RITUAL GEOMETRY IN INDIA AND ITS PARALLELISM
IN OTHER CULTURAL AREAS

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The ritual origin of Indian geometry has been well established. The perpetual daily sacred fires and the optional fires were placed on altars of various shapes. These altars were constructed on a specified area having normally five layers, each made of fixed number of bricks. The tradition is very old in India, and important details of information are available in the Samhitas, Brahmanas and the class of writings called Sulbasutras. As to the reason which might have induced the ancient Indians to devise all these strange shapes, Rigveda says, 'May the altars be raised for our happiness', Taittiriya Samhita says, 'He, who desires heaven, may construct falcon-shaped altar, for falcon is the best flyer among the birds'. These may appear to be superstitious fancies but led to important contribution in geometry and mathematics, because of their conviction in social value system. The construction of altars having drawn on a base of different figures — square, circle, semi-circle, isosceles trapezium, triangles, rhombus, falcon or tortoise shape and others led to the development of various geometrical figures, their transformations and calculation of areas involving many Pythagorean relations with rational and irrational numbers leading to its general statement, approximation of the value of $\sqrt{2}$ and others. Tackling of geometrical and mathematical problems with irrational numbers was indeed a unique achievement of the early Indians. They had not only developed the technical terms like dvikarani ($\sqrt{2}$), dvityakarani ($1/\sqrt{2}$), tr-karani ($\sqrt{3}$), triyakaharani ($1/\sqrt{3}$), pahcaka-karani ($1/\sqrt{5}$) and pahcama-karani ($1/\sqrt{5}$) etc. but actually understood the significance. Their handling of the problems for addition, subtraction, multiplication and division with these numbers shows that both rational and irrational numbers were tackled on the same footing keeping them on a number line. The Greeks, Egyptians, Babylonians and the Chinese were also concerned with rectangles, triangles, circles, pyramids and others. The main purpose of the paper is to discuss the background and antiquity of Indian tradition, the origin of some of the important concepts and their parallelism with those developed in other cultural areas on a broader perspective.

1. INTRODUCTION

Thibaut in 1876 and Burk in 1901 referred to the Sacred Book of the East and the Sulbasutras containing the use of large number of triples in connection with the construction of altars and hinted at the non-Greek origin of the Pythagorean geometry. Neugebauer kept silent on geometry of the Indian till 1945, when he discovered a set of Pythagorean triples in the Babylonian cuneiform text and remarked 'What is Pythagorean in the Greek tradition had better be called Babylonian. The geometry of the East had found no place even in Van der Waerden's book, Science Awakening
(1954; new edition 1961). As to the reason for this omission, Seidenberg (1962) pointed out, “We are all subject to the ancient Greek rationalist ideology which is apt to scorn priestly works, and this may have caused the general neglect of the Sulbasūtras. It is true that those who maintained the priority of Indian geometry may have claimed too much when they said that Greek geometry came from India. What they should have said was that Indian geometry and the Greek geometry derive from a common source”. Van der Waerden⁵ in 1980 while making an analysis of the Chinese triples made exactly the same recommendation. He mentioned, “If one compares the Nine Chapters with Babylonian texts, one sees that the Chinese text is more systematic and richer in geometrical content. Several problems and methods which are present in the Chinese texts are absent from the numerous extant Babylonian texts. For instance, the Chinese text tells us by what method Pythagorean triples can be found, whereas in Babylonian texts we find a list of such triples. Therefore it seems reasonable to assume that the common source is not Babylonian, but Pre-Babylonian”. Seidenberg has explained the psyche behind the suppression or omission but at the same time hints at a common source. Van der Waerden picks up this hypothesis and suggests a common source i.e. Pre-Babylonian. What has actually been attempted is an hypothetical reconstruction to prove that they follow from a common source on the basis of Neolithic monuments found in England, Scotland and Ireland. It does not appear to be meaningful and a happy trend. If this is allowed to put forth in India and China similar reconstruction would not be difficult. What would have been interesting in history of science, to my mind, was further analysis of important records of all the old cultures which have made them distinct and to search for new records if any. My intention in this appear is to show some of the characteristic features of early ritual geometry and how the concept of irrational numbers had been developed in India in tackling the problems of triples involving length, breadth and diagonal of right triangle alongwith general statement. Lastly, a parallelism of approaches will be made to appreciate the Indian contribution which is distinct from those of others.

