New interactions in neutrino oscillations with three light flavors

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Abstract

If one assumes solar and LSND neutrino oscillations to explain the corresponding data, then the atmospheric neutrino deficit cannot be accommodated within the Standard Model with three light flavors, unless one ignores the data's zenith-angle dependence. We propose a novel solution to this problem by postulating large anomalous diagonal ν_τ-quark interactions which affect $\nu_\mu - \nu_\tau$ oscillations traversing the Earth and induce the observed zenith-angle dependence.

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Three flavors of massless lefthanded weakly interacting neutrinos occur in the Standard Model. Experimental studies at the Large Electron Positron (LEP) collider have definitively established the number of light weak neutrino flavors to be three. However, the masslessness of any neutrino is not predicted on fundamental grounds. Beyond the Standard Model, theoretical arguments exist showing how neutrinos could acquire tiny Majorana [1] or Dirac [2] masses. On the experimental front, there is indirect evidence of small nonvanishing neutrino masses from three different kinds of phenomena pertaining to neutrino oscillations. (1) The observed depletion [3] of the solar neutrino flux from the prediction of the standard solar model in different segments of the solar neutrino energy spectrum, (2) the claimed discovery [4] of $\bar{\nu}_\mu - \bar{\nu}_e$ and $\nu_\mu - \nu_e$ oscillations by the Liquid Scintillator Neutrino Detector (LSND) experiment and (3) the deficit [5] in the atmospheric neutrino flux, measured on the ground in terms of the ratio of ratios $R \equiv (\nu_\mu + \bar{\nu}_\mu) : (\nu_e + \bar{\nu}_e)_{\text{expt.}} / (\nu_\mu + \bar{\nu}_\mu) : (\nu_e + \bar{\nu}_e)_{MC}$, MC standing for the Monte-Carlo expectation – all point to nonzero neutrino masses. The canonical best fits to the data from the above three different experimental studies cannot be accommodated within the hypothesis of only three light neutrino flavors because of the three nonoverlapping ranges of δm^2 involved. This and other considerations from astrophysics and cosmology have led to the speculation of the existence [6] of a fourth sterile (i.e. electroweak singlet) light neutrino as a possible way of reconciling all of the known data. On the other hand, if only three neutrino flavors are assumed, two of the three possible neutrino oscillations can be explained and the question is to what extent the third can be accommodated. Previous studies have chosen the exception to be either the atmospheric data [7] or the solar data [8]. A recent general analysis [9] shows that the former hypothesis is in fact favored.

It seems to us that the results of the solar neutrino and the LSND experiments are quite unambiguous, assuming the absence of unknown sources of systematic error. In contrast, the detailed conclusions from the atmospheric neutrino experiment seem to depend sensitively

on the intricacies of the Monte-Carlo simulations [5] used. It may, therefore, be more profitable to consider three-flavor scenarios which naturally explain the solar neutrino and the LSND data and then to explore the observed atmospheric neutrino anomaly within those. That leads one naturally to the scheme of Ref. [7]. This scheme, in which the interactions are exclusively those of the Standard Model, leads to a universal value of R that can give acceptable fits to the sub-GeV and multi-GeV data, integrating over all zenith angles. However, it disallows any measurable dependence of the data on the zenith angle predicting an essentially flat distribution.

As in Ref. [7], we too start with only three light neutrino flavors and Standard Model interactions, but then we extend the latter to include the possibility of anomalous neutrino interactions [10]. Specifically, we allow ν_τ to have any large nonstandard diagonal four-fermion (effectively contact) interaction with quarks. This is motivated by two facts: (1) the reported observation [11] of anomalous e^+-quark interactions at HERA, which suggests the possibility of anomalous lepton-quark interactions in general; (2) among the three known neutrinos, $\nu_{\mu,e}$ are strongly forbidden by experimental constraints to have such large interactions while there exist essentially no restrictions on ν_τ. (In particular, there could be a heavy vector boson coupling only to leptons of the third generation but to the light quarks as well). As we show below, this will result in the novel possibility of an induced zenith-angle dependence for atmospheric neutrino oscillations, which appears to be favored by the data.

