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CP-violating Anomalous $WW\gamma$ Couplings in e^+e^- Collisions

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Abstract

We investigate the sensitivity of future linear collider experiments to CP violating $WW\gamma$ couplings in the process $e^+e^- \to \nu\bar{\nu}\gamma$. We consider several sets of machine parameters: centre of mass energies $\sqrt{s} = 350$, 500 and 800 GeV and operating at different luminosities. From an analysis of the differential cross-section the following 95% C.L. limits $|\tilde{\kappa}_\gamma| < 0.18$, $|\tilde{\lambda}_\gamma| < 0.069$ are estimated to be obtained at a future 500 GeV LC with an integrated luminosity of $125\,\mathrm{fb}^{-1}$, a great improvement as compared to the LEP2 reach, where a senstitivity of order 2 for both couplings is found.

1 Introduction

The ongoing LEP run at energies above the W^+W^- threshold has made it possible to study directly the non-Abelian structure of the electroweak Standard Model (SM) in the clean environment of e^+e^- collisions. LEP2 has not only confirmed the existence of triple gauge-boson vertices, inferred either indirectly [1] or from observations at $p\bar{p}$ colliders [2], but also established constraints on them [3]. The goal of the present studies at LEP2, and at future colliders, is to test the structure of the bosonic sector with a precision comparable to that achieved for the fermion-vector boson couplings. Such precise measurements of gauge vector-boson couplings will not only provide stringent tests of the gauge structure of the SM, but also probe for new physics.

Within the Standard Model, triple and quartic vector-boson interactions are intimately related to the gauge structure of the model and therefore are completely determined. Of course, radiative corrections within the SM modify the tree-level couplings. However, such corrections are quite small. In particular for the CP-violating couplings they are expected to be exteremely small and unmeasurable in near future since *e.g.* the electric dipole moment of the W boson vanishes to two loops[4]. On the other hand, any theory incorporating new physics may conceivably induce much larger (already at the one-loop) deviations in some of the couplings. Corrections at a permille level can be expected in multi-Higgs or supersymmetric extensions[5]. Models with dynamical breaking of electroweak symmetry by new strong forces, could produce even larger corrections [6]. The concomitant CP violation could then manifest itself in non-zero CP-violating gauge boson couplings, observation of which would be a clear signal of beyond-SM physics.

Owing to the general perception that the CP-violating couplings are severely constrained by the data on the neutron electric dipole moment (EDM) [7], in phenomenological analyses of the physics potential of future colliders, their sensitivity to these couplings has received less attention than that accorded to the CP-conserving ones. However, with these constraints being the subject to naturalness assumption, there is no substitute for a direct measurement. Furthermore, since they depend on different combinations of anomalous couplings, the direct measurements are complementary to the indirect analyses. In the present paper we wish to study the sensitivity of future e^+e^- linear colliders (LC) to CP-violating couplings, in particular we will consider the process $e^+e^- \to \nu\bar{\nu}\gamma$ as a means to test $WW\gamma$ interactions. The motivation for our study, of course, is that the origin of CP violation remains unexplained and it should be experimentally investigated wherever possible. We find that at an e^+e^- 500 GeV linear collider with an integrated luminosity of $\mathcal{L} = 125\,\mathrm{fb}^{-1}$, an analysis of the differential cross-section allows us to derive the following 95% C.L. limits $|\tilde{\kappa}_\gamma| < 0.18$, $|\tilde{\lambda}_\gamma| < 0.069$. With a high luminosity option of 500 fb^{-1}, more stringent limits $|\tilde{\kappa}_\gamma| < 0.13$, $|\tilde{\lambda}_\gamma| < 0.049$ can be established. For comparison, we find that the LEP2 experiments at $\sqrt{s} = 192$ GeV and $\mathcal{L} = 2\,\mathrm{fb}^{-1}$ can reach a sensitivity of the order $|\tilde{\kappa}_\gamma| \sim |\tilde{\lambda}_\gamma| \sim 2$

2 Anomalous Couplings

It is convenient to describe the phenomenology at scales well below the scale of new physics in a model independent way. It is, by now, standard to introduce an effective low-energy Lagrangian that contains only SM fields. This assumes that the physics responsible for any deviations is not directly observable, but can manifest itself through virtual corrections. This formalism provides a simple parametrization of the triple gauge-boson couplings (TGC). In purely phenomenological terms, the effective Lagrangian for the $WW\gamma$ and the WWZ interactions can be expressed in terms of seven parameters each [8] *viz.*

