

Brane Dynamics in the Randall-Sundrum model, Inflation and Graceful Exit

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We study the averaged action of the Randall-Sundrum model with a time dependent metric ansatz. It can be reformulated in terms of a Brans-Dicke action with time dependent Newton's constant. We show that the physics of early universe, particularly inflation, is governed by the Brans-Dicke theory. The Brans-Dicke scalar, however, quickly settles to its equilibrium value and decouples from the post-inflationary cosmology. The deceleration parameter is negative to start with but changes sign before the Brans-Dicke scalar settles to its equilibrium value. Consequently, the brane metric smoothly exits inflation. We have also studied the slow-roll inflation in our model and investigated the spectra of the density perturbation generated by the radion field and find them consistent with the current observations.

I. INTRODUCTION

The quest to explore physics beyond the Standard Model (SM) has, in the past couple of years, been dominated by the idea that space time is of a dimension larger than four and that we are essentially confined to a 4-dimensional hypersurface thereof. Although such suggestions go back to the work of Akama [1] and Rubakov and Shaposhnikov[2, 3], the present interest has been occasioned by the recognition that such solitonic hypersurfaces (or branes) abound in string theory. While an exact and viable low-energy realization of string theory is still some way off, considerable excitement has been aroused by the prospect of explaining the embarrassing gap between the electroweak scale M_W and the Planck scale (M_{pl} , $M_{pl}/M_W \sim 10^{16}$) by invoking the presence of such extra dimensions[4, 5, 6, 7, 8]. What is particularly new of such models is that the ordinary matter is confined to our four-dimensional brane while the gravity propagates in the whole spacetime.

These models can be broadly divided into two sets. The first, stimulated primarily by the papers of Arkani-Hamed *et al.*[4, 5, 6] (ADD)(see also [9]), invokes large (possibly sub-mm) new spatial dimensions transverse to the SM brane, with the higher-dimensional metric being factorizable and essentially flat. With such a construction in place, it can be shown that the fundamental gravitational scale could be close to the TeV scale, with the aforementioned hierarchy being explained by the large volume of the compactified dimensions.

Subsequently, Randall and Sundrum [7, 8] introduced an important variation, wherein the directions parallel and transverse to the branes do not factorize into a product space. An immediate consequence is that the 4-dimensional graviton wavefunction has a non-trivial dependence on the transverse coordinate(s). If this dependence is of the 'correct' form, the bulk (*i.e.* higher dimensional) graviton would be localized away from the brane that we live on, resulting in a suppression of gravitational interactions in our world. While such 'warped' compactifications had already been considered in the literature[3, 10] it was only after RS incorporated the modern brane idea, that the activity started in right earnest.

The fact that the RS models have far less striking phenomenological consequences (especially in collider experiments or in astrophysical contexts) as compared to the those for ADD models could have been expected to lead a gradual dissolution of interest in such models. That this has not been the case is, to a great extent, due to the fact that RS-models are perhaps more easily embeddable in supergravity and superstring compactifications[11, 12]. A particular example is afforded by the heterotic M theory, whose field theory limit is the 11-dimensional supergravity compactified on S_1/Z_2 with Yang-Mills, albeit supersymmetric, fields living on the two boundaries[11]. Furthermore, on account of their warped geometry, the RS models are able to resolve the SM hierarchy problem without needing to introduce large dimensions (a hierarchy problem in itself). At a more prosaic level, it has been argued that the early universe evolution of the RS models is dramatically different from that of standard FRW cosmology. It is this aspect that we intend to focus on in this article.

As is well known, Big Bang cosmology, while successful in answering many questions, leaves many others unanswered. Prominent amongst these are the flatness problem, the observed low density of monopoles and the horizon problem. Theories with inflation (a period in the distant past characterized by an exponential growth of the scale factor) offer solutions to all of these, and in fact are the only ones to do so. Inflation, however, is associated with its own set of problems. Primary amongst these is the generation of a correct (scalar) potential that would drive inflation, and equally importantly, the mechanism of a noncontrived exit of the universe from an inflationary phase. This *graceful exit* problem has been plaguing both cosmologists and string theorists with no simple solution in sight. Naturally, questions such as this reside at the core of our investigations.

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The cosmological implications of models with extra dimensions have been discussed by many authors [13, 14, 15]. Inflationary solutions were obtained for both flat bulk geometry [16], as well as for a AdS bulk geometry [17, 18, 19, 20, 21]. Inflation, in our model, occurs though in a novel way. To be specific, we use the dynamics of the warp factor to drive inflation. Starting from the effective four-dimensional action, recast in terms of the variables of the negative tension brane, we find that, after some field redefinitions, it could be interpreted as an action for scalar-tensor gravity. Solving the consequent equations of motion, we find that the evolution is governed by an initial exponential inflationary phase followed by a decelerating phase at the end of which the radion stabilises at its present day value. In other words, the exit is indeed graceful. Since the warp factor itself is the ‘inflaton’ in this theory, neither the adequate growth of the scale factor nor the exit demands any unnatural tuning of parameters.

