,0}(M_Z^2)$, α_S, M_Z and G_F, one can write these Z-pole observables, as well as the total decay width Γ_Z, as functions of S,T and V, as shown in the Appendix. In contrast, the ratio of any two partial widths of the Z — *e.g.*, $\Gamma(Z \to {\rm hadrons})/\Gamma(Z \to \ell\bar{\ell})$ — does not contain V, which is also true for all Z-pole asymmetries. Thus, these ratios and asymmetries put bounds on S and T, while the absolute values of the total and partial decay widths of Z contain V also. Now one can use eq. (49) to put a bound on U. This is somewhat of a loose bound at the moment, but once M_W is known with an accuracy comparable to that of M_Z — from the Tevatron and from LEP-II — the bound on U would become quite stringent.

We now consider the W-width, which is also expected to be measured quite accurately at LEP-II. The one-loop corrected leptonic decay width of the W is given in the $\star$ scheme by

$$\Gamma_W^\ell \equiv \Gamma(W \to \ell\bar{\nu}) = Z_{W^\star}(M_W^2)\frac{\alpha_{\star,0}(M_W^2)(M_W/M_Z)}{12s_\star^2(M_W^2)}M_Z. \quad (55)$$

The corresponding total width is

$$\begin{aligned}\Gamma_W &= 3\Gamma_W^\ell + 3\{1+\delta_{\rm QCD}\}\Gamma_W^\ell\{|V_{ud}|^2+|V_{cd}|^2+|V_{us}|^2+|V_{cs}|^2\} \\ &= [3+3.123(1+|V_{tb}|^2)]\Gamma_W^\ell. \end{aligned} \tag{56}$$

$\alpha_{*,0}(M_W^2)$ comes out as 128.89 ± 0.12 from logarithmic running. Further, for the evolution from M_Z^2 to M_W^2, the use of the quadratic approximation [7] may not be too bad. This yields

$$s_\star^2(M_W^2) = \sin^2\theta_W|_Z + \frac{\alpha}{c^2-s^2}\left(\frac{1}{4}S - c^2s^2T\right) - \alpha s^2 X. \tag{57}$$

The expressions for the total and the partial widths of the W, as shown in the Appendix, contain S,T,U,W and X.

Finally, we come to observables related to low-q^2 experiments. For deep inelastic neutrino scattering the quark couplings are

$$g_L^2 \equiv [\rho_\star(0)]^2[(g_{L\star}^u)^2 + (g_{L\star}^d)^2], \tag{58}$$

$$g_R^2 \equiv [\rho_\star(0)]^2[(g_{R\star}^u)^2 + (g_{R\star}^d)^2]. \tag{59}$$

The ratios of the neutral to charged current cross sections in the neutrino and antineutrino cases can be written in terms of the above couplings as

$$R_\nu = g_L^2 + r^0 g_R^2, \tag{60}$$

$$R_{\bar\nu} = g_L^2 + \frac{1}{\bar r^0} g_R^2, \tag{61}$$

where the numerical parameters r^0 and $\bar r^0$ are 0.383 ± 0.014 and 0.371 ± 0.014 respectively. Explicit expressions for g_{LR}^2 and $R_{\nu,\bar\nu}$, in terms of the oblique parameters S, T and X, appear in the Appendix. Last, but not the least, is the atomic weak charge of cesium which is determined by the parity-violation experiment on cesium vapour:

$$Q_W({}^{133}_{55}Cs) = -\rho_\star(0)[78 - (1-4s_\star^2(0))55]. \tag{62}$$

Like other experimental quantities measured at low energies, this also is a linear combination of S,T and X, as given in the last line of the Appendix.

V. Constraints on the New Oblique Parameters

One can put constraints on the oblique parameters, both old and new, as follows. Compare the analytical expressions shown in the previous section, and their numerical counterparts given in the Appendix, with the experimental data. First, choose the data for only those quantities whose expressions contain S,T and U, but not any of the new oblique parameters. They include $\rho_*(0)$, M_W/M_Z, and various ratios and asymmetries measured on the Z-mass. Since the SM and the BSM contributions add linearly and since [1], for $m_t = 175$ GeV and $m_H = 100$ GeV,

$$S_{SM} = 0.60, \quad T_{SM} = 0.79, \quad U_{SM} = 0.95, \tag{63}$$

we constrain the BSM contributions, denoted by a tilde overhead, to be

$$\tilde{\mathrm{S}} = -0.04 \pm 0.26, \quad \tilde{\mathrm{T}} = -0.04 \pm 0.31, \quad \tilde{\mathrm{U}} = -0.63 \pm 0.61. \tag{64}$$

Within 1σ error, these are perfectly consistent with zero (no observable BSM physics). The fact that the allowed ranges for $\tilde{\mathrm{S}}$, $\tilde{\mathrm{T}}$ and $\tilde{\mathrm{U}}$ are nearly the same as those obtained using the linear approximation proves the robustness of the linear approximation.

