AHARGANA AND WEEKDAYS AS PER MODERN SŪRYASIDDHĀNTA

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The rule of ahargana for calculating number of days for a specified date with reference to an original epochal date, as per modern Sūryasiddhānta, is discussed. The Sūryasiddhānta epoch was started on Sunday, 1,955,880,000 years before Kali epoch. The Kali epoch begins on Friday, 1st Cātra, -3179 Śaka and Brahmagupta epoch on Sunday, 587 Śaka. The rule of ahargana of modern Sūryasiddhānta agrees very well with these facts. The selection of so early an epoch in the modern Sūryasiddhānta is suggestive of the fact that it wanted to fit these data into a single simple general rule for calculating mean positions of planets.

Keywords: ahargana, Brahmagupta epoch, Julian day epoch, Kali ahargana, Kali epoch, modern Sūryasiddhānta epoch, Varāhamihira epoch, weekdays.

INTRODUCTION

Ahargana (ahan = day, gana = sum) up to a specified day is counted from the number of days elapsed since the beginning of an epoch. The epoch may be any convenient date from which the counting of number of days begins. The main purpose of ahargana was to find the mean position of sun, moon and planets on the arbitrary date with reference to a basic epochal position. The epoch was also considered important since the astronomer selected it to mark his own time. A few astronomers of course

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have stuck to Kali era epoch following traditional practice. Schmidt,\(^1\) and Sen\(^2\) have given outlines of the rules of \textit{ahargaṇa} in general but have discussed neither the rules of modern \textit{Sūryasiddhānta} in detail nor verified the correctness of its rules. Sen\(^3\) has also made some mistakes while making the translation. Burgess has also given translation of the rules of \textit{ahargaṇa} with example which has been found to be erroneous by Schmidt.\(^4\) Obviously my objective in the paper is to find the epochal date of the modern \textit{Sūryasiddhānta}, its rules of \textit{ahargaṇa} and to test the correctness of these rules regarding the names of weekdays, as referred to in other texts before \textit{Sūryasiddhānta}. Or in other words, we will verify the following facts from the rules of modern \textit{Sūryasiddhānta} and show that these are connected and are not isolated facts.

These are:

(i) The modern \textit{Sūryasiddhānta} selected an epoch on first Caitra, 1,955,880,000 years before the Kali era which was Sunday.\(^5\)

(ii) The Kali era epoch began on Friday, first Caitra 3179 years before the Śaka era.\(^6\)

(iii) The epoch 427 Śaka first Caitra was Tuesday. This was introduced by Varāhamihira\(^7\) (505 AD), and

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\(^3\) This may be seen when the translation of the rule of \textit{ahargaṇa} as per \textit{Sūryasiddhānta} is discussed.
\(^4\) \textit{Sūryasiddhānta}, tr. by E. Burgess, 1860, reprint Calcutta University, 1985. The illustration used by Burgess (p. 34) for finding the \textit{ahargaṇa}, according to Schmidt, (op. cit., p.142), "does not mention that the date should be given in mean calender, most likely he used the true calendar, and his illustration is therefore misleading."
\(^5\) Vide verification of \textit{Sūryasiddhānta} Epoch (i).
\(^6\) \textit{Mahābhāskariya} (1.4-6) of Bhāskara I, ed. and tr. by Kripa Shankara Shukla, pp. 2-5, Lucknow University, 1960.
\(^7\) \textit{Pañcasiddhāntikā} (1.8-10) of Varāhamihira, ed. and tr. by Thibaut and Dvivedi (Benares, 1889) gives the day as Monday (\textit{Somadiwasa}), but in the critical apparatus Thibaut also gives Tuesday (\textit{bhaumadiwasa}). Schmidt (op. cit., p.150) has shown that

\textit{Cont. on next page}
The epoch of 587 Śaka began on first Caitra, Sunday. It was introduced in *Khaṇḍakāhādyaka* by Brahmagupta (628 AD).