In section II, we will show that the averaged four dimensional action obtained by integrating out the fifth dimension y [21], can be written as an action for a generalised Brans-Dicke theory [22]. The function $\omega(\phi)$ in this theory turns out to be

$$\omega(\phi) = -\frac{3}{2} \frac{\alpha\phi}{1 + \alpha\phi}. \quad (\text{I.1})$$

In section III, we obtain a solution to the Brans-Dicke equations of motion. Solution to the equations of motion is such that the Brans-Dicke (BD) scalar rapidly approaches its equilibrium value. The universe on the visible brane, which is also known as the standard model brane, undergoes exponential inflation during this time. As the scalar settles down to its equilibrium value, its equations of motion decouples and we effectively get four dimensional Einstein equations. We also study the behaviour of the deceleration parameter. We find that it is negative at early times but that its sign changes well before the BD scalar stabilizes. This, in turn, implies that the universe on the visible brane exits the inflationary era.

II. BRANS-DICKE GRAVITY ON THE BRANE

The Randall-Sundrum model that we will be interested in this paper consists of five dimensional Einstein gravity with negative cosmological constant and two 3-branes located at the fixed points of the orbifold S^1/Z_2 . We parametrize the orbifold direction y in such a way that the orbifold fixed points are at $y = 0$ and at $y = 1/2$ where the positive tension and negative tension branes are respectively located. The action[29] is given by [7, 8]

$$S = 2 \int d^4x \int_0^{1/2} dy \sqrt{-G} (M_5^3 R - \Lambda + \mathcal{L}_R) + \int d^4x \sqrt{-g^{(+)}} (L^+ - V^+) + \int d^4x \sqrt{-g^{(-)}} (L^- - V^-) \quad (\text{II.1})$$

where G_{MN} ($M, N = \mu, y$) is the five dimensional bulk metric, R is the five dimensional Ricci scalar and Λ is the cosmological constant in bulk, M_5 is five dimensional Planck mass. \mathcal{L}_R is some non-specified bulk dynamics responsible for generating a potential for the radion. The four dimensional metrics $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$, relevant to the positive and negative tension brane respectively, can be expressed in terms of G_{MN} as

$$g_{\mu\nu}^{(+)} = G_{\mu\nu}(x^\mu, y = 0), \quad g_{\mu\nu}^{(-)} = G_{\mu\nu}(x^\mu, y = 1/2). \quad (\text{II.2})$$

This restriction can be implemented through delta-functions which fix the locations of two 3-branes. While L^+ and L^- are the Lagrangians for the matter fields confined to the positive and negative tension branes respectively, V^\pm correspond to the associated brane tensions.

Equations of motion, in the absence of matter on either brane, are solved for the metric[7, 8]

$$ds^2 = e^{-2m_0 r_c |y|} (\eta_{\mu\nu} dx^\mu dx^\nu) + r_c^2 dy^2, \quad (\text{II.3})$$

provided the brane tensions and the bulk cosmological constant are related through

$$V^+ = -V^- = 12m_0 M_5^3, \quad \Lambda = -12m_0^2 M_5^3. \quad (\text{II.4})$$

This metric, as is well known by now, gives rise to an exponential hierarchy between the natural mass scale on the positive tension brane and that on the negative tension brane. For an appropriate choice of $m_0 r_c$, it is possible to generate the Planck mass M_{pl} on the negative tension brane to be of the order of 10^{19} GeV starting from a bulk energy scale of about a few TeV. What is of prime interest is that the resolution of the hierarchy (between the four dimensional Planck mass and the electroweak scale) does not necessitate choosing unnaturally small or big values for any of the parameters in the theory.

We would like to see what kinds of cosmological models we can accommodate in this scenario. Since our purpose is to study cosmological evolution, we will consider a new ansatz for the metric where the metric components are functions of the coordinate y as well as functions of time t . We will consider only a minimal modification of the RS static brane ansatz to accommodate temporal dependence[21]. In other words, we now have

$$ds^2 = e^{-2m_0 b(t)|y|} [g_{\mu\nu} dx^\mu dx^\nu] + b^2(t) dy^2 \quad (\text{II.5})$$

where

Clearly, this choice is such that when $b(t) \rightarrow \text{const.} = b_0$ and $a(t) \rightarrow \text{const.} = 1$, we recover the static RS solution.