$$\begin{aligned}\mathcal{L}_{\mathit{eff}}^{WWV} = \; & -ig_V \Big[(1+\Delta g_1^V)\left(W^\dagger_{\mu\nu}W^\mu - W^{\dagger\mu}W_{\mu\nu}\right)V^\nu + (1+\Delta\kappa_V)W^\dagger_\mu W_\nu V^{\mu\nu} \\ & + \frac{\lambda_V}{M_W^2}W^\dagger_{\mu\nu}W^\nu{}_\sigma V^{\sigma\mu} - ig_5^V\epsilon^{\mu\nu\rho\sigma}(W^\dagger_\mu\partial_\rho W_\nu - W_\nu\partial_\rho W^\dagger_\mu)V_\sigma \\ & + ig_4^V W^\dagger_\mu W_\nu(\partial^\mu V^\nu + \partial^\nu V^\mu) + \tilde{\kappa}_V W^\dagger_\mu W_\nu \tilde{V}^{\mu\nu} + \frac{\tilde{\lambda}_V}{m_W^2}W^\dagger_{\lambda\mu}W^\mu{}_\nu\tilde{V}^{\nu\lambda}\Big] ,\end{aligned} \tag{1}$$

where V^μ is a neutral vector boson field, i.e. either the γ or the Z field. The W^μ ($W^{\mu\dagger}$) stands for the $W^-(W^+)$ field, respectively, and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $\tilde{V}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}V^{\rho\sigma}$. The overall normalization is such that the coupling g_V is defined as in the SM, *viz.*,

$$g_\gamma = e, \qquad g_Z = e\cot\theta_W \; , \tag{2}$$

with θ_W the weak mixing angle. In the SM we have, at the tree level,

$$\Delta g_1^V = \Delta\kappa_V = \lambda_V = \tilde{\kappa}_V = \tilde{\lambda}_V = g_4^V = g_5^V = 0 \tag{3}$$

Non zero values of the above, usually called anomalous gauge couplings, would indicate new physics. The three couplings, Δg_1^V, $\Delta\kappa_V$ and λ_V, are even under both C and P transformations. Of the remaining four, two $\tilde{\kappa}_V$ and $\tilde{\lambda}_V$ violate P but conserve C, g_4^V respects P but violates C, and g_5^V violates both P and C.

Eq. (1) represents the most general WWV Lagrangian consistent with Lorentz- and gauge-invariance. Higher derivative terms can be absorbed into the above couplings provided these are treated as form factors and not constants. It is thus important to bear in mind the fact that the strength of the various terms in the vertex would vary (in general, independently) with the momentum scales of the process being considered. The imaginary parts of the form factors are essentially the absorptive parts of the WWV vertex functions, and, as such, are small in the SM or MSSM. Although absorptive parts that arise from the same sector of new physics as the anomalous couplings themselves (as for example in the models of Ref. [6]) need not be suppressed, we will assume here that the anomalous couplings are real.

To date, the only direct limits on CP-violating $WW\gamma$ couplings are
(*i*) $-0.92 < \tilde{\kappa}_\gamma < 0.92$, $-0.31 < \tilde{\lambda}_\gamma < 0.30$ from an analysis of $p\bar{p} \to W\gamma + X$ events done by the D0 Collaboration at Tevatron [9], and
(*ii*) $\tilde{\kappa}_\gamma = 0.11^{+0.71}_{-0.88} \pm 0.09$, $\tilde{\lambda}_\gamma = 0.19^{+0.28}_{-0.41} \pm 0.11$ from the analysis of $e^+e^- \to W^+W^-$ and $We\nu$ data collected by DELPHI Collaboration [10].

A competitive indirect limit $|\tilde{\kappa}_\gamma| < 0.6$ has been derived recently [11] from the $b \to s\gamma$ CLEO data [12], whereas indirect constraints based on neutron electric dipole moment (EDM) put a very strong limit $|\tilde{\kappa}_\gamma| \lesssim 2 \times 10^{-4}$ [7]. By investigating $e^+e^- \to W^+W^-$ at a future 500 GeV linear collider, it has been shown that the constraint $|\tilde{\kappa}_\gamma| \leq 0.1$ can be established [13]. Similar conclusion has been reached for the process $p\bar{p} \to W\gamma$ at an upgraded Tevatron [14]. Better limits of the order of 10^{-3} can be reached in the $e\gamma$ and $\gamma\gamma$ modes of the linear colliders with the polarized Compton back-scattered photon beams [15]. As for WWZ couplings, the reaction $e^+e^- \to \nu\bar{\nu}Z$ has been examined [16] with the resulting limit of the order $|g_4^Z| < 0.1$.

Given the tight (but subject to theoretical assumptions) indirect bound from EDM, it seems unlikely that a non-zero CP-violating couplings will be observed directly at future colliders. Nevertheless, the EDM and the direct observables that we study in this paper depend on different combinations of the anomalous couplings and consequently provide complementary information. We argue therefore that experimental searches, wherever possible, should be attempted, if only to overdetermine the system.

