Late-time acceleration in Higher Dimensional Cosmology

Isha Pahwa, Debajyoti Choudhury, T.R. Seshadri

Department of Physics and Astrophysics, University of Delhi,
Delhi 110007, India.

E-mail: ipahwa@physics.du.ac.in, debajyoti.choudhury@gmail.com, trs@physics.du.ac.in

Abstract. We investigate late time acceleration of the universe in higher dimensional cosmology. The content in the universe is assumed to exert pressure which is different in the normal and extra dimensions. Cosmologically viable solutions are found to exist for simple forms of the equation of state. The parameters of the model are fixed by comparing the predictions with supernovae data. While observations stipulate that the matter exerts almost vanishing pressure in the normal dimensions, we assume that, in the extra dimensions, the equation of state is of the form $P \propto \rho^{1-\gamma}$. For appropriate choice of parameters, a late time acceleration in the universe occurs with q_0 and z_{tr} being approximately -0.46 and 0.76 respectively.

Keywords: Cosmology, Dark Energy, Late Time Acceleration, Extra Dimensions

ArXiv ePrint: 1104.1925

Contents

1 Introduction

By now, several observations have confirmed that our universe is in a phase of accelerated expansion. Further, it is also clear that this acceleration had set in at a relatively recent time in the cosmological calendar. Starting with observations of Type 1A supernovae[1, 2], this feature of the Universe is now strongly suggested by observations of cosmic microwave background (CMB)[3], large-scale structures[4], baryon acoustic oscillations[5] and gravitational lensing[6]. Although observational confirmations are piling up, a convincing theoretical framework is still lacking. Several interesting mechanisms have been suggested to explain this feature of the universe, such as cosmological constant[7], quintessence[8], modified gravity[9–11], chaplygin gas[12, 13] and many others. However, these models have their own shortcomings. For example, models with a non-zero cosmological constant need a high degree of fine tuning [8, 14–19] whereas potentials required for quintessence models are unnatural in the context of particle physics[20]. Appealing to higher dimensional cosmological models is another promising mechanism to explain this mysterious phenomenon. This is the line of approach we adopt in this paper.

There are various possible constructs in the extra dimensional context including (but not limited to) brane world models [21–23]. Here, we consider a particular simple model akin to that used in Ref[24]. Whereas Ref[24] invoked extra dimensions to solve the horizon problem in early universe, using an anisotropic fluid residing in $1+D_1+D_2$ dimensions, we adapt the formalism to produce a late time acceleration instead.

Motivated by observations, we assume that the universe is filled with a uniform density matter. However, the pressure exerted by the matter in the normal dimensions is different from that in the compact dimensions, while being isotropic within each subspace. As we will argue, observations severely constrain the functional dependence of the pressure on the density. Within this constraint, however, a very simple form of the equation of state gives an excellent agreement with data. Although pressureless matter would, normally, decelerate the expansion of the universe rather than accelerating it, it is the interplay with the hidden dimensions that provides the impetus for this expansion.

It should be clarified at this stage that our construct is not a brane-world scenario and that we do not attempt to address issues such as the hierarchy problem in the Standard Model of Particle Physics. We, rather, make the simplifying assumption that these extra dimensions are compactified to a scale small enough to play essentially no direct role at the TeV scale. In this sense, the spirit is closer to more canonical scenarios defined in dimensions larger than four (an example could be a generic model derived from String Theory). Possible phenomenological manifestations of such models are postponed to future discussions.

The rest of the paper is constructed as follows. In section 2, the formalism of the model is developed, and the equation of state argued for. In the subsequent section, we present the solutions to the ensuing evolution equations. In Section 4, we compare the predictions of the model with data and infer the preferred values of the parameters. And, finally, we conclude in Section 5.

2 Evolution Equations

We start with a spacetime which has, in addition to one temporal and three normal spatial dimensions, D extra spatial dimensions. This $1+3+D$ dimensional spacetime is described by the line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-k_1 r^2} + r^2 d\Omega\right) + b^2(t)\left(\frac{dR^2}{1-k_2 R^2} + R^2 d\Omega_{D-1}\right). \tag{2.1}$$

We reserve the super(sub)script '0' for the time dimension. Whereas lower-case Roman indices ($i, j = 1, 2, 3$) denote the normal spatial dimensions, upper case Roman indices denote the extra dimensions and take the values $I, J = 4, 5, ..., D+3$. Here, D is a parameter which takes integral values and is to be fixed by comparing with observations.