The metrics on the two branes can be readily obtained from this ansatz:

$$g_{\mu\nu}^{(+)} = g_{\mu\nu}, \quad g_{\mu\nu}^{(-)} = e^{-m_0 b(t)} g_{\mu\nu}. \quad (\text{II.7})$$

It is worth pointing out that the metric on the negative tension brane is related to that on the positive tension brane by a conformal transformation. This has an interesting consequence for brane inflation, which we will discuss in the next section. We can now use this ansatz to determine equations of motion for $a(t)$ and $b(t)$. To get these equations let us first substitute (II.5) and (II.6) into (II.1). After performing the y integration we get the four dimensional effective action [21]

$$S_{eff} = \frac{3}{\kappa^2 m_0} \int d^4 x a^3 \left[(1 - \Omega_b^2) \frac{\dot{a}^2}{a^2} + m_0 \Omega_b^2 \frac{\dot{b}}{a} - \frac{m_0^2}{4} \Omega_b^2 \dot{b}^2 - V_r(b) \right] + \int d^4 x a^3 L^- \Omega_b^4, \quad (\text{II.8})$$

where $\kappa^2 = 1/2M_5^3$, $\Omega_b \equiv e^{-m_0 b(t)/2}$ and $a^3 V_r(b) = -b(t) \int dy \Omega_b^4 \mathcal{L}_R$. We have ignored the matter term in the positive tension brane but have considered one on the negative tension where our observable universe resides. A straightforward calculation shows that the action (II.8) can now be written as

$$S_{eff} = \frac{1}{2\kappa^2 m_0} \int d^4 x a^3 \left[(1 - \Omega_b^2) R_4(a) + \frac{3}{2} m_0^2 \Omega_b^2 \dot{b}^2 - 6V_r(b) \right] + S_M, \quad (\text{II.9})$$

where

$$R_4(a) = -6 \frac{\ddot{a}}{a} - 6 \frac{\dot{a}^2}{a^2}, \quad (\text{II.10})$$

and $S_M = \int d^4 x a^3 L^- \Omega_b^4$ is the matter action on the negative tension brane. Notice that $R_4(a)$ is a Ricci scalar derived from the metric $g_{\mu\nu}$ given in equation (II.6). From equation (II.7), it is, therefore, obvious that $R_4(a) = R_4^{(+)}(a)$. The four dimensional effective action on the positive tension brane can then be easily read out from (II.9)[21], viz.,

$$S_{eff}^{(+)} = \int d^4 x \sqrt{-g^{(+)}} \left[\frac{R_4^{(+)}(a)}{16\pi G^{(+)}} + \frac{3m_0}{4\kappa^2} \Omega_b^2 \dot{b}^2 - \frac{3}{\kappa^2 m_0} V_r(b) \right] + S_M, \quad (\text{II.11})$$

where $16\pi G^{(+)} = \frac{2\kappa^2 m_0}{1 - \Omega_b^2} = [M_{pl}^{(+)}]^{-2}$ corresponds to the Newton's constant on the positive tension brane. This action governs evolution of the positive tension brane. We, however, are interested primarily in our (negative tension) brane. To study the cosmology in our universe, we need to determine the effective four dimensional action on the negative tension brane. Let us parametrize the metric on the negative tension brane by[21]

$$g_{\mu\nu}^{(-)} = [-1, Y^2(\tau), Y^2(\tau), Y^2(\tau)], \quad (\text{II.12})$$

where τ is the new time coordinate on the negative tension brane. From eq.(II.7), it is easy to see that τ is related to the time t on the positive tension brane through

$$d\tau = e^{-m_0 b/2} dt, \quad Y = e^{-m_0 b/2} a. \quad (\text{II.13})$$

Using these relations in (II.8), we get the effective action on the negative tension brane to be

$$S_{eff}^{(-)} = \frac{3}{\kappa^2 m_0} \int d^3 x d\tau Y^3(\tau) \left[\frac{(1 - \Omega_b^2)(\tau) Y'^2(\tau)}{\Omega_b^2(\tau) Y^2(\tau)} + \frac{\Omega_b'^2(\tau)}{\Omega_b^4(\tau)} - 2 \frac{Y'(\tau) \Omega_b'(\tau)}{Y(\tau) \Omega_b^3(\tau)} - \frac{V_r(b)}{\Omega_b^4} \right] + S_M, \quad (\text{II.14})$$

where $S_M = \int d^3 x d\tau Y^3(\tau) L^-$ is the action on the negative tension brane in the new coordinate system and prime denotes differentiation with respect to the argument.

This action can be brought to the Brans-Dicke form. To see this, we define a new variable ϕ by

$$\Omega_b^2 = \frac{1}{1 + \alpha\phi}, \quad (\text{II.15})$$

where $\alpha = m_0/16M_5^3\pi$. It follows, after a straightforward calculation, that the action (II.14) can be recast in the form

$$S_{eff}^{(-)} = \int \frac{1}{16\pi} \sqrt{-g^{(-)}(\tau)} d^3 x d\tau \left[\phi(\tau) R_4^{(-)}(\tau) - \frac{\omega(\phi(\tau))}{\phi(\tau)} \phi'^2(\tau) - V(\phi) \right] + S_M^{(-)}, \quad (\text{II.16})$$