3 Why $e^+e^- \to \nu\bar{\nu}\gamma$?

The process $e^+e^- \to W^+W^-$ necessarily involves both $WW\gamma$ and WWZ vertices and consequently all 14 couplings. On the other hand, the reaction [17]

$$e^-(p_1) + e^+(p_2) \longrightarrow \nu(p_3) + \bar{\nu}(p_4) + \gamma(p_5), \tag{4}$$

with the Feynman diagrams shown in Fig. 1, has the advantage[1] that only the $WW\gamma$ vertex is present. Therefore the process with "a photon + missing energy" in e^+e^- annihilation can probe $WW\gamma$ couplings independently of WWZ, reducing greatly the number of unknown couplings to be determined experimentally. Moreover, the number of unknown couplings is further reduced to $\Delta\kappa_\gamma$, λ_γ, $\tilde{\kappa}_\gamma$ and $\tilde{\lambda}_\gamma$, since electromagnetic gauge invariance requires that, for on-shell photons, $\Delta g_1^\gamma = g_4^\gamma = g_5^\gamma = 0$, though these can assume other values for off-shell photons, a fact often missed in the literature.

For the process (4), being formally of higher order in the electroweak coupling than W pair production, one may expect a reduced sensitivity. However, since the total cross section for this process increases[2] with incoming energy [18] until fairly large $\sqrt{s}$, while that for W^+W^- decreases strongly with $\sqrt{s}$, the reduction in sensitivity is less and less severe at higher energy. Coupled with W^+W^- and $W\gamma$ measurements, performed at the different momentum transfers, the process (4) would allow the possibility of studying the form factor nature of anomalous couplings.

The sensitivity of $e^+e^- \to \nu\bar{\nu}\gamma$ to C and P conserving anomalous couplings ($\Delta\kappa_\gamma$ and λ_γ) at future linear colliders[3], has been recently studied in detail in Ref. [18]. Here we

[1]The same is also true for both $p\bar{p} \to W\gamma$ and $e^\pm\gamma \to W^\pm\nu$.

[2]Actually the rapid rise of the cross section allows for generous kinematical cuts to suppress possible backgrounds, as we will demonstrate in this paper.

[3]First experimental results from this process at LEP2 have been published recently [19].

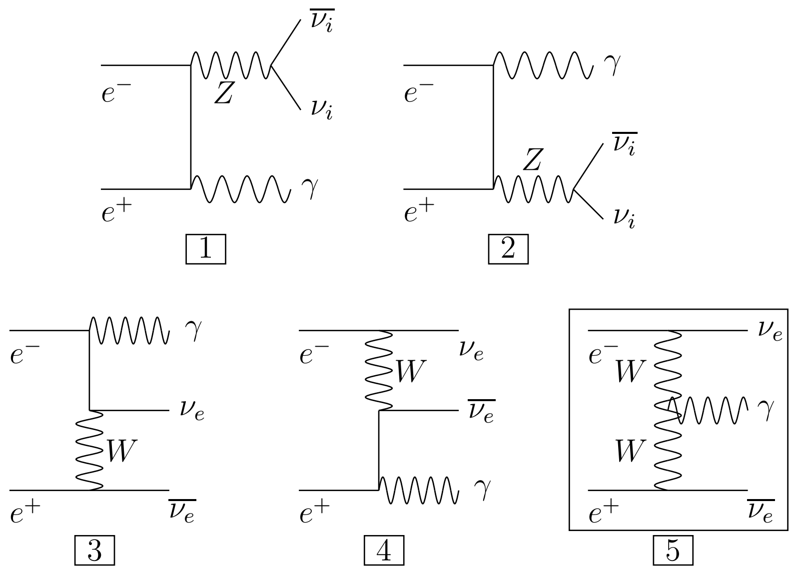

Figure 1: *The Feynman diagrams responsible for* $e^+e^- \to \bar{\nu}\nu\gamma$.

will study the sensitivity to CP-violating couplings, namely to $\tilde{\kappa}_\gamma$ and $\tilde{\lambda}_\gamma$. In the static limit they are related to the electric dipole moment $d_W = e(\tilde{\kappa}_\gamma + \tilde{\lambda}_\gamma)/2m_W$ and magnetic quadrupole moment $Q_W = -e(\tilde{\kappa}_\gamma - \tilde{\lambda}_\gamma)/m_W^2$ of the W^+.

Since, in the process $e^+e^- \to \nu\bar{\nu}\gamma$, the only kinematical variables at our disposal are the energy E_γ and the polar angle θ_γ of the produced photon, no truly CP-odd observables can be constructed. In the absence of the phases of $\tilde{\kappa}_\gamma$ and $\tilde{\lambda}_\gamma$ we can only look for the quadratic effects that the CP-violating anomalous couplings induce in the differential distribution of photons. It is therefore possible to exploit the same CP-conserving observables and to follow the methods of Ref. [18] to derive limits on CP-violating photon couplings at future e^+e^- linear colliders.