**PROBLEMS OF LUNI-SOLAR SYSTEM**

The counting of days between two dates is not easy, since Siddhāntic astronomy follows a luni-solar system, which consisted of artificial *saura* year, *saura* months, intercalary months (*adhimāsa*) which are to be adjusted with lunar months, while lunar days (*tithis*) and omitted lunar days (*ksaya tithis*) are to be adjusted with civil days (*savana dinas*). The main problem arises from the fact that the *saura* year (period of time in which the sun travels through 12 signs of the zodiac) does not contain an integral number of lunar months (1 year = 12 lunar months of 354 days or 13 lunar months of 374 days). Further, a lunar month has 30 *tithis* but 29.5 civil days, and the lunar months are pegged with the seasons. As a result a big period like *Mahāyuga* of 4320000 years or *Kalpa* of 4320000000 years had to be conceived, and the parameters in these periods like number of *saura* months, *saura* days, lunar months, intercalary months, lunar days, intercalary days, omitted lunar days etc. are given. This appears to be simple computational manipulation though the correct process is not known. The values for shorter or longer periods were obviously calculated by the method of ratio-proportion.

The elements of the modern *Sūryasiddhānta* (i. 30-38) in a period of 4320000 years may be given with modern symbols for our future reference as follows:

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Tuesday is correct. Neugebauer and Pingree in their edition (København, 1970) of the *Pañcasiddhāntikā* (pt. I, pp. 8, 29) accepted Tuesday as correct reading. Kuppanna Sastri and K. V. Sarma in their edition (Madras, 1993 of the same text I.8-10), however read it as ‘Monday’ and gives an hypothetical example towards its explanation without assessing the facts from other sources.

8. *Brāhmaśphutasiddhānta* (I. 29-30) gives the rule of *ahargaṇa* with epoch at the beginning of Kali era. However in his *Khaṇḍākhādyaka* (I.3-5), the epoch is arbitrarily selected as 587 Śaka as Sunday.
\[ M_S = \text{no. of saura months} = 12 \times 4320000 = 51,840,000 \]
\[ M_L = \text{no. of lunar months} \]
\[ = \text{no. of revolutions of Moon} - \text{no. of revolutions of Sun} \]
\[ = 57,753,336 - 4320,000 \]
\[ = 53,433,336 \]
\[ M_I = \text{intercalary months} = M_L - M_S = 1,593,336 \]
\[ D_S = \text{no. of saura days} = 51,840,000 \times 30 = 1,555,200,000 \]
\[ D_L = \text{no. of lunar days} = 53,433,336 \times 30 = 1,603,000,80 \]
\[ D_O = \text{no. of omitted days} = \text{no. of lunar days} - \text{civil days} \]
\[ = 1,603,000,080 - 1,577,917,828 = 25,082,252. \]

**The Rules of Ahargaṇa**

Let us now examine the rules of ahargaṇa as prescribed in the *Sūryasiddhānta* (1.46-50):

\[ \text{sūryābdasamkhyaśā jñeyāḥ kṛtasyānte gataḥ amī /} \\
\[ \text{khacatuskayamādīryagnisaranandaniśākarāḥ} // \\
\[ \text{ata ārdhvam amī yuktā gatakālābdasamkhyaśā /} \\
\[ \text{māsikṛtā yutā māsaīr madhusuklādībhir gataīḥ //} \\
\[ \text{prthakṣṭhās te dhināsaghnāḥ sūryamānasaviḥḥājitaḥ /} \\
\[ \text{labdhādhināsakair yuktā dinikṛtya dinānvitāḥ //} \\
\[ \text{dvīṣās tithikṣayābhhyastās candravāsaraḥbhājitaḥ /} \\
\[ \text{labdhonarātrirahitā laṅkāyām ārdharātrikāḥ //} \\
\[ \text{sāvano dyuṅgānāḥ sūryād dinamāśābdapās tataḥ /} \\
\[ \text{saptabhiḥ kṣayitaḥ śeṣaḥ sūryādyo vāsareśvaraḥ //} \\
\]