where

$$\sqrt{-g^{(-)}(\tau)} = Y^3(\tau), \quad (\text{II.17})$$

$$R_4^{(-)}(\tau) = -6 \frac{Y''(\tau)}{Y(\tau)} - 6 \frac{Y'^2(\tau)}{Y^2(\tau)}, \quad (\text{II.18})$$

$$V(\phi) = V_r(b)(1 + \alpha\phi)^2, \quad (\text{II.19})$$

The action (II.14), rewritten as in (II.16), corresponds to a generalised BD theory [22], where the BD parameter ω is a function of the BD scalar field ϕ . A few comments are in order at this point. Recall that the function Ω_b is instrumental in providing the resolution of the large mass hierarchy between the electroweak scale and the four dimensional Planck scale. This can be achieved only if Ω_b takes very small value (of $o(10^{-16})$). It is, therefore, clear from (II.15) that typical values that $\alpha\phi$ should take, to conform to the expected behaviour of Ω_b , will be quite large (of $o(10^{16})$). An immediate implication of this is that $\omega(\phi)$ stays mostly in the vicinity of $-3/2$. This, though, seems to be in direct conflict with astronomical observations (particularly the Solar system experiments [23]) which require $|\omega| > 3000$. We will, however, see in the next section that our solution to the equations of motion are such that the Brans-Dicke scalar stabilizes to its equilibrium value well within the inflationary epoch[30].

III. INFLATION AND GRACEFUL EXIT

The four dimensional averaged action for the RS field, as we saw in the previous section, gives us the Brans-Dicke action with a specific form for $\omega(\phi)$ given in (II.20). In this section, we will write down the equations of motion and obtain a solution to these equations of motion. As mentioned in the last section, the solution we wish to seek is the one which, in the static limit, reduces to the Randall-Sundrum solution. While preserving the solution to the hierarchy problem on the visible brane, this still leaves open the possibility of interesting phenomena in early cosmology. Before we venture to obtain the solution to the equations of motion, we would like to point out that, as in the Brans-Dicke theory, the effective Newton's constant in this case equals the inverse of the Brans-Dicke scalar field ϕ . Therefore, using our definition of ϕ as in (II.15), we can write

$$\phi = \frac{1}{G^{(-)}} = \frac{1}{\alpha} \frac{(1 - \Omega_b^2)}{\Omega_b^2}. \quad (\text{III.1})$$

Viewing gravity on the branes as a four-dimensional theory, the corresponding 'Planck masses' are then given by

$$M_{pl}^{(-)2} = \frac{1}{2\kappa^2 m_0} \frac{(1 - \Omega_b^2)}{\Omega_b^2} = e^{m_0 b} M_{pl}^{(+2)}. \quad (\text{III.2})$$

The relation between $M_{pl}^{(\pm)}$ is the same as obtained by RS and serves to explain the hierarchy between the energy scales. To have the right size for this ratio, one needs have $m_0 b_0 \sim 70$ where b_0 is the stabilised value of the radion b . In other words, for the stabilised situation, Ω_{b_0} is a very small number. We will ignore the matter terms on the negative tension brane. However, we will keep the bulk matter fields in our analysis. This assumption is justified because time evolution of the bulk metric is expected to be much slower than that on the brane which is undergoing exponential inflation due to warp factor. Therefore, the brane matter is getting inflated away while the bulk matter, which we will take to be independent of brane spatial coordinates, is not.

The equations of motion, as obtained from the action in (II.16), are [22]

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{\phi} + \frac{\omega}{\phi^2} (\phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi_{,\alpha} \phi^{,\alpha}) + \frac{1}{\phi} [\phi_{,\mu;\nu} - g_{\mu\nu} \square\phi] - g_{\mu\nu} \frac{V(\phi)}{2\phi}, \quad (\text{III.3})$$

$$(2\omega + 3)\square\phi = T + \left[\phi \frac{dV(\phi)}{d\phi} - 2V(\phi) \right], \quad (\text{III.4})$$

where T is the trace of energy momentum tensor $T_{\mu\nu}$ of the matter fields on the brane. Neglecting the matter fields on the brane, the independent set of equations of motion are

$$\begin{aligned} 3H^2 &= \frac{\omega(\phi)}{2} \frac{\phi'^2(\tau)}{\phi^2(\tau)} - 3H \frac{\phi'(\tau)}{\phi(\tau)} + \frac{V(\phi)}{2\phi} \\ 2H' + 3H^2 &= -\frac{\omega(\phi)}{2} \frac{\phi'^2(\tau)}{\phi^2(\tau)} - \frac{\phi''(\tau)}{\phi(\tau)} - 2H \frac{\phi'(\tau)}{\phi(\tau)} + \frac{V(\phi)}{2\phi} \\ \phi''(\tau) + 3H\phi'(\tau) &= -\frac{1}{2\omega(\phi) + 3} \left[\phi'^2(\tau) \frac{d\omega(\phi)}{d\phi} + \phi \frac{dV(\phi)}{d\phi} - 2V(\phi) \right] \end{aligned} \quad (\text{III.5})$$

where $H = \frac{Y'(\tau)}{Y(\tau)}$. It can be easily seen that only two of these three equations are independent. For example, if we choose to work with the first two, then it can be shown that the wave equation for the BD scalar field (III.4) is satisfied identically.

