# $b \rightarrow s \gamma$ confronts $B$-violating scalar couplings: $R$-parity violating supersymmetry or diquarks 

Debrupa Chakraverty ${ }^{1}$ and Debajyoti Choudhury ${ }^{2}$<br>Mehta Research Institute, Chhatnag Road, Jhusi, Allahabad 211 019, India.<br>E-mail: ${ }^{1}$ rupa@mri.ernet.in, ${ }^{2}$ debchou@mri.ernet.in


#### Abstract

We investigate the possible role that baryon number violating Yukawa interactions may take in the inclusive decay $B \rightarrow X_{s} \gamma$. The constraints, derived using the experimental results of the CLEO collaboration, turn out, in many cases, to be more stringent than the existing bounds.


The Standard Model (SM) of the strong, weak and electromagnetic interactions is in very good agreement with almost all present experimental data, even though a few important predictions have not yet been tested. Still, most physicists would readily admit that the SM cannot be the final theory, both on aesthetic grounds as well as on account of certain well-founded technical objections. As a result, numerous attempts have been and are being made in the quest of a more fundamental theory. Experimentally, there have been two main strategies to probe new physics. On the one hand, we attempt to directly produce, and observe, new particles at high energy colliders. On the other, we look for virtual effects of such particles and/or interactions in various low and intermediate energy processes. The decay $b \rightarrow s \gamma$ is an excellent candidate for the latter option [14]. Experimentally, the branching ratio for the inclusive decay $B \rightarrow X_{s} \gamma$ have been measured by CLEO [15] and ALEPH [16] to be

$$
B R\left(B \rightarrow X_{s} \gamma\right)= \begin{cases}(3.15 \pm 0.93) \times 10^{-4} & (\mathrm{CLEO})  \tag{1}\\ (3.11 \pm 1.52) \times 10^{-4} & (\mathrm{ALEPH})\end{cases}
$$

The above are in good agreement with each other and with the SM prediction [17] of $B R\left(B \rightarrow X_{s} \gamma\right)=(3.29 \pm 0.33) \times 10^{-4}$. While a small window for the contribution of new physics does remain, this agreement can obviously be used to constrain deviations from the SM.

In this paper, we investigate the influence that a scalar diquark may have on the above decay ${ }^{[7}$. Diquarks abound in many grand unified theories (with or without supersymmetry) and even in composite models [18]. While vector diquarks are constrained to be superheavy (with masses of the scale of breaking of the additional gauge symmetry), no such restrictions apply to the masses of scalar diquarks. Consequently, such particles can be as light as the electroweak scale. For example, a diquark like behaviour can be found even in a low energy theory like the Minimal Supersymmetric Standard Model (MSSM), albeit in the version with broken $R$-parity.

| Diquark Type | Coupling | $S U(3)_{c} \times S U(2)_{L} \times U(1)_{Y}$ |
| :---: | :---: | :---: |
| $\Phi_{1}$ | $h_{i j}^{(1)}\left(\bar{Q}_{L i}\right)^{c} Q_{L j} \Phi_{1}$ | $\left(\overline{6}, 3,-\frac{2}{3}\right)$ |
| $\Phi_{2}$ | $h_{i j}^{(2)}\left(\bar{Q}_{L i}\right)^{c} Q_{L j} \Phi_{2}$ | $\left(3,3,-\frac{2}{3}\right)$ |
| $\Phi_{3}$ | $\left[h_{i j}^{(3)}\left(\bar{Q}_{L i}\right)^{c} Q_{L j}+\tilde{h}_{i j}^{(3)}\left(\bar{u}_{R i}\right)^{c} d_{R j}\right] \Phi_{3}$ | $\left(\overline{6}, 1,-\frac{2}{3}\right)$ |
| $\Phi_{4}$ | $\left[h_{i j}^{(4)}\left(\bar{Q}_{L i}\right)^{c} Q_{L j}+\tilde{h}_{i j}^{(4)}\left(\bar{u}_{R i}\right)^{c} d_{R j}\right] \Phi_{4}$ | $\left(3,1,-\frac{2}{3}\right)$ |
| $\Phi_{5}$ | $h_{i j}^{(5)}\left(\bar{u}_{R i}\right)^{c} u_{R j} \Phi_{5}$ | $\left(\overline{6}, 1,-\frac{8}{3}\right)$ |
| $\Phi_{6}$ | $h_{i j}^{(6)}\left(\bar{u}_{R i}\right)^{c} u_{R j} \Phi_{6}$ | $\left(3,1,-\frac{8}{3}\right)$ |
| $\Phi_{7}$ | $h_{i j}^{(7)}\left(\bar{d}_{R i}\right)^{c} d_{R j} \Phi_{7}$ | $\left(\overline{6}, 1, \frac{4}{3}\right)$ |
| $\Phi_{8}$ | $h_{i j}^{(8)}\left(\bar{d}_{R i}\right)^{c} d_{R j} \Phi_{8}$ | $\left(3,1, \frac{4}{3}\right)$ |

Table 1: Gauge Quantum Numbers and Yukawa Couplings of Scalar Diquarks ( $Q_{e m}=$ $\left.T_{3}+\frac{Y}{2}\right)$.