In eq. (2.1), $a(t)$ denotes the scale factor in the normal (3)- dimensions and $b(t)$ represents the scale factor in the extra dimensions. Since we consider the entire $(1+3+D)$-dimensional universe to be homogeneous, the two scale factors a and b are functions only of the time t. As is well known, the visible universe is well described by a vanishing spatial flatness ($k_1 = 0$) and we shall assume the situation to be so. For reasons of simplicity as well as symmetry with the observed sector, we shall assume $k_2 = 0$ as well.

For this line-element, the components of the Einstein tensor $G_{\mu\nu}$ (with $\mu, \nu = 0, 1, \ldots, D+3$) read

$$G^0_0 = -3D\frac{\dot{a}}{a}\frac{\dot{b}}{b} - 3\frac{\dot{a}^2}{a^2} - \frac{D(D-1)}{2}\frac{\dot{b}^2}{b^2} \tag{2.2}$$

$$G^i_i = -2\frac{\ddot{a}}{a} - D\frac{\ddot{b}}{b} - 2D\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{\dot{a}^2}{a^2} - \frac{D(D-1)}{2}\frac{\dot{b}^2}{b^2} \qquad \forall\, i \tag{2.3}$$

$$G^I_I = (1-D)\frac{\ddot{b}}{b} - 3\frac{\ddot{a}}{a} + 3(1-D)\frac{\dot{a}}{a}\frac{\dot{b}}{b} + (D-1)(1-\frac{D}{2})\frac{\dot{b}^2}{b^2} - 3\frac{\dot{a}^2}{a^2} \qquad \forall I\,. \tag{2.4}$$

The energy-momentum tensor is assumed to be of the form

$$T^\mu_\nu = diag(-\rho, P_a, P_a, P_a, P_b, \ldots, P_b) \tag{2.5}$$

where ρ is the energy density of the fluid and P_a (P_b) is the pressure exerted in normal (extra) dimensions. This form of energy momentum implies that there is isotropy within the subspace associated with the normal dimensions and also within the orthogonal subspace spanned by the extra dimensions. However, the pressures in the two subspaces are different. Note that it is the observed large-scale isotropy of the universe that prompts one to consider an isotropic matter distribution. No such restriction applies to the pressure exerted in the extra dimensions, and thus, we could as easily have considered more elaborate structure for T^I_J. However, other than adding more freedom to the model, this would not have resulted in any particular qualitative improvement to the scenario. Hence, we desist from adopting such a course and adopt eq. (2.5).

The fact that T^μ_ν needs to be divergenceless ($T^\mu_{\nu;\mu} = 0$) implies

$$\frac{d}{dt}(\rho a^3 b^D) + P_a b^D \frac{d}{dt}a^3 + P_b a^3 \frac{d}{dt}b^D = 0\,. \tag{2.6}$$

In standard (3-dimensional) cosmology, the constituents of the universe today are dark energy (essentially in the form of a cosmological constant), dark matter and baryons. (The radiation energy density has substantially redshifted and hence, has negligible contribution to the energy density of the universe.) Of these, both dark matter and baryons are very well approximated by pressureless matter. Indeed, the nature of the dark matter can be inferred very well as compared to the dark energy. Consequently, we start by assuming that the matter does not exert any pressure in the three visible directions and $P_a = 0$. On the other hand, in the extra dimensions, it does exert a pressure of the form $P_b = P_b(\rho)$. Note that this implies that it is a *single fluid* that exerts such an anisotropic pressure. A mechanical analogy would be that of gas molecules filling a space but constrained to move only along a subspace. We could, of course, have adopted a scenario with two fluids, each inhabiting a subspace. However, this would only have increased the degrees of freedom in the theory without adding qualitatively to our understanding. Hence we desist from doing this, although such a course of action may well be necessitated when one attempts to construct a microscopic theory.

The dark energy is, thus, directly 'visible' only in the extra dimensions. The effect (late time acceleration) in the normal dimensions is through the evolution of extra dimensions. An infinite variety of equation of states for this dark energy are possible. For simplicity, we will assume a monomial form, viz. $P_b = w_b \rho$ with

$$w_b = \frac{w}{\bar{\rho}^{\gamma}} \,, \tag{2.7}$$

where w and γ are parameters of the model to be chosen so as to reproduce the observational data and $\bar{\rho}$ is defined in eq. 3.1. Once again, this choice (reminiscent of a generalized Chaplygin gas [25]) also serves to minimize the number of free parameters in the theory.