Let us now perform the transformation

$$\tilde{g}_{\mu\nu} = \phi g_{\mu\nu}, \quad \tilde{Y}^2 = \phi Y^2, \quad d\tilde{\tau}^2 = \phi d\tau^2, \quad (\text{III.6})$$

followed by a field redefinition

$$(d\psi)^2 = \frac{2\omega(\phi) + 3}{\phi} d\phi^2 \quad (\text{III.7})$$

In other words,

$$\psi = \pm \frac{\sqrt{3}}{2} \ln \left(\frac{\sqrt{1 + \alpha\phi} + 1}{\sqrt{1 + \alpha\phi} - 1} \right), \quad (\text{III.8})$$

and for the sake of convenience, we shall work with the positive branch. Also, from now on, prime denotes differentiation w.r.t. $\tilde{\tau}$.

The above serve to simplify the equations of motion considerably rendering them identical to those for a scalar coupled minimally to gravity:

$$3\tilde{H}^2(\tilde{\tau}) = \psi'^2(\tilde{\tau}) + V(\psi), \quad (\text{III.9})$$

$$2\tilde{H}'(\tilde{\tau}) + 3\tilde{H}^2(\tilde{\tau}) = -\psi'^2(\tilde{\tau}) + V(\psi), \quad (\text{III.10})$$

$$\psi''(\tilde{\tau}) + 3\tilde{H}(\tilde{\tau})\psi'(\tilde{\tau}) = -\frac{1}{2} \frac{dV}{d\psi}. \quad (\text{III.11})$$

The above set of equations involve three unknowns, viz., $\psi(\tilde{\tau})$, $\tilde{H}(\tilde{\tau})$ and $V(\psi)$. However, only two of the equations are independent. The system is thus underdetermined[31], and consequently we are forced to fix one of them by making a suitable Ansatz. A convenient way of parametrizing the same is to posit that

$$\frac{d\psi}{d\tilde{\tau}} = K(\psi) \quad (\text{III.12})$$

where $K(\psi)$ is an as yet undetermined function of ψ . $K(\psi)$ can also be an explicit function of $\tilde{\tau}$. However, if ψ is a monotonic function of $\tilde{\tau}$, which is a sensible assumption at least for the inflationary era, then explicit $\tilde{\tau}$ dependence of $K(\psi)$ can be reexpressed in terms of its dependence on ψ . As we will see below this is precisely what we observe. Even before we choose a specific form for $K(\psi)$, we may further simplify our equations of motion. Subtracting (III.9) from (III.10) and using (III.12) gives

$$\frac{d\tilde{H}}{d\psi} = -K(\psi), \quad V(\psi) = 3\tilde{H}^2(\psi) - K^2(\psi) \quad (\text{III.13})$$

We are now in a position to specify an Ansatz for $K(\psi)$. Let us start with the simplest of relations, namely a monomial form

$$K(\psi) = \beta\psi^\gamma, \quad (\text{III.14})$$

where β and γ are constants. This immediately leads to

$$\tilde{H}(\psi) = \begin{cases} -\beta \left[\frac{\psi^{\gamma+1}}{\gamma+1} + H_0 \right] & \gamma \neq -1 \\ -\beta [\ln \psi + H_0] & \gamma = -1 \end{cases} \quad (\text{III.15})$$

and

$$\psi = \begin{cases} \psi_0 e^{\beta(\tilde{\tau} - \tilde{\tau}_0)} & \gamma = 1 \\ \left[\beta(1 - \gamma)(\tilde{\tau} - \tilde{\tau}_0) + \psi_0^{1-\gamma} \right]^{1/(1-\gamma)} & \gamma \neq 1 \end{cases} \quad (\text{III.16})$$

where H_0 and ψ_0 are constants of integration. It follows then that, for $\gamma \neq -1$,

$$V(\psi) = \beta^2 \left\{ 3 \left[\frac{\psi^{\gamma+1}}{\gamma+1} + H_0 \right]^2 - \psi^{2\gamma} \right\}. \quad (\text{III.17})$$

A potential bounded from below is achieved only for $\gamma > 0$ and this is a condition that we need to impose. Such a potential has extrema at the origin as well as at ψ_m , which is a solution of

$$\frac{\psi_m^{\gamma+1}}{\gamma+1} - \frac{\gamma}{3} \psi_m^{\gamma-1} + H_0 = 0. \quad (\text{III.18})$$

