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Fermionic un-particles, gauge interactions and the β function

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Abstract

The dynamics of fermionic unparticles is developed from first principles. It is shown that any unparticle, whether fermionic or bosonic, can be recast in terms of a canonically quantized field, but with non-local interaction terms. We further develop a possible gauge theory for fermionic unparticles. Computing the consequent contribution of un-fermions to the β function of the theory, it is shown that this can be viewed as the sum of two contributions, one fermion-like and the other scalar-like. However, if full conformal invariance is imposed, the latter vanishes identically. We discuss the consequences thereof as well as some general phenomenological issues.

1 Introduction

The idea of elementary fields with non-integer scaling dimension as proposed in [1] and ‘deconstructed’ in [2] has received considerable attention. Such fields can, perhaps, be best motivated in a nontrivial scale invariant theory with an infrared fixed point (examples being afforded by a vector-like non-abelian gauge theory with a large number of massless fermions as studied by Banks and Zaks (BZ) [3], or certain nonlinear sigma models [4]). Manifesting itself in the existence of asymptotic states that are not particle-like (in the sense of not having a well-defined mass, but rather a continuous mass spectrum), such a field has been termed an “unparticle” [1]. In an interacting theory, the aforementioned non-integer scaling dimension (though it has been shown in [2] that the unparticle can be looked upon as an infinite ladder of ordinary particles in the limit of a vanishing energy gap) leads to curious effects in its propagation as well as its kinematics. Such effects are illustrated if the unparticle is allowed to couple to standard model (SM) currents through an effective interaction and this has given rise to a whole body of work that have looked at various phenomenological implications of the existence of such particles.

Direct phenomenological issues aside, unparticles are unusual, to say the least [5]. As we shall demonstrate below, the unparticle field can be redefined to yield a canonical massless field (thus preserving the signature scale invariance of this sector). The price to be paid is the transmutation of any (local) interactions with normal particles into non-local ones. In other words, unparticles provide us with a concrete realisation of the structure of non-local field theories. Thus, even as a pure field theoretical problem, it is interesting to study the behavior of unparticles in their interactions with other matter and gauge particle.

Even though phenomenological consequences have been discussed at length, some of the more basic theoretical issues have not found too much space in the literature. In particular, changes in the dynamics of a theory (e.g. the Standard Model) due to the presence of unparticles has not quite been carefully delineated. Since this can have significant effects on the running of the coupling constants, and hence to the overall behavior of the theory at different length scales, it is important to understand the nature of the β function in a theory with unparticles.

There has been some discussion in the literature on the effect of scalar unparticles on a gauge theory [6]. There has also been some discussion regarding fermionic unparticles [7]. However, the propagator proposed in the latter set of papers do not go over to the usual fermionic propagator when the dimension of the field is taken to 3/2. As a result, the conclusions of the papers are somewhat suspect.

In order to clarify all these issues we start from a Lagrangian of fermionic unparticles, gauge it with the usual prescription of introducing a Wilson line and proceed to calculate the β function contribution coming from these unparticles. For this we use the result of [8] which treats the general case of local and non local Lagrangians.

In a recent paper, Grinstein et al. [9] have pointed out certain important issues regarding the compatibility of the propagators for unparticles used in the literature with conformal field theory (CFT). We use in this paper, a propagator invariant under scale transformations, Lorentz transformations, and translations but not under special conformal transformations. We thereafter comment on the effect that special conformal transformations have on the result. In order to clarify these somewhat subtle matters which have been insufficiently dealt with in the literature, we explicitly show in Appendix B the transformation of the fermionic 2-pt function under the conformal group. The form of the 2 point function for fermions is different depending on whether we demand invariance under the full conformal group or not. It turns out that the β function obtained gets contribution from a spinor and a scalar part which would lead to a different evolution of the coupling from that in a fully conformally invariant theory where the β function gets contribution only from the spinor part as for a normal fermion. This is essentially because in a fully conformally invariant theory we are forced to choose a fermion propagator to be the same as a ‘normal’ fermion.

In Section II we demonstrate the derivation of the fermionic unparticle propagator. In Section

III we show that a redefinition of the fermion fields allows us to rewrite the Lagrangian in terms of a local and non local part and thereby calculate the one loop fermionic corrections to the scalar propagator. Thereafter in Section IV we repeat this case for the corrections to the gauge field propagator due to un-fermionic loops. In this case we discuss the issue of gauge invariance and provide a prescription for gauging the theory. In Section V, we calculate the full one loop two point function for the non abelian gauge field due to un-fermion and thence the correction to the β function of the theory due to un-fermionic modes. The consequences to phenomenology are discussed in a subsequent Section VI.

Finally in the Appendices we tie up various loose ends left in the paper. In particular we show that rescaling of the fields as done for fermions could equally well have been done for scalars and we use conformal field theory considerations to derive the form of the fermionic propagator.

2 Fermionic unparticles

Consider an unparticle fermion field $\Psi_{\mathcal{U}}(x)$ of dimension d . The two-point correlator

$$\mathcal{T} \equiv \int d^4x e^{iP \cdot x} \langle 0 | T[\Psi_{\mathcal{U}}(0) \bar{\Psi}_{\mathcal{U}}(x)] | 0 \rangle \quad (2.1)$$

should then be describable as a coherent superposition of a continuum of single particle propagators with an appropriate density of states. In other words,

$$\begin{aligned} \mathcal{T} &= \int_0^\infty \frac{dM^2}{2\pi} \rho(M^2) \frac{\not{P} + M}{P^2 - M^2 + i\epsilon} \\ \rho(M^2) &= \frac{3}{2} A_{2d/3} (M^2)^{d-5/2} \\ A_d &\equiv \frac{16\pi^{5/2}}{(2\pi)^{2d}} \frac{\Gamma(d+1/2)}{\Gamma(d-1)\Gamma(2d)} \end{aligned} \quad (2.2)$$

where the exponent $(d - \frac{5}{2})$ is determined purely from dimensional arguments and A_d , as before, is the phase space for the emission of d massless particles¹. Note that $\rho(M^2)$ should have $A_{2d/3}$ (and not A_d as in Ref.[7]) since the emission of a (composite) fermion of mass-dimension d is equivalent to emission of $2d/3$ canonical fermions (which have mass dimension $3/2$). This has crucial implications as indicated below. On performing the integral explicitly, one obtains

$$\mathcal{T} = \frac{-3}{4 \cos(\pi d)} A_{2d/3} (-P^2 - i\epsilon)^{d-5/2} \left[\not{P} - \cot(\pi d) \sqrt{-P^2 - i\epsilon} \right]. \quad (2.3)$$

This, then, defines the propagator for a fermionic unparticle. Clearly, in the limit $d \rightarrow \frac{3}{2}^+$, the propagator reduces to that for a canonical massless fermion, as it rightly should. This is in marked contrast to the propagator as obtained in Ref.[7]. Furthermore, it should be easy to see that eq.(2.3) satisfies the constraints imposed by scale invariance, namely, that the two point correlator of a field of dimension d should go as $|x_1 - x_2|^{-2d}$ where x_i denote the space-time coordinates.