4 The SM expectations for $e^+e^- \to \nu\bar{\nu}\gamma$

The SM cross-section for the process (4) is best calculated using helicity amplitudes and the relevant expressions can be found in Ref. [20]. Experimentally, the signal comprises a single energetic photon plus missing momentum carried by invisible neutrinos. The energy and direction of the photon can be measured with high accuracy. Note however, that only diagram $\boxed{5}$ of Fig. 1 can contribute to the signal, while all diagrams (including $\boxed{5}$) contribute to the background. By applying simple kinematical cuts, some of these contributions can be suppressed and the sensitivity to TGC enhanced significantly. For example, diagrams $\boxed{3}$ and $\boxed{4}$ are responsible for an enhancement of the cross-section at both small photon energy and small emission angles. To eliminate these, we impose

$$25^\circ < \theta_\gamma < 155^\circ \tag{5}$$

as well as[4]

$$E_\gamma > 0.1\sqrt{s}\ . \tag{6}$$

Note that cut (6) is different from that of Ref. [18], wherein a $\sqrt{s}$–independent cut of $E_\gamma > 25$ GeV was imposed. This modification obviously implies that the selected events must have higher transverse momentum thus avoiding potential background contributions from processes such as $e^+e^- \to \gamma\gamma$ where one photon disappears down the beam pipe. This is especially true for larger $\sqrt{s}$.

Similarly, to eliminate events where an on-shell Z boson decays to a $\nu\bar{\nu}$ pair (diagrams $\boxed{1}$ and $\boxed{2}$ with radiative Z-return), we require here that the photon energy satisfies

$$\left| E_\gamma - \frac{s - m_Z^2}{2\sqrt{s}} \right| > 5\Gamma_Z\ , \tag{7}$$

where M_Z and Γ_Z are the mass and the width of the Z boson. With these cuts, the SM cross section (summed over neutrino flavours) is

$$\sigma_{\rm SM}(\bar{\nu}\nu\gamma) = \begin{cases} 0.469 \text{ pb} & \sqrt{s} = 350 \text{ GeV} \\ 0.437 \text{ pb} & \sqrt{s} = 500 \text{ GeV} \\ 0.361 \text{ pb} & \sqrt{s} = 800 \text{ GeV}\ . \end{cases} \tag{8}$$

For CM energy in the range (200–1000 GeV), the cross-section falls almost linearly (see Fig. 2). This is in marked contrast to Fig. (2a) of Ref. [18] where the cross-section was shown to *increase* with $\sqrt{s}$. The difference obviously lies in the stronger form of the

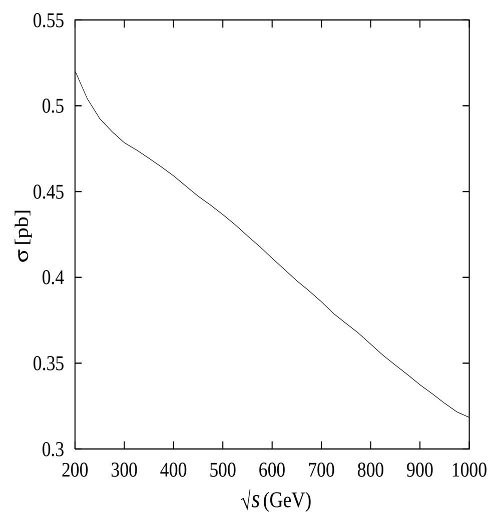

Figure 2: *The energy dependence of the SM cross section.*

energy cut (6) that we use. It might seem that the loss of statistics that such a cut entails might reduce the sensitivity. However, as we shall show in section 6, this is not really the case.

[4]The same energy cut has been used in the recent analysis by ALEPH [19].

5 The anomalous contribution

A non-zero value for any one of the anomalous couplings in eq. (1) would imply additional terms in the matrix element arising from diagram $\boxed{5}$. It is easy to see that the contributions due to $\tilde{\kappa}_\gamma$ and $\tilde{\lambda}_\gamma$ do not interfere with the SM amplitude. This is but a reflection of the fact that one cannot construct a CP-odd observable for the process of eq. (4). Denoting $d_{ij} \equiv p_i \cdot p_j$, the anomalous contribution to the spin-summed (averaged) matrix-element-squared is then

$$\begin{aligned} \left(\frac{eg^2}{P_{13}P_{24}}\right)^{-2} |\mathcal{M}|^2_{\rm anom} &= \tilde{\kappa}^2\mathcal{C} + \frac{4\tilde{\lambda}^2}{m_W^4} d_{13}d_{24}\left\{\mathcal{C} + (d_{13}-d_{24})(d_{15}d_{35}-d_{25}d_{45})\right\} \\ &+ \frac{2\tilde{\kappa}\tilde{\lambda}}{m_W^2}\Big\{(d_{13}+d_{24})\mathcal{C} + (d_{13}-d_{24})(d_{24}d_{15}d_{35}-d_{13}d_{25}d_{45}) \\ &\qquad -2d_{15}d_{25}d_{35}d_{45}\Big\} , \end{aligned} \tag{9}$$

where we have suppressed the subscript γ in $\tilde{\kappa}$ and $\tilde{\lambda}$ and

$$\begin{aligned} P_{13} &\equiv 2d_{13} + m_W^2 \\ P_{24} &\equiv 2d_{24} + m_W^2 \\ \mathcal{C} &\equiv d_{14}d_{25}d_{35} + d_{23}d_{15}d_{45} . \end{aligned}$$