A generic diquark is a scalar or vector particle that couples to a quark current with a net baryon number $B= \pm 2 / 3$. Clearly, this may transform as either a $S U(3)_{c}$ triplet or sextet. Concentrating on the scalars (for reasons mentioned above), the generic Yukawa term in the Lagrangian can be expressed as

$$
\begin{equation*}
\mathcal{L}_{Y}^{(A)}=h_{i j}^{(A)} \bar{q}_{i}^{c} P_{L, R} q_{j} \Phi_{A}+h . c . \tag{2}
\end{equation*}
$$

where $i, j$ denote quark flavours, $A$ denotes the diquark type and $P_{L, R}$ reflect the quark chirality. Standard Model gauge invariance demands that a scalar diquark

[^0]transform either as a triplet or as a singlet under $S U(2)_{L}$ and that it have a $U(1)$ hypercharge $|Y|=\frac{2}{3}, \frac{4}{3}, \frac{8}{3}$. The full list of quantum numbers is presented in Table 1 . It is clear that the couplings $h_{i j}^{(1)}, h_{i j}^{(4)}, h_{i j}^{(5)}$ and $h_{i j}^{(7)}$ must be symmetric under the exchange of $i$ and $j$ while $h_{i j}^{(2)}, h_{i j}^{(3)}, h_{i j}^{(6)}$ and $h_{i j}^{(8)}$ must be antisymmetric. For the other two, viz. $\tilde{h}_{i j}^{(3)}$ and $\tilde{h}_{i j}^{(4)}$, there is no particular symmetry property. Note that the quantum numbers of $\Phi_{2,4,6}$ allow them to couple to a leptoquark (i.e. a quarklepton) current as well. This implies that these particular diquarks could also mediate lepton-number ( $L$ ) violating processes. Clearly, such leptoquark couplings need to be suppressed severely so as to prevent rapid proton decay.

We make a brief interlude here to discuss the MSSM [19]. Whereas $B$ and $L$ are (accidentally) preserved in the SM (at least in the perturbative context), it is not so within the MSSM. Supersymmetry and gauge invariance, together with the field content, allow terms in the superpotential that violate either $B$ or $L$ [20]. Catastrophic rates for proton decay can be avoided though by imposing a global $Z_{2}$ symmetry 21] under which the quark and lepton superfields change by a sign, while the Higgs superfields remain invariant. Representible as $R \equiv(-1)^{3 B-L+2 S}$, where $S$ is the spin of a field, this " $R$-parity" is positive for the SM fields and negative for all the supersymmetric partners. However, while this symmetry is useful in preventing phenomenologically unacceptable terms, it has no theoretical foundation and is entirely ad hoc in nature. Hence, it is of interest to examine the consequences of violating this symmetry, not in the least because it plays a crucial role in the search for supersymmetry. In our study, we shall restrict ourselves to the case where only the $B$-violating terms are non-zero. Such scenarios can be motivated from a class of supersymmetric GUTs as well [22]. The corresponding terms in the superpotential can be parametrized as

$$
\begin{equation*}
W_{R R}=\lambda_{i j k}^{\prime \prime} \bar{U}_{R}^{i} \bar{D}_{R}^{j} \bar{D}_{R}^{k} \tag{3}
\end{equation*}
$$

where $U_{R}^{i}$ and $D_{R}^{i}$ denote the right-handed up-quark and down-quark superfields respectively. The couplings $\lambda_{i j k}^{\prime \prime}$ are antisymmetric under the exchange of the last two indices. The corresponding Lagrangian can then be written in terms of the component fields as:

$$
\begin{equation*}
\mathcal{L}_{R}=\lambda_{i j k}^{\prime \prime}\left(u_{i}^{c} d_{j}^{c} \tilde{d}_{k}^{*}+u_{i}^{c} \tilde{d}_{j}^{*} d_{k}^{c}+\tilde{u}_{i}^{*} d_{j}^{c} d_{k}^{c}\right)+\text { h.c. } \tag{4}
\end{equation*}
$$

Thus, a single term in the superpotential corresponds to three different diquark interactions, namely two of type $\tilde{h}_{i j}^{(4)}$ and one of type $h_{i j}^{(8)}$.