It is, then, a straightforward exercise to construct an appropriate Lagrangian density for a free fermionic unparticle, namely

$$\mathcal{L}_{\Psi} = \frac{4 \cos(\pi d) \sin^2(\pi d)}{3 A_{2d/3}} \bar{\Psi}_{\mathcal{U}}(x) (\partial_\mu \partial^\mu)^{3/2-d} \left[i\not{\partial} + \cot(\pi d) \sqrt{\partial_\nu \partial^\nu} \right] \Psi_{\mathcal{U}}(x). \quad (2.4)$$

We believe that the above should be used for doing phenomenology with fermionic unparticles rather than that of Ref. [7] as has been done in the literature.

Certain important features should be noted at this stage:

¹In general, the time ordered product for the fermionic case may be defined by two independent spectral functions. Here, for simplicity, we shall choose them to be identical. We return to this point in Appendix B

- The un-fermion, as described above, must have a scaling dimension $d \geq 3/2$.
- The propagator \mathcal{T} , while reducing to the canonical form for $d \rightarrow \frac{3}{2}^+$, is ill-defined for any $d = n + \frac{1}{2}, \forall n > 1, n \in \mathbb{Z}$.
- On the other hand, \mathcal{T} is quite well-defined for $d = \frac{3}{2}$, belying the naive expectation that multi-particle cuts should not be expressible in terms of a single propagator.
- The Lagrangian density of eq.(2.4) has the curious form of being the sum of two Lagrangians: a fermionic one and a scalar one, both unparticle-like. As a corollary, so does \mathcal{T} .

As would be obvious from the discussions in the subsequent sections, the last point above is a very crucial one in the understanding of the fermionic unparticle described by eq.(2.4) and the penultimate point is but a consequence of this.

3 A redefinition of fields

It is instructive to reformulate the fermionic unparticle in terms of a different field. Effecting a field redefinition, viz.

$$\begin{aligned} \Psi_U(x) \rightarrow \chi(x) &\equiv \sqrt{\mathcal{A}} (\partial_\mu \partial^\mu)^{3/4-d/2} \Psi_U(x) \\ \mathcal{A} &\equiv \frac{4 \cos(\pi d) \sin^2(\pi d)}{3 A_{2d/3}} \end{aligned} \quad (3.5)$$

we may rewrite the Lagrangian density of eq.(2.4) as

$$\mathcal{L}_\Psi = \mathcal{L}_\chi = \bar{\chi}(x) \left[i\partial^\mu + \cot(\pi d) \sqrt{\partial_\nu \partial^\nu} \right] \chi(x) . \quad (3.6)$$

Thus, $\chi(x)$ behaves like a canonically quantized field. The presence of the extra term may seem baffling at first, but a closer inspection shows that the operator

$$\Sigma \equiv \cot(\pi d) (\partial_\mu \partial^\mu)^{1/2}$$

is equivalent to a non-local mass term [8] for $\chi(x)$. Indeed, the propagator can be expressed as

$$\begin{aligned} S(p) &= \frac{i}{p + \Sigma_0(p)} = i \sin^2(\pi d) \frac{p - \Sigma_0(p)}{p^2 + i\epsilon} \\ \Sigma_0(p) &\equiv \cot(\pi d) (-p^2)^{1/2} . \end{aligned} \quad (3.7)$$

Since Ψ_U and χ represent quite different entities, the two theories would result in very different predictions for asymptotic unfermion states. On the other hand, if the unfermions are to appear only as virtual states, the two theories clearly would be equivalent, as long as all its interactions are non-derivative in nature. (Note that the above considerations are valid for redefinition of a general field, scalar or spinor. The effects are similar to the above. This has been explicitly shown in Appendix A). To illustrate this, let us consider a Yukawa interaction between an un-fermion with a normal real scalar ϕ , viz.

$$\begin{aligned} \mathcal{L} &= \frac{4 \cos(\pi d) \sin^2(\pi d)}{3 A_{2d/3}} \bar{\Psi}_U(x) (\partial_\mu \partial^\mu)^{3/2-d} \left[i\partial^\mu + \cot(\pi d) \sqrt{\partial_\nu \partial^\nu} \right] \Psi_U(x) \\ &+ y \bar{\Psi}_U(x) \Psi_U(x) \phi(x) + \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{m^2 \phi^2}{2} . \end{aligned} \quad (3.8)$$

The one-loop correction to the ϕ propagator is then given by the diagram of Fig.1(a) and amounts

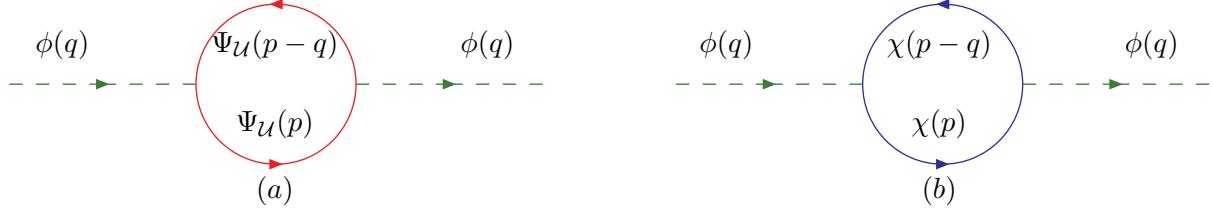


Figure 1: *The one-loop correction to a scalar propagator for a scalar coupling to an un-fermion. Panels (a) and (b) correspond to the Lagrangians of eq.(3.8) and eq.(3.10) respectively.*

to

$$\begin{aligned}
i\Pi(q^2) &= -Tr \int \frac{d^4 p}{(2\pi)^4} (i y)^2 \frac{-3 i}{4 \cos(\pi d)} A_{2d/3} (-p^2)^{d-5/2} \left[\not{p} - \cot(\pi d) \sqrt{-p^2} \right] \\
&\quad \frac{-3 i}{4 \cos(\pi d)} A_{2d/3} [-(p-q)^2]^{d-5/2} \left[(\not{p} - \not{q}) - \cot(\pi d) \sqrt{-(p-q)^2} \right] \quad (3.9) \\
&= \frac{-9 y^2 A_{2d/3}^2}{4 \cos^2(\pi d)} \int \frac{d^4 p}{(2\pi)^4} [p^2 (p-q)^2]^{d-5/2} [p \cdot (p-q) + \Sigma_0(p) \Sigma_0(p-q)] .
\end{aligned}$$