The aforementioned energy momentum tensor governs the evolution of the 1+3+D dimensional spacetime. Let us now consider the Einstein equations, namely

$$G^{\mu}_{\nu} = \kappa T^{\mu}_{\nu}, \tag{2.8}$$

with $\kappa = 8\pi G$. The '00' component can be expressed as

$$\frac{\dot{a}^2}{a^2} + D\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{D(D-1)}{6}\frac{\dot{b}^2}{b^2} = \frac{8\pi G\rho}{3} \,. \tag{2.9}$$

Solving for $\dot{a}/a$ and bringing it to a form close to the familiar form of the FRW equations, we have

$$\frac{\dot{a}^2}{a^2} = \frac{8\,\pi\,G\,\rho}{3} + \frac{D\,(2D+1)}{6}\,\frac{\dot{b}^2}{b^2} \mp \frac{D}{2}\,\frac{\dot{b}}{b}\sqrt{\frac{D\,(D+2)}{3}\,\frac{\dot{b}^2}{b^2} + \frac{32\,\pi\,G\rho}{3}} \tag{2.10}$$

In the absence of the last two terms on the R.H.S., this equation would, understandably, reduce to the standard form. In other words, the $\dot{b}/b$ dependent terms act as an effective dark energy source[1].

Since the scale factor, $b(t)$, corresponding to the extra dimensions enters the equations only through $\dot{b}/b$ $(= d\ln b/dt)$, what is relevant for the evolution of $a(t)$ is not the absolute value of b but only the ratio by which $b(t)$ changes with time. This is because in our model, spatial curvature in the hidden dimension $k_2 = 0$. In other words, it is not the size of the hidden world that matters, but its fractional rate of change (compression or expansion). It is this rate of compression of the extra dimensions that effectively acts like a dynamical dark energy source for the visible universe. Note that, for a non-zero $\dot{b}/b$, the scale-factor in our world, $a(t)$, evolves non-trivially even in the absence of any matter ($\rho = 0$). This is not unexpected, because gravity does couple the two subspaces and the contraction (expansion) of one can lead to the expansion (contraction) of the other. To be specific, $\rho = 0$ leads to a power-law evolution of the two scale-factors (the exponents being determined by D) wherein one of them increases with time and the other decreases.

As is well known, usual $(1+3)$-dimensional deSitter cosmologies admit both expanding and contracting solutions. We choose one of the solutions, namely, the expanding one, because we observe that the universe is expanding and not contracting. The situation here is a little more subtle. As far the '00' component of the Einstein equations goes, there is still a generalized symmetry of the form $(\dot{a}, \dot{b}) \leftrightarrow (-\dot{a}, -\dot{b})$. However, as the form of the other components of $G_{\mu\nu}$ shows, this symmetry is not manifest. A consequence of this and the preceding discussion is that a pressureless fluid (even in a higher-dimensional world) would not admit 'late time acceleration', although uniformly accelerating/decelerating solutions are possible[26].

3 Cosmological Solutions

The evolution of the universe is governed by the Einstein equations along with eq. (2.6 & 2.7). Not all of these are independent, though. For example, using the constraint equation ($G^0_0 = 8\pi G\rho$) and the continuity equation ($T^{\mu}_{\nu;\mu} = 0$) we may eliminate $\ddot{b}$. Before we do so, it is convenient to rescale the variables in terms of dimensionless quantities, namely

$$\begin{aligned} t &\equiv \frac{\tau}{H_0} \qquad & A' &\equiv \frac{a'}{a} = \frac{\dot{a}}{a\,H_0} \\ \bar{\rho} &\equiv \frac{\rho}{\rho_c} \qquad & B' &\equiv \frac{b'}{b} = \frac{\dot{b}}{b\,H_0} \,, \end{aligned} \tag{3.1}$$

[1] It is worth pointing out that, in models of non-minimal coupling, terms that are linear in $\dot{a}/a$ do appear on the right hand side of the FRW equations as in eq. (2.9)

where $1/\rho_c = 8\pi G/(3H_0^2)$ and primes denote derivative with respect to τ. In terms of these variables, the equations of motion now read

$$
\begin{aligned}
0 &= (D+2)\,A'' + 3(D+1)\,A'^2 + \frac{D\,(1-D)}{2}\,B'^2 + D\,(D-1)\,A'\,B' + 3\,D\,\Omega_0\,\bar{\rho}\,w_b \\
\bar{\rho}' &= -\bar{\rho}\,\left[3\,A' + D\,(1+w_b)\,B'\right] \\
B' &= (D-1)^{-1}\left[-3\,A' \pm \sqrt{3\,D^{-1}\left\{(D+2)\,A'^2 + 6\,\frac{(D-1)}{D}\,\bar{\rho}\Omega_0\right\}}\right] .
\end{aligned}
\tag{3.2}
$$