As already pointed out, a resolution of the hierarchy problem needs $\Omega_b \simeq o(10^{-16})$, or, in other words, $m_0 b_0 \simeq 70$ at the present epoch. Since $\psi_m \sim \sqrt{3}\Omega_b$, it immediately follows that we can neglect the first term in the above equation, and

$$H_0 \simeq \frac{\gamma}{3} \psi_m^{\gamma-1}. \quad (\text{III.19})$$

The magnitude and sign of the vacuum energy at the minimum in question depends crucially on H_0 and γ . As observations support a small positive value for the cosmological constant, this may be used to eliminate large regions of

leads to a very large and negative value for H_0 (as long as we want a small ψ_m) and is therefore severely disfavoured. For $\gamma = 0$ (when the potential is quadratic and hence seems to resemble the chaotic inflation scenario), we again get a negative, albeit small, value for H_0 . For $0 < \gamma < 1$, on the other hand, we have a reversal of the earlier situation. Now H_0 to be positive but very large positive which, again, is not a desirable feature. The point $\gamma = 1$ is special as this is associated with $H_0 \sim 1/3$ which implies small positive vacuum energy. And finally, for $\gamma > 1$ it is easy to see that the extremum of the potential at a small ψ_m is actually a maximum and hence does not represent a real vacuum. Thus, phenomenological considerations restrict us to the case of $\gamma = 1$, and we now examine it in greater detail.

Substituting $\gamma = 1$ in eqs.(III.15–III.17), we have

$$\begin{aligned}\psi(\tilde{\tau}) &= \psi_0 e^{\beta(\tilde{\tau}-\tilde{\tau}_0)}, \\ H(\psi) &= -\beta[\psi^2/2 + H_0], \\ V(\psi) &= \beta^2[3H_0^2 + (3H_0 - 1)\psi^2 + \frac{3}{4}\psi^4].\end{aligned}\tag{III.20}$$

Also from (III.19) one gets $H_0 \approx 1/3$.

While we may continue to work with the field ψ and eqs (III.9)-(III.11), the calculation of the different parameters characterizing inflation and the comparison with the observational data is better achieved if we recast the physics in terms of a (conventional) dimensionful field χ . We define,

$$\chi = \sqrt{2}M\psi\tag{III.21}$$

where $M = M_{pl}/\sqrt{8\pi} \sim 0.2M_{pl}$ is the reduced Planck mass. At the minimum, then, $\chi_m \sim 5.3 \times 10^2$ GeV. In terms of χ , we may rewrite the evolution equations as

$$\begin{aligned}3H^2 &= \frac{1}{M^2} \left[\frac{\chi'^2}{2} + V(\chi) \right], \\ 2H' + 3H^2 &= \frac{1}{M^2} \left[-\frac{\chi'^2}{2} + V(\chi) \right], \\ \chi'' + 3H\chi' &= -\frac{dV}{d\chi}.\end{aligned}\tag{III.22}$$

In the above, the potential has been redefined to be

$$V(\chi) \equiv M^2V(\psi).\tag{III.23}$$

Suppressing, for the moment, the quadratic terms, the potential now reads

$$V(\chi) \approx 3M^2\beta^2 \left[H_0^2 + \frac{1}{16} \frac{\chi^4}{M^4} \right].\tag{III.24}$$

Thus, only one unknown parameter, viz. β , remains in the potential and this we will fix by comparing our results with observations.

Apart from the Hubble parameter, cosmological expansion is also characterized by the deceleration parameter q :

$$q = - \left[\frac{H'}{H^2} + 1 \right].$$

For an inflating universe, $q < 0$ and inflation stops at $q = 0$. One could also investigate inflation in terms of the slow roll parameters which are defined as

$$\epsilon = \frac{M^2}{2V^2} \left(\frac{dV}{d\chi} \right)^2, \quad \eta = \frac{M^2}{V} \frac{d^2V}{d\chi^2}.\tag{III.25}$$

For slow roll inflation to occur, both $|\epsilon| \ll 1$ and $|\eta| \ll 1$ need to be satisfied. If the slow roll conditions were valid, eqns.(III.22) can be approximated to read

$$3H^2 = \frac{1}{M^2}V(\chi), \quad -3H\chi' = \frac{dV}{d\chi}.\tag{III.26}$$

Within the region of validity of the above equations, one then has

$$q \approx \epsilon - 1,\tag{III.27}$$

and hence $q \leq 0$ is nearly equivalent to $\epsilon \leq 1$. Henceforth we will investigate the evolution of the universe in terms of ϵ as it renders easier comparison with different observational parameters.