Rewriting the theory in terms of the field $\chi(x)$, we have

$$\begin{aligned}
\mathcal{L} &= \bar{\chi}(x) \left[i\not{\partial} + \cot(\pi d) \sqrt{\partial_\nu \partial^\nu} \right] \chi(x) + \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{m^2 \phi^2}{2} \\
&\quad + \frac{3 A_{2d/3} y}{4 \cos(\pi d) \sin^2(\pi d)} \left[(\partial^2)^{d/2-3/4} \bar{\chi}(x) \right] \left[(\partial^2)^{d/2-3/4} \chi(x) \right] \phi(x) . \quad (3.10)
\end{aligned}$$

The correction to the ϕ propagator is now given by Fig.1(b) and is

$$i\Pi(q^2) = -Tr \int \frac{d^4 p}{(2\pi)^4} \left[\frac{3 A_{2d/3} y}{4 \cos(\pi d) \sin^2(\pi d)} \right]^2 \left[(-p^2)^{d/2-3/4} \{-(p-q)^2\}^{d/2-3/4} \right]^2 \\
[\not{p} + \Sigma(p)]^{-1} [(\not{p} - \not{q}) + \Sigma(p-q)]^{-1} .$$

This is easily seen to be the same as eq.(3.9), thus vindicating our assertion that as long as the unfermion is to be treated only as virtual states, the Ψ_U and the χ theories would yield identical results.

Before we end this section, note that the scaling dimension d appears only as a (periodic) parameter in the χ theory. For $d \rightarrow \frac{3}{2}^+$, it once again reduces to a canonical fermion as it must, while for integral values of d it behaves purely as an un-scalar.

4 Gauging the un-fermion

Although most of the unparticle literature assumes unparticles to be gauge singlets, it is interesting to consider the possibility that these have gauge interactions [11, 6]. While it is quite conceivable that the unparticles may transform non-trivially under a gauge group orthogonal to the SM, nothing, in principle, forbids them from having nonzero SM quantum numbers too.

The act of gauging the unfermion Lagrangian raises an important issue, namely whether to gauge the Ψ_U theory or the χ theory. A “minimal” coupling for Ψ_U is not the same as that for χ and vice-versa. As is quite apparent, our previous assertion (of the two theories being equivalent when restricted to only virtual unfermions) does not hold any longer. This, though, is not unexpected and is related to the derivative coupling typical of gauge interactions. A similar situation would be faced for ordinary (SM) particles as well, were it not for the fact that experiments point us to the right choice.

In the absence of any phenomenological input, there is no *a priori* reason to prefer one theory over the other, and we must make a choice as to the “right” field to gauge. In this paper, we are guided by simplicity and choose to couple the field $\chi(x)$ to a nonabelian gauge field staying as close to minimal substitution as possible. Indeed, the formalism developed by Terning [8] can be readily used in this context. As is expected, the presence of the $\sqrt{\partial_\nu \partial^\nu}$ term in the Lagrangian leads to an infinite number of interaction terms of the generic form $A^n \bar{\chi} \chi$. For our purpose, it suffices to consider only the three-point and four-point vertices. Defining

$$\begin{aligned}\Sigma_1(p; q) &\equiv \frac{\Sigma_0(p+q) - \Sigma_0(p)}{(p+q)^2 - p^2}, \\ \Sigma_2(p; q_1, q_2) &\equiv \frac{\Sigma_1(p; q_1 + q_2) - \Sigma_1(p; q_1)}{(p+q_1+q_2)^2 - (p+q_1)^2},\end{aligned}\tag{4.11}$$

the three point function is given by [8]

$$\Gamma^\mu(p, q, p+q) = i g T_a [\gamma_\mu + (2p+q)_\mu \Sigma_1(p; q)]\tag{4.12}$$

where p ($p+q$) is the momentum of the incoming (outgoing) unfermion field χ and q is the momentum of the incoming gluon. Similarly, the 4-point function is seen to be

$$\begin{aligned}\Gamma^{\mu\nu}(p, q_1, q_2, p+q_1+q_2) &= i g^2 \left[\{T_a, T_b\} g^{\mu\nu} \Sigma_1(p; q_1 + q_2) \right. \\ &\quad + T_a T_b (2p+q_2)^\nu [2(p+q_2) + q_1]^\mu \Sigma_2(p; q_2, q_1) \\ &\quad \left. + T_b T_a (2p+q_1)^\mu [2(p+q_1) + q_2]^\nu \Sigma_2(p; q_1, q_2) \right]\end{aligned}\tag{4.13}$$

where p ($p+q_1+q_2$) is the momentum of the incoming (outgoing) fermion field χ and $q_{1\mu}$ and $q_{2\nu}$ are the momenta of the incoming gluons (with color indices a and b respectively). The Lorentz structure of the extra piece in Γ^μ and the first term in $\Gamma^{\mu\nu}$ are both reflective of a scalar-gauge field interaction. This is not unexpected as both these terms owe their origin to the scalar-like $\sqrt{\partial_\nu \partial^\nu}$ term in the Lagrangian. Indeed, the non-local “mass” term can be thought of as a scalar inextricably intertwined with the fermion. Consequently, $\Sigma_0(p)$ contributes to the propagator, while the first differential $\Sigma_1(p; q)$ is the coefficient of the scalar-gluon coupling resultant from the first fermion-scalar-fermion intermingling. The higher differentials (Σ_2 onwards) can be thought of similarly.

Before we end this section, note that

$$q_\mu \Gamma^\mu(p, q, p+q) = i g T_a [S^{-1}(p+q) - S^{-1}(p)]\tag{4.14}$$

thus satisfying the Ward identity.