The best direct bound on diquark type couplings comes from the analysis of dijet events by the CDF collaboration [23]. Considering the process $q_{i} q_{j} \rightarrow \Phi_{A} \rightarrow q_{i} q_{j}$, an exclusion curve in the $\left(m_{\Phi_{A}}, h_{i j}^{(A)}\right)$ plane can be obtained from this data. Two points need to be noted though. At a $p \bar{p}$ collider like the Tevatron, the $u u$ and $d d$ fluxes tend to be small and hence the bounds are relatively weak. This is even more true for quarks of the second or third generation (which are relevant for the couplings that we are interested in). Secondly, such an analysis needs to make assumptions regarding
the branching fraction of $\Phi_{A}$ into quark pairs, a point that is of particular importance in the context of $R$-parity violating supersymmetric models.

There also exist some constraints derived from low energy processes. Third generation couplings, for example, can be constrained from the precision electroweak data at LEP [24] or, to an extent, by demanding perturbative unitarity to a high scale [25]. Couplings involving the first two generations, on the other hand, are constrained ${ }^{3}$ by the non-observance of neutron-antineutron oscillations or from an analysis of rare nucleon and meson decays [26, 27]. While many of these individual bounds are weak, certain of their products are much more severely constrained by the data on neutral meson mixing and $C P$-violation in the $K$-sector [28]. It is our aim, in this article, to derive analogous but stronger bounds.


Figure 1: Feynman diagrams that determine the one loop $b \rightarrow s \gamma$ decay amplitude.
Within the SM, the quark level transition $b \rightarrow s \gamma$ is mediated, at the lowest order, by electromagnetic penguin diagrams shown in Fig. [1 (a-d). While only the top-quark diagrams have been shown, for consistency's sake, other charge $2 / 3$ quarks should also be included. However, these contributions are negligible on two counts: $(i)$ the small mixing angles and (ii) the corresponding loop integrals being suppressed to a great extent due to the smallness of the light quark masses. The matrix element for this process is then governed by the dipole operator:

$$
\begin{equation*}
-\frac{4 G_{F}}{\sqrt{2}} V_{t s}^{*} V_{t b}\left(\frac{e}{32 \pi^{2}}\right) C_{7}^{S M}\left(m_{W}\right) \bar{s} \sigma_{\mu \nu} F^{\mu \nu}\left[m_{b}\left(1+\gamma_{5}\right)\right] b \tag{5}
\end{equation*}
$$

The QCD corrections to this process are calculated via an operator product expansion

[^1]based on the effective Hamiltonian
\[

$$
\begin{equation*}
H_{\mathrm{eff}}=-\frac{4 G_{F}}{\sqrt{2}} V_{t s}^{*} V_{t b} \sum_{i=1}^{8} C_{i}(\mu) O_{i}(\mu) \tag{6}
\end{equation*}
$$

\]

which is then evolved from the electroweak scale down to $\mu=\mu_{b}$ through renormalisation group (RG) equations. A large correction owes itself to the chromomagnetic operator $b \rightarrow s G$ ( $G$ being a gluon)

$$
\begin{equation*}
-\frac{4 G_{F}}{\sqrt{2}} V_{t s}^{*} V_{t b}\left(\frac{g_{s}}{32 \pi^{2}}\right) C_{8}^{S M}\left(m_{W}\right) \bar{s}_{\alpha} \sigma_{\mu \nu} G_{\alpha \beta}^{\mu \nu}\left[m_{b}\left(1+\gamma_{5}\right)\right] b_{\beta}, \tag{7}
\end{equation*}
$$

which arises from the diagrams of Fig. $\mathrm{Z}^{(\mathrm{a}-\mathrm{c}) \text { ), The Wilson coefficients } C_{7}^{S M}\left(m_{W}\right) ~}$ and $C_{8}^{S M}\left(m_{W}\right)$ can be evaluated perturbatively [29 31 at the $W$ scale where the matching conditions are imposed. The explicit expressions are

$$
\begin{align*}
& C_{7}^{S M}\left(m_{W}\right)=x\left[\frac{7-5 x-8 x^{2}}{24(x-1)^{3}}+\frac{x(3 x-2)}{4(x-1)^{4}} \ln x\right], \\
& C_{8}^{S M}\left(m_{W}\right)=x\left[\frac{2+5 x-x^{2}}{8(x-1)^{3}}-\frac{3 x}{4(x-1)^{4}} \ln x\right], \tag{8}
\end{align*}
$$

where $x=m_{t}^{2} / m_{W}^{2}$.