With an anomalous ν_τ-quark interaction, the survival probability R may vary with the zenith angle in atmospheric neutrino oscillations despite a large δm^2 chosen at around 0..25 eV^2 to satisfy the LSND data. Since the interaction cross section (in the detector) of neutrinos is roughly 3 times that of antineutrinos, and the neutrino flux is somewhat larger than the antineutrino flux [12] at higher energies, such a variation is potentially able to explain the multi-GeV atmospheric data. The extra ν_τ interactions inside the sun are offset by a

δm^2 much larger than it would be for the canonical matter-enhanced effect, as explained below.

We start with the following approximate mass eigenstates: $\nu_1 \sim \nu_e$, $\nu_2 \sim c_0\nu_\mu + s_0\nu_\tau$, $\nu_3 \sim -s_0\nu_\mu + c_0\nu_\tau$, where $c_0 \equiv \cos\theta_0$, $s_0 \equiv \sin\theta_0$, and θ_0 is not small. We choose $m_1 \sim 0$, $m_2 \sim 10^{-2}$ eV, and $m_3 \sim 0.5$ eV. We then allow ν_1 to mix with ν_3 with a small angle θ' and the new ν_1 to mix with ν_2 with a small angle θ. The exact mass eigenstates are then

$$\nu_1 = cc'\nu_e + cs'(-s_0\nu_\mu + c_0\nu_\tau) - s(c_0\nu_\mu + s_0\nu_\tau), \tag{1}$$

$$\nu_2 = c(c_0\nu_\mu + s_0\nu_\tau) + sc'\nu_e + ss'(-s_0\nu_\mu + c_0\nu_\tau), \tag{2}$$

$$\nu_3 = c'(-s_0\nu_\mu + c_0\nu_\tau) - s'\nu_e. \tag{3}$$

Let us now turn one-by-one to the three sets of neutrino oscillation data.

LSND: $\bar{\nu}_\mu - \bar{\nu}_e$ oscillations, as probed in this experiment, are controlled by $\delta m_{31}^2 \sim 0.25$ eV^2. Any significantly higher value chosen for δm_{31}^2 will be in contradiction with restrictions imposed by the search for ν_μ disappearance in the CDHS experiment [13] for large angles (which will be needed later in explaining the atmospheric neutrino effect). On the other hand, for such a value of δm^2, the LSND data [4] imply a mixing angle χ with $\sin 2\chi \simeq 0.19$. These numbers are just about compatible with the constraints of the Bugey experiment [14]. Comparing with (3), we find $2s_0 s' c' \simeq 0.19$.

Solar neutrino data: The canonical solution for solar neutrino oscillations takes $\nu_1 \sim \nu_e$ and $\nu_2 \sim$ a linear combination of ν_μ and ν_τ, with $m_2 > m_1$ and some mixing between ν_1 and ν_2. In its passage through the sun, ν_e gets an extra induced mass because of its forward scattering with the electrons. The matching of this mass with δm_{21}^2 produces the well-known MSW effect [15]. Here we have new extra diagonal ν_τ-quark interactions. Consequently, a larger δm_{21}^2 is needed to cancel against the induced ν_τ mass, which should be negative in this case [16].

Let us examine in detail the effect of ν_τ-quark interactions on the passage of electron neutrinos through the sun. We write this new interaction as [10]

$$\mathcal{L}_{\text{new}} = -\sqrt{2}\bar{\nu}_{\tau L}\gamma_\mu \nu_{\tau L}(G^q_{\tau\tau V}\bar{q}\gamma^\mu q + G^q_{\tau\tau A}\bar{q}\gamma^\mu \gamma_5 q) \tag{4}$$

for all quarks q. Note that the τ neutrino and the τ antineutrino are known from τ-decay properties to be lefthanded and righthanded respectively. Therefore, all possible four-fermion interactions involving them and quarks can be brought into the form (4) by Fierz transformations. Only the vector coupling $G^q_{\tau\tau V}$ contributes to the potential relevant to forward scattering while its contributions for the neutrino and antineutrino cases are equal in magnitude but opposite in sign. We define $\epsilon'_q \equiv G^q_{\tau\tau V}/G_F$ as in Ref. [10]. Note that we do not require any flavor-changing interactions which would have been necessary to obtain oscillations if $\delta m^2 = 0$.