The numerical value of the extra contribution can be obtained by integrating $|\mathcal{M}|^2_{\rm anom}$ over the appropriate phase space volume. In Fig. 3, we display this quantity as a function of $\sqrt{s}$ for unit values of $\tilde{\kappa}$ and $\tilde{\lambda}$ (and $n = 0$; for $n > 1$ see below). The generalization to arbitrary values is trivial. In contrast to the SM case (Fig. 2), the anomalous contribution *grows* with the CM energy, the effect being more pronounced for dimension 6 operator, *i.e.* the non-zero $\tilde{\lambda}$ coupling (note the different scales on vertical axes). This is but a consequence of the lack of unitarity for such theories.

Tree-level unitarity may be restored by postulating a form-factor behaviour for $\tilde{\kappa}$ and $\tilde{\lambda}$. To wit,

$$\tilde{\kappa} = \tilde{\kappa}_0 \left[\frac{\Lambda^4}{(\Lambda^2 + 2d_{13})(\Lambda^2 + 2d_{24})}\right]^n \tag{10}$$

and similarly for $\tilde{\lambda}$ [7]. In eqn.(10), n is an integer and Λ is the scale where new physics manifests itself. While $n \geq 1$ ensures that the cross-section falls off for sufficiently high $\sqrt{s}$, the effect is not so marked for the regime of interest, even for relatively low values of Λ (see Fig. 3).

A related consequence of this lack of partial wave unitarity is that the high-energy end of the photon spectrum gets disproportionately populated. In fact, the strongest dependence on anomalous TGC appears towards the end of the photon energy spectrum, *i.e.*, in the region which is seriously affected by the cut (7) designed to eliminate γZ events. Fortunately though, this overpopulation persits for somewhat lower values of E_γ as well thus allowing us to draw relatively strong constraints on such couplings.

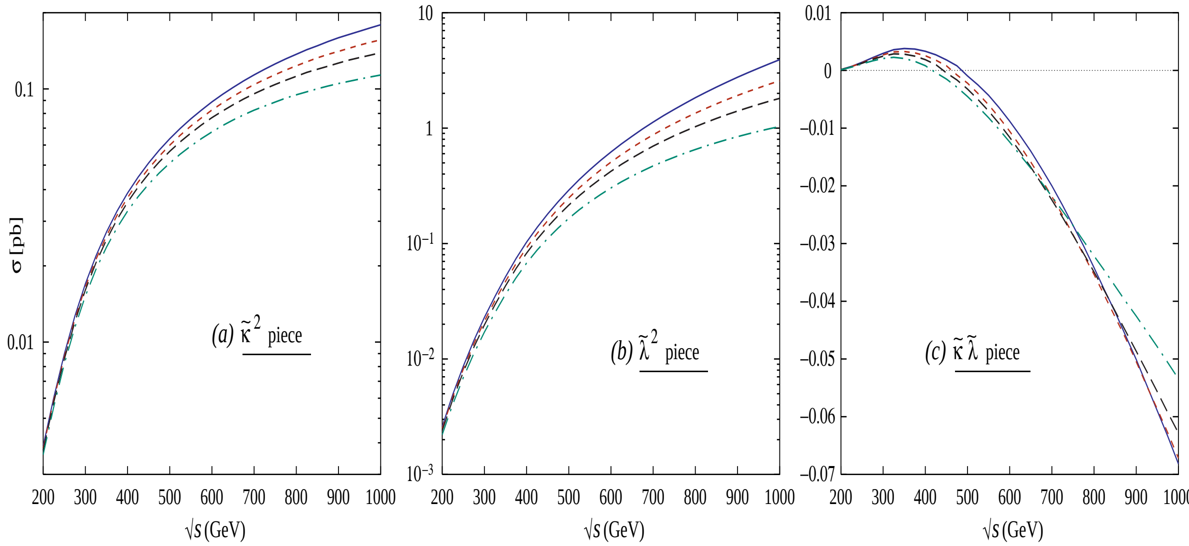

Figure 3: *The energy dependence of the* extra *pieces in the cross section for unit values of the anomalous couplings. The solid, short-dashed, long-dashed and dot-dashed curves correspond to* $n = 0, 1, 2, 4$ *in eqn.(10). The scale* (Λ) *of new physics has been assumed to be* 1 TeV.