Now, with the potential of eqn.(III.24), one has

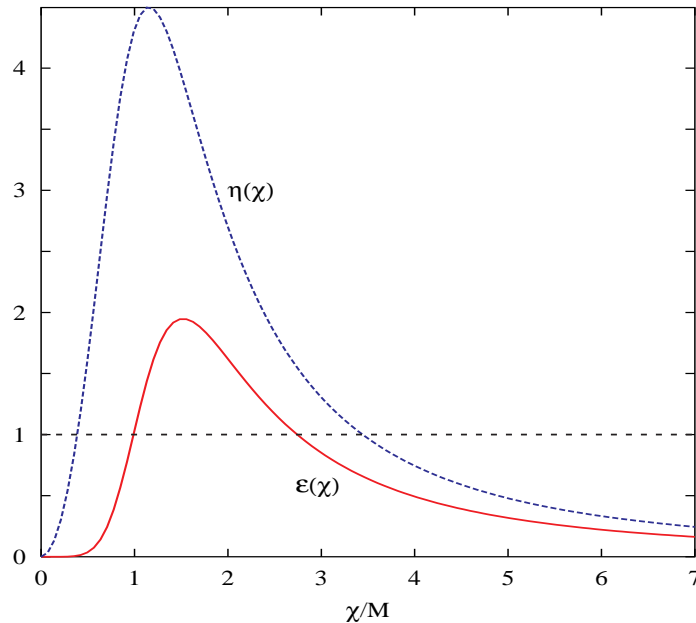


FIG. 1: The slow roll parameters η (dashed line) and ϵ (solid line) as functions of the field χ . The constant H_0 has been set to $1/3$.

Clearly, slow roll is possible for both $\chi \ll M$ and $\chi \gg M$. In the former case, the field χ has to increase from a value $< 5.3 \times 10^2 \text{ GeV}$ and β needs to be positive. But from the expression of H in eqn.(III.20), one concludes that, in such a case the universe is actually collapsing. In other words, the scalar field in this model can not increase from a small value and yet be consistent with observations.

What if χ is actually decreasing from a large value? For this to occur, $\beta < 0$ and eqn.(III.20) immediately implies that the universe is always expanding. In Fig.1, we present the deceleration parameter as a function of the field χ . For a large initial value χ_i , the universe starts with an inflationary period which ends around $\chi_f \sim 3.44M = 0.69M_{pl}$. The total amount of inflation is, of course, decided by the initial value χ_i and thus can be used to determine the latter. This quantity is usually expressed as the number of e-foldings N_e of inflation and can be quantified by

$$N_e = -\frac{1}{M^2} \int_{\chi_i}^{\chi_f} d\chi V(\chi) \left(\frac{dV}{d\chi} \right)^{-1} \simeq \frac{\chi_i^2 - \chi_f^2}{8M^2} \quad (\text{III.29})$$

The minimal value of N_e , required for a satisfactory solution of the horizon and flatness problems, depends crucially on the energy scale of the inflationary expansion. Assuming that our inflation is a GUT scale one (an assumption we shall justify later), we must have had at least 60 e-foldings of inflation. Using this minimal criterion (and the already determined value $\chi_f \sim 3.44M$), one obtains $\chi_i \sim 4.42M_{pl}$. This superPlanckian starting point is quite reminiscent of the chaotic inflationary scenario.

Does our model lead to the right density perturbation spectrum? To answer this question, we need to consider two quantities, namely the spectral tilt n_s

$$n_s = 1 - 6\epsilon + 2\eta \quad (\text{III.30})$$

which describes the scale dependence of the perturbation and the ratio r of the amplitude of tensor and scalar perturbations

$$r \simeq 4\pi A_t^2 / A_s^2. \quad (\text{III.31})$$

Both these quantities are to be evaluated at the instant when the comoving scale equals the Hubble radius ($k = aH$). For a GUT scale inflation with prompt reheating, this happens at around 55 e-foldings. Within our model, these quantities turn out to be

$$n_s \approx 0.95, \quad \text{and} \quad r \approx 0.12 \quad (\text{III.32})$$

and are very much consistent with the observational bounds inferred by the BOOMERANG [26], MAXIMA [27], and DASI [28] collaborations, namely

$$0.8 < n_s < 1.05, \quad r < 0.3. \quad (\text{III.33})$$

. Note also that the spectral tilt is very similar to that for the Harrison-Zeldovich spectrum ($n_s = 1$).

Before we end our discussion of the phenomenological aspects of the model, we must comment on the as yet undetermined parameter β . It can be related to the amplitude of scalar perturbation:

$$\beta \simeq \frac{512\pi V^3}{(dV/d\chi)^2}$$

Observational data estimates $A_s \sim 2 \times 10^{-5}$ leading to a small value of $\beta \sim -9.32 \times 10^{-8} M_{pl}$. Using this value of β we have $V(\chi_f)^{1/4} \simeq 10^{15}$ GeV, where $V(\chi_f)$ is the vacuum energy at the end of the inflation. This justifies our implicit assumption of GUT scale inflation.

Could there be a second phase of inflation? A naive reading of Fig. 1 would certainly seem to suggest that inflation could restart once $(\chi/M) \lesssim 1$. But, in reality, this is not the case. Once the original phase of inflation ends, matter creation starts in right earnest. The presence of the matter terms in the r.h.s. of eqns.(III.5) alters the dynamics radically thereby negating the possibility of a late inflation.