5 The β function

With the un-fermion having gauge interactions, the renormalization group evolution of the gauge coupling constant g would receive contributions from unfermion loops. While each of vacuum polarization, un-fermion self-energy, unfermion gauge vertices (single gluon as well as multiple gluons) as well as the gauge boson self-interactions receive corrections, the generalized Ward identities ensure that, as far as the β -function is concerned, one needs to consider only the additional contribution to the vacuum polarization. The corresponding diagrams, to one-loop order, are displayed in Fig.2.

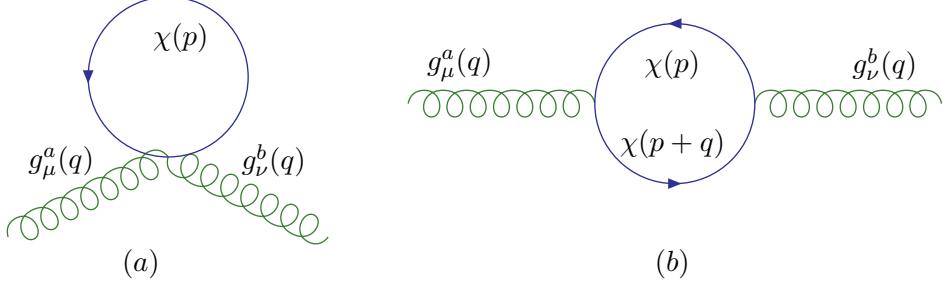


Figure 2: The one-loop contributions to the vacuum polarization from un-fermions.

The first diagram (involving the 4-pt vertex) gives

$$\begin{aligned}
i \Pi_{\mu\nu}^{ab}(q; 4pt) &= -Tr \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p + \Sigma_0(p)} i \Gamma_{\mu\nu}(p, q, -q, p) \\
&= g^2 Tr \int \frac{d^4 p}{(2\pi)^4} \frac{p - \Sigma_0(p)}{p^2 - \Sigma_0^2(p)} \\
&\quad \left\{ \{T_a, T_b\} g_{\mu\nu} \Sigma_1(p; 0) + T_a T_b (2p - q)_\nu (2p - q)_\mu \Sigma_2(p; -q, q) \right. \\
&\quad \left. + T_b T_a (2p + q)_\mu (2p + q)_\nu \Sigma_2(p; q, -q) \right\}. \tag{5.15}
\end{aligned}$$

Noting that $\Pi_{\mu\nu}^{ab}$ is invariant under $q \leftrightarrow -q$, the second piece in the integral is identical to the third. Thus,

$$\begin{aligned}
i \Pi_{\mu\nu}^{ab}(q; 4pt) &= -2g^2 tr(T_a T_b) \sin^2(\pi d) Tr(1) \\
&\quad \int \frac{d^4 p}{(2\pi)^4} \frac{\Sigma_0(p)}{p^2} \left\{ g_{\mu\nu} \Sigma_1(p; 0) + (2p + q)_\mu (2p + q)_\nu \Sigma_2(p; q, -q) \right\}
\end{aligned}$$

where $\Sigma_1(p; 0) = \Sigma_0(p)/(2p^2)$ and Tr represents the Dirac algebra trace. Some straightforward algebra results in

$$\begin{aligned}
i \Pi_{\mu\nu}^{ab}(q; 4pt) &= -2g^2 tr(T_a T_b) \sin^2(\pi d) Tr(1) \\
&\quad \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \left\{ \frac{\cot^2(\pi d)}{2} \left[\frac{(2p + q)_\mu (2p + q)_\nu}{2p \cdot q + q^2} - g_{\mu\nu} \right] \right. \\
&\quad \left. + \Sigma_0(p) \Sigma_1(p; q) \frac{(2p + q)_\mu (2p + q)_\nu}{2p \cdot q + q^2} \right\}.
\end{aligned}$$

The first piece $[\dots]$ gives a contribution identical to the *total* contribution from a canonically quantized scalar in the same representation [6], and we finally have

$$\begin{aligned}
i \Pi_{\mu\nu}^{ab}(q; 4pt) &= -\frac{Tr(1)}{2} \cos^2(\pi d) \left[i \Pi_{\mu\nu}^{ab} \right]_{\text{normal scalar}} \\
&\quad - g^2 tr(T_a T_b) \sin^2(\pi d) Tr(1) \int \frac{d^4 p}{(2\pi)^4} \frac{C_{\mu\nu}^{(4)}}{p^2 (p + q)^2}, \tag{5.16} \\
C_{\mu\nu}^{(4)} &\equiv 2 \Sigma_0(p) \Sigma_1(p; q) (p + q)^2 \frac{(2p + q)_\mu (2p + q)_\nu}{2p \cdot q + q^2}.
\end{aligned}$$

Note that the entire contribution is proportional to $\cos^2(\pi d)$ and vanishes identically for $d \rightarrow \frac{3}{2}^+$ as it should (for then the unfermion goes over to a canonical fermion). The extra proportionality constant $[-Tr(1)/2]$ ($= 2$ in 4 dimensions) is reflective of the fact that this contribution emanates from a fermion (and hence the negative sign) which obviously has twice the degrees of freedom compared to a (canonically quantized) scalar in the same representation.

The contribution from the second diagram in Fig.2 (involving the 3-pt vertices) is

$$i \Pi_{\mu\nu}^{ab}(q; 3pt) = -tr(T_a T_b) Tr \int \frac{d^4 p}{(2\pi)^4} \Gamma_\mu(p, q, p+q) S(p) \Gamma_\nu(p+q, -q, p) S(p+q) . \quad (5.17)$$

Using $\Sigma_1(p+q; -q) = \Sigma_1(p; q)$, this can be reduced to

$$\begin{aligned} i \Pi_{\mu\nu}^{ab}(q; 3pt) &= -g^2 tr(T_a T_b) \sin^4(\pi d) \int \frac{d^4 p}{(2\pi)^4} [p^2 (p+q)^2]^{-1} \\ &\quad \left\{ Tr[\gamma_\mu \not{p} \gamma_\nu (\not{p} + \not{q})] + Tr(1) C_{\mu\nu}^{(3)} \right\}, \\ C_{\mu\nu}^{(3)} &\equiv (2p+q)_\mu (2p+q)_\nu \Sigma_1^2(p; q) [\Sigma_0(p) \Sigma_0(p+q) + p \cdot (p+q)] \\ &\quad - [p_\mu (2p+q)_\nu + (2p+q)_\mu p_\nu] \Sigma_1(p; q) \Sigma_0(p+q) \\ &\quad - \Sigma_1(p; q) \Sigma_0(p) [(2p+q)_\mu (p+q)_\nu + (2p+q)_\nu (p+q)_\mu] \\ &\quad + \Sigma_0(p) \Sigma_0(p+q) g_{\mu\nu} . \end{aligned} \quad (5.18)$$

