

# Testing Anomalous $W$ Couplings in $e^-e^-$ Collisions

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## Abstract

We analyze the influence of anomalous gauge couplings in the reaction  $e^-e^- \rightarrow e^-W^-\nu_e$  at a 500 GeV linear collider. The limits imposed by this process on deviations from the standard model of electro-weak interaction, are competitive with those inferred from other high energy experiments. Furthermore, the allowed domain in the parameter space is quite different, and hence such an experiment would more than complement the other direct searches.

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The success of the LEP experiments in measuring the  $Z$ -boson mass and its couplings to fermions to an accuracy level of better than 1% has led to a dramatic confirmation of the predictions of the  $SU(2)_L \otimes U(1)_Y$  theory of electroweak interactions as embedded in the standard model (SM). A measure of this success is reflected in the recent effort to further probe the SM in the gauge sector so as to be able to experimentally establish it as ruled by a gauge principle. Indeed, some progress has already been achieved through the observation of the process  $p\bar{p} \rightarrow e\nu\gamma X$  at the Collider Detector at Fermilab (CDF) [1] and at UA2 at CERN [2]. Interpreting this to be a signal of  $W\gamma$  production and subsequent decay of the  $W$ , limits were put on possible deviations of the  $WW\gamma$  vertex from its SM structure [3]. These limits are very weak though, especially when compared with the contributions from the one-loop corrections in the SM [4]. Recently, this has led to a lot of work in identifying better signals for such deviations especially in the context of LEP-200, LHC and SSC [5, 6] as well as as HERA [7]. Several studies have also been performed for the case of linear  $e^+e^-$  colliders [8, 9] as well as for  $e\gamma$  and  $\gamma\gamma$  colliders [9], where a high energy photon beam is obtained by back-scattering an intense laser ray off a electron beam at a linear collider.

In this Letter, we advocate the use of electron-electron collisions to probe the gauge sector of the SM. Such beams can in principle be easily obtained at a linear collider of the next generation (CLIC, JLC, NLC, TESLA, VLEPP, ...). We concentrate here on a machine operating at 500 GeV and able to accumulate 1 to 10 fb<sup>-1</sup> of integrated luminosity.

In principle, any deviation of a gauge coupling from its SM value would contribute significantly in the estimation of quantum corrections. This argument was used to constrain these couplings using LEP data as well as other low energy measurements [4, 6, 10]. However, it was pointed out [11] that most of these calculations made an improper use of the cut-off procedure and as a result had grossly overestimated such constraints. Subsequently, a number of studies were made both in the context of a decoupling Lagrangian [12] as well as the case of a non-linearly realized  $SU(2)_L \otimes U(1)_Y$  symmetry with an unspecified Higgs sector [13], to obtain various constraints. All such efforts suffer though from one of two

pitfalls: either a model dependence or crucial assumptions about the relative magnitudes of various effects, and hence are no substitute for direct measurements.

The most general triple electroweak vector boson (TEVB) coupling can be parametrised in the form of an effective Lagrangian with seven parameters for each of the neutral vector bosons [14]. We shall be more restrictive, though. Since the upper limit on the neutron electric dipole moment restrict (barring large cancellations between different contributions) the  $CP$  violating parameters to be less than  $O(10^{-4})$ , we choose to neglect them altogether. Furthermore, we demand individual  $C$  and  $P$  invariance for the TEVB vertices. The Lagrangian for the  $WWV$  ( $V = \gamma/Z$ ) vertex can then be expressed as

$$\mathcal{L}_{eff}^V = -ig_V \left[ g_1^V (W_{\alpha\beta}^\dagger W^\alpha - W^{\dagger\alpha} W_{\alpha\beta}) V^\beta + \kappa_V W_\alpha^\dagger W_\beta V^{\alpha\beta} + \frac{\lambda_V}{M_W^2} W_{\alpha\beta}^\dagger W^{\beta\sigma} V^{\sigma\alpha} \right] \quad (1)$$

where  $V_{\alpha\beta} = \partial_\alpha V_\beta - \partial_\beta V_\alpha$  and  $W_{\alpha\beta} = \partial_\alpha W_\beta - \partial_\beta W_\alpha$ . In (1),  $g_V$  measures the  $WWV$  coupling strength in the SM with  $g_\gamma = e$  and  $g_Z = e \cot \theta_W$ . Whereas electromagnetic gauge invariance forces  $g_1^\gamma = 1$ , the other couplings are model dependent and the tree level SM values are  $g_1^Z = \kappa_\gamma = \kappa_Z = 1$  and  $\lambda_\gamma = \lambda_Z = 0$ .