"Time elapsed in solar years (from the end of creation) to end of Kṛtayuga (Golden age) is 1,953,720,000 years. To this add the required number of years elapsed since the end (of the Golden Age). Reduce the sum to months and add to it the number of months elapsed since the light half of Caitra of the current year. Set the result down in two places. Multiply the result (at one place) by the number of intercalary months (in a Mahāyuga) and divide it by the number of saura months (in a Mahāyuga). The number of intercalary

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9 *Sūryasiddhānta*, ed. with commentary of Parameśvara by Kripa Shankar Shukla, Lucknow University, 1957.
months thus obtained is to be added to the last result (kept separate at another place). Reduce the sum to days, and add to it the number of days elapsed of the current month. Set the result down (again) in two places; (at one place) it is multiplied by the omitted lunar days (in a Mahāyuga) and divided by the number of lunar days (in a Mahāyuga). The number of omitted lunar days thus obtained (for the required period) is deducted from the last result (kept separate in another place). The remainder is the number of civil days at the midnight on the meridian of Laṅkā. From this may be found the lord of the day (i.e. the day of the week), months and the year counting from Sunday. If the number of civil days is divided by seven, the remainder marks the lord of the day (i.e. day of the week) beginning with Sunday."

This first line within paranthesis has been translated by Sen that the period is counted from the beginning of the creation is not correct.\(^\text{10}\)

The following steps are indicated in the rules:-

\[
\begin{align*}
m_S & = 12 \, Y + m \, \text{months}, \\
m_L & = m_S \cdot M_I/M_S, \\
d_L & = 30 \cdot (m_S + m_L) + d \, \text{days},
\end{align*}
\]

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\(^{10}\) Sen (op.cit., p. 150) points out that the period is counted from the beginning of creation whereas modern Śūryasiddhānta (i.45) says without any doubt it has counted from the end of creation. The creation time has indeed been subtracted from the time under consideration in a Kalpa to get the period of 1,953,720,000 years. According to Śūryasiddhānta, new Kalpa begins with destruction of all that exists. The present new Kalpa has already passed through Dawn of the current Kalpa (equal to Kṛtayuga period), six Manvantaras, twenty-seven Mahāyugas and the present Kṛtayuga (SS.i.22). Now one Mahāyuga = 4320000 years and one Manvantaras = 71 Mahāyugas + one twilight = 306,720,000 years + 1,728,000 years = 308,448,000 years. The Kṛtayuga (Golden age), Tretā (Silver age), Dvāpara (Brazen age) and Kali (Iron age) cover a period in the ratio 4:3:2:1 of the tenth part of Mahāyuga. Hence one Kṛtayuga = 1,728,000 years. Obviously present Kalpa has already passed through 1,728,000 + 6 x 306, 720,000 + 27 x 1,320,000 + 1,728,000 = 1,970,784,000 years. From this, the time of creation 17,064,000 years (equivalent to 47,400 Divine years) are subtracted (projjhya syṣṭes tatāh kālam purvoktām divyasaṁkhyavā) which gives, 1,970,784,000 − 17,064,000 = 1,953,720,000 years. This is the period mentioned by the modern Śūryasiddhānta in the rules of ahargana. This has also been correctly explained by Burgess (p.13). The concept of (syṣṭikāla) appears to be a mathematical reconstruction based on epochal period. However, its needs further deeper study.
\[ d_o = d_L \cdot D_O / D_L, \] and
\[ a = d_L - d_o \]

Here \( Y \) denotes the number of specified period in years, \( m_s = \text{saura} \) months, \( M_L = \text{lunar} \) months, \( m_I = \text{intercalary} \) months, \( d_L = \text{lunar} \) days, \( d_o = \text{omitted lunar} \) days, and \( a = \text{ahargaṇa} \) for the specified period. The symbols \( M_s, M_I, D_L \) and \( D_o \) are values in a mahāyuga, as explained earlier. Here \( m \) months and \( d \) days have significant values if the required date is other than Caitra 1 on the meridian of Laṅkā or Ujjain.\(^{11}\) On Caitra I, \( m = 0, d = 0. \) Further, the values of \( m_I \) and \( d_o \) for a specified period are obtained from the proportion, \( m_I / m_S = M_I / M_S, d_O / d_L = D_O / D_L. \) Only integer values for these ratios were used in actual practice.