We now proceed to consider V,W and X. First note that the SM values of V,W and X are of the order of 10^{-3}. More precisely, for $m_t = 175$ GeV and $m_H = 100$ GeV, $V_{SM} = -0.004$, $W_{SM} = -0.003$ and $X_{SM} = 0.001$. Thus the constraints obtained on these new oblique parameters are essentially those on $\tilde{\mathrm{V}}$, $\tilde{\mathrm{W}}$ and $\tilde{\mathrm{X}}$. The constraint on V can be obtained from the total decay width of Z, and comes out to be

$$\tilde{\mathrm{V}} = 0.30 \pm 0.38. \tag{65}$$

Similarly, the constraint on W can be obtained from the total decay width of the W-boson, or its partial width to leptons. One finds that

$$\tilde{\mathrm{W}} = 0.11 \pm 4.70. \tag{66}$$

Because of the greater inaccuracy in the present data on the W, relative to those on the Z, this constraint on W is one order of magnitude weaker as compared to that on V. Coming to X, we constrain it from observables at $q^2 = 0$ as follows:

$$\tilde{\mathrm{X}} = 0.38 \pm 0.59. \tag{67}$$

As expected, all of these new oblique parameters are compatible with zero, and central values are of the same order as those of $\tilde{\mathrm{S}}$, $\tilde{\mathrm{T}}$ and $\tilde{\mathrm{U}}$.

In the $\star$ scheme explicit expressions for the oblique parameters can be written in terms of observable quantities. For S,T,U, such expressions were already given by Peskin and Takeuchi [3]. We write those for V,W,X below.

$$V = 162.0 + 55.0\Gamma_Z - 7.8R_l - 137.0\rho_*(0), \tag{68}$$

$$W = 582.0 - 15.52R_l + 9.89\rho_*(0) - 461.72\frac{M_W}{M_Z} + 0.137Q_W({}^{133}_{55}Cs) + 66.34\Gamma_W, \tag{69}$$

$$X = 163.0 - 7.90R_l + 46.90\rho_*(0) + 0.62Q_W({}^{133}_{55}Cs). \tag{70}$$

Here $R_l \equiv \Gamma(Z \to \text{hadrons})/\Gamma(Z \to \ell\bar{\ell})$. Of course, V,W and X can be written in terms of other observables too, but we have chosen the set with the least experimental errors and hence maximum sensitivity to any possible BSM contributions.

VI. **Summary and Discussion**

Three salient features of our work can be summarized as follows.

- Electroweak oblique parameters S,T,U,V,W and X can be consistently defined at the one-loop level with the help of Π, Π' and Π'' functions and the q^2-derivative of the Π'' function — without any q^2 expansion approximation. The symmetry content of S,T and U are manifest in our definitions.

- The new oblique parameters V and W appear only in the wavefunction renormalizations of the Z and the W boson respectively, and through them, in the expressions for the total and partial decay widths of these bosons. X appears only in $s_*^2(0)$ and through it, in other observables (such as Moller asymmetry [14]) at $q^2 = 0$. The physics contents of these new parameters are quite distinct.

- The constraints on the new oblique parameters are as follows: $V = 0.30 \pm 0.38$, $W = 0.11 \pm 4.70$, $X = 0.38 \pm 0.59$, and there is practically no difference here between those with tilde and those without.

What is really the new element in our work? We have given definitions of oblique parameters with correct symmetry properties. We have also shown that ratios of LEP and Tevatron observables, that are independent of the renormalization constants $Z_{W,Z}$, involve only S,T and U, and give no information on the new oblique parameters V,W and X. We have further considered LEP and Tevatron observables that involve Z_Z and Z_W; we have been able to relate Z_Z and Z_W to the oblique parameters and to derive numerical bounds on V and W. On the other hand, constraints on X follow from low-energy measurements. We have also shown that the linear approximation of Peskin and Takeuchi is, in fact, quite robust. This is because the bounds on S,T,U depend very little on whether one uses the linear approximation or not. It is nonetheless interesting to explore the new parameters V,W and X as they can be significant if there is relatively light BSM physics. With that objective, we have expressed V,W and X as functions of those experimentally measured quantities which are being measured most accurately. These can provide suitable precision probes for BSM physics with respect to better data in future. Our stress in this paper, however, has been more on formal issues concerning oblique parameters and less on precision testing. The numbers quoted are, therefore, correct only within the framework of the $\star$ scheme; one-loop non-oblique corrections as well as significant two-loop effects involving the top quark have not been taken into account. A precision analysis should include those non-oblique correction, in view of the highly accurate results on the Z-peak. Nevertheless, the general definitions of oblique parameters given in this work as well as the accompanying discussion of the expansion approximation together with the suggestive evidence for the robustness of the linear expansion will hopefully be useful in clarifying the broad picture.