Figure 2: Feynman diagrams that determine the one loop $b \rightarrow s G$ decay amplitude.
The leading order results for the Wilson coefficients at $\mu_{b}$, the $B$-meson scale, is given by

$$
\begin{equation*}
C_{7}\left(\mu_{b}\right)=\eta^{16 / 23}\left[C_{7}^{S M}\left(m_{W}\right)-\frac{8}{3} C_{8}^{S M}\left(m_{W}\right)\left[1-\eta^{-2 / 23}\right]+\frac{232}{513}\left[1-\eta^{-19 / 23}\right]\right] \tag{9}
\end{equation*}
$$

with $\eta \equiv \alpha_{s}\left(m_{W}\right) / \alpha_{s}\left(\mu_{b}\right)$, calculated using the leading $\mu$ dependence of $\alpha_{s}$, and the present world average value of the strong coupling constant viz. $\alpha_{s}\left(m_{Z}\right)=0.118 \pm$ 0.005 . To this order, then,

$$
\begin{equation*}
\Gamma(b \rightarrow s \gamma)=\frac{\alpha G_{F}^{2} m_{b}^{5}}{32 \pi^{4}}\left|V_{t b} V_{t s}^{*} C_{7}^{S M}\left(\mu_{b}\right)\right|^{2} \tag{10}
\end{equation*}
$$

where $\alpha$ is the fine structure constant. As the above decay rate suffers from large uncertainties due to $m_{b}$ and the CKM matrix elements, it is prudent to normalise it against the measured semileptonic decay rate of the $b$ quark

$$
\begin{equation*}
\Gamma\left(b \rightarrow c e \bar{\nu}_{e}\right)=\frac{G_{F}^{2} m_{b}^{5}}{192 \pi^{3}} \kappa(z) g(z)\left|V_{c b}\right|^{2}, \tag{11}
\end{equation*}
$$

where $z=m_{c}^{2} / m_{b}^{2}$ and

$$
g(z) \equiv 1-8 z+8 z^{2}-z^{4}-12 z^{2} \ln z
$$

is the phase space factor. The analytic expression for $\kappa(z)$, the one loop QCD correction to the semileptonic decay, can be found in Ref. [32]. The explicit dependence on $m_{b}^{5}$ is thus removed, while the ratio of the CKM elements in the scaled decay rate viz.,

$$
\begin{equation*}
\left|\frac{V_{t s}^{*} V_{t b}}{V_{c b}}\right|=0.976 \pm 0.010 \tag{12}
\end{equation*}
$$

is much better known than the individual elements.
An updated next to leading order (NLO) analysis [32] of the $B \rightarrow X_{s} \gamma$ branching ratio with QED corrections has been presented in Ref. 17. Incorporating both the NLO QCD and the resummed QED corrections, the Wilson coefficient $C_{7}^{\mathrm{eff}}\left(\mu_{b}\right)$ in SM can be expanded as

$$
\begin{equation*}
C_{7}^{\mathrm{eff}}\left(\mu_{b}\right)=C_{7}\left(\mu_{b}\right)+\frac{\alpha_{s}\left(\mu_{b}\right)}{4 \pi} C_{7}^{(1)}\left(\mu_{b}\right)+\frac{\alpha}{\alpha_{s}\left(\mu_{b}\right)} C_{7}^{(e m)}\left(\mu_{b}\right) . \tag{13}
\end{equation*}
$$

For brevity's sake, we do not give here the expressions for $C_{7}^{(1)}\left(\mu_{b}\right)$ and $C_{7}^{(e m)}\left(\mu_{b}\right)$ as these can be found in Ref. [32] and Ref. [17] respectively. The inclusion of the NLO and QED corrections in the $b \rightarrow s \gamma$ decay rate has significantly reduced the large uncertainty present in the previous LO calculation. From the quark level $b \rightarrow s \gamma$ decay rate, it is possible to infer the $B$ meson inclusive branching ratio $B R\left(B \rightarrow X_{s} \gamma\right)$ by including the nonperturbative $1 / m_{b}$ and $1 / m_{c}$ corrections. These bound state corrections also have been taken into account in Ref. [17.

Having delineated the formalism, it now remains to calculate the additional contributions due to the possible presence of non-zero diquark couplings. At the one-loop level, the only new contributions to $b \rightarrow s \gamma$ and $b \rightarrow s G$ arise from the diagrams of

Fig. $\mathbb{1}(e-h)$ and Fig. $Z^{( }(d-g)$ respectively . In the generic case, apart from modifications in the coefficients of $O_{7,8}$, two additional operators arise. Denoted as

$$
\begin{align*}
& \tilde{O}_{7}=\frac{e}{32 \pi^{2}} \bar{s} \sigma_{\mu \nu} F^{\mu \nu}\left[m_{b}\left(1-\gamma_{5}\right)\right] b  \tag{14}\\
& \tilde{O}_{8}=\frac{g_{s}}{32 \pi^{2}} \bar{s}_{\alpha} \sigma_{\mu \nu} G_{\alpha \beta}^{\mu \nu}\left[m_{b}\left(1-\gamma_{5}\right)\right] b_{\beta}
\end{align*}
$$

they differ from their SM counterparts in their chirality structure].
To keep the analysis simple, we shall assume that only one diquark multiplet is light and that all the fields within a multiplet are degeneratd. With this simplifying assumption, the new contributions, at the electroweak scale, are given by