Solar neutrino oscillations occur between ν_e and $\nu_\alpha = c_0\nu_\mu + s_0\nu_\tau$ with angle θ. Since $m_3^2 \gg m_2^2$, $\nu_\beta = -s_0\nu_\mu + c_0\nu_\tau$ effectively decouples. Hence the relevant evolution equation can be written, after rotating away a common phase, as [10]

$$4iE_\nu \frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} \simeq \begin{pmatrix} 0 & m_2^2\sin 2\theta \\ m_2^2\sin 2\theta & 2m_2^2\cos 2\theta + 4\sqrt{2}G_F E_\nu(s_0^2\epsilon'_q N_q - N_e) \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix}. \tag{5}$$

In (5), $N_q\epsilon'_q \equiv N_u\epsilon'_u + N_d\epsilon'_d$ and $N_{e,u,d}$ is the number of (electrons, u-quarks, d-quarks) per unit solar volume. The coefficient $s_0^2 \equiv \sin^2\theta_0$ of the $\epsilon'_q N_q$ term in the second diagonal matrix element originates from the $3 \times 3 \to 2 \times 2$ flavor matrix reduction. In order to have a large ϵ'_q and yet satisfy the resonance condition for solar-neutrino flavor conversion, we see that m_2 should be larger than its canonical value of 2.45×10^{-3} eV, and ϵ'_q should be negative. [If ϵ'_q comes from R-parity violating squark exchange, then it must be positive; but if it comes from vector exchange, then it may be of either sign.]

We now assume as a crude approximation that $N_q \simeq 4N_e$ in the sun. It then follows from

(5) that the effective mixing angle for $\nu_e - \nu_\alpha$ oscillations in solar matter is given by

$$\tan 2\theta_m^S = \frac{\sin 2\theta}{\cos 2\theta + 2\sqrt{2}G_F m_2^{-2} E_\nu (4s_0^2 \epsilon_q' - 1)N_e} \tag{6}$$

and the MSW resonance condition [15] is

$$m_2^{-2}(-4s_0^2 \epsilon_q' + 1) = \cos 2\theta (2\sqrt{2}G_F N_e E_\nu)^{-1}. \tag{7}$$

In the canonical MSW solution, the left-hand side is $(6 \times 10^{-6} \mathrm{eV}^2)^{-1}$. It can thus be matched with the requirement

$$s_0^2 \epsilon_q' \simeq -3.92 = -4.17(m_2^2/10^{-4}\mathrm{eV}^2) + 0.25. \tag{8}$$

Our seemingly arbitrary choice of $\delta m_{21}^2 \sim 10^{-4}$ eV^2 is now seen as a reasonable value so that ϵ_q' can be large enough to be relevant for the following discussion on the atmospheric neutrino data. Note that the same range of θ, $i.e.$ near 0.04, works here as well as in the standard MSW solution for solar neutrino oscillations.

Atmospheric neutrino data: The depletion in the flux of muon neutrinos and antineutrinos, produced in the earth's upper atmosphere, is caused by $\nu_\mu - \nu_\tau$ flavor oscillations which occur between the physical states ν_2 and ν_3 with $\delta m_{32}^2 \sim 0.25$ eV^2 and angle θ_0. The oscillation wavelength "in vacuo" is $\lambda \sim 4\pi E_\nu/\delta m_{32}^2 \sim 10(E_\nu/\mathrm{GeV})$ km, E_ν being the neutrino energy. Hence, for $E_\nu < 10$ GeV, several oscillations occur in the earth's atmosphere. Consequently, one obtains the classical survival probability $P_0 = c_0^4 + s_0^4 = 1 - \frac{1}{2}\sin^2 2\theta_0$. The choice of $s_0 \simeq 0.47$ yields $R = P_0 \simeq 0.66$ for the ratio of ratios, in reasonable agreement with the data for downward going neutrinos and antineutrinos. This implies $s' \simeq 0.21$.

