

Muon anomalous magnetic moment confronts exotic fermions and gauge bosons

Debajyoti Choudhury ^{*}, Biswarup Mukhopadhyaya [†], and Subhendu Rakshit [‡]

Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad 211 019, India

Abstract

We investigate the status of models containing exotic fermions or extra Z -like neutral gauge bosons in the light of the recent data on anomalous magnetic moment of muon. We find that it is possible to extract interesting bounds on the parameters characterizing such models. The bounds are particularly strong if the new flavour-changing neutral currents are axial vectorlike.

The recent measurement of the anomalous magnetic moment (a_μ) of the muon by the E821 experiment [1] at BNL may very well act as an Occam's razor in constraining physics beyond the Standard Model (SM). The data seem to indicate a 2.6σ deviation from theoretical predictions based on the SM. More precisely, the measured value $a_\mu \equiv (g_\mu - 2)/2$ lies in the range

$$a_\mu^{\text{exp}} = (11\,659\,202 \pm 14 \pm 6) \times 10^{-10} \quad (1)$$

in units of the Bohr magneton ($e/2m_\mu$). Accumulation of fresh data is likely to reduce the errors even further. Comparing this with the SM prediction [2] of

$$a_\mu^{\text{SM}} = (11\,659\,159.7 \pm 6.7) \times 10^{-10}, \quad (2)$$

one has a rather tantalizing allowance for contributions from a number of non-standard options [3–6]. At the same time, it is possible to constrain new theories with potential extra contributions to $(g_\mu - 2)$. This is especially true, as is evident from equations (1) and (2), for scenarios where the additional contributions to the magnetic moment can be negative. One such set of possibilities consists of models with exotic fermions and/or extra neutral gauge bosons [7]. In this note we shall examine constraints on parameters of these models, such as the couplings and the masses of these extra particles, that can be imposed from the new measurement of the muon anomalous magnetic moment.

^{*}E-mail address: debchou@mri.ernet.in

[†]E-mail address: biswarup@mri.ernet.in

[‡]E-mail address: srakshit@mri.ernet.in

1 Exotic Leptons

Let us first discuss models with exotic leptons. On demanding that they have SM gauge interactions with the usual SM leptons, our choice gets restricted to vectorlike doublets or singlets and mirror fermions. For the last named ones, it may be noted that anomaly cancellation requires the presence of entire mirror families consisting of quarks and leptons. Constraints on ordinary–exotic fermion mixing from electroweak observables have been already discussed in the literature [8]. For example, LEP results have imposed a model independent lower bound of 93.5 GeV on the mass of a charged exotic lepton [9]. A characteristic feature of all these scenarios is the existence of flavour-changing couplings of the Z -boson with fermions either in the left or in the right-handed sector (or both). These exotic fermions contribute to muon anomalous magnetic moment via one loop diagrams shown in Fig. 1(a) where we generically represent the exotic lepton by F .

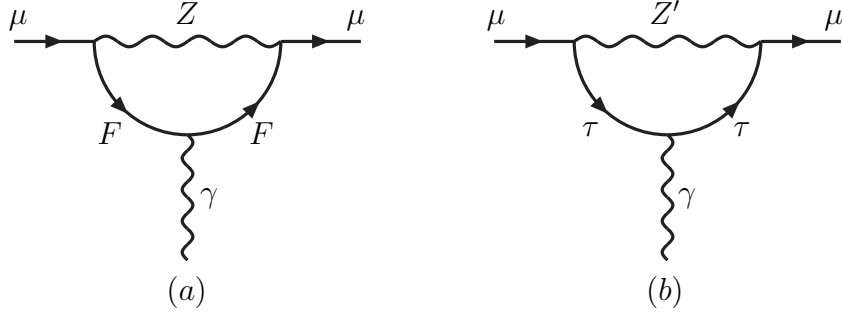


Figure 1: *Feynman diagrams contributing to a_μ . Diagram (a) corresponds to theories with exotic fermions while diagram (b) appears in theories with an exotic Z' .*

