# Estimates of Long-distance Contributions to the $B_{s} \rightarrow \gamma \gamma$ DECAY 

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#### Abstract

We present first calculations of new long-distance contributions to $B_{s} \rightarrow \gamma \gamma$ decay due to intermediate $D_{s}$ and $D_{s}^{*}$ meson states. The relevant $\gamma$ vertices are estimated using charge couplings and transition moment couplings. Within our uncertainties, we find that these long-distance contributions could be comparable to the known short-distance contributions. Since they have different Cabibbo-KobayashiMaskawa matrix-element factors, there may be an interesting possibility of observing CP violation in this decay.


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[^0]A new era of experiments on rare $B$ decays is about to dawn. The large data sets already obtained at LEP and CESR will be dwarfed by those provided by the $e^{+} e^{-} B$ factories, and by the hadron experiments HERA-B, CDF, D0 and LHC-B. Among the rare $B$ decays with particularly clean experimental signatures is $B_{s} \rightarrow \gamma \gamma$, whose branching ratio presently has the experimental upper limit $\mathcal{B}\left(B_{s} \rightarrow \gamma \gamma\right)<1.48 \times 10^{-4}$ [1] ]. Higher-sensitivity measurements of $B_{s}$ decays are not among the highest priorities of the $e^{+} e^{-} B$ factories, whose physics programmes are focussed initially on searches for CP violation in $B_{d}$ decays, but the hadron experiments cannot avoid being sensitive to $\mathcal{B}\left(B_{s} \rightarrow \gamma \gamma\right)$ to levels several orders of magnitude below the present experimental upper limit [1].

The lowest-order short-distance contributions to the $B_{s} \rightarrow \gamma \gamma$ decay arise from two sets of graphs: (i) box diagrams, and (ii) triangle diagrams with an external photon leg. These have been calculated [2] , and yield, for $m_{t} \approx 175 \mathrm{GeV}$, $\mathcal{H}\left(B_{s} \rightarrow \gamma \gamma\right) \sim 3.8 \times 10^{-7}$, the next generation of experiments. Interestingly, the branching fraction can be enhanced substantially in extensions of the Standard Model such as a generic 2-Higgs scenario [5]. Emboldened by the fact that the short-distance QCD corrections are not too big [5] [7], one may then even attempt to use such data as may be forthcoming as probes for new physics.

However, it is not immediately obvious that this decay is dominated by such short-distance contributions. Estimates have been made of some long-distance contributions to the decay amplitudes, for example via intermediate charmonium states $J / \psi$ and $\psi^{\prime}[8]$. These were found to be very small, indicating that the short-distance contribution might be correct in order of magnitude. But, before concluding this to be the case, one must examine other possible long-distance contributions.

In this paper we study such contributions due to intermediate $D_{s}$ and $D_{s}^{*}$ states via the diagrams shown in Figures 1 and 2. These include loops of $D_{s}$ mesons alone, loops of $D_{s}^{*}$ mesons alone, and diagrams involving radiative $D_{s} \rightarrow D_{s}^{*}$ transitions. The vertices are estimated using data on $B \rightarrow D D, D D^{*}$ and $D^{*} D^{*}$ decays, the electromagnetic charge couplings of the $D_{s}$ and $D_{s}^{*}$, and phenomenological estimates of the $D_{s}^{*} \rightarrow D_{s}+\gamma$ decay rate for the inelastic transition-matrix element. The imaginary part of the diagrams can be calculated easily while the determination of the real part requires the use of dispersion relations.

We find that these long-distance contributions to $B_{s} \rightarrow \gamma \gamma$ decay may be comparable to the short-distance contributions calculated previously, within the considerable uncertainties inherent to the phenomenological inputs we use. We find no evidence that $\mathcal{B}\left(B_{s} \rightarrow \gamma \gamma\right)$ decay should occur at a rate very close to the present experimental upper limit, but even this possibility cannot be ruled out. Since the short- and long-distance contributions have different Cabibbo-Kobayashi-Maskawa matrix-element factors, the fact that they may be of comparable magnitudes offers the interesting possibility of observing CP violation in this decay mode. However, a detailed exploration of this possibility lies beyond the scope of this paper.