$$
\begin{align*}
C_{7}^{D}\left(m_{W}\right) & =\frac{N_{c}}{\mathcal{A}}\left[\left\{Q_{\Phi} F_{1}(y)+Q_{t} F_{3}(y)\right\} l_{b} l_{s}^{*}+\frac{m_{t}}{m_{b}}\left\{Q_{t} F_{4}(y)-Q_{\Phi} F_{2}(y)\right\} r_{b} l_{s}^{*}\right] \\
\tilde{C}_{7}^{D}\left(m_{W}\right) & =\frac{N_{c}}{\mathcal{A}}\left[\left\{Q_{\Phi} F_{1}(y)+Q_{t} F_{3}(y)\right\} r_{b} r_{s}^{*}+\frac{m_{t}}{m_{b}}\left\{Q_{t} F_{4}(y)-Q_{\Phi} F_{2}(y)\right\} l_{b} r_{s}^{*}\right], \\
C_{8}^{D}\left(m_{W}\right) & =\mathcal{A}^{-1}\left[\left\{C_{\Phi} F_{1}(y)+C_{t} F_{3}(y)\right\} l_{b} l_{s}^{*}+\frac{m_{t}}{m_{b}}\left\{C_{t} F_{4}(y)-C_{\Phi} F_{2}(y)\right\} r_{b} l_{s}^{*}\right] \\
\tilde{C}_{8}^{D}\left(m_{W}\right) & =\mathcal{A}^{-1}\left[\left\{C_{\Phi} F_{1}(y)+C_{t} F_{3}(y)\right\} r_{b} r_{s}^{*}+\frac{m_{t}}{m_{b}}\left\{C_{t} F_{4}(y)-C_{\Phi} F_{2}(y)\right\} l_{b} r_{s}^{*}\right], \\
\mathcal{A} & \equiv-4 \sqrt{2} G_{F} V_{t s}^{*} V_{t b} m_{\Phi}^{2} \tag{15}
\end{align*}
$$

where

$$
\begin{align*}
& F_{1}(y)=\frac{1}{12(y-1)^{4}}\left[6 y^{2} \ln y-2 y^{3}-3 y^{2}+6 y-1\right] \\
& F_{2}(y)=\frac{1}{2(y-1)^{3}}\left[1-y^{2}+2 y \ln y\right]  \tag{16}\\
& F_{3}(y)=\frac{1}{12(y-1)^{4}}\left[2+3 y-6 y^{2}+y^{3}+6 y \ln y\right], \\
& F_{4}(y)=\frac{1}{2(1-y)^{3}}\left[3-4 y+y^{2}+2 \ln y\right]
\end{align*}
$$

with $y=m_{t}^{2} / m_{\Phi}^{2}$. The color factor $N_{c}$ is -1 and 2 for triplet and $\overline{6}$ scalar respectively. $Q_{t}$ and $Q_{\Phi}$ are the charges of top quark and the diquark respectively. The color factors $C_{t}$ and $C_{\Phi}$ - for the diagrams of Fig. $2 f$ and Fig. $2 g$-are given in Table 2.

Note that, once again, we consider only such contributions, as involve the top quark. As is easy to ascertain from eq.(15), for other quarks in the loop, the corresponding integrals are too small to be of any consequence. Thus, any coupling to

[^2]| Diquark Type | $C_{\Phi}$ | $C_{t}$ | $l_{b}$ | $l_{s}$ | $r_{b}$ | $r_{s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Phi_{1}$ | $-5 / 2$ | $1 / 2$ | $h_{33}^{(1)} / \sqrt{2}$ | $h_{32}^{(1)} / \sqrt{2}$ | 0 | 0 |
| $\Phi_{2}$ | $1 / 2$ | $1 / 2$ | $h_{33}^{(2)} / \sqrt{2}$ | $h_{32}^{(2)} / \sqrt{2}$ | 0 | 0 |
| $\Phi_{3}$ | $-5 / 2$ | $1 / 2$ | $h_{33}^{(3)} / \sqrt{2}$ | $h_{32}^{(3)} / \sqrt{2}$ | $\tilde{h}_{33}^{(3)}$ | $\tilde{h}_{32}^{(3)}$ |
| $\Phi_{4}$ | $1 / 2$ | $1 / 2$ | $h_{33}^{(4)} / \sqrt{2}$ | $h_{32}^{(4)} / \sqrt{2}$ | $\tilde{h}_{33}^{(4)}$ | $\tilde{h}_{32}^{(4)}$ |

Table 2: Values of the coefficients $C_{\Phi}, C_{t}, l_{b}, l_{s}, r_{b}, r_{s}$ for different types of Scalar Diquarks.
the diquarks $\Phi_{7,8}$, for example, would not be constrained to an appreciable degree by radiative $b$ decays. In Table 2, we also display the relevant chiral Yukawa couplings (to the $b$ - and $s$-quarks) for different choices of the diquark.