Parametrizing the flavour changing neutral current vertex by

$$\Gamma_{\text{FCNC}}^\mu = \frac{g}{2 \cos \theta_W} \gamma^\mu (a_L P_L + a_R P_R), \quad (3)$$

it is a straightforward exercise to calculate the additional contribution to a_μ . Under the simplifying assumption that only one species of such exotic fermion (F) exists, we have [10]

$$\begin{aligned} a_\mu &= \left(\frac{g}{2\pi \cos \theta_W} \right)^2 \frac{m_\mu}{M_Z} r \left[(a_R + a_L)^2 \left\{ \frac{m_\mu}{M_Z} f_1(r) + \frac{m_F}{M_Z} f_2(r) \right\} \right. \\ &\quad \left. + (a_R - a_L)^2 \left\{ \frac{m_\mu}{M_Z} f_1(r) - \frac{m_F}{M_Z} f_2(r) \right\} \right] \\ f_1(x) &= \left(\frac{5}{6} - \frac{5}{2}x + x^2 + (x^3 - 3x^2 + 2x) \ln \frac{x-1}{x} \right) \\ &\quad + \frac{x-1}{2x} \left(\frac{5}{6} + \frac{3}{2}x + x^2 + (x^2 + x^3) \ln \frac{x-1}{x} \right) \\ f_2(x) &= \left(2x - 1 + 2(x^2 - x) \ln \frac{x-1}{x} \right) + \frac{1-x}{2x} \left(\frac{1}{2} + x + x^2 \ln \frac{x-1}{x} \right) \end{aligned} \quad (4)$$

with $r \equiv (1 - m_F^2/M_Z^2)^{-1}$.

It should be mentioned here that the presence of exotic fermions also implies flavour-changing Yukawa couplings. Therefore, additional contributions accrue from Higgs-mediated loops as well. Numerically, however, these are relatively less significant, particularly for large Higgs masses.

The functions $rf_{1,2}(r)$, exhibited in Fig. 2 are seen to assume constant asymptotic values for both very small and very large r . Since these functions themselves are of the same order (with $rf_2(r)$ actually being the larger of the two), clearly the term in eqn.(4) proportional to m_F would tend to dominate, particularly because we are considering m_F on the order of the electroweak scale or above. This ceases to be the case if $a_L = 0$ or $a_R = 0$. (*i.e.* for pure left- or right-handed couplings). In such cases, the chirality structure of the interaction term itself rules out any non-zero contribution corresponding to a mass insertion in the internal fermion line. Consequently, any enhancement due to large m_F is at best logarithmic and the ensuing constraints from a_μ are not significant.

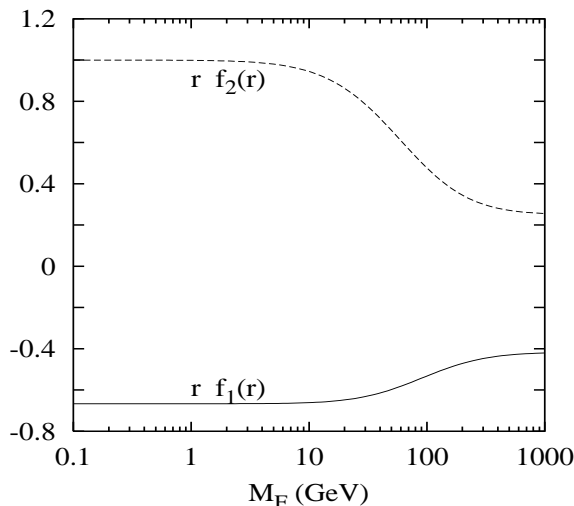


Figure 2: The functions $rf_{1,2}(r)$ that appear in eqn.(4) for $r = (1 - m_F^2/m_Z^2)^{-1}$.

The situation is more complicated for the general case where both left- and right-handed couplings are present. In such a case, one has to constrain a 3-dimensional parameter space spanned by (m_F, a_L, a_R) . We prefer to illustrate the issues involved by considering the two extremes, namely, $a_L = \pm a_R$. The results for a more general ratio between the two couplings lie in between the two extremes and can be deduced by considering an appropriate combination of the same. For $a_L = -a_R$ (a purely axial coupling), the additional contribution is negative. Since the data allows for only a very small range for $\delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} < 0$, (*i.e.* for values of the a_μ lower than the SM prediction), this immediately translates to a significant constraint in the a_R - m_F plane (see Fig. 3). On the other hand, $a_L = a_R$ (vectorlike current) leads to a positive contribution to a_μ . Since the available allowance here is much larger, the resulting constraints are weaker.