At the quark level, the operators responsible for the $\mathcal{B}\left(B_{s} \rightarrow \gamma \gamma\right)$ decay are $\bar{b} \gamma_{5} s F_{\mu \nu} F^{\mu \nu}$

[^1]and $\bar{b} \gamma_{5} s F_{\mu \nu} \widetilde{F}^{\mu \nu}$, where $F$ is the electromagnetic fileld strength tensor, and $\widetilde{F}$ is its dual. The matrix element for the decay $B_{s} \rightarrow \gamma\left(q_{1 \mu}\right) \gamma\left(q_{2 \nu}\right)$ can then be parametrized as
\[

$$
\begin{align*}
\mathcal{M} & =\alpha \mathcal{G}\left(\mathcal{R}_{1} S_{\mu \nu}+i \mathcal{R}_{2} P_{\mu \nu}\right) \\
S_{\mu \nu} & \equiv q_{1 \nu} q_{2 \mu}-q_{1} \cdot q_{2} g_{\mu \nu} \\
P_{\mu \nu} & \equiv \epsilon_{\mu \nu \alpha \beta}^{\alpha} q_{1}^{\alpha} q_{2}  \tag{1}\\
\mathcal{G} & =\frac{G_{F}}{\sqrt{2}} V_{c b} V_{c s}^{*}
\end{align*}
$$
\]

where $\mathcal{R}_{1}$ and $\mathcal{R}_{2}$ are the yet-to-be-determined hadronic matrix elements, and $(\alpha \mathcal{G})$ has been factored out in anticipation of the results to be derived later in the text. The partial width is then given by

$$
\begin{equation*}
\Gamma\left(B_{s} \rightarrow \gamma \gamma\right)=\frac{(\alpha \mathcal{G})^{2}}{64 \pi} m_{B}^{3}\left(\left|\mathcal{R}_{1}\right|^{2}+\left|\mathcal{R}_{2}\right|^{2}\right) \tag{2}
\end{equation*}
$$

Since $\Gamma\left(B_{s}\right) \approx 5 \times 10^{-10} \mathrm{MeV}$, we have, for $\left|V_{c b}\right|=0.04$,

$$
\begin{equation*}
\operatorname{Br}\left(B_{s} \rightarrow \gamma \gamma\right) \approx 10^{-7}\left(\left|\frac{\mathcal{R}_{1}}{100 \mathrm{MeV}}\right|^{2}+\left|\frac{\mathcal{R}_{2}}{100 \mathrm{MeV}}\right|^{2}\right) \tag{3}
\end{equation*}
$$

It can be argued that the long-distance contributions should be dominated by two-meson intermediate states. Now, the quark-level transitions with the least Cabibbo suppression are $b \rightarrow c \bar{c} s$ and $b \rightarrow c \bar{u} d$. The second process obviously leads to very dissimilar mesons which may rescatter into two photons only through higher-order terms. Consequently, we concentrate on the $b \rightarrow c \bar{c} s$ case, with the corresponding weak Hamiltonian being given by

$$
\begin{align*}
\mathcal{H}_{\mathrm{wk}} & =\mathcal{G} J_{\mu} j^{\mu} \\
J_{\mu} & =\bar{c} \gamma_{\mu}\left(1-\gamma_{5}\right) b  \tag{4}\\
j^{\mu} & =\bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) c
\end{align*}
$$

where $V$ denotes the CKM matrix. The relevant intermediate states are then $D_{s}^{+} D_{s}^{-}, D_{s}^{*+} D_{s}^{*-}$, $D_{s}^{*+} D_{s}^{-}$, etc. ${ }^{\top}$. To calculate the $B_{s} \rightarrow \gamma \gamma$ decay amplitude exactly, we need to determine the corresponding matrix elements of (74), a non-trivial task. However, as a first approximation, we may assume naive factorization ${ }^{\beta}$. The $H_{\text {wk }}$ matrix elements are then expressible in terms of