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Appendix

In this appendix, we give the numerical expressions for observables measured at the Z- (or the W) peak, and at $q^2 = 0$. First we quote those observables which can be expressed as functions of S,T and U only. As already noted in the text, these expressions are only valid within the framework of the $\star$ scheme. Thus, we do not qoute any error margins in these expressions, as the precision of the $\star$ scheme can be questioned where two-loop effects, threshold effects and other non-oblique corrections are significant.

$$
\begin{aligned}
s_*^2(M_Z^2) &= 0.231[1+0.014S-0.010T], && \text{(A.1)}\\
\rho_*(0) &= 1+7.29\times 10^{-3}T, && \text{(A.2)}\\
R_l &\equiv \Gamma_h/\Gamma_{\ell\bar\ell} = 20.787[1-2.78\times 10^{-3}S+1.97\times 10^{-3}T], && \text{(A.3)}\\
R_b &\equiv \Gamma_b/\Gamma_h = 0.216[1+0.54\times 10^{-3}S-0.41\times 10^{-3}T], && \text{(A.4)}\\
R_c &\equiv \Gamma_c/\Gamma_h = 0.171[1-1.17\times 10^{-3}S+0.79\times 10^{-3}T], && \text{(A.5)}\\
\sigma_h^0 &= 41.464[1+0.59\times 10^{-3}S-0.25\times 10^{-3}T], && \text{(A.6)}\\
M_W/M_Z &= 0.877[1-1.19\times 10^{-3}S+3.68\times 10^{-3}T+4.06\times 10^{-3}U], && \text{(A.7)}\\
A_{LR} &= 0.148[1-0.175S+0.122T] = -P_\tau, && \text{(A.8)}\\
A_{FB}^l &= 0.75(A_{LR})^2 = 0.016[1-0.350S+0.245T], && \text{(A.9)}\\
A_{FB}^b &= 0.104[1-0.177S+0.124T], && \text{(A.10)}\\
A_{FB}^c &= 0.073[1-0.192S+0.134T]. && \text{(A.11)}
\end{aligned}
$$

Next, those expressions which contain V,W and X, apart from S,T and U, are listed.

$$
\begin{aligned}
s_*^2(0) &= 0.2312[1+0.014S-0.010T-0.032X], && \text{(A.12)}\\
\Gamma(Z\to\nu\bar\nu) &= 0.166[1-0.35\times 10^{-3}S+7.10\times 10^{-3}T+7.29\times 10^{-3}V], && \text{(A.13)}\\
\Gamma(Z\to\ell\bar\ell) &= 0.084[1-1.65\times 10^{-3}S+8.45\times 10^{-3}T+7.29\times 10^{-3}V], && \text{(A.14)}\\
\Gamma(Z\to u\bar u) &= \Gamma(Z\to c\bar c)\\
&= 0.297[1-5.6\times 10^{-3}S+11.2\times 10^{-3}T+7.29\times 10^{-3}V], && \text{(A.15)}\\
\Gamma(Z\to d\bar d) &= \Gamma(Z\to s\bar s)\\
&= 0.383[1-3.8\times 10^{-3}S+10.0\times 10^{-3}T+7.29\times 10^{-3}V], && \text{(A.16)}\\
\Gamma(Z\to b\bar b) &= 0.375[1-3.9\times 10^{-3}S+10.0\times 10^{-3}T+7.29\times 10^{-3}V],
\end{aligned}
$$

$$\tag{A.17}$$

$$\begin{aligned}\Gamma_h &= 2\Gamma_u + 2\Gamma_d + \Gamma_b \\ &= 1.735[1 - 4.4\times 10^{-3}S + 10.4\times 10^{-3}T + 7.29\times 10^{-3}V],\end{aligned}\tag{A.18}$$

$$\begin{aligned}\Gamma_Z &= \Gamma_{\nu\bar{\nu}} + \Gamma_{\ell\bar{\ell}} + \Gamma_h \\ &= 2.484[1 - 3.3\times 10^{-3}S + 9.6\times 10^{-3}T + 7.29\times 10^{-3}V],\end{aligned}\tag{A.19}$$

$$\begin{aligned}\Gamma(W\to\ell\bar{\nu}) &= 0.224[1 - 0.010S + 0.015T + 0.012U \\ &\qquad + 7.29\times 10^{-3}W - 1.6\times 10^{-4}X],\end{aligned}\tag{A.20}$$