6 An estimate of the sensitivity

In assessing the sensitivity of a future LC we will use the double differential distribution $\mathrm{d}^2\sigma/\mathrm{d}E_\gamma\,\mathrm{d}\cos\theta_\gamma$ with the phase space divided into a number of bins. Choosing a simple χ^2 test to derive 95% C.L. boundaries in the two-parameter space of $(\tilde{\kappa}_\gamma, \tilde{\lambda}_\gamma)$, we define

$$\chi^2 = \sum_i^{\#\ \text{of bins}} \left| \frac{N_{SM}(i) - N_{AN}(i)}{\Delta N_{SM}(i)} \right|^2 , \tag{11}$$

with $N_{SM}(i)$ and $N_{AN}(i)$ being the number of events in the bin i expected within the SM and a theory with the anomalous TGC, respectively. The error ΔN_{SM} is defined as a combination of statistical and systematic errors (*cf.* [18])

$$\Delta N_{SM} = \sqrt{(\sqrt{N})^2 + (\delta_{syst} N)^2} \, . \tag{12}$$

For our numerical analysis, we use a few sets of machine parameters (*i.e.* luminosities and CM energies) considered in the current ECFA-DESY workshop on future LC [21],

$$\mathcal{L} = \left\{ \begin{array}{lllllll} 25, & 75, & 100 & \& & 300 & \mathrm{fb}^{-1} & \sqrt{s} = 350\ \mathrm{GeV} \\ 75, & 125, & 300 & \& & 500 & \mathrm{fb}^{-1} & \sqrt{s} = 500\ \mathrm{GeV} \\ 125, & 200, & 500 & \& & 800 & \mathrm{fb}^{-1} & \sqrt{s} = 800\ \mathrm{GeV} \end{array} \right. . \tag{13}$$

For comparison, we also consider the LEP2 environment with $\sqrt{s} = 192$ GeV and $\mathcal{L} = 0.5$ and 2 fb^{-1}.

We divide the entire range of angular acceptance (see eq. (5)) into 26 equal-sized bins of 5° each, while, for E_γ, we assume uniform bins of 10 GeV each. It might seem counterintuitive to use more bins for large $\sqrt{s}$ in view of the smaller SM cross-section. However, this decrease in cross-section is more than compensated for by the large increase in the proposed luminosity.