[^2]simpler quantities given by
\[

$$
\begin{align*}
\left\langle D_{s}^{-}(q)\right| j^{\mu}|0\rangle & =-i f q^{\mu} \\
\left\langle D_{s}^{*-}(k, \epsilon)\right| j^{\mu}|0\rangle & =m_{*} f_{*} \epsilon^{* \mu} \\
\left\langle D_{s}^{-}(k)\right| J_{\mu}\left|B_{s}(p)\right\rangle & =0 \\
\left\langle D_{s}^{*-}(k, \epsilon)\right| J_{\mu}\left|B_{s}(p)\right\rangle & =0 \\
\left\langle D_{s}^{+}(k)\right| J_{\mu}\left|B_{s}(p)\right\rangle & =f^{+}\left(q^{2}\right)(p+k)_{\mu}+f^{-}\left(q^{2}\right) q_{\mu}  \tag{5}\\
\left\langle D_{s}^{*+}(k, \epsilon)\right| J_{\mu}\left|B_{s}(p)\right\rangle & =\frac{\mathcal{V}}{m_{*}} \epsilon_{\mu \nu \alpha \beta} \epsilon^{* \nu} p^{\alpha} k^{\beta} \\
& +\frac{i}{m_{*}}\left[m_{B}^{2} \mathcal{A}_{1} \epsilon_{\nu}^{*}-\epsilon^{*} \cdot p\left\{\mathcal{A}_{2}(k+p)_{\mu}+\frac{m_{*}^{2}}{q^{2}} \mathcal{A}_{3} q_{\mu}\right\}\right]
\end{align*}
$$
\]

where $q=p-k$ in the last two terms, and $m_{*} \equiv m_{D^{*}}$. The quantities $\mathcal{V}$ and $\mathcal{A}_{i}$ are related to the usual form factors through

$$
\begin{align*}
\mathcal{V} & =\frac{2 m_{*}}{m_{B}+m_{*}} V_{B_{s} D_{s}^{*}}\left(q^{2}\right) \\
\mathcal{A}_{1} & =\frac{m_{*}\left(m_{B}+m_{*}\right)}{m_{B}^{2}} A_{B_{s} D_{s}^{*}}^{1}\left(q^{2}\right)  \tag{6}\\
\mathcal{A}_{2} & =\frac{m_{*}}{m_{B}+m_{*}} A_{B_{s} D_{s}^{*}}^{2}\left(q^{2}\right) \\
\mathcal{A}_{3} & =2\left[A_{B_{s} D_{s}^{*}}^{3}\left(q^{2}\right)-A_{B_{s} D_{s}^{*}}^{0}\left(q^{2}\right)\right]
\end{align*}
$$

Finally, we have

$$
\begin{align*}
\left\langle D_{s}^{+}(k) D_{s}^{-}(q)\right| \mathcal{H}_{\mathrm{wk}}\left|B_{s}(p)\right\rangle= & -i \mathcal{G} f\left[\left(p^{2}-k^{2}\right) f^{+}\left(q^{2}\right)+q^{2} f^{-}\left(q^{2}\right)\right] \\
\left\langle D_{s}^{*+}(k) D_{s}^{*-}(q)\right| \mathcal{H}_{\mathrm{wk}}\left|B_{s}(p)\right\rangle= & \mathcal{G} f_{*} \epsilon_{+}^{* \mu} \epsilon_{-}^{* \nu}\left[\mathcal{V}\left(q^{2}\right) \epsilon^{\nu \mu \alpha \beta} p_{\alpha} k_{\beta}\right. \\
& \left.\quad+i\left\{\mathcal{A}_{1}\left(q^{2}\right) p^{2} g^{\mu \nu}-2 \mathcal{A}_{2}\left(q^{2}\right) p^{\mu} p^{\nu}\right\}\right]  \tag{7}\\
\left\langle D_{s}^{*+}(k, \epsilon) D_{s}^{-}(q)\right| \mathcal{H}_{\mathrm{wk}}\left|B_{s}(p)\right\rangle= & \frac{\mathcal{G} f}{m_{*}} \epsilon^{*} \cdot p\left[p^{2} \mathcal{A}_{1}\left(q^{2}\right)-\left(p^{2}-k^{2}\right) \mathcal{A}_{2}-m_{*}^{2} \mathcal{A}_{3}\left(q^{2}\right)\right] \\
\left\langle D_{s}^{+}(k) D_{s}^{*-}(q, \epsilon)\right| \mathcal{H}_{\mathrm{wk}}\left|B_{s}(p)\right\rangle= & 2 \mathcal{G} m_{*} f_{*} f^{+}\left(q^{2}\right) \epsilon^{*} \cdot p .
\end{align*}
$$

Within the factorization approximation, the relations in (7) can be interpreted as a parametrization of the $B D_{s}^{(*)} D_{s}^{(*)}$ vertices.