The first term in the integral, apart from the $\sin^4(\pi d)$ factor, is readily seen to be identical to the contribution of a canonical fermion to the vacuum polarization[6]. Shifting $p \rightarrow -(p+q)$, the second line in $C_{\mu\nu}^{(3)}$ can be seen to be equivalent to the third line. Next, concentrating on the second term of $C_{\mu\nu}^{(3)}$, we see

$$\begin{aligned} &\int \frac{d^4 p}{(2\pi)^4} \frac{p \cdot (p+q)}{p^2 (p+q)^2} (2p+q)_\mu (2p+q)_\nu \Sigma_1^2(p; q) \\ &= - \int \frac{d^4 p}{(2\pi)^4} \frac{p \cdot (p+q)}{(2p \cdot q + q^2)^2} (2p+q)_\mu (2p+q)_\nu \left[\cot^2(\pi d) \left\{ \frac{1}{(p+q)^2} + \frac{1}{p^2} \right\} + \frac{2 \Sigma_0(p+q) \Sigma_0(p)}{p^2 (p+q)^2} \right] \end{aligned}$$

and the first piece above can be seen to be equal to the second one by shifting $p \rightarrow -(p+q)$. Similarly, the first term of $C_{\mu\nu}^{(3)}$ can be recast as

$$\begin{aligned} &\int \frac{d^4 p}{(2\pi)^4} \frac{(2p+q)_\mu (2p+q)_\nu}{p^2 (p+q)^2} \Sigma_1^2(p; q) \Sigma_0(p) \Sigma_0(p+q) \\ &= -\cot^2(\pi d) \int \frac{d^4 p}{(2\pi)^4} \frac{(2p+q)_\mu (2p+q)_\nu}{2p \cdot q + q^2} \Sigma_1(p; q) \left[\frac{\Sigma_0(p)}{p^2} - \frac{\Sigma_0(p+q)}{(p+q)^2} \right] \\ &= -2 \cot^2(\pi d) \int \frac{d^4 p}{(2\pi)^4} \frac{(2p+q)_\mu (2p+q)_\nu}{2p \cdot q + q^2} \Sigma_1(p; q) \frac{\Sigma_0(p)}{p^2} . \end{aligned}$$

Thus, finally,

$$\begin{aligned} i \Pi_{\mu\nu}^{ab}(q; 3pt) &= \sin^4(\pi d) \left[i \Pi_{\mu\nu}^{ab} \right]_{\text{normal fermion}} \\ &\quad - g^2 tr(T_a T_b) \sin^4(\pi d) Tr(1) \int \frac{d^4 p}{(2\pi)^4} [p^2 (p+q)^2]^{-1} \tilde{C}_{\mu\nu}^{(3)} \\ \tilde{C}_{\mu\nu}^{(3)} &\equiv (2p+q)_\mu (2p+q)_\nu [p \cdot (p+q) \Sigma_1^2(p; q) - 2 \cot^2(\pi d) \Sigma_1(p; q) \Sigma_0(p) (p+q)^2] \\ &\quad - 2 [p_\mu (2p+q)_\nu + (2p+q)_\mu p_\nu] \Sigma_1(p; q) \Sigma_0(p+q) \\ &\quad + \Sigma_0(p) \Sigma_0(p+q) g_{\mu\nu} . \end{aligned} \quad (5.19)$$

The first term in $\tilde{C}_{\mu\nu}^{(3)}$ has the same structure as $C_{\mu\nu}^{(4)}$ allowing us to make a crucial cancellation. In other words, adding eqs.(5.16&5.19), we now have

$$\begin{aligned}
i\Pi_{\mu\nu}^{ab}(q) &= -\frac{Tr(1)}{2} \cos^2(\pi d) \left[i\Pi_{\mu\nu}^{ab} \right]_{\text{normal scalar}} + \sin^4(\pi d) \left[i\Pi_{\mu\nu}^{ab} \right]_{\text{normal fermion}} \\
&\quad - 2g^2 \text{tr}(T_a T_b) \sin^4(\pi d) \text{Tr}(1) \hat{\Pi}_{\mu\nu} \\
\hat{\Pi}_{\mu\nu} &\equiv \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \left[G_{\mu\nu}^{(1)} + 2 \cot^2(\pi d) G_{\mu\nu}^{(2)} \right] \\
G_{\mu\nu}^{(1)} &\equiv \frac{\Sigma_0(p) \Sigma_0(p+q)}{(p+q)^2} \left\{ \frac{2q \cdot (p+q)}{(2p \cdot q + q^2)^2} (2p+q)_\mu (2p+q)_\nu \right. \\
&\quad \left. + 2 \frac{[p_\mu (2p+q)_\nu + (2p+q)_\mu p_\nu]}{2p \cdot q + q^2} + g_{\mu\nu} \right\} \\
G_{\mu\nu}^{(2)} &\equiv \frac{1}{(2p \cdot q + q^2)} \left\{ [p_\mu (2p+q)_\nu + (2p+q)_\mu p_\nu] - p \cdot q \frac{(2p+q)_\mu (2p+q)_\nu}{(2p \cdot q + q^2)} \right\}.
\end{aligned} \tag{5.20}$$