These are two first order differential equations in A' and $\bar{\rho}$ with the last of the three being an algebraic

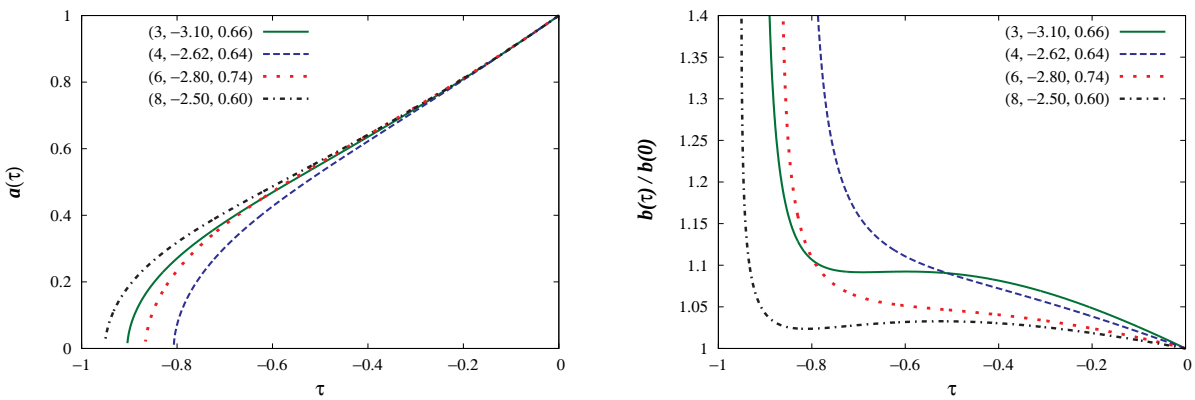

Figure 1. Behaviour of the scale factors $a(\tau)$ [left panel] and $b(\tau)$ [right panel] with the rescaled time τ. The numbers in the parentheses refer to (D, w, γ).

relation. Note that the Einstein equations can only determine $A'(\tau)$ and $B'(\tau)$ and not the scale factors themselves, a situation exactly analogous to the $(1+3)$-dimensional case. To solve eq. (3.2) for $A'(\tau)$ and $\rho(\tau)$, we require two initial conditions. Since we know the conditions in the present universe relatively precisely, we will prescribe the conditions today. In other words, with $\tau = 0$ referring to the present epoch, we evolve these equations back in time with the following 'initial' conditions

$$
\begin{aligned}
\left.\frac{\dot{a}}{a}\right|_{\tau=0} &= H_0 \implies A'|_{\tau=0} = 1 \\
\left.\frac{\rho}{\rho_c}\right|_{\tau=0} &= 1 \quad \implies \bar{\rho}|_{\tau=0} = 1 .
\end{aligned}
\tag{3.3}
$$

We may now numerically solve the two coupled first order differential equations. Two such solutions exist, one for each sign in the last of eq. (3.2). We reject here the branch with the '$-$' sign as it leads to an accelerated expansion for all times rather than a transition from a decelerated phase to an accelerated one.

The model is characterized by three parameters namely D, γ and w. In figure 1, we present the solutions for the two scale factors for some representative values of these parameters. For ease of comparison, we have rescaled the solutions [2] so that

$$
\begin{aligned}
a|_{\tau=0} &= 1 \qquad \implies \qquad A|_{\tau=0} = 0 \\
b|_{\tau=0} &= 1 \qquad \implies \qquad B|_{\tau=0} = 0 .
\end{aligned}
$$

We have a whole class of solutions in which a, the scale factor for our universe starts from 1 at $\tau = 0$ and decreases monotonically for negative values of τ. A word of caution is in place here. Since we have neglected radiation completely, the equations are valid only as long as the universe is matter dominated. The redshift corresponding to matter-radiation equality is about $z_{eq} \sim 2.9 \times 10^4 \Omega_m h^2$. For the measured values

[2] This is not to say that the two scale factors are indeed the same in the present epoch, but reflects the fact that the scale factors are arbitrary upto a constant.

of $\Omega_m(\sim 0.27)$ and $h(\sim 0.72)$, we have $z_{eq} \sim 3000$. In fact, even before we go as far back as $z = z_{eq}$, the approximation breaks down as the radiation density can no longer be neglected. We have checked, though, that the inclusion of the radiation component does not change the evolution drastically for $z \gtrsim z_{eq}$. And, since we are primarily interested in the evolution of the universe in relatively recent times, the inclusion of radiation does not affect the results in any discernible way.