**VERIFICATION OF SŪRYASIDDHĀNTA EPOCH**

The Sūryasiddhānta epoch was started at the end of the creation.

Let \( t_o = \) epoch of Sūryasiddhānta which started at the end of creation on Sunday at midnight on the meridian Lanka.

\[ t_1 = \] Kali era epoch which begins 3179 years before the Śaka era on 1st Caitra, Friday

\[ t_2 = \] Śaka era 427 begins on 1st Caitra, Tuesday and

\[ t_3 = \] Śaka era 587 begins on 1st Caitra, Sunday.

These are facts and are cited by different authorities.

(i) To derive \( t_1 \) from \( t_0 \) by the rule ahargaṇa of the modern Sūryasiddhānta

Here, \( Y = \) number of years from the end of creation to the beginning of Kali era = time period from the end of creation to the end Kṛtayuga (Golden Age) + Tretāyuga (Silver Age) + Dwāparayuga (Brazen Age)

\(^{11}\) Laṅkā of Sūryasiddhānta is described as a great city situated on an island to the south of Bhārata-varṣa (India). The island of Ceylon which bears the name Laṅkā, however, is not the astronomical Laṅkā, as the former is about six degrees to the north of the equator. The meridian line passing through Ujjain, in Siddhāntic astronomy, meets the equator at Laṅkā.
\[= (1,953,720,000 + 1,296,000 + 864,000) \text{ years} = 1,955,880,000 \text{ years, then}
\]
\[m_s = 1,955,880,000 \times 12 = 23,470,560,000,
\]
\[m_l = m_s \times M_1/M_s = 23,470,560,000 \times 1,593,336/51,840,000 = 721,382,873,
\]
\[d_l = 30 (m_s + m_l) = 30 \times 24, 191,942,873 = 725,758,286,190,
\]
\[d_o = d_l \times D_o/D_l = 725,758,286,190 \times 25,082,280/1,603,000,080 = 11,356,002,268, \text{ and ahargana } a = d_l - d_o = 714,402,283,922. \text{ Now 714,402,283,922 } \div 7 \text{ leaves a remainder } r_1 = 5.
\]

(ii) To derive \( t_2 \) from \( t_1 \) by the \textit{ahargaṇa} rule of Sūryasiddhānta,

Here, \( Y = \) epoch 427 Śaka from the Kali era \( = 3179 + 427 = 3606 \) years, then
\[m_s = 3606 \times 12 = 43,272,
\]
\[m_l = 43,272 \times 1,593,336/51,840,000 = 1330,
\]
\[d_l = 44,602 \times 30 = 1,338,060
\]
\[d_o = 1,338,060 \times 25,082,280/1,603,000,080 = 20936, \text{ and}
\]
\[a = 1,338,060 - 20,936 = 1,317,124. \text{ Hence Kali ahargaṇa (KA) number } = 1,317,124.
\]

Now 1,317,124 \( \div 7 \) leaves the remainder \( r_2 = 4 \).

(iii) To derive \( t_3 \) from \( t_1 \) by the \textit{ahargaṇa} rule of Sūryasiddhānta.