The form factors themselves are nonperturbative quantities. Apart from lattice calculations [9], the best arenas for determining these are phenomenological approaches such as the BSW model [10] or heavy-quark effective theory [11. Some of these parameters are also measured experimentally [12], although a somewhat large spread persists in the data. Typically, though, we have $f, f_{*} \sim 200 \mathrm{MeV}, f_{ \pm}, \mathcal{V} \sim 0.6$ and $\mathcal{A}_{1,2} \sim 0.25$. On the other hand, $\mathcal{A}_{3}$ is very small.

As a next step, we need to determine the meson-photon couplings. Since we are dealing here with charged mesons, the simplest course is to assume minimal substitution. This then
fixes the charge coupling uniquely for each angular-momentum state of the ( $c \bar{s}$ ) system. Of course, the fact that these mesons are not fundamental particles means that we should ideally discuss a series of form factors, including charge radii as well as higher moments. As a first approximation, however, we neglect such aspects of the meson-photon coupling.

The one-loop contribution due to the $D_{s}$ alone can then be expressed in terms of the diagrams in Fig. 1]. Computing these diagrams, the first one gives


Figure 1: One-loop contribution to the $B_{s} \rightarrow \gamma \gamma$ amplitude due to $D_{s}$ mesons alone.

$$
\begin{aligned}
G_{\mu \nu}^{(1)}=\int & \frac{d^{4} k}{(2 \pi)^{4}}(-i \mathcal{G}) f\left[\left(p^{2}-k^{2}\right) f_{+}+(p-k)^{2} f_{-}\right] \frac{i}{k^{2}-m^{2}}(i e)\left[2 k+q_{1}\right]_{\mu} \\
& \frac{i}{(k-q)^{2}-m^{2}}(i e)\left[2 k-q_{1}-p\right]_{\mu} \frac{i}{(k-p)^{2}-m^{2}}
\end{aligned}
$$

whilst the third gives

$$
G_{\mu \nu}^{(3)}=\int \frac{d^{4} k}{(2 \pi)^{4}}(-i \mathcal{G}) f\left[\left(p^{2}-k^{2}\right) f_{+}+(p-k)^{2} f_{-}\right] \frac{i}{k^{2}-m^{2}} \frac{i}{(k-p)^{2}-m^{2}}\left(2 i e^{2} g_{\mu \nu}\right) .
$$

In each of the above, the form-factors $f_{ \pm}$are to be evaluated at $(p-k)^{2}$.
We note that an exact calculation of the above integrals needs knowledge of the momentum dependence of the form factors. Furthermore, the integrals are formally divergent, and to calculate the real parts we would need to consider a cutoff scale that sets the limit of validity of a theory with 'fundamental' meson fields. Rather than attempt this, we take recourse to the optical theorem and calculate only the absorptive parts of the diagrams above. On using the Cutkosky rules, the sum of the three diagrams reduces to

$$
\begin{align*}
\operatorname{Im}\left(G_{\mu \nu}^{\left(D_{s}\right)}\right) & =2 \alpha \mathcal{G} \tilde{m}_{1} f\left[\left(1-\tilde{m}_{1}\right) f^{+}\left(m^{2}\right)+\tilde{m}_{1} f^{-}\left(m^{2}\right)\right] I_{111} S_{\mu \nu} \\
I_{i j k} & =\ln \frac{1+2 \tilde{m}_{k}-\tilde{m}_{i}-\tilde{m}_{j}+\lambda_{i j}}{1+2 \tilde{m}_{k}-\tilde{m}_{i}-\tilde{m}_{j}-\lambda_{i j}}  \tag{8}\\
\lambda_{i j} & =\left[\left(1-\tilde{m}_{i}-\tilde{m}_{j}\right)^{2}-4 \tilde{m}_{i} \tilde{m}_{j}\right]^{1 / 2}
\end{align*}
$$

Here $\tilde{m}_{i}=m_{i}^{2} / m_{B}^{2}$ with $m_{1}=m \equiv m_{D_{s}}$ and $m_{2}=m_{*} \equiv m_{D_{s}^{*}}$. The form of (8) is a testimonial to the fact that the three diagrams of Fig. [1] together form a gauge-invariant set. Thus, if $D_{s}$ were the only meson that could contribute, $\mathcal{R}_{1,2}$ would be determined by (8), and

$$
\begin{align*}
\operatorname{Im} \mathcal{R}_{1}\left(D_{s}\right) & =0.39 f\left[f^{+}\left(m^{2}\right)+0.16 f^{-}\left(m^{2}\right)\right]  \tag{9}\\
\operatorname{Im} \mathcal{R}_{2}\left(D_{s}\right) & =0
\end{align*}
$$

The CP-violating form factor is thus identically zero.
What about the real parts of the amplitudes $\mathcal{R}_{1}$ ? These may be computed using dispersion relations [14], and, for the case in hand, are seen to be smaller than the imaginary parts. Substituting for the form factors in the above and using (3), we then have

$$
\begin{equation*}
B r\left(B_{s} \xrightarrow{2 D_{s}} \gamma \gamma\right) \sim 2.5 \times 10^{-8} . \tag{10}
\end{equation*}
$$

This, by itself, is small compared to the short-distance contribution [6], though the interference term can be significant.