As can be easily discerned, the relative evolution in $b(\tau)$ is small. In the time interval that $a(\tau)$ has increased by nearly a factor of 3000, $b(\tau)$ has decreased by $\sim 29\%$. While the opposing signs of the evolution was predicted even in a matter-free universe (see discussion in the preceding section), the large difference in the magnitude of the evolution is but a consequence of the difference in the pressure exerted by matter in the two worlds.

The epoch of matter–radiation equality, $\tau_{eq} \equiv \tau(z = z_{eq})$, has a considerable dependence on the parameter choice. For a given value of D and γ, a smaller w shifts τ_{eq} further into the past [for example, with $(D, \gamma) = (6, 0.59)$, we have $\tau_{eq} \approx -0.95, -0.97, -0.99$ for $w = -2.58, -2.6, -2.62$] thus increasing

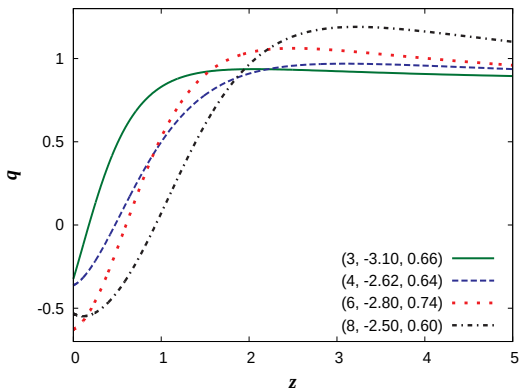

Figure 2. Evolution of the deceleration parameter q with the redshift z. The numbers in the parentheses refer to (D, w, γ).

the present-day age of the universe. Similarly, for a given value of D, and w, as we decrease γ, once again τ_{eq} shifts further into the past [$(D, w) = (6, -2.8)$ leads to $\tau_{eq} = -0.91, -0.88, -0.86$ for $\gamma = 0.7, 0.72, 0.74$]. Note, though, that the ranges of parameters are restricted (and correlated). An arbitrary set would tend to destroy either late time acceleration or shift τ_{eq} to unacceptable values.

As figure 1 shows, the curves for $a(\tau)$ have a slight upward concavity for $\tau \gtrsim -0.3$. This is but a reflection of late time acceleration. Prior to this epoch, the universe was in a decelerated phase, as attested to by the prominent upward convexity at $\tau \lesssim -0.5$. This becomes clearer when we plot the deceleration parameter q as a function of the redshift (see figure 2).

4 Observational Constraints

Having established that the model, for some choice of parameters, does lead to correct late time acceleration, we now seek to confront it with other observational data. The most important such data relates to Type Ia supernovae. The very comprehensive Union2 data set [27] lists the distance modulus μ as well as the redshift for 557 such supernovae. As the distance modulus $\mu \equiv 5 \log d_L + 25$ is nothing but a rephrasing of the luminosity $d_L(z)$, defined through

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')} , \tag{4.1}$$

we, then, need to calculate $d_L(z)$, given our determination of $H(z)$ for a particular choice of parameters. We define a χ^2–test through

$$\chi^2(D, \gamma, w) = \sum_{i=0}^{i=n} \frac{[\mu_{obs}(z_i) - \mu_{th}(D, \gamma, w; z_i)]^2}{\sigma_i^2} , \tag{4.2}$$

where μ_{th} defines the value expected in our model for a particular choice of parameters, whereas μ_{obs} and σ are the observational value and the associated root-mean-squared error. We may, now, determine the best-fit value of the parameters by minimizing the χ^2. In table 1, we list such best fit values for some choices of D. The results are analogous for other choices. Note that, with an increase in D, both $|w|$ and γ decrease. This is quite understandable as the ensuing smaller extra-dimensional pressures would now have an enhanced effect in the normal world owing to the larger effective coupling between $\dot{a}(t)$ and $\dot{b}(t)$. Thus, if we were to admit very large D values, without any concern for the microscopic theory, the fluid would tend to a normal one.