Much criticism has been levelled at the lagrangian (1) for its apparent lack of gauge invariance. It has been shown recently [11], however, that any Lorentz and  $U(1)_{em}$  gauge invariant Lagrangian, containing  $W's$  and  $Z's$ , automatically obeys  $SU(2)_L \otimes U(1)_Y$  gauge invariance, realized nonlinearly in general. Though the deviations of the above parameters from their SM values, as viewed in the context of effective field theories, are expected to be tiny, in some scenarios they can be very large indeed [15]. We shall not comment on the possible source of such deviations, but rather concentrate on their measurability. Note that the use of (1), rather than the equivalent full chiral lagrangian [16] is well-justified in such a phenomenological study.

Having established the formalism, we now turn to the laboratory. The process we consider here is

$$e^- e^- \longrightarrow e^- W^- \nu_e . \quad (2)$$

The final state is thus exceedingly simple: an electron, a  $W^-$  boson and missing momen-

tum. Note that unlike in most signals in hadronic or  $e^+e^-$  collisions, there is no  $s$ -channel process here. At tree level the contributing diagrams, shown in Fig. 1, are of two types : (i) a  $t$  or  $u$ -channel exchange of a vector boson with a “ $W$ -Bremsstrahlung” off one of the four fermions, or (ii) those involving a trilinear coupling. Only the diagrams pertaining to the second class are of course sensitive to anomalous couplings.

The analytical expressions for the matrix elements were obtained as a function of the five anomalous parameters  $g_1^Z$ ,  $\kappa_\gamma$ ,  $\kappa_Z$ ,  $\lambda_\gamma$  and  $\lambda_Z$ , with the aid of the software system COMPHEP [17] and by an independent helicity-amplitude calculation. The Monte-Carlo routine VEGAS [18] was used for the numerical integration over phase space.

For the electron to be visible, we required that its transverse momentum be greater than 5 GeV and that its absolute rapidity should not exceed 3. These requirements also eliminate a very large fraction of the contributions originating from the  $W$ -Bremsstrahlung diagrams alone, thus enriching the signal to background ratio. In addition, the rapidity cut ensures that the internal photon never comes close to mass shell and hence provides an easily integrable matrix element squared. Because the  $W$  boson undergoes a further decay we have imposed no kinematical cut on its momentum. Explicit computations show though that similar cuts on the  $W$ -momentum have only a marginal effect. Since a  $e^+e^-$  machine suffers very little background hadronic activity, the reconstruction of a single  $W$  should thus be straightforward and we shall assume, in this study, an efficiency of 100%. In the SM limit the total cross-section amounts to  $\sigma_{SM} = 1.49$  pb [19].

An effort to deal simultaneously with all five parameters in of the lagrangian (1) is bound to lead to a great deal of confusion. We therefore first present in Fig. 2 the dependence of the difference between the SM and anomalous cross-sections on each of the five parameters with the others assuming their tree-level SM values. This yields some idea about the detectability of the TEVB couplings at such a collider. In addition it offers a check of our calculations in that the dependence on  $\kappa_\gamma$  and  $\lambda_\gamma$  is similar to that obtained in ref. [7] for the analogous process  $ep \rightarrow eW^\pm$  jets at HERA.

The dependence on the  $WW\gamma$  couplings is much more pronounced than that on the  $WWZ$  coupling. This is easily traced back to the ratio of the  $\gamma$ - and the  $Z$ -couplings both to the fermions and the  $W$  and is in marked contrast to the case of their contributions to the precision electroweak parameters [12, 13]. The different sensitivity there arises from the fact that the most accurate electroweak measurements are those dealing with the properties of the  $Z$ . Furthermore, the dependence on  $\lambda_{\gamma,Z}$  is weaker than that on  $\kappa_{\gamma,Z}$ . However, since  $\lambda_{\gamma,Z}$  represent interaction terms of dimension 6 or higher, the sensitivity to these parameters grows faster with the interaction energy and hence they would be more easily detectable at machines operating at a higher energy, *e.g.* the LHC.