What about the other long-distance effects? Of the ones calculated in the literature, $B_{s} \rightarrow$ $\phi \gamma \rightarrow \gamma \gamma$ has an amplitude somewhat smaller than the one calculated here, whilst $B_{s} \rightarrow$ $J / \psi \phi \rightarrow \gamma \gamma$ is seen to contribute only at the $1 \%$ level [8]. We are thus in a situation where the $2 D_{s}$ intermediate state may, in fact, give the largest long-distance contribution to the decay amplitude. It is interesting, at this stage, to compare the $B_{s} \rightarrow \gamma \gamma$ case with that for $K_{S} \rightarrow \gamma \gamma$ decay. Whereas there it is the $2 \pi$ intermediate state that dominates the decay amplitude [15], the analogous contribution to the $B_{s}$ decay falls well short of the short-distance amplitude.

We now turn our attention to the next higher state that can contribute to this process, namely the $D_{s}^{*}$. As long as we neglect any $D_{s}^{*} D_{s} \gamma$ coupling, the additional diagrams involving the $D_{s}^{*}$ field are exact analogues of those in Fig. [1] , and lead to

$$
\begin{align*}
\operatorname{Im} \mathcal{R}_{1}\left(D_{s}^{*}\right)= & \frac{f_{*}}{8 \tilde{m}_{2}^{2}}\left[\mathcal{A}_{1}\left(m_{*}^{2}\right)\left\{\left(1-4 \tilde{m}_{2}\right) \lambda_{22}+\tilde{m}_{2}\left(1-12 \tilde{m}_{2}+48 \tilde{m}_{2}^{2}\right) I_{222}\right\}\right. \\
& \left.\quad-\mathcal{A}_{2}\left(m_{*}^{2}\right)\left\{\left(\tilde{m}_{2}-5\right) \lambda_{22}+\tilde{m}_{2}\left(1-10 \tilde{m}_{2}+32 \tilde{m}_{2}^{2}\right) I_{222}\right\}\right] \\
= & f_{*}\left[1.57 \mathcal{A}_{1}\left(m_{*}^{2}\right)+15.3 \mathcal{A}_{2}\left(m_{*}^{2}\right)\right]  \tag{11}\\
\operatorname{Im} \mathcal{R}_{2}\left(D_{s}^{*}\right)= & \frac{f_{*} \mathcal{V}\left(m_{*}^{2}\right)}{8 \tilde{m}_{2}}\left\{\left(12 \tilde{m}_{2}-1\right) \lambda_{22}+\left(4 \tilde{m}_{2}-32 \tilde{m}_{2}^{2}\right) I_{222}\right\} \\
= & 0.13 f_{*} \mathcal{V}\left(m_{*}^{2}\right)
\end{align*}
$$

Two new features confront us. One is that the $C P$-violating amplitude is non-zero, though small. More relevantly for our present purpose, we find $\operatorname{Im} \mathcal{R}_{1}\left(D_{s}^{*}\right) \sim 800 \mathrm{MeV}$, on the basis of which (3) then gives us

$$
\begin{equation*}
\operatorname{Br}\left(B_{s} \xrightarrow{2 D_{s}^{*}} \gamma \gamma\right) \sim 6.5 \times 10^{-6} \tag{12}
\end{equation*}
$$

This, of course, is much larger than the short-distance contribution, and suggests that this particular decay should be observable at the very first run of the hadronic machines!

A few objections could possibly be raised against this conclusion. For one thing, the validity of the factorization approximation as applied to decays into two vector mesons is not apparent. Thus, instead of using (5), it would be preferable to use the data to parametrize the $B_{s} D_{s}^{*} \bar{D}_{s}^{*}$ vertex. Unfortunately, though, this exclusive mode is undetected so far. Once it is measured, it will be a straightforward task to reexpress our result as a prediction of the ratio of the two decay modes.