For neutrinos and antineutrinos coming downward, the density of the atmosphere is negligible for the new diagonal ν_τ-quark interactions to be of any importance. For upward moving ones, the density of the earth turns out to be in the right range for them to make a difference. Since ν_e gets effectively decoupled from the $\nu_\mu - \nu_\tau$ oscillation problem, we now

have

$$4iE_\nu \frac{d}{dt} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} \simeq \begin{pmatrix} 0 & m_3^2 \sin 2\theta_0 \\ m_3^2 \sin 2\theta_0 & 2m_3^2 \cos 2\theta_0 + 4\sqrt{2}G_F E_\nu \epsilon_q' N_q \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix}. \tag{9}$$

For the earth we estimate an average $N_q \sim 9 \times 10^{30}$ m^{-3}. Thus if one chooses to define the parameter $X \equiv \epsilon_q' E_\nu/(10 \text{ GeV})$, the effective mixing angles in terrestrial matter (a) θ_m^E between ν_μ and ν_τ and (b) $\bar{\theta}_m^E$ between $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ are respectively given by

$$\tan 2\theta_m^E = \frac{\sin 2\theta_0}{\cos 2\theta_0 + 0.091X}, \quad \tan 2\bar{\theta}_m^E = \frac{\sin 2\theta_0}{\cos 2\theta_0 - 0.091X}, \tag{10}$$

with $\cos 2\theta_0 = \mp 0.091X$ as the resonance conditons.

For sub-GeV neutrinos, the X term is insignificant, but for multi-GeV neutrinos it may become large enough for the resonance condition to be satisfied. Assuming adiabaticity, the neutrino and antineutrino flavor survival probabilities are described well by the formulae [17]

$$P(\nu_\mu \to \nu_\mu) = \frac{1}{2}(1 + \cos 2\theta_0 \cos 2\theta_m^E), \tag{11}$$

$$\bar{P}(\bar{\nu}_\mu \to \bar{\nu}_\mu) = \frac{1}{2}(1 + \cos 2\theta_0 \cos 2\bar{\theta}_m^E), \tag{12}$$

where $\cos 2\theta_m^E$ and $\cos 2\bar{\theta}_m^E$ are computed from (11). Although the conditions for adiabaticity may not be satisfied, our purpose is to try to find the maximum effect for a given magnitude of the anomalous interaction which is of course unknown. Any nonadiabaticity would only tend to diminish this effect.

Owing to the opposite signs of the media contributions to neutrino and antineutrino oscillations, matter effects will get somewhat diluted. However, there are two important factors to be considered. First, the initial ν_μ flux is larger than the $\bar{\nu}_\mu$ flux for multi-GeV neutrinos. In the upper atmosphere, $\nu_\mu(\bar{\nu}_\mu)$ is produced together with $\mu^+(\mu^-)$ from $\pi^+(\pi^-)$ decay. The subsequent decay of $\mu^+(\mu^-)$ to $\bar{\nu}_\mu(\nu_\mu)$ will equalize the total number of ν_μ and $\bar{\nu}_\mu$, but there is an energy dependence and given that the μ^+/μ^- ratio is larger than one [12], we allow a factor of $r =$ ratio of the ν_μ to $\bar{\nu}_\mu$ flux in our following discussion. Second, in the

detection of atmospheric neutrinos, there is no experimental measurement of the charge of the resulting leptons. Specifically, μ-like events include both μ^- and μ^+, but they are not separated. Now $\sigma_\nu \simeq 3\sigma_{\bar{\nu}}$ for an isoscalar target, hence the measured probability is weighted:

$$P_m \simeq \frac{3rP + \bar{P}}{3r + 1}. \tag{13}$$

In the absence of media effects, $P = \bar{P}$ even with oscillations (assuming CP conservation), hence $P_m = P = \bar{P}$. In the presence of media effects, $P \neq \bar{P}$, so the above expression for P_m should be used.