For example, for $a_L = -a_R$ with an exotic fermion mass of 400 GeV, the maximum

value of a_R turns out to be about 1.0×10^{-2} . Reverting to Fig. 2, note that the function $rf_2(r)$ assumes the constant value of $1/4$ for large m_F . In the expression for a_μ^{exotic} , this asymptotic value is multiplied though by m_F/m_Z . Hence, the exotic fermion contribution to a_μ grows with larger m_F . This is but a reflection of the non-decoupling character of the exotic sector [11] so long as we do not correlate the exotic fermion mass and its mixing angle with the ordinary fermion(s) but rather treat the two as independent phenomenological parameters. The tightening of the bound on a_R with increasing m_F is amply demonstrated by Fig. 3.

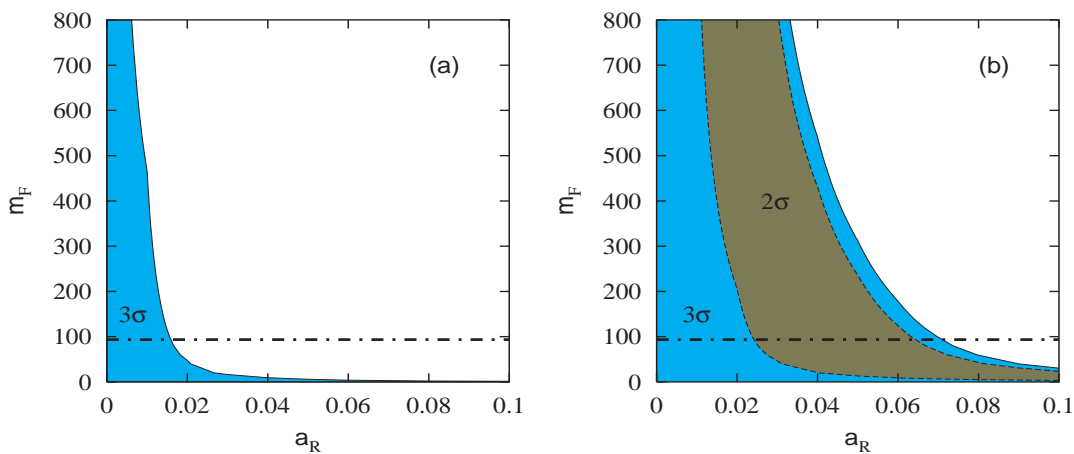


Figure 3: *The region of parameter space allowed by the a_μ data at different confidence levels for models containing an exotic lepton. The light-shaded region is allowed at the 3σ level, whereas the darker one is allowed at 2σ level. The region below the dash-dotted line is ruled out by LEP data [9]. Fig. (a) corresponds to $a_L = -a_R$, and while Fig. (b), to $a_L = a_R$. Case (a) is inconsistent with the data at the 2σ level.*

If we examine specific models [12] like those based on the E_6 gauge group, some additional features may reveal themselves. The fundamental (27-dimensional) representation of E_6 contains 16, 10 and 1-plets of $SO(10)$. The 10-plet of $SO(10)$ contains a vector-like lepton isodoublet (N_E, E) and the 1-plet is an isosinglet neutrino N . In addition, the 16-plet contains a right-handed neutrino (ν_R) which is singlet under the entire electroweak gauge group. In this case there is an extra contribution coming from another diagram where the photon couples to two W bosons and the N_E sits inside the loop. The corresponding contribution has a sign opposite to that of the case we considered earlier and can be sizable. As a result the total additional contribution to a_μ may turn out to be positive, especially if we take the $W\overline{N}_E\mu$ coupling to be the same as $Z\overline{F}\mu$ coupling. However, this is not the case in general, since ν_μ could mix with all three of ν_E , N and ν_R . This extra mixing in the neutrino sector prevents an exact or near-exact cancellation between the charged current loop and the corresponding Z -mediated loop. Thus, if one has non-decoupling contributions from both Z and W loops, the constraint from $(g_\mu - 2)$ has to be on a parameter space of larger dimensionality. Any projection on the

two dimensional subspace spanned by (a_R, m_μ) would be weaker than those in Fig. 3.