Here \( Y = \) epoch 587 Śaka from the beginning of Kali era
\[= 3179 + 587 = 3766 \text{ years, then}
\]
\[m_s = 3766 \times 12 = 45,192,
\]
\[m_l = 45,192 \times 1,593,336/51,840,000 = 1389,
\]
\[d_l = 46,581 \times 30 = 1,397,430,
\]
\[d_o = 1,397,430 \times 25,082,280/1,603,000,080 = 21865, \text{ and}
\]
\[a = 1,397,430 - 21,865 = 1,375,565. \text{ Hence Kali ahargaṇa (KA) number } = 1,375,565.
\]

The number \( 1,375,565 \div 7 \) leaves the remainder \( r_3 = 2 \).
(iv) Table of remainders for weekdays (verification):

<table>
<thead>
<tr>
<th>Sūryasiddhānta epoch</th>
<th>Kali epoch</th>
<th>Kali epoch</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1 = 5$ from (i)</td>
<td>$r_2 = 4$ from (ii)</td>
<td>$r_3 = 2$ from (epoch 587 Śaka)</td>
</tr>
<tr>
<td>(Kali epoch)</td>
<td>(epoch 427 Śaka)</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0- Sunday</td>
<td>0-Friday</td>
<td>0-Friday</td>
</tr>
<tr>
<td>1- Monday</td>
<td>1- Saturday</td>
<td>1-Saturday</td>
</tr>
<tr>
<td>2- Tuesday</td>
<td>2- Sunday</td>
<td>2- Sunday</td>
</tr>
<tr>
<td>3- Wednesday</td>
<td>3-Monday</td>
<td></td>
</tr>
<tr>
<td>4- Thursday</td>
<td>4-Tuesday</td>
<td></td>
</tr>
<tr>
<td>5- Friday</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The first column of the Table of Remainders shows that the Sūryasiddhānta epoch begins on Sunday, 1,955,880,000 years before the Kali epoch and ends with the beginning of Kaliyuga on Friday. The second column shows that the Kaliyuga begins on Friday and ends with epoch 427 Śaka beginning on Tuesday. Likewise, the third column begins with Kaliyuga on Friday and ends with beginning of epoch 587 Śaka on Sunday. These results are obtained from the rules of ahargaṇa, as prescribed in the Sūryasiddhānta.

**Concluding Remarks.**

The rules of Sūryasiddhānta are quite general and tested before it is applied. It agrees quite well with Kali, Varāhamihira and Brahmagupta epochs. Now the question arises, why has it chosen so early an epoch? It appears that the author of the modern Sūryasiddhānta wanted to frame an ahargaṇa rule which could accommodate all the epochal data before him, and at the same time could use a simple general formula, thereby avoiding separate formula for each planet for finding mean positions.

The astronomical Kali epoch is placed by all Siddhānta-writers at – 3179 Śaka year (elapsed). According to Julian calendar, the year is 3102
BC or -3101 AD. The Julian day (JD) number\textsuperscript{12} of the beginning of Kaliyuga is fixed at 588465. Hence the Kali epoch (ardharātrika system) falls on 3102 BC Feb. 17 (ended), Thursday, Ujjain mean midnight. According to audayika system, Thursday in question would end on Friday morning at 6AM, Ujjain mean morning for which the Julian day number must be increased by one. Hence the date of this Kali epoch is 3102 BC Feb. 18 Friday, Ujjain mean time 6AM, and the JD number for this date is 588466. Obviously the Kali ahargaṇa (KA) number is zero. The date 1st Caitra, 427 Śaka, Tuesday obviously corresponds to March 22, 505 AD (Christian date), its KA number = 11317124 (shown in verification ii), and JD number =1905590. The date 1st Caitra 587 Śaka, Sunday likewise corresponds to March 23, 665 AD, its KA number =1375565 (shown in verification iii), and JD number =1964031.

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\textsuperscript{12}The reckoning of Julian days (possibly adopted by Julius Ceasar- 1st century BC) started from the mean noon (GMT) on January 1, 4713 BC, Monday. On that day at the mean noon (GMT), JD=0. This was revised by Pope Gregory XIII in 1582 AD. According to Julian calendar October 4, 1582 was a Thursday. Pope ordered that the next day would be October 15, 1582, Friday on the advice of his astronomer. However, the continuity of the weekdays was not disturbed. For JD number of the beginning of Kaliyuga, see Introduction of Siddhāntaśekhara of Śrīpati by P.C. Sengupta, p. ix, part II, ed, Babuaji Misra, University of Calcutta, 1947.