In estimating the effects of scalar diqaurk couplings, it is useful to consider the ratios (17] $\xi_{7,8}$ with

$$
\begin{equation*}
\xi_{7} \equiv 1+\frac{C_{7}^{D}\left(m_{W}\right)}{C_{7}^{S M}\left(m_{W}\right)}, \tag{17}
\end{equation*}
$$

and similarly for $\xi_{8}$. For the new operators $\tilde{O}_{7}$ and $\tilde{O}_{8}$, we define,

$$
\begin{equation*}
\tilde{\xi}_{7}=\frac{\tilde{C}_{7}^{D}\left(m_{W}\right)}{C_{7}^{S M}\left(m_{W}\right)} \tag{18}
\end{equation*}
$$

and $\tilde{\xi}_{8}$ in an analogous fashion. With these definitions, the $B \rightarrow X_{s} \gamma$ branching ratio can be written as,

$$
\begin{align*}
B R\left(B \rightarrow X_{s} \gamma\right) & =B_{22}(\delta)+\left(\xi_{7}^{2}+\tilde{\xi}_{7}^{2}\right) B_{77}(\delta)+\left(\xi_{8}^{2}+\tilde{\xi}_{8}^{2}\right) B_{88}(\delta)+\xi_{7} B_{27}(\delta) \\
& +\xi_{8} B_{28}(\delta)+\left(\xi_{7} \xi_{8}+\tilde{\xi}_{7} \tilde{\xi}_{8}\right) B_{78}(\delta) \tag{19}
\end{align*}
$$

In a parton level analysis, the photon would be monochromatic, with $E_{\gamma}=E_{\gamma}^{\max }=$ $m_{b} / 2$. However, once the gluon Bremsstrahlung contribution is included, the photon spectrum becomes nontrivial and, for experimental purposes, one needs to make an explicit demand on the photon energy, namely

$$
\begin{equation*}
E_{\gamma}>(1-\delta) E_{\gamma}^{\max } \tag{20}
\end{equation*}
$$

where $\delta$ is the fraction of the spectrum above the cut. The values of $B_{i j}(\delta)$ are listed in ref. [17] for different choices of the renormalisation scale $\mu_{b}$ and the cut off parameter on the photon energy $\delta$. As is well known, some ambiguities exist in the choice of
$\mu_{b}$ which should, typically, lie in the region $m_{b} / 2$ to $2 m_{b}$. For our analysis, we used $\mu_{b}=m_{b}$ and $\delta=0.9$. We have checked that other values of $\delta$ do not change the bound significantly.

Since the new physics becomes operative only above the electroweak scale, the additional contributions to the operators $O_{7}$ and $O_{8}$ will only serve to change the Wilson coefficients at $m_{W}$. Of course, the additional operators $\tilde{O}_{7}$ and $\tilde{O}_{8}$ would influence the RG equations for $C_{7}$ and $C_{8}$ as well. However, since we are primarily interested in small $C_{7,8}^{D}\left(m_{W}\right)$, it is safe to neglect any term in the RG equations involving these coefficients.


Figure 3: The partial width for $B \rightarrow X_{s} \gamma$ as a function of the product of the diquark and/or $R$ parity violating couplings for a fixed diquark mass of 100 GeV , The shaded region represents the $1 \sigma$ limits of the experimentally observed value. The curves for $h_{33}^{(3)} h_{32}^{(3)}$ and $h_{33}^{(4)} h_{32}^{(4)}$ are identical to those for $h_{33}^{(1)} h_{32}^{(1)}$ and $h_{33}^{(2)} h_{32}^{(2)}$ respectively.

In the absence of an $L$-violating coupling, these diquarks clearly do not influence the semileptonic decay modes of the $B$-meson. Thus, we may continue to normalize the radiative $b$-decay against $b \rightarrow c e \bar{\nu}_{e}$ in order to avoid the severe dependence on $m_{b}$. In Fig. 3, we plot the branching ratio $B R\left(B \rightarrow X_{s} \gamma\right)$ in presence of a diquark (multiplet) of mass 100 GeV . We continue to work under the assumption that only one pair of couplings are non-zero. Furthermore, we assume the said couplings to be reall. That the curves should be parabolic in the product of the two couplings in question is obvious. To appreciate the fact that many of these curves have their minima lying on the SM value, one needs to consider the chirality structure of the corresponding diquark couplings (see Table (2). For example, combinations involving either of $\tilde{h}_{32}^{(3)}$ or $\tilde{h}_{32}^{(4)}$, imply that there is no left-handed coupling to the strange quark. From eqs.(15), it is then easy to see that the SM Wilson coefficients $C_{7,8}$ remain unaffected.