D	χ^2_{min}	w	γ	q_0	z_{tr}
2	538.92	$-3.71^{+0.25}_{-0.63}$	$0.72^{+0.23}_{-0.08}$	$-0.49^{+0.10}_{-0.26}$	$0.75^{+0.23}_{-0.27}$
3	539.18	$-3.10^{+0.18}_{-0.55}$	$0.66^{+0.25}_{-0.07}$	$-0.47^{+0.09}_{-0.27}$	$0.78^{+0.22}_{-0.32}$
6	539.33	$-2.50^{+0.13}_{-0.47}$	$0.59^{+0.27}_{-0.06}$	$-0.44^{+0.08}_{-0.29}$	$0.77^{+0.20}_{-0.31}$
10	539.42	$-2.31^{+0.12}_{-0.38}$	$0.57^{+0.14}_{-0.06}$	$-0.46^{+0.08}_{-0.36}$	$0.79^{+0.19}_{-0.33}$
35	539.44	$-2.06^{+0.35}_{-0.11}$	$0.53^{+0.26}_{-0.05}$	$-0.45^{+0.09}_{-0.26}$	$0.79^{+0.20}_{-0.33}$
100	539.74	$-1.98^{+0.07}_{-0.38}$	$0.51^{+0.28}_{-0.05}$	$-0.43^{+0.06}_{-0.30}$	$0.80^{+0.19}_{-0.34}$

Table 1. The values of w and γ corresponding to the best fit for a given choice of D. The error bars correspond to the projections of the 95% C.L. ellipses on the two axes. Also shown are the corresponding values of q_0 and z_{tr}.

As the χ^2–values listed in table 1 show, the fits are excellent. To further compare the shape of the theoretical spectrum with the Union2 data set, we also performed a Kolmogorv-Smirnov test. For each of the cases the K-S statistic was found to be smaller than 1.8×10^{-3} reflecting an extremely good fit. In fact, so good are the fits, that the current data is unable to differentiate between these choices of parameters.

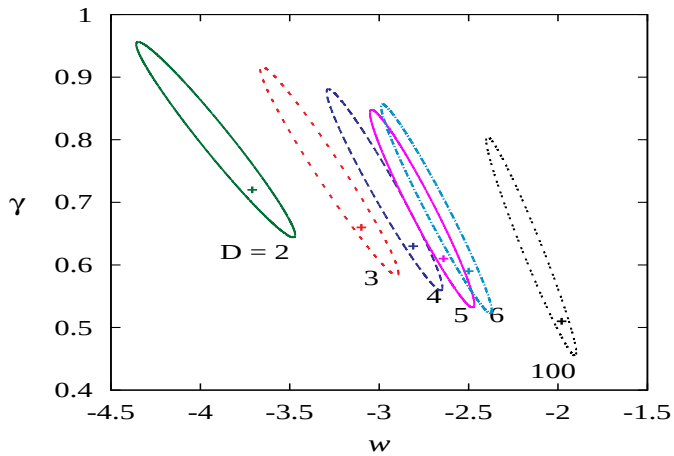

Figure 3. 95% C.L. contours in the $\gamma - w$ plane for different values of D. The points represent the best fit values for each D.

Since D assumes only discrete values, we refrain from treating it as a free parameter for the rest of the analysis. Rather, for a given D, we consider the γ-w plane as a two-dimensional parameter space. We may, then, attempt to define 95% C.L. contours in this plane by considering $\Delta\chi^2$. These are displayed in figure 3. As is evident, there is a strong negative correlation between the two parameters. Note, furthermore, that positive w (hence, positive pressure) is essentially ruled out. Similarly, integral values of γ are also essentially ruled out. Further, this is true even if we consider values of D far larger than those preferred by microscopic theories of high energy physics. As can be deduced from table 1, each of the marked points in figure 3 denotes essentially a global minimum of χ^2, with the position of minima getting increasingly closer as one increases D arbitrarily.

Once $a(\tau)$ and, hence, $H(z)$ is determined in a model, one may also calculate both the present deceleration parameter q_0 as well as z_{tr}, the redshift corresponding to the epoch of transition from the decelerated to the accelerated phase. Note that these values are not uniquely determined by the data alone as the cosmological model has a strong bearing on this determination. Also shown, in table 1, are the values of q_0 and z_{tr} as determined within our model.