Suppose we now make the same choice of $s_0 \simeq 0.47$ which leads to a suppression probability $P_0 = 0.66$ for the downward travelling neutrinos. Then, for an optimistically large and negative value of $X = -15$, we get $P = 0.31$ and $\bar{P} = 0.76$, hence P_m is lowered to 0.39 if $r = 1.5$ in Eq. (14) or 0.42 if $r = 1.0$. This is a potential explanation of the observation of a smaller R for atmospheric neutrinos going through the earth (zenith angle $> \pi/2$) where $R = P_m$ than for those coming down through only the atmosphere (zenith angle $< \pi/2$) where $R = P_0$.

There have been studies [18] of R-parity violating squark interactions, scalar and vector leptoquarks, as well as contact interactions of neutrinos. Whereas these may be related to our proposed effective interaction, they are all restricted to be small. We consider instead a vector boson B which couples to $\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d - \bar{\nu}_\tau\gamma^\mu\nu_\tau - \bar{\tau}_L\gamma^\mu\tau_L$ with coupling g_B. This interaction would result in a negative X as desired. Using the result of a previous model [19] where B couples to baryon number, and allowing for the fact that here B couples to u and d but not s, c, and b, we find $\alpha_B \equiv g_B^2/4\pi \leq 0.057$. We now require $g_B^2/m_B^2 = 15G_F$, hence $m_B = 64$ GeV is allowed. The deviation from $e - \mu - \tau$ universality in Z decay is then given by

$$\frac{\alpha_B}{2\pi}\left(\frac{g_L^2}{g_L^2 + g_R^2}\right)_\tau (F_1 + F_2) = \frac{0.057}{2\pi}(0.5735)(1.32) = 0.0069, \tag{14}$$

where F_1 and F_2 are well-known functions [19] of the ratio m_B^2/M_Z^2 which is about 0.5 for $m_B = 64$ GeV. The above is exactly two standard deviations from the experimental data [20], taking into account the kinematical correction due to m_τ in $Z \to \tau^+\tau^-$. The deviation in the total invisible width of Z is 0.012 versus the standard-model value of 3, which is again two standard deviations from the data [20]. This is then a possible explicit model for our scenario.

Our scenario will have the following consequences for the forthcoming experiments. Solar neutrino experiments will confirm the MSW solution, but the interpretation of δm^2 is subject to the ambiguity that it could be $\delta m^2/(1 - 4s_0^2\epsilon_q')$ instead. However, the anomalous ν_τ-quark interactions will be observable at SNO, thereby resolving this ambiguity. Both $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\tau$ conversion experiments will measure a δm^2 at around 0.25 eV^2, but $\sin^2 2\theta_{\mu,e} \sim 0.036$ while $\sin^2 2\theta_{\mu,\tau} \sim 0.69$. The former is outside the region being probed by reactor experiments ($\bar{\nu}_e$ disappearance) such as Chooz and Palo Verde; the latter is outside that being probed by short-baseline accelerator experiments (ν_τ appearance) such as CHORUS and NOMAD. On the other hand, both regions are covered by all the proposed long-baseline experiments (either through ν_μ disappearance, or ν_e and ν_τ appearances) such as MINOS, K2K (KEK-PS/Super-Kamiokande) and CERN-SPS/ICARUS.

More immediately, the new data from Super-Kamiokande, Soudan 2, and MACRO on $\nu_\mu + \bar{\nu}_\mu$ events through the earth will be sensitive to the anomalous ν_τ-quark interactions. There should be an energy dependence as well as a zenith-angle dependence. In particular, the zenith-angle dependence should be absent or much smaller for sub-GeV data. For a zenith angle near zero, our proposal is easily distinguishable from the $\delta m^2 \sim 10^{-2}$ eV^2 oscillation interpretation because we have $R = P_0$ whereas the latter would require $R \sim 1$, owing to the short distance between production and detection in that case. To test our hypothesis further, the detection and acceptance efficiencies of neutrinos versus antineutrinos have to be

understood in more detail. Better yet, the capability of these experiments for distinguishing neutrinos from antineutrinos should be explored [21].

Acknowledgements

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