To assess the resolving power of the process (2), we require the absolute difference between the SM and anomalous cross-sections  $\Delta\sigma = |\sigma_{\text{SM}} - \sigma_{\text{anom}}|$  to be large enough to provide a deviation from the SM prediction exceeding the Poisson fluctuations by  $N$  standard deviations *i.e.*, the event numbers ( $n = \text{luminosity} \times \sigma$ ) should satisfy the relation  $\Delta n > N\sqrt{n_{\text{SM}}}$ . The required luminosity  $\mathcal{L}$  to reach this goal is then

$$\mathcal{L} > \frac{N^2 \sigma_{\text{SM}}}{\Delta\sigma^2}. \quad (3)$$

To compare the resolving power of this reaction with the projections for conventional linear colliders, we present the 90% C.L. ( $N = 1.64$  in Eq. (3)) limits to which it can constrain these parameters for an integrated luminosity of  $10 \text{ fb}^{-1}$ . Under the similar assumption that only one of the parameters may be non-zero, these are then

$$\begin{aligned} \kappa_\gamma & : (0.982, 1.018) & \kappa_Z & : (0.89, 1.09) \\ \lambda_\gamma & : (-0.08, 0.09) & \lambda_Z & : (-0.13, 0.15) \\ & & g_Z^1 & : (0.90, 1.075) \end{aligned} \quad (4)$$

These results are comparable to those obtained in Refs [6, 8, 9]. The slightly tighter constraints these authors obtain for some of the parameters, can be traced back to a more detailed analysis of the angular distributions, which goes beyond the scope of this Letter. Without such an analysis (*e.g.* Kalyniak *et al.* in [8]), the limits are weaker than what we achieve from a straightforward comparison of the total cross sections.

Although this analysis provides us with grounds for optimism, yet some circumspection is

called for. Indeed, it is rather unlikely that, if at all, then only one parameter assume an anomalous value while all other remain at their SM values. Even if one were to neglect the effects of interference between the different contributions on grounds of their representing operators of higher dimension (though such an argument is invalid in a phenomenological analysis such as the present), we still would have to worry about incoherent addition. The best approach then is to obtain contour plots. As we are still plagued by the plethora of parameters, we choose to present only two combinations (in each of which the other parameters are assumed to have their SM values).

Assuming only  $\kappa_\gamma - 1$  and  $\kappa_Z - 1$  to be non-zero, Fig. 3 shows the  $3\sigma$  band (according to Eq. (3), with  $N = 3$ ) to which these parameters can be restricted to with a total integrated luminosity of  $1 \text{ fb}^{-1}$ . Clearly, the interference between the  $\kappa_\gamma$  and the  $\kappa_Z$  contributions can be large, thus rendering even a large value of either unobservable. An increase in luminosity would not change this fact in any way, for we would still have a band of the same average dimensions, albeit narrower. One may argue however, that from a theoretical standpoint, such large cancellation are unlikely. We have hence concentrated more on the region around the SM point (Fig. 4) and show the reachability for a integrated luminosity of both 1 and  $10 \text{ fb}^{-1}$ . In Fig. 5, we show similar contours for the  $\lambda_\gamma - \lambda_Z$  pair of parameters. Unlike the  $\kappa$ -case, here we do not observe a “mexican hat” band but a simple well centered around the SM values 0. As expected, the sensitivity to the  $\lambda$ 's is much weaker than that to the  $\kappa$ 's.

To conclude, we demonstrate the considerable power of a  $e^-e^-$  collider as a tool for unravelling the self-interactions of the electroweak gauge bosons. In fact, the limits that can be obtained from a simple minded analysis of the total cross-sections alone compares quite favourably with those deduced from a sophisticated analysis of the decay distributions in a more conventional collider. However, what accords even greater value to such an experiment is the significant difference in the region of the parameter space that it would probe. We believe, hence, that it would prove to be of great import when used in conjunction with the other experiments.

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## Figure Captions

1. Lowest order Feynman diagrams contributing to the process in Eq. (2).
2. Difference in cross-sections of the SM and anomalous processes of Eq. (2) as a function of each of the five anomalous couplings in Eq. (1), when all others assume their SM values. The SM cross-section amounts to 1.49 pb.
3. Contours of detectability at the  $3\sigma$  level ( $N = 3$  in Eq. (3)) of the  $\kappa_\gamma$  and  $\kappa_Z$  parameters for an integrated luminosity of  $1 \text{ fb}^{-1}$ . All other parameters assume their SM value.
4. Blowup of Fig. 3 in the neighbourhood of the SM values. The contours for  $10 \text{ fb}^{-1}$  are also shown.
5. Same as Fig. 4 for the  $\lambda_\gamma$  and  $\lambda_Z$  parameters.

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