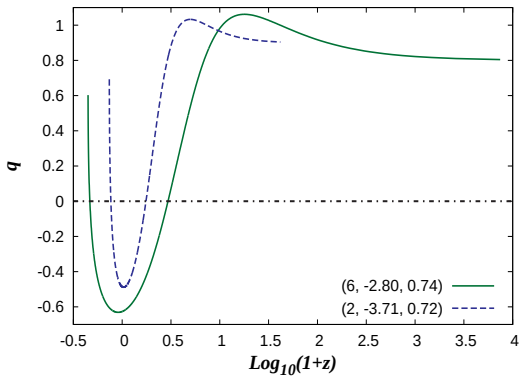

Figure 4. Evolution of the deceleration parameter q with the redshift. The numbers in the parentheses refer to (D, w, γ).

5 Conclusions and Discussion

Several approaches have been explored in the literature to arrive at a late-time acceleration of the Universe. However, none of the models are completely satisfactory. Most models rely on a mysterious type of matter (Dark Energy) which gives an effective repulsive gravity which is supposed to provide the observed accelerated expansion. This mysterious source term is perceived only through its effect on cosmological expansion.

In this paper, we have followed an approach using higher dimensions. The mysterious behaviour of matter (*a la* the Dark Energy) is manifested directly only in the extra dimensions. The accelerated expansion of the scale factor in normal dimensions is produced only indirectly by the source term through its effect on the extra dimensions. Thus, in our model, the issue of invoking strange properties for the dark energy fluid does not arise as it exhibits its unusual property only in the extra dimensions.

The particularly simple scenario we consider here is able to produce the requisite late time acceleration. The matter content acts as a pressureless gas in the normal dimensions and has a monomial equation of state as far as the extra dimensions are concerned. Just this simple ansatz leads to not only a late time acceleration, but also to a very moderate contraction of the extra dimensions since the epoch of radiation-matter equality although the normal dimensions have expanded by a factor of ~ 3000 in the same time period. With the size of the extra dimensions hardly changing, the role of any fields confined to the extra-dimensions as also those of any possible Kaluza-Klein towers of the SM fields in the low energy limit has remained essentially unaltered in this epoch.

For a significantly wide range of parameters, the model shows excellent agreement with the observational data on Type Ia supernovae, as attested to by both a low χ^2 per degree of freedom as well as the Kolmogorov-Smirnov statistic. We still need a negative pressure nonetheless, albeit limited to the extra dimensions. Whether a more complicated scenario, involving a non-isotropic extra dimensional subspace and/or multiple fluids, obviates this restriction is yet to be seen.

An interesting feature of our model is that the present phase of accelerated expansion is, generically, a transient one. In figure 4, we plot $q(z)$ against $\log_{10}(1+z)$. Note that the future is defined by $\log_{10}(1+z) < 0$. For example, $(D, w, \gamma) = (2, -3.71, 0.72)$, namely one of our best fit points, the universe will transit to a decelerating phase at $z = -0.23$. Similarly, for $(D, w, \gamma) = (6, -2.80, 0.74)$, which is somewhat away from a best fit (but within the 95% C.L region), this transition would occur at $z = -0.28$.

One aspect which we have not dwelt with in this paper is the rate of growth of perturbations. While a detailed analysis is beyond the scope of this paper and would be addressed elsewhere, let us make a few

comments. With the dynamics presented here being the dominant driving mechanism post matter-radiation equality, changes in structure formation would be confined to this epoch. In fact, the quantum of difference from the Λ-CDM scenario would be of the same approximate size as in a large class of theories with a dynamical Dark Energy source. This is supported by the fact that q_0 in this model (see table 1) agrees (within error bars) with -0.6, the value it assumes in the standard Λ-CDM scenario. Indeed, a careful analysis of this aspect could prove of value in further narrowing down of the parameter space, with larger D being less preferred.

Acknowledgments

The authors thank the anonymous referee for important suggestions that led to an improvement in the presentation. DC thanks the Department of Science and Technology, India for assistance under the project DST-SR/S2/HEP-043/2009. IP acknowledges the CSIR, India for assistance under grant 09/045(0908)/2009-EMR-I. Authors acknowledge facilities provided by Inter University Center For Astronomy and Astrophysics, Pune, India through IUCAA Resource Center(IRC) at Department of Physics and Astronomy, University of Delhi, New Delhi, India.