2 Extended gauge sector

Another set of potentially interesting contributions come from diagrams with an extra Z -type boson (Z'). The breaking of E_6 results in more than one additional $U(1)$ symmetries, at least one of which may survive down to the TeV scale in many scenarios, leading to an extra neutral gauge boson of phenomenological significance. The ensuing contributions to $(g_\mu - 2)$ will involve a graph similar to the one shown in the previous figure, but with the muon replacing F , and Z' replacing Z . As has already been shown [3], the smallness of the muon mass in the loop makes the contribution rather small, and one hardly obtains a bound of any significance.

If however, one considers a model where the Z' has *flavour-changing couplings* to the known charged leptons, then the data on a_μ can subject it to more stringent limits. The constraints on the mass of such a boson is model-dependent [9]. For example if one extends the SM gauge group by an additional $U(1)$, with respect to which the μ and the τ have different charges, then the corresponding Z' has an unsuppressed gauge coupling with a $\bar{\mu}\tau$ current [13]. In such cases, one has contributions to the muon anomalous magnetic moment via one-loop diagram in Fig. 1(b). Once again eqn.(3) gives the generic structure of the flavour changing neutral current vertex.

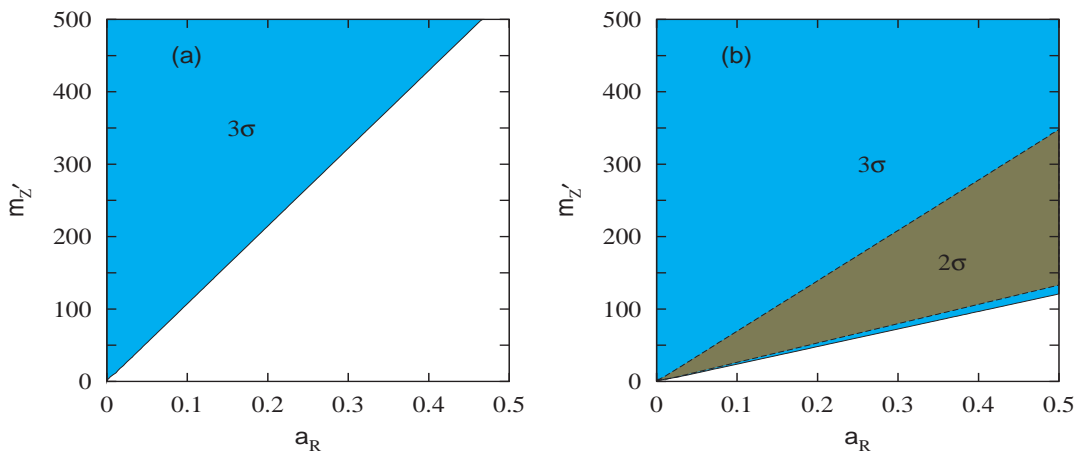


Figure 4: The region of parameter space allowed by the a_μ data at different confidence levels for models with an extra Z' . The light-shaded region is allowed at the 3σ level, whereas the darker one, to 2σ level. Fig. (a) corresponds to $a_L = -a_R$, and while Fig. (b), to $a_L = a_R$. Case (a) is inconsistent with the data at the 2σ level.

For the sake of convenience, we assume the gauge coupling associated with the Z' to be the same as that for the Z and absorb any deviation thereof into $a_{L,R}$. These two couplings, along with $M_{Z'}$, then describe the theory. To reduce the number of parameters,

we examine again cases with $a_L = a_R$ and $a_L = -a_R$. Fig. 4 shows the area in the $a_R - M_{Z'}$ plane excluded by the a_μ data for each of these two cases. Once again, the term proportional to $\frac{m_\tau}{M_{Z'}}$ dominates the new contribution. Note, however, the considerable difference from the case of the exotic lepton. The ratio $\frac{m_\tau}{M_{Z'}}$, being much smaller than the typical value of $\frac{m_F}{M_Z}$ that we considered, results in much smaller contribution in this case. This immediately translates to a relaxation (see Fig. 3) on the bounds on $a_{L,R}$ by a factor $\sim (\frac{m_F M_{Z'}}{m_\tau M_Z})^{1/2}$. Corrections to this naive estimate arise from the difference in values of the function $f_2(r)$ in the two cases.