Secondly, we have, until now, neglected quite a few possible contributions to the process. For example, the contributions due to the higher excited states could in principle be comparable in magnitude, and might interfere destructively, thus reducing the total long-distance contribution. However, in the absence of extensive data on these states, it is almost impossible to estimate such contributions to any degree of reliability. We can only hope that this lack of information does not invalidate the results presented here.

Finally, there is another class of contribution that we have neglected so far, involving the $D_{s}^{*} D_{s} \gamma$ vertex. Whilst this transition moment can, in principle, be calculated within a given model for the mesons, it is easier to work in terms of an effective Lagrangian, the relevant part of which can be expressed as

$$
\begin{equation*}
\mathcal{L}_{\mathrm{eff}}=i \frac{e \mathcal{D}}{m_{*}} \epsilon_{\rho \sigma \alpha \beta} q^{\alpha} p^{\beta} . \tag{13}
\end{equation*}
$$

We shall assume that $\mathcal{D}$ is real, i.e. there is no absorptive part associated with this vertex. The new contributions are given by the diagrams of Fig. 2 , along with the crossed ones, and the resultant shifts $\delta \mathcal{R}_{1,2}$ given by

$$
\begin{aligned}
4 \tilde{m}_{2} \operatorname{Im} \delta \mathcal{R}_{1}= & |\mathcal{D}|^{2} f\left[\left(1-\tilde{m}_{1}\right) f^{+}\left(m^{2}\right)+\tilde{m}_{1} f^{-}\left(m^{2}\right)\right]\left\{\lambda_{11}-\tilde{m}_{2} I_{112}\right\} \\
- & |\mathcal{D}|^{2} f_{*}\left[\begin{array}{l}
\mathcal{A}_{1}\left(m_{*}^{2}\right)\left\{\lambda_{12}-\tilde{m}_{1} I_{221}\right\} \\
\\
\\
\left.\quad+\mathcal{A}_{2}\left(m_{*}^{2}\right)\left\{\left(\tilde{m}_{2}-\tilde{m}_{1}-0.5\right) \lambda_{12}+\left(\left(\tilde{m}_{2}-\tilde{m}_{1}\right)^{2}+\tilde{m}_{1}\right) I_{221}\right\}\right]
\end{array}\right.
\end{aligned}
$$

$8 \tilde{m}_{2} \operatorname{Im} \delta \mathcal{R}_{2}=\mathcal{D} f_{*} f^{+}\left(m_{*}^{2}\right)\left\{2\left(\tilde{m}_{1}^{3}-\tilde{m}_{1}^{2}-4 \tilde{m}_{1}^{2} \tilde{m}_{2}+2 \tilde{m}_{1} \tilde{m}_{2}+5 \tilde{m}_{1} \tilde{m}_{2}^{2}-2 \tilde{m}_{2}^{3}+3 \tilde{m}_{2}^{2}-\tilde{m}_{2}\right) I_{122}\right.$

$$
\left.+8 \tilde{m}_{1} \tilde{m}_{2} I_{121}+3\left(\tilde{m}_{1}-\tilde{m}_{1}^{2}+\tilde{m}_{2}^{2}-3 \tilde{m}_{2}\right) \lambda_{12}\right\}
$$

$$
+\mathcal{D} f\left[\mathcal{A}_{1}\left(m^{2}\right)+\left(\tilde{m}_{2}-1\right) \mathcal{A}_{2}\left(m^{2}\right)-\tilde{m}_{2} \mathcal{A}_{3}\left(m^{2}\right)\right]
$$

$$
\left\{4 \tilde{m}_{1} I_{211}+\left(1-3 \tilde{m}_{1}\right) I_{112}+\left(2 \tilde{m}_{2}-\tilde{m}_{1}-3-\frac{\tilde{m}_{1}-\tilde{m}_{1}^{2}}{\tilde{m}_{2}}\right) \lambda_{12}\right\}
$$

$$
+|\mathcal{D}|^{2} f_{*} \mathcal{V}\left(m_{*}^{2}\right)\left\{\left(\tilde{m}_{2}^{2}-\tilde{m}_{1}^{2}\right) I_{221}+\left(\tilde{m}_{2}+\tilde{m}_{1}-0.5\right) \lambda_{12}\right\}
$$