References

[1] S. Perlmutter et al., *Measurements of and from 42 High-Redshift Supernovae*, *Astrophys. J.* **517** (1999) 565 [astro-ph/9812133]

[2] A. G. Riess et al., *Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant*, *Astrophys. J.* **116** (1998) 1009 [astro-ph/9805201]

[3] D. N. Spergel et al. [WMAP collaboration], *Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology*, *Astrophys. J. Suppl.* **170** (2007) 377 [astro-ph/0603449]

[4] U. Seljak et al. [SDSS collboration], *Cosmological parameter analysis including SDSS Ly-alpha forest and galaxy bias: constraints on the primordial spectrum of fluctuations, neutrino mass, and dark energy*, *Phys. Rev. D* **71** (2005) 103515 [astro-ph/0407372]

[5] D. J. Eisenstein et al. [SDSS collboration], *Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies*, *Astrophys. J.* **633** (2005) 560 [astro-ph/0501171]

[6] B. Jain and A. Taylor, *Cross-correlation Tomography: Measuring Dark Energy Evolution with Weak Lensing*, *Phys. Rev. Lett.* **91** (2003) 141302 [astro-ph/0306046]

[7] S. Weinberg, *The cosmological Constant Problem*, *Rev. Mod. Phys.* **61** (1989) 1

[8] E. J. Copeland, M. Sami and S. Tsujikawa, *Dynamics of Dark Energy*, *Int. J. Mod. Phys. D* **15** (2006) 1753 [hep-th/0603057]

[9] T. P. Sotiriou and V. Faraoni, *$f(R)$ theories of gravity*, *Rev. Mod. Phys.* **82** 451 (2010)

[10] I. Thongkool and M. Sami and R. Gannouji and S. Jhingan, *Constraining $f(R)$ gravity models with disappearing cosmological constant*, *Phys. Rev. D* **80** (2009) 043523

[11] A. Starobinsky, *Disappearing cosmological constant in $f(R)$ gravity* , *JETP Letters* **86** (2007) 183

[12] A. Kamenshchik and U. Moschella and V. Pasquier, *An Alternative to Quintessence*, *Physics Letters B* **511** (2001) 265

[13] M. C. Bento and O. Bertolami and A. A. Sen, *Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification*, *Phys. Rev. D* **66** (2002) 043507

[14] V. Sahni and A. A. Starobinsky, *The Case for a Positive Cosmological Λ-term*, *Int. J. Mod. Phys. D* **9** (2000) 373

[15] T. Padmanabhan, *Cosmological constantthe weight of the vacuum*, *Phys. Rep.* **380** (2003) 235

[16] E. V. Linder, *Title: The Dynamics of Quintessence, The Quintessence of Dynamics*, [arXiv:0704.2064]

[17] J. Frieman, M. Turner and Huterer, *Dark Energy and the Accelerating Universe*, [arXiv:0803.0982]

[18] R. Caldwell and M. Kamionkowski, *The Physics of Cosmic Acceleration*, [arXiv:0903.0866]

[19] A. Silvestri and M. Trodden, *Approaches to Understanding Cosmic Acceleration*, [arXiv:0904.0024]

[20] S. M. Carroll, *Quintessence and the Rest of the World: Suppressing Long-Range Interactions*, *Phys. Rev. Lett.* **81** (1998) 3067

[21] B. Huang and S. Li and Y. Ma, *Five-dimensional metric f(R) gravity and the accelerated universe*, *Phys. Rev. D* **81** (2010) 064003

[22] L. Qiang and Y. Ma and M. Han and D. Yu, *Five-dimensional Brans-Dicke theory and cosmic acceleration*, *Phys. Rev. D* **71** (2005) 061501

[23] M. R. Setare and E. N. Saridakis, *Braneworld models with a non-minimally coupled phantom bulk field: a simple way to obtain the -1-crossing at late times*, [arXiv:0811.4253]

[24] D. Sahdev, *Perfect-fluid higher-dimensional cosmologies*, *Phys. Rev. D* **30** (1984) 2495

[25] R. Jackiw and V. P. Nair and S. Y. Pi and A. P. Polychronakos, *Perfect Fluid Theory and its Extensions*, [hep-ph/0407101]

[26] P. K. Townsend and M. N. Wohlfarth, *Accelerating Cosmologies from Compactification*, *Phys. Rev. Lett.* **91** (2003) 061302 [hep-th/0303097]

[27] Supernova Cosmology Project, http://supernova.lbl.gov/Union/