It then remains to calculate only $\hat{\Pi}_{\mu\nu}$. Rather than calculating it explicitly, note that it can always be expressed as

$$\hat{\Pi}_{\mu\nu}(q) = \frac{g_{\mu\nu}}{n-1} \hat{\Pi}_{\alpha\beta} \left[g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2} \right] + \frac{q_\mu q_\nu}{n-1} \hat{\Pi}_{\alpha\beta} \left[\frac{-g^{\alpha\beta}}{q^2} + \frac{n}{q^4} q^\alpha q^\beta \right]$$

where n is the dimension of spacetime. Now,

$$\int \frac{d^4 p}{(2\pi)^4} q^\mu q^\nu \frac{1}{p^2} G_{\mu\nu}^{(2)} = \int \frac{d^4 p}{(2\pi)^4} \frac{p \cdot q}{p^2} = 0$$

whereas

$$\int \frac{d^4 p}{(2\pi)^4} q^\mu q^\nu \frac{1}{p^2} G_{\mu\nu}^{(1)} = 3 \int \frac{d^4 p}{(2\pi)^4} \frac{\Sigma_0(p) \Sigma_0(p+q)}{p^2 (p+q)^2} q \cdot (2p+q) = 0$$

as the integrand is odd under the $p \rightarrow -(p+q)$ shift. In other words, $\hat{\Pi}_{\mu\nu}$ is proven to be gauge invariant and it remains only to calculate $g^{\mu\nu} \hat{\Pi}_{\mu\nu}$. Now,

$$\begin{aligned}
g^{\mu\nu} G_{\mu\nu}^{(1)} &= \frac{\Sigma_0(p) \Sigma_0(p+q)}{(p+q)^2} \left\{ \frac{q^2 (2p+q)^2}{(2p \cdot q + q^2)^2} + n - 2 \right\} \\
g^{\mu\nu} G_{\mu\nu}^{(2)} &= \frac{1}{(2p \cdot q + q^2)} \left\{ \frac{-p \cdot q}{(2p \cdot q + q^2)} (2p+q)^2 + 2p \cdot (2p+q) \right\}.
\end{aligned} \tag{5.21}$$

Defining

$$y \equiv 2p \cdot q + q^2, \tag{5.22}$$

we may write

$$g^{\mu\nu} \frac{1}{p^2} G_{\mu\nu}^{(2)} = \frac{2q^2}{y^2} + q^2 \frac{p \cdot q}{p^2 y^2} + \frac{2}{y}. \tag{5.23}$$

Since $y \rightarrow -y$ when $p \rightarrow -(p+q)$, the integral of y^{-1} vanishes identically, and we have

$$g^{\mu\nu} \frac{1}{p^2} G_{\mu\nu}^{(2)} \longrightarrow \frac{2q^2}{y^2} + q^2 \frac{p \cdot q}{p^2 y^2}. \tag{5.24}$$

To calculate the corresponding integral for $G_{\mu\nu}^{(1)}$, we first affect a Taylor expansion for $\Sigma_0(p+q)$, namely

$$\Sigma_0(p+q) = \Sigma_0(p) \left[1 + \frac{y}{2p^2} - \frac{y^2}{8p^4} + \dots \right]. \quad (5.25)$$

This is permissible since the β -function is only sensitive to ultraviolet physics. With this,

$$g^{\mu\nu} \frac{1}{p^2} G_{\mu\nu}^{(1)} = -\cot^2(\pi d) \frac{1}{(p+q)^2} \left\{ \frac{q^2 (2p+q)^2}{y^2} + n-2 \right\} \left[1 + \frac{y}{2p^2} - \frac{y^2}{8p^4} + \dots \right].$$

A bit of algebra, along with repeated use of the $p \rightarrow -(p+q)$ shift to throw away terms leads us to

$$g^{\mu\nu} \frac{1}{p^2} G_{\mu\nu}^{(1)} \longrightarrow -\cot^2(\pi d) \left\{ \frac{4q^2}{y^2} - \frac{q^4}{p^2 y^2} + \frac{q^2}{2p^2 (p+q)^2} + (n-2) \left[\frac{1}{p^2} - \frac{q^2}{8p^4} \right] \right\} + \dots \quad (5.26)$$

where the ellipses contain terms $\mathcal{O}(p^{-5})$ which only lead to finite contributions. And, finally,

$$\frac{g^{\mu\nu}}{p^2} \left[G_{\mu\nu}^{(1)} + 2 \cot^2(\pi d) G_{\mu\nu}^{(2)} \right] = \cot^2(\pi d) \left\{ \frac{q^2}{p^2 y} - \frac{q^2}{2p^2 (p+q)^2} - (n-2) \left[\frac{1}{p^2} - \frac{q^2}{8p^4} \right] \right\}. \quad (5.27)$$

Once again, the use of $p \rightarrow -(p+q)$ leads to a vanishing contribution from the first two terms, leaving only the last two. However, note that, in dimensional regularization, each of these vanish identically! Thus, $\widehat{\Pi}_{\mu\nu} = 0$ and

$$i \Pi_{\mu\nu}^{ab}(q) = \sin^4(\pi d) \left[i \Pi_{\mu\nu}^{ab} \right]_{\text{normal fermion}} - \frac{Tr(1)}{2} \cos^2(\pi d) \left[i \Pi_{\mu\nu}^{ab} \right]_{\text{normal scalar}}. \quad (5.28)$$

Several points are apparent at this stage:

- We do expect a form like this as
 - the χ propagator is essentially a sum of a local fermion propagator and a non-local scalar propagator;
 - the $\bar{\chi}\chi g$ vertex is sum of a local fermion-fermion-gluon and a (non-local) scalar-scalar-gluon vertex;
 - the coefficients are essentially a measure of the scalar and vector fractions.
- For $d = 3/2$, it does reduce to the canonical fermion case.
- The relative sign between the fermion-like and scalar-like contributions was expected:
 - the contribution from the \not{p} part of the propagator should behave like the canonical fermion contribution;
 - note that the canonical scalar contribution has a sign opposite to that due to a canonical fermion.
- The factor $Tr(1)/2 (= 2)$ is a reminder of the fact that each fermion mode is now associated with a scalar.
- Note that this contribution to the β -function can now have either sign depending on the value of d . This is not unexpected for a change in d results in varying the relative content of fermion and scalar in χ .
- The above relation holds for both abelian and non-abelian theories. With a proper choice of gauge representations for the unparticles [$SU(3)$, $SU(2)$ as well as hypercharges], one can easily ensure gauge-coupling unification if one were so inclined to.

6 Phenomenology

At this stage, it is necessary to consider phenomenological consequences of having an unparticle with nontrivial SM quantum numbers. For example, what would the rate of production of a charged unparticle in e^+e^- collisions be and what would its signature be? Had we gauged the original field $\Psi_{\mathcal{U}}$, this question, perhaps, would be best answered in the deconstructed framework [2]. With the unparticle now corresponding to an infinite tower (with a vanishing mass gap) of particles each coupled identically (and non-minimally) to the photon, an e^+e^- collision would now result in pair production of particles that are identical in electromagnetic properties. In other words, we would be faced with pair production characterized by a continuum in the mass of the particle, which, in the presence of magnetic fields, would translate to a continuum in track curvature. (It should be remembered that the unparticle cannot decay.) The absence of such signals would indicate a very small coupling constant/charge, the nature of which continues to be an unresolved problem in unparticles.