Numerically, then,

$$
\begin{align*}
\operatorname{Im} \delta \mathcal{R}_{1} & =|\mathcal{D}|^{2}\left\{0.077 f\left[f^{+}\left(m^{2}\right)+0.16 f^{-}\left(m^{2}\right)\right]+f_{*}\left[-0.67 \mathcal{A}_{1}\left(m_{*}^{2}\right)+0.15 \mathcal{A}_{2}\left(m_{*}^{2}\right)\right]\right\} \\
\operatorname{Im} \delta \mathcal{R}_{2} & =0.15 \mathcal{D} f\left[\mathcal{A}_{1}\left(m^{2}\right)-0.845 \mathcal{A}_{2}\left(m^{2}\right)-0.155 \mathcal{A}_{3}\left(m^{2}\right)\right] \\
& -0.098 \mathcal{V}\left(m_{*}^{2}\right) f_{*}|\mathcal{D}|^{2}-0.12 \mathcal{D} f_{*} f^{+}\left(m_{*}^{2}\right) \tag{14}
\end{align*}
$$

One may, in principle, estimate $\mathcal{D}$ from the partial decay width of $D_{s}^{*}$ :

$$
\begin{equation*}
\Gamma\left(D_{s}^{*} \rightarrow D_{s} \gamma\right)=\frac{\alpha}{24}|\mathcal{D}|^{2} m_{*}\left(1-\frac{m^{2}}{m_{*}^{2}}\right)^{3}=\left(1.46 \times 10^{-3} \mathrm{MeV}\right)|\mathcal{D}|^{2} \tag{15}
\end{equation*}
$$



Figure 2: One-loop contribution to the $B_{s} \rightarrow \gamma \gamma$ amplitude involving the $D_{s}^{*} D_{s} \gamma$ vertex. The crossed diagrams are not shown.

Unfortunately, this decay mode is not yet well measured. All we know is that $\Gamma\left(D_{s}^{*}\right)<1.9 \mathrm{MeV}$ and that $\left(D_{s}+\gamma\right)$ and $\left(D_{s}+\pi\right)$ are the only decay modes seen. This implies that

$$
|\mathcal{D}|^{2}<1300 \operatorname{Br}\left(D_{s}^{*} \rightarrow D_{s} \gamma\right)
$$

A branching ratio of even $1 \%$ could then lead to an order of magnitude change in $\Gamma\left(B_{s} \rightarrow \gamma \gamma\right)$ ! However, a stricter bound for $\mathcal{D}$ can be obtained if one relates it to the corresponding form factor for the $S=0$ charm meson. Noting that $\Gamma\left(D^{*}\right)<0.131 \mathrm{MeV}$ and $\operatorname{Br}\left(D^{*+} \rightarrow D^{+} \gamma\right) \approx 1.7 \%$ [13], a relation analogous to (15) leads to

$$
\left|\mathcal{D}_{D^{*} D \gamma}\right|^{2}<0.95
$$

A value of $\mathcal{D}$ of this order obviously cannot negate the conclusions of (12). Examining (14) closely, we see that the contribution due to a pair of on-shell $D_{s}$ exchanging a $D_{s}^{*}$ is actually much smaller than that of (9). This result is similar again to the case of $K_{S} \rightarrow \gamma \gamma$ decay [14], where the analogous contributions due to the flavour-octet vector mesons are small. The other diagrams, where at least one $D_{s}^{*}$ is on shell, can, however, compete with the $2 D_{s}$ contribution.

To conclude, we have estimated the long-distance contributions to the $B_{s} \rightarrow \gamma \gamma$ decay arising from charmed-meson intermediate states. We find that the $2 D_{s}$ contribution, by itself, is larger than the other long-distance contributions calculated hitherto in the literature [8]. It is, however, still smaller than the short-distance amplitude. More interestingly, the $2 D_{s}^{*}$ contribution is much larger, and could enhance the branching fraction by more than an order of magnitude. This would lead to a very striking signal at the hadronic $B$ factories. Moreover, if it is indeed comparable to the short-distance amplitude, the different Cabibbo-KobayashiMaskawa structures might offer interesting prospects for observing CP violation.

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[^1]:    ${ }^{1}$ Over a wide range of the top-quark mass, the partial width grows linearly with $m_{t}$ \|.

[^2]:    ${ }^{2}$ Since the contributions due to charmonia have already been investigated [8], we shall not consider these here.
    ${ }^{3}$ Based on experience elsewhere, we may hope that the associated errors are $<\mathcal{O}(10 \%)$.