$$\begin{aligned}\Gamma_W &= 2.065[1 - 0.010S + 0.015T + 0.012U \\ &\qquad + 7.29\times 10^{-3}W - 1.6\times 10^{-4}X],\end{aligned}\tag{A.21}$$

$$g_L^2 = 0.298[1 - 0.83\times 10^{-2}S + 2.04\times 10^{-2}T + 1.82\times 10^{-2}X],\tag{A.22}$$

$$g_R^2 = 0.030[1 + 2.88\times 10^{-2}S - 0.56\times 10^{-2}T - 6.32\times 10^{-2}X],\tag{A.23}$$

$$R_\nu = 0.310[1 - 6.94\times 10^{-3}S + 19.45\times 10^{-3}T + 15.19\times 10^{-3}X],\tag{A.24}$$

$$R_{\bar{\nu}} = 0.378[1 - 0.45\times 10^{-3}S + 14.92\times 10^{-3}T + 0.98\times 10^{-3}X],\tag{A.25}$$

$$Q_W({}^{133}_{55}Cs) = -73.855[1 + 9.9\times 10^{-3}S + 0.4\times 10^{-3}T - 21.7\times 10^{-3}X].\tag{A.26}$$

As already discussed in the text, we have used the quadratic approximation [7] in deriving (A.20) and (A.21), since these expressions involve $s_*^2(M_W^2)$ which cannot be evaluated without such an approximation. At any arbitrary q^2, s_*^2 can then be written as

$$s_*^2(q^2) = \sin^2\theta_W|_Z + \frac{\alpha}{c^2 - s^2}\Big[\frac{1}{4}S - c^2 s^2 T\Big] - \alpha\Big(1 - \frac{q^2}{M_Z^2}\Big)X.\tag{A.27}$$

(A.27) assumes that $\Pi'_{\gamma Z}$ has a constant slope from $q^2 = 0$ to $q^2 = M_Z^2$ — which is the quadratic approximation. At $q^2 = 0$ or $q^2 = M_Z^2$, no such approximation is needed; the expression is exact. However, at $q^2 = M_W^2$, the quadratic-approximated (A.27) yields

$$s_*^2(M_W^2) = \sin^2\theta_W|_Z + \frac{\alpha}{c^2 - s^2}\Big[\frac{1}{4}S - c^2 s^2 T\Big] - \alpha s^2 X.\tag{A.28}$$

References

[1] G. Altarelli and R. Barbieri, Phys. Lett. **B253**, 161 (1991); G. Altarelli, R. Barbieri and S. Jadach, Nucl. Phys. **B369**, 3 (1992); G. Altarelli, R. Barbieri and F. Caravaglios, *ibid*, **B405**, 3 (1993).

[2] F. Jegerlehner, in *Radiative Corrections in* $SU(2)_L \times U(1)_Y$, eds. B.W. Lynn and J.F. Wheater (World Scientific, 1984); W.F.L. Hollik, Fortschr. Phys. **38**, 165 (1990).

[3] M.E. Peskin and T. Takeuchi, Phys. Rev. **D46**, 381 (1992). Earlier discussions may be found in the articles by B.W. Lynn, M.E. Peskin and R.G. Stuart in *Physics at LEP*, LEP Jamboree, Geneva 1985, eds. J. Ellis and R. Peccei, Report No. CERN-86-02, and by M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 2963 (1990). See also B. Holdom, Phys. Lett. **B259**, 329 (1991).

[4] J.Erler and P. Langacker, Phys. Rev. **D52**, 441 (1995).

[5] W.J. Marciano and J.L. Rosner, Phys. Rev. Lett. **65**, 2963 (1990); D.C. Kennedy and P. Langacker, *ibid*, **65**, 2967 (1990).

[6] G. Bhattacharyya, S. Banerjee and P. Roy, Phys. Rev. **D45**, 729 (1992); E. Ma and P. Roy, Phys. Rev. Lett. **68**, 2879 (1992).

[7] R. Barbieri, in *Particle Physics at the Fermi Scale*, eds. Y. Pang, J. Qiu and Z. Qiu (Gordon and Breach, 1994).

[8] I. Maksymyk, C.P. Burgess and D. London, Phys. Rev. **D50**, 529 (1994); C.P. Burgess *et al*, Phys. Lett. **B326**, 276 (1994).

[9] R. Barbieri, M. Frigeni and F. Caravaglios, Phys. Lett. **B279**, 169 (1992).

[10] See Hollik, ref. [2].

[11] Review of Particle Properties, Phys. Rev. **D50**, 1173 (1994).

[12] D.C. Kennedy and B.W. Lynn, Nucl. Phys. **B322**, 1 (1989).

[13] B.W. Lynn, M.E. Peskin and R.G. Stuart, ref. [3].

[14] A. Czarnecki and W.J. Marciano, Phys. Rev. **D53**, 1066 (1996).