More importantly, the theory with an extra Z' is a *decoupling one*. This has the immediate consequence of the bounds relaxing as $M_{Z'}$ grows. In other words, the $(g_\mu - 2)$ measurement is progressively insensitive to a very heavy Z' , quite unlike the case of an exotic lepton.

In conclusion, we have explored the phenomenological consequences of the models containing extra gauge bosons or exotic fermions in the light of the recent data on the anomalous magnetic moment of the muon from the E821 experiment at BNL. We find that the constraints on the model parameters can be significant if the flavour-changing couplings for these scenarios are dominantly of the axial vector type. While the parameter space of vectorlike fermion mixing with the muon gets rather strongly restricted by the recent data, somewhat weaker but non-negligible constraints also arise for a Z' if the latter has flavour-changing interactions in the leptonic sector.

References

- [1] The Muon $(g - 2)$ Collaboration, [hep-ex/0102017](#).
- [2] A. Czarnecki and W.J. Marciano, *Nucl. Phys. (Proc. Supp.)* **B76**, 245 (1999); P.J. Mohr and B.N. Taylor, *Rev. Mod. Phys.* **72**, 351 (2000).
- [3] A. Czarnecki and W.J. Marciano, [hep-ph/0102122](#).
- [4] K. Lane, [hep-ph/0102131](#); L. Everett, G. Kane, S. Rigolin and L. Wang, [hep-ph/0102145](#); J.L. Feng and K.T. Matchev, [hep-ph/0102146](#); E.A. Baltz and P. Gondolo, [hep-ph/0102147](#); U. Chattopadhyay and P. Nath, [hep-ph/0102157](#); U. Mahanta, [hep-ph/0102176](#); D. Chakraverty, D. Choudhury and A. Datta, [hep-ph/0102180](#).
- [5] Some of the earlier works are: R. Barbieri and L. Maiani, *Phys. Lett.* **B117**, 203 (1982); D.A. Kosower, L.M. Krauss and N. Sakai, *Phys. Lett.* **B133**, 305 (1983); R. Arnowitt, A.H. Chamseddine and P. Nath, *Z. Physik* **C26**, 407 (1984); J.L. Lopez, D.V. Nanopoulos and X. Wang, *Phys. Rev.* **D49**, 366 (1994); C. Arzt, M.B. Einhorn and J. Wudka, *Phys. Rev.* **D49**, 1370 (1994); U. Chattopadhyay and P. Nath, *Phys. Rev.* **D53**, 1648 (1996); T. Moroi, *Phys. Rev.* **D53**, 6565 (1996); M. Carena, G.F. Giudice and C.E. Wagner, *Phys. Lett.* **B390**, 234 (1997); P. Nath and M. Yamaguchi, *Phys. Rev.* **D60**, 116006 (1999); M. L. Graesser, *Phys. Rev.* **D61**, 074019

- (2000); U. Mahanta and S. Rakshit, *Phys. Lett.* **B480**, 176 (2000); U. Chattopdhyay, D.K. Ghosh and S. Roy, *Phys. Rev.* **D62**, 115001 (2000); R. Casadio, A. Gruppuso and G. Venturi, *Phys. Lett.* **B495**, 378 (2000); H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Phys. Lett.* **B493**, 135 (2000).
- [6] A. Czarnecki and W.J. Marciano, [hep-ph/0010194](#) and references therein.
 - [7] D.A. Morris, *Phys. Rev.* **D37**, 2012 (1988).
 - [8] P. Langacker and D. London, *Phys. Rev.* **D38**, 886 (1988); E. Nardi, E. Roulet and D. Tommasini, *Phys. Rev.* **D46**, 3040 (1992).
 - [9] D.E. Groom *et al.*, *Europhys J.* **C15** 1 (2000).
 - [10] J.P. Leveille, *Nucl. Phys.* **B137**, 63 (1978).
 - [11] T. Appelquist and J. Carazzone, *Phys. Rev.* **D11**, 2856 (1975).
 - [12] D. London, [hep-ph/9303290](#); J.L. Rosner, [hep-ph/9907438](#) and the references therein.
 - [13] X.G. He, G.C. Joshi, H. Lew and R.R. Volkas, *Phys. Rev.* **D43**, 22 (1991); *Phys. Rev.* **D44**, 2118 (1991); G. Dutta, A.S. Joshipura and K.B. Vijaykumar, *Phys. Rev.* **D50**, 2109 (1994).