[^3]Consequently, the new contribution adds incoherently with the SM amplitude. For the rest of the combinations, though, the interference term is non-negligible leading to a shift in the minimum. Hence, unlike those for the first set of combinations (those involving $\tilde{h}_{32}^{(3,4)}$ ), the branching ratios corresponding to these sets are in agreement with the experimental numbers for two non-contiguous ranges of the product.

In each of eqs.(15), the second term, whenever allowed, is clearly the dominant piece. This enhancement by the factor of $m_{t} / m_{b}$ comes into play only when the diquark couplings to the bottom and the strange quarks have pieces with opposite chirality. In other words, for a diquark of the type $\Phi_{3}$ (or $\Phi_{4}$ ), the simultaneous presence of both the allowed types of couplings is severely constrained by the data on $B \rightarrow X_{s} \gamma$.

| Products of Couplings | Bounds from $B \rightarrow X_{s} \gamma$ |  |
| :---: | :---: | :---: |
|  | $1 \sigma$ | $2 \sigma$ |
| $\begin{array}{r} h_{33}^{(1)} h_{32}^{(1)} \\ \text { or } h_{33}^{(3)} h_{32}^{(3)} \end{array}$ | $\begin{gathered} {[-1.0,-0.85]} \\ {\left[-9.3 \times 10^{-2}, 5.8 \times 10^{-2}\right]} \end{gathered}$ | $\begin{gathered} {[-1.1,-0.74]} \\ {[-0.2,0.12]} \end{gathered}$ |
| $\begin{array}{r} h_{33}^{(2)} h_{32}^{(2)} \\ \text { or } h_{33}^{(4)} h_{32}^{(4)} \end{array}$ | $\begin{gathered} {[-0.16,0.25]} \\ {[2.6,3.0]} \end{gathered}$ | $\begin{gathered} {[-0.33,0.53]} \\ {[2.3,3.2]} \end{gathered}$ |
| $h_{33}^{(3)} \tilde{h}_{32}^{(3)}$ | $\left[-9.2 \times 10^{-4}, 9.2 \times 10^{-4}\right]$ | $\left[-1.4 \times 10^{-3}, 1.4 \times 10^{-3}\right]$ |
| $\tilde{h}_{33}^{(3)} h_{32}^{(3)}$ | $\begin{gathered} {\left[-2.2 \times 10^{-4}, 3.5 \times 10^{-4}\right]} \\ {\left[3.3 \times 10^{-3}, 3.9 \times 10^{-3}\right]} \end{gathered}$ | $\begin{gathered} {\left[-4.5 \times 10^{-4}, 7.3 \times 10^{-4}\right]} \\ {\left[2.9 \times 10^{-3}, 4.1 \times 10^{-3}\right]} \end{gathered}$ |
| $\tilde{h}_{33}^{(3)} \tilde{h}_{32}^{(3)}$ | [-0.12, 0.12] | [-0.18, 0.18] |
| $h_{33}^{(4)} \tilde{h}_{32}^{(4)}$ | $\left[-2.5 \times 10^{-3}, 2.5 \times 10^{-3}\right]$ | $\left[-3.7 \times 10^{-3}, 3.7 \times 10^{-3}\right]$ |
| $\tilde{h}_{33}^{(4)} h_{32}^{(4)}$ | $\begin{gathered} {\left[-1.1 \times 10^{-2},-9.3 \times 10^{-3}\right]} \\ {\left[-9.4 \times 10^{-4}, 5.9 \times 10^{-4}\right]} \end{gathered}$ | $\begin{gathered} {\left[-1.2 \times 10^{-2},-8.3 \times 10^{-3}\right]} \\ {\left[-2.0 \times 10^{-3}, 1.2 \times 10^{-3}\right]} \end{gathered}$ |
| $\tilde{h}_{33}^{(4)} \tilde{h}_{32}^{(4)}$ | $[-0.35,0.35]$ | [-0.51, 0.51] |

Table 3: Limits on the Scalar Diquark Couplings for $m_{\Phi_{i}}=100 \mathrm{GeV}$.
In Table 3, we capture the essence of Fig. 3 in the form of actual limits that can be set on such products of couplings, for a diquark mass of 100 GeV . Understandably, the $2 \sigma$ bounds are weaker than the $1 \sigma$ ones. Similarly, the color-sextet couplings are more severely constrained than the color-triplet ones. It should be noted that the
structure of the interaction terms $h_{i j}^{(1)}$ are the same as those for $h_{i j}^{(3)}$. Consequently, the bounds are exactly the same. A similar story obtains for $h_{i j}^{(2)}$ and $h_{i j}^{(4)}$. As discussed above, for a few of the products there are two non-contiguous bands allowed. For the combinations $h_{33}^{(2)} h_{32}^{(2)}$ and $h_{33}^{(4)} h_{32}^{(4)}$, though, the second window (both at the $1 \sigma$ and $2 \sigma$ levels) lies beyond the perturbative limit and, hence, are phenomenologically uninteresting.