Gauging the Lagrangian for the χ -field changes the situation considerably. As Sec.4 shows, we now have a single massless field (albeit with a non-local propagator) and with non-local interactions with the electromagnetic field. The definition of asymptotic states is straightforward and does not need deconstruction. A non-zero charge for χ would result in copious production of χ -pairs. With the effective (non-local) mass of the each χ essentially given by $E_{\text{c.m.}}/2$, the electromagnetic radiation is an interesting issue in itself. However, the absence of any such nonstandard signal in an electromagnetic calorimeter would indicate the nonexistence of a light charged unparticle.

One resolution of this problem would be to dispense with the masslessness of the χ field. With the unparticle coupling to the SM, the lack of scale invariance in the latter would also manifest itself in the unparticle sector as a result of quantum corrections. In other words, once such effects are taken into account in constructing the effective theory for unparticles, the integral in eq.(2.2) would be characterized by an infrared cutoff Λ_{IR} , the magnitude of which would be determined by the nature of the interaction with the SM. Apart from a resultant complicated propagator, the χ field would still be characterized by a minimal mass $\sim \mathcal{O}(\Lambda_{IR})$ as well as a non-local term in its propagator. Phenomenologically, still, the aforementioned problems would persist unless Λ_{IR} is sufficiently large.

What if the unparticle is an $SU(2) \otimes U(1)$ singlet, but is coloured? In an e^+e^- collider, χ production would now occur only as a higher order process, say as $e^+e^- \rightarrow q\bar{q}\chi\bar{\chi}$, $g\chi\bar{\chi}$. The latter being a loop process, the ensuing contribution to the 3-jet rate is relatively small and well below the detection level. The 4-jet cross section, on the other hand, could be significant and worth comparing with the LEP observations of the slight excess that was seen at ALEPH. At a hadronic collider, though, the $q\bar{q}, gg \rightarrow \chi\bar{\chi}$ rate is quite large. Note, though, that the χ 's would give rise to un-hadrons/un-jet which could contain various stable exotic bound states. Non-observation of such states would once again require that Λ_{IR} is significantly large. A large Λ_{IR} essentially means that the SM β -functions would stay unaffected until $Q^2 \approx \Lambda_{IR}^2$ and the unparticle contribution(s) would manifest themselves only thereafter.

Finally, a simple way of avoiding the above mentioned phenomenological constraints, is to make the unparticle gauge group different from the SM ones. Any interaction with the SM particles would then proceed strictly through a messenger sector as originally proposed by Georgi. The un-gauge is then a part of a shadow (mirror) world with its own dynamics and phenomenology (including candidates for dark matter etc.) and the exercise undertaken in this paper being applicable only towards an understanding of the same.

7 Summary

In this paper, we start by expressing the two point correlator of a fermionic unparticle as a coherent superposition of a continuum of single particle propagators convoluted with the appropriate density of states. The corresponding propagator has the correct limiting properties (quite unlike the correlator considered so far in the literature). This, then, allows us to write the correct free-field Lagrangian for such states.

Unparticles have, so far, been described by a non-canonical kinetic energy term with an effective local Lagrangian describing interactions with Standard Model fields. We show here, though, that a field redefinition allows us to rewrite the theory in terms of a canonically quantized field but with a nonlocal interaction Lagrangian. This holds for all unparticles, whether bosonic or fermionic. That the two related theories would give identical results for cases where the unparticle appears only as virtual states is easy to appreciate. We demonstrate this through explicit calculations as well. Of course, for asymptotic unparticle states, the two theories do give different results, as they indeed should.

We next consider a field theory consisting of fermionic unparticles coupled to a gauge (gluon) field. Using standard methods of field theory, we have calculated the contribution to the β function of the theory coming from this fermionic unparticle-gauge particle coupling. To carry out this procedure, it becomes necessary to decide whether to impose full conformal invariance on the theory, or only demand scale invariance. As we show in this paper, the consequences are quite different. Demanding full conformal invariance produces for us a contribution which is exactly the same as that for an ordinary fermion, modulo an overall constant. On the other hand, demanding only scale invariance, allows us a different form of the fermion propagator and consequently a contribution to the β function which is a combination of a scalar and a fermionic part.

Since the dynamics of the theory depends on the β function, it is important to understand the issue of conformal invariance *vis a vis* a theory of unparticles. The differences arising from these issues would have consequences for phenomenology in any theory that contains couplings of unparticles to other particles. Some of these issues have been touched upon in the paper.

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A Appendix A

In this appendix we show that the rescaling done for fermionic unparticles could equally well have been done for scalar unparticles. Starting from the Lagrangian ²

$$\mathcal{L} = \phi^* (\partial_\mu \partial^\mu)^{2-d} \phi + \frac{\lambda}{\Lambda^{4(d-1)}} (\phi^* \phi)^2$$

we redefine, by introducing the field ω

$$\phi \rightarrow \omega \equiv (\partial_\mu \partial^\mu)^{(1-d)/2} \phi$$

The Lagrangian, in terms of ω , becomes

$$\mathcal{L} = \omega^* \partial_\mu \partial^\mu \omega + \frac{\lambda}{\Lambda^{4(d-1)}} \left[\omega^* (\partial_\mu \partial^\mu)^{d-1} \omega \right]^2.$$

²Note that for a d dimensional field ϕ for $d \neq 1$ the interaction term is not conformally invariant, though it can be made so by changing the power of the $\phi^* \phi$ term. However, this is not our primary concern in this Appendix. Similarly, the kinetic energy term would, in principle, need an A_d term.

Clearly, this rescaling results in a non-local interaction term which can be dealt along the lines of Ref. [8]

B Appendix B

In this Appendix, we discuss the connection between Conformal Invariance (CI) and the form of the fermion propagator. Although most of the analysis in this section is well known, we place it here for the record and in the context of fermionic unparticles.