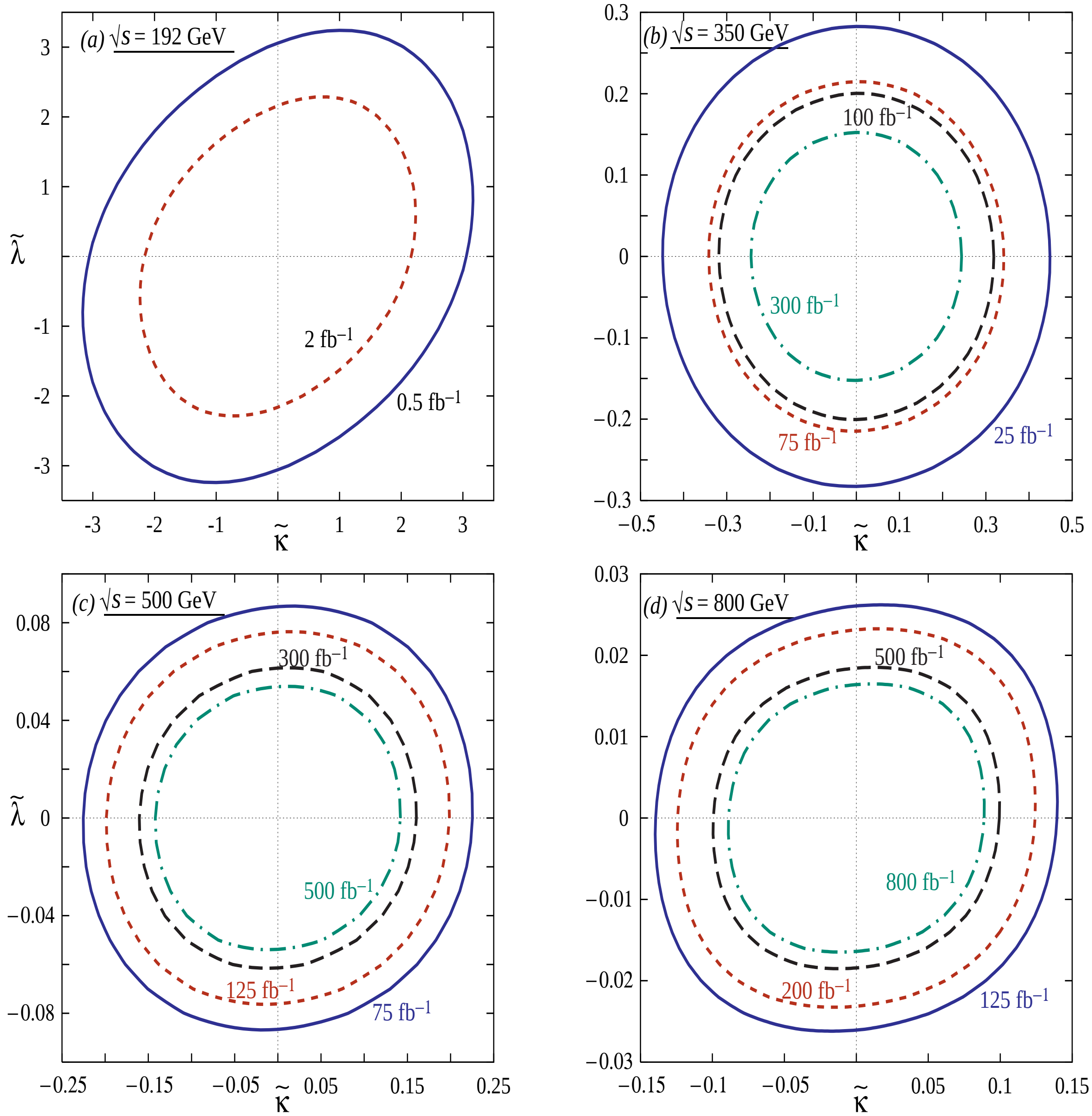

Figure 4: *95% exclusion contours for different energies and luminosities. It is assumed that there is no form-factor suppression for the couplings ($n = 0$ or $\Lambda \to \infty$ in eq. (10)).*

The systematic error δ_{syst} arises mainly from the detector parameters (e.g. uncertainty in the luminosity measurement and the detector efficiency). We take δ_{syst} to be 2%, which is commonly considered as a fairly conservative assumption. As it turns out, the error in ΔN_{SM} is dominated by statistical errors and the results are insensitive to small changes in δ_{syst}.

For a theory with 2 variables, 95% C.L. corresponds to $\chi^2 = 6$ and the corresponding contours are shown in Fig. 4. Clearly, a marked improvement accompanies an increase in luminosity. This reflects the already stated fact of δ_{syst} in eq. (12) being dominated by the statistical errors. Also easy to discern is that a higher CM energy results in stronger constraints even for the same luminosity. This is expected since such couplings lead to a rapid growth in the number of events with $\sqrt{s}$ (see Fig. 3). By the same reasoning, the improvement along the $\tilde{\lambda}$ axis is more pronounced than that along the $\tilde{\kappa}$ axis.

The contours in Fig. 4 are nearly elliptical and with very little tilt. This of course implies that there is only a small correlation between the constraints on $\tilde{\kappa}$ and $\tilde{\lambda}$, a feature that we should have expected from a comparison of the relative strengths of the $\tilde{\kappa}^2$, the $\tilde{\lambda}^2$ and the $\tilde{\kappa}\tilde{\lambda}$ pieces in the cross-section (see Fig. 3). If one of the coupling is known to be identically zero (or determined by another experiment), then the individual bounds on the other can be obtained by determining $\chi^2 = 1$ (68.3% C.L.) or $\chi^2 = 3.8$ (95% C.L.) as the case may be. These bounds are summarised in Table 1.

$\sqrt{s}$ (GeV)	Luminosity (fb^{-1})	Individual Bounds	
		68.3% C.L.	95% C.L.
192	0.5	$\lvert\tilde{\kappa}\rvert < 1.95,\ \lvert\tilde{\lambda}\rvert < 1.96$	$\lvert\tilde{\kappa}\rvert < 2.74,\ \lvert\tilde{\lambda}\rvert < 2.74$
	2	$\lvert\tilde{\kappa}\rvert < 1.38,\ \lvert\tilde{\lambda}\rvert < 1.38$	$\lvert\tilde{\kappa}\rvert < 1.94,\ \lvert\tilde{\lambda}\rvert < 1.94$
350	75	$\lvert\tilde{\kappa}\rvert < 0.22,\ \lvert\tilde{\lambda}\rvert < 0.14$	$\lvert\tilde{\kappa}\rvert < 0.31,\ \lvert\tilde{\lambda}\rvert < 0.19$
	300	$\lvert\tilde{\kappa}\rvert < 0.16,\ \lvert\tilde{\lambda}\rvert < 0.098$	$\lvert\tilde{\kappa}\rvert < 0.22,\ \lvert\tilde{\lambda}\rvert < 0.14$
500	125	$\lvert\tilde{\kappa}\rvert < 0.13,\ \lvert\tilde{\lambda}\rvert < 0.049$	$\lvert\tilde{\kappa}\rvert < 0.18,\ \lvert\tilde{\lambda}\rvert < 0.069$
	500	$\lvert\tilde{\kappa}\rvert < 0.091,\ \lvert\tilde{\lambda}\rvert < 0.035$	$\lvert\tilde{\kappa}\rvert < 0.13,\ \lvert\tilde{\lambda}\rvert < 0.049$
800	200	$\lvert\tilde{\kappa}\rvert < 0.079,\ \lvert\tilde{\lambda}\rvert < 0.015$	$\lvert\tilde{\kappa}\rvert < 0.11,\ \lvert\tilde{\lambda}\rvert < 0.021$
	800	$\lvert\tilde{\kappa}\rvert < 0.057,\ \lvert\tilde{\lambda}\rvert < 0.010$	$\lvert\tilde{\kappa}\rvert < 0.079,\ \lvert\tilde{\lambda}\rvert < 0.015$

Table 1: *Expected individual constraints on $\tilde{\kappa}_\gamma$ and $\tilde{\lambda}_\gamma$ in the event that the other coupling is already known.*

Until now, we have sidestepped a few issues, namely the role of beam polarization, possible form-factor dependence of the couplings and the role of the minimum energy cut. The first is easy to estimate. Since the signal receives contributions only from left-handed electrons, polarizing the beam so is expected to help. But this would also increase the background by almost as much[5]. Consequently, the improvement is akin to that resulting from a somewhat higher luminosity.