As discussed earlier, $\tilde{h}_{i j}^{(4)}$ is analogous to the trilinear $R$-parity violating coupling $\lambda_{i j k}^{\prime \prime}$. Thus the constraints on $\tilde{h}_{33}^{(4)} \tilde{h}_{32}^{(4)}$ are equivalent to those on the product $\lambda_{3 j 2}^{\prime \prime} \lambda_{3 j 3}^{\prime \prime}$. For each of these couplings, the best individual bound comes from the precision measurements at the $Z$ pole [24, 33], and amounts to $\lambda_{3 j 2}^{\prime \prime}, \lambda_{3 j 3}^{\prime \prime}<0.50$ at the $1 \sigma$ level. We thus do not do very well as far as this particular combination is concerned. This can be attributed to both the chirality and the color structure of the operator, each of which is "unfavourable" as far the $b \rightarrow s \gamma$ decay is concerned. For most of the other combinations though, we do significantly better than the product of individual bounds [24.


Figure 4: The partial width for $B \rightarrow X_{s} \gamma$ as a function of diquark mass. (a) For each curve, the associated product of diquark and/or $R$ parity violating couplings is held to be 1, while all other couplings are set to be vanishingly small. The shaded region represents the $1 \sigma$ limits of the observed value. The curves for $h_{33}^{(3)} h_{32}^{(3)}$ and $h_{33}^{(4)} h_{32}^{(4)}$ are identical to those for $h_{33}^{(1)} h_{32}^{(1)}$ and $h_{33}^{(2)} h_{32}^{(2)}$ respectively. (b) As in (a), but the non-zero products of diquark and/or $R$ parity voilating couplings are held to be 0.005.

In our effort to compare with the results available in the literature, we have, until now, held the diquark mass to be 100 GeV and varied the strength of its coupling. In reality, though a diquark is more likely to be somewhat heavier. For the sake of completeness, we next investigate the dependence on the diquark mass (Figs. (4), while holding the product fixed. As is expected, the extra contribution falls off with $m_{\Phi}$. The fall-off is somewhat slower than $m_{\Phi}^{-2}$ (see the expressions for $F_{i}(y)$ in eq.(16))
and the effects persist till $\sim 3 T e V$. The different rates of fall-off are governed by the dominant $F_{i}(y)$ in each case. The case for the combination $\tilde{h}_{33}^{(3)} h_{32}^{(3)}$ looks somewhat nontrivial. However, the shape is just a consequence of accidental cancellations between various terms of eq.(19). As the exact nature of these cancellations depend crucially on the value of the diquark couplings, not much should be read into the shape in general or the minimum in particular.

The two dependences ( $m_{\Phi}$ and coupling strength) that we have studied can be combined to rule out parts of the phase space. In Figs. 5 , we exhibit this for two particular combinations. In each, the shaded regions of the parameter space are in agreement with the experimental results at the designated level. For $h_{33}^{(1)} h_{32}^{(1)}$, the second allowed region is beyond the perturbative limit.


Figure 5: The region of the parameter space allowed by the data when all other couplings are set to zero. The lightly shaded area agrees with the data at $1 \sigma$ level, whereas the encompassing darker region agrees at $2 \sigma$.

In summary, we have studied the effects of the scalar diquark and/or R prity violating coupling to the branching ratio $B \rightarrow X_{s} \gamma$. Among the possible new contributions, the scalar diquark mediated diagram yield promising effects. The precise measurement of this branching ratio at the upcoming $B$ factories in near future and the reduction of theoretical uncertainty will improve the limits on the product combination of different scalar diquark and/or R parity violating couplings, we obtained.

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[^0]:    ${ }^{1} \mathrm{~A}$ brief discussion on the sensitivity of the branching ratio $B \rightarrow X_{s} \gamma$ to scalar diquark-top contribution has been presented in 17.
    ${ }^{2} \mathrm{We}$ do not consider the case of non-gauged vector diquarks as such theories are nonrenormalizable.

[^1]:    ${ }^{3}$ Although many of these analyses have been done for the case of $R$-parity violating models, clearly similar bounds would also apply to nonsupersymmetric diquark couplings as well.

[^2]:    ${ }^{4}$ Clearly, to this order, none of $\Phi_{5,6}$ can mediate either of these processes and hence we shall not consider such fields any further.
    ${ }^{5}$ In all of these four operators, contributions proportional to the strange quark mass have been neglected.
    ${ }^{6}$ Large splittings within a multiplet is disfavoured by the electroweak precision data.

[^3]:    ${ }^{7}$ The extension to complex couplings is straightforward. The imaginary parts, however, can be better constrained from an analysis of the $C P$ violating decay modes.