The conformal algebra in 4 dimensions has 15 generators, P_μ (4 translations), $M_{\alpha\beta}$ (6 Lorentz transformations), D (1 scale transformation) and K_μ (4 special conformal transformations) with the following algebra:

$$i[P_\mu, P_\nu] = 0, i[M_{\alpha\beta}, P^\mu] = \delta_{[\alpha}^\mu P_{\beta]}, i[M_{\alpha\beta}, M^{\mu\nu}] = \delta_{[\alpha}^\mu M_{\beta]}^\nu$$

and

$$\begin{aligned} i[D, P_\mu] &= P_\mu, i[D, K_\mu] = -K_\mu, i[D, M_{\alpha\beta}] = 0, \\ i[K_\mu, K_\nu] &= 0, i[M_{\alpha\beta}, K^\mu] = \delta_{[\alpha}^\mu K_{\beta]}, i[P_\mu, K_\nu] = 2g_{\mu\nu}D - 2M_{\mu\nu} \end{aligned}$$

The basic transformations under translations, Lorentz transformations, scaling and special conformal transformations are as follows:

I. Translations:

$x^\mu \rightarrow x'^\mu = x^\mu + \epsilon$ under which a generic field transforms as

$$\phi(x) \rightarrow \phi'(x') = \phi(x). \quad (\text{B.1})$$

Thus

$$\begin{aligned} \phi'(x) &= \phi(x - \epsilon) = (1 - \epsilon \cdot \partial) \phi(x) \\ &= \phi(x) - i\epsilon [P_\mu, \phi(x)]. \end{aligned} \quad (\text{B.2})$$

II. Lorentz transformations:

$x^\mu \rightarrow x'^\mu = \Lambda^\mu_\nu x^\nu \simeq (\delta^\mu_\nu + \theta^\mu_\nu) x^\nu$ under which a generic field transforms as

$$\phi(x) \rightarrow \phi'(x') = M(\theta) \phi(x). \quad (\text{B.3})$$

Thus,

$$\phi'(x') = M(\theta) \phi(x) = (1 + \theta^{\mu\nu} M_{\mu\nu}) \phi(x), \quad (\text{B.4})$$

where $M_{\mu\nu}$ is the spin part of the Lorentz generator, which for scalar field $\phi(x)$, vector field A^α and spinor field $\psi(x)$ has the property

$$M_{\mu\nu} \phi(x) = 0, \quad M_{\mu\nu} A^\alpha(x) \equiv \delta_{[\mu}^\alpha \delta_{\nu]}^\beta A_\beta = \delta_{[\mu}^\alpha A_{\nu]}.$$

and

$$M_{\mu\nu} \psi(x) = \frac{1}{4} [\gamma_\mu, \gamma_\nu] \psi(x).$$

III. Scale Transformations:

$$\phi(x) \rightarrow \phi'(x') = e^{d\lambda} \phi(x) \simeq (1 + d\lambda) \phi(x) \quad (\text{B.5})$$

IV. Special Conformal Transformation:

$$x^\mu \rightarrow x'^\mu \equiv g(a) x^\mu = \frac{x^\mu + a^\mu x^2}{1 + 2a \cdot x + a^2 x^2} \simeq (1 - 2a \cdot x) x^\mu + x^2 a^\mu. \quad (\text{B.6})$$

Thus, for a scalar field $\phi(x)$ of mass dimension d ,

$$\phi(x) \rightarrow \phi'(x') = [1 + 2d a \cdot x + 2 a^\mu x^\nu M_{\mu\nu}] \phi(x) . \quad (\text{B.7})$$

We are now equipped to consider the invariance properties of the various two point functions. Since in this paper we are concerned with the fermion propagator, we will discuss only this case. The cases of the scalar and vector propagators can be similarly calculated.

For a fermion field of mass dimension d

$$\begin{aligned} \psi'(x') &= (1 + 2d a \cdot x + 2 a^\mu x^\nu M_{\mu\nu}) \psi(x) \\ &= (1 + 2da \cdot x + \frac{1}{2}[\not{a}, \not{x}]) \psi(x) \\ \bar{\psi}'(x') &= \bar{\psi}(x)(1 + 2d a \cdot x - \frac{1}{2}[\not{a}, \not{x}]) , \end{aligned} \quad (\text{B.8})$$

leading to

$$\langle \psi'(0) \bar{\psi}'(x') \rangle = (1 + 2d a \cdot x) \langle \psi(0) \bar{\psi}(x) \rangle - \frac{1}{2} \langle \psi(0) \bar{\psi}(x) \rangle [\not{a}, \not{x}] . \quad (\text{B.9})$$

Now, from scaling arguments, the two-point function has the form

$$\langle \psi(0) \bar{\psi}(x) \rangle = \frac{A \not{x}}{(x^2)^{d+\frac{1}{2}}} + \frac{B}{(x^2)^d} \quad (\text{B.10})$$

where A and B are dimensionless quantities, presumably dependent on d .

Therefore, using this form on the LHS and RHS of eq.(B.9)

$$\frac{A \not{x}'}{(x'^2)^{d+\frac{1}{2}}} + \frac{B}{(x'^2)^d} = (1 + 2d a \cdot x) \left(\frac{A \not{x}}{(x^2)^{d+\frac{1}{2}}} + \frac{B}{(x^2)^d} \right) - \frac{1}{2} \left(\frac{A \not{x}}{(x^2)^{d+\frac{1}{2}}} + \frac{B}{(x^2)^d} \right) [\not{a}, \not{x}] . \quad (\text{B.11})$$

Using $x'^2 = x^2(1 - 2a \cdot x)$, we get for the LHS, expanding to linear power in a

$$\begin{aligned} &\frac{A[(1 - 2a \cdot x)\not{x} + x^2 \not{a}]}{(x^2)^{d+\frac{1}{2}}} (1 - 2a \cdot x)^{-d-\frac{1}{2}} + \frac{B}{(x^2)^d} (1 - 2a \cdot x)^{-d} \\ &= \frac{A[1 + (2d - 1)a \cdot x]\not{x} + x^2}{(x^2)^{d+\frac{1}{2}}} + \frac{B}{(x^2)^d} (1 + 2da \cdot x) . \end{aligned} \quad (\text{B.12})$$

On the other hand, the RHS of eq.(B.9) reduces to

$$\frac{A[1 + (2d - 1)a \cdot x]\not{x} + x^2}{(x^2)^{d+\frac{1}{2}}} + \frac{B}{(x^2)^d} \left(1 + 2da \cdot x - \frac{1}{2}[\not{a}, \not{x}] \right) . \quad (\text{B.13})$$

Thus for the Green's function to be conformally invariant, we would need $B = 0$. On the other hand, if we dispense with the requirement of full conformal invariance and demand only scale invariance (along with invariance under Lorentz transformation and translations) we are allowed to keep both the terms.

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