As for the form-factor dependence, clearly the anomalous contribution decreases with increasing n (or smaller Λ). More importantly, the high-energy part of the photon spectrum gets depleted faster. This implies weaker constraints as evinced by Fig. 5(a). Again, the effect is more pronounced along the $\tilde{\lambda}$ axis.

Finally, we come to the role of the cut on photon energy eq. (6). As we commented earlier, one role of this cut is to eliminate contributions from diagrams $\boxed{3}$ and $\boxed{4}$ of

[5] Right-handed electrons participate only in diagrams $\boxed{1}$ and $\boxed{2}$ of Fig. 1. But these are almost totally eliminated by the cut of eq. (7).

Fig. 1. Strictly speaking, this is not necessary as the χ^2 function would simply assign a low weight to such bins. In fact, in the absence of other backgrounds, this cut (as also the other two) only succeeds in rejecting additional small but positive contributions to the χ^2. Fortunately, this is not a severe loss as Fig. 5(b) demonstrates clearly. More importantly, eq. (6) serves to eliminate other backgrounds such as $e^+e^- \to \gamma\gamma$ where one of the photons is missed by the electromagnetic calorimeter. In a real experimental setup, the precise nature and utility of such cuts would be dictated by the detector design and hence it is premature to dwell on it at further detail.

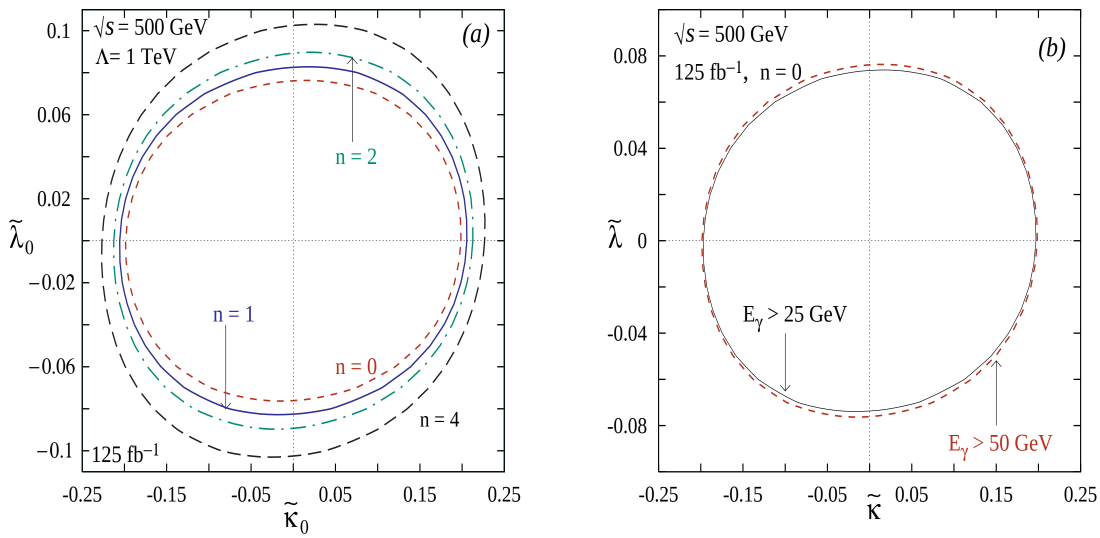

Figure 5: *(a) The effect of form-factor behaviour (see eq. (10)) on the exclusion contour for a given energy and luminosity. (b) The effect of the minimum energy requirement (see eq. (6)) on the exclusion contour for a given energy and luminosity. Backgrounds other than those from eq. (4) are deemed to be absent.*

7 Conclusions

We have investigated the process $e^+e^- \to \nu\bar{\nu}\gamma$ as a means to derive limits on CP-violating couplings $\tilde{\kappa}_\gamma$, $\tilde{\lambda}_\gamma$ at a future linear e^+e^- collider. Being sensitive only to the $WW\gamma$ couplings, it permits their independent evaluation. In addition, probing different kinematical configurations (real photon, space-like W) as compared to that of $e^+e^- \to W^+W^-$ (real W, time-like photon), it allows us to probe the form factor behaviour of the couplings over a distinct region of the momentum transfer. We have shown that using the differential distribution $d\sigma/dE_\gamma\, d\cos\theta_\gamma$ and appropriate kinematical cuts, constraints comparable with those expected from $W\gamma$ production at Tevatron or W^+W^- production at a 500 GeV LC can be obtained. As this study makes use of all attainable physical information, a potential improvement of the results can be only achieved by applying other statistical methods of testing the consistency with the SM.

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