It is interesting to consider the dynamics of the field $b(\tilde{\tau})$. Using eqns.(III.1), (III.8) and (III.16), we have, for $\gamma = 1$,

$$m_0 b(\tilde{\tau}) = -2 \ln \tanh \left[\frac{2\sqrt{\pi}\chi(\tilde{\tau})}{\sqrt{3}M_{pl}} \right] = -2 \ln \tanh \left[\frac{2\sqrt{\pi}\chi_i e^{\beta\tilde{\tau}}}{\sqrt{3}M_{pl}} \right] \quad (\text{III.35})$$

where we denote the field at the initial instant of time by χ_i . Note that while the first equality in eqn.(III.35) is but a definition (and hence valid for all times and all γ 's), the second equality is valid only when our approximation of neglecting the matter terms on the brane is a good one. While this approximation is an excellent one during the inflationary era, it certainly is not so when matter creation and reheating effects become important. Thus, although the ultimate settling down of the warp-factor to its current value is ensured, the time scale for the process cannot be determined from this analysis alone. It is also interesting to note that the warp factor remains close to unity (and hence gravity on our brane is not suppressed in the least) during almost the entire inflationary era. Examining this issue quantitatively, although $b(t)$ changes by nearly 7 orders of magnitude during the inflationary era (from $m_0 b(\chi_i) \simeq 5.5 \times 10^{-8}$ to $m_0 b(\chi_f) \simeq 0.24$), this change is approximately only linear (vide eqn.III.35) in time[32]. In other words, the expansion of the fifth dimension during inflation is far less than that of the observable brane. Consequently, the matter energy density on the brane, which, to begin with, was already much smaller than that in the bulk, is inflated away to a far greater degree than the latter. The asymmetric inflationary expansion of the higher dimensional world, thus, provides a further justification for ignoring the matter terms in the brane but not those in the bulk. Furthermore, the relatively slow variation of the radion field $b(t)$ also explains the nearly scale-invariant nature of the primordial density perturbation which, in our model, arises primarily due to the fluctuations of the radion field.

We now return to another important issue of our scenario. The result $|\omega| \lesssim 3/2$ seems to run counter to the findings of the Solar system experiments [23]. However, it needs to be realized that this model exits the Brans-Dicke phase at early times itself. With ψ , and hence ϕ , assuming a constant value very quickly with time, it essentially decouples from the equations of motion (see III.3). And also as our BD field is massive, it no longer mediates any long range force—at least classically—and hence plays no role in observables such as those considered in Ref. [23] where one considered the massless BD field.[33]. It might be argued though that this line of reasoning depends crucially on our having ignored the matter energy density and pressure. Once the variation of the BD scalar field ϕ becomes negligible with time and particle creation processes take over, such terms may well become dominant. Of course, the actual process of matter creation, which takes place primarily through the processes of reheating and cooling, depends on the details of the radion potential as well as the radion couplings to the SM particles. The latter, however, become relevant only at very late times, and even then are not of a sufficiently large magnitude to significantly alter the aforementioned results.

IV. SUMMARY

To conclude, we exhibit that a five-dimensional world with a warped geometry (a time-dependent analog of the Randall-Sundrum metric ansatz), supports an inflationary evolution on the brane with the desirable property of a graceful exit mechanism. The key ingredient is the radion field whose coupling to the gravity on the negative tension brane is as that of a scalar field in scalar tensor gravity. By a field transformation, we have recast the action in a Brans-Dicke-like form. The advantage of our approach over canonical models of inflation is that we do not need to introduce any extra scalar (the inflaton). Rather, the radion itself drives inflation. Even more interestingly, the radion evolution exits the inflationary phase in a completely natural way, thus providing for a graceful exit. We have also made a detailed analysis of the cosmological implications of our inflationary model by investigating the density perturbations generated by the radion. For this we have assumed a GUT scale inflation which has also been justified by comparing the amplitude of the scalar perturbation in our model with the COBE normalized value. Two concrete predictions of our model are the values of the spectral index n_s and the ratio of the tensor and scalar perturbations r which are both well within the range predicted by current CMB observations. As a number of new missions (MAP and PLANCK) of CMB observations is under way, one can get a more constrained range for n_s and r . Since $\gamma = 1$ is the only case of successful inflation in this model, brane inflation based on RS type model will be tested more accurately by future observations.

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- [31] Of course, if the stabilising potential were specified, so would $V(\psi)$ be. And then the system is no longer underdetermined.
- [32] Subsequent evolution increases $b(t)$ by an additional factor of nearly 300 to reach $m_0 b_0 \simeq 70$.
- [33] For other interesting cosmological solutions to the Brans-Dicke gravity with this behaviour of the scalar and the scale factor and with the late time acceleration in the presence of a potential for the BD scalar, see ref.[25].