2. Altars or Citiis

The ritual connection of Indian geometry, as elaborated by Thibaut and Burk has been intensely discussed by Datta,⁶ Seidenberg,⁷ Sen & Bag⁸ and others. The ceremonies were performed on the top of the altars either in sacrificer’s house or on a nearby plot of ground. The altar is a specified raised area, generally made of bricks for keeping the fire. The fire altars were of two types, namely: the perpetual (nitya, daily) and optional (kāmya, wish fulfillment). The perpetual fires were constructed on a smaller area of one sq. puruṣa and the optional fires were constructed on a bigger area of 7½sq. puruṣa or more, each having minimum of five layers of bricks. The perpetual fires had 21 bricks and optional fires had 200 bricks in each layer in the first construction, and the number of bricks became more in the subsequent constructions with various other restrictions. The measurement of area and the construction of bricks for optional fires were a laborious process and the ceremonies continued for a long time. The whole family of the organiser had to reside by the side of the optional fire altar, for which another class of structure known as mahāvedi and other related vedis
were made. The tradition is very old in India and details of information are available from the *Samhitās, Brāhmaṇas* and the class of writings called *Śulbasūtras*. The information available in different *Śulbasūtras* are tabulated below:

<table>
<thead>
<tr>
<th>Altar</th>
<th>Shape/Horizontal section</th>
<th>Area</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Perpetual Fire Altar:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i) Āhavanīya</td>
<td>Square</td>
<td>One sq. Vyāyāma</td>
<td>BŚI. 3.1-5; 7.4; 7.5</td>
</tr>
<tr>
<td>ii) Gārhapatya</td>
<td>Circles, square</td>
<td>-</td>
<td>ĀŚI. 4.4</td>
</tr>
<tr>
<td>iii) Dakṣināgni</td>
<td>Semi-circle</td>
<td>-</td>
<td>KŚI. 1.11</td>
</tr>
<tr>
<td>II. Vedis:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i) Mahāvedi or Saumikyā vedi</td>
<td>Isosceles trap.</td>
<td>972 sq. padas</td>
<td>BŚI. 4.3</td>
</tr>
<tr>
<td></td>
<td>a = face = 24 padas</td>
<td></td>
<td>BŚI. 3.11-13</td>
</tr>
<tr>
<td></td>
<td>b = base = 30 padas</td>
<td></td>
<td>ĀŚI. 5.1-8</td>
</tr>
<tr>
<td></td>
<td>c = height = 36 padas</td>
<td></td>
<td>KŚI. 2.11; 2.12</td>
</tr>
<tr>
<td></td>
<td>Isosceles trap.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ii) Sautrāmaṇi vedi</td>
<td>(a = 24/3, b = 30/3, c = 36/3)</td>
<td>324 sq. padas</td>
<td></td>
</tr>
<tr>
<td>iii) Paitṛkī vedi</td>
<td>i) Isosceles trap.</td>
<td>108 sq. padas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a = 8, b = 10, c = 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii) a square having four corners in four coordinate directions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iv) Prāgvaṇśa</td>
<td>Rectangle</td>
<td>192 or 120 sq. prakramas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>16 × 12 or 12 × 10</td>
<td></td>
<td></td>
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</tbody>
</table>
### III. Optional Fire Altars:

<table>
<thead>
<tr>
<th>Altar</th>
<th>Shape/Horizontal section</th>
<th>Area</th>
<th>Reference¹⁰</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Caturasra-syenacit</td>
<td>Hawk bird with sq. body, sq. wings and sq. tail.</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 8.1-19, 9.1-10; Åśl. 8.2-15.7</td>
</tr>
<tr>
<td>ii) Vakrapakṣa-vyastapuccha śyena</td>
<td>Hawk bird with bent wings and out-spread tail.</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 10.1-20; 11.1-13; Bṣ. 11.1-13</td>
</tr>
<tr>
<td>iii) Kahkacit</td>
<td>Hawk bird with curved wings &amp; tail.</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 12.1-8; Åśl. 21.1-21</td>
</tr>
<tr>
<td>iv) Alajacit</td>
<td>Alaja bird with curved wings and tail.</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 13.1-6; Åśl. 21.1</td>
</tr>
<tr>
<td>v) Prauga</td>
<td>Triangle</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 14.1-8; Åśl. 12.4-12</td>
</tr>
<tr>
<td>vi) Ubbhayataprauga</td>
<td>Rhombus</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 15.1-6; Åśl. 12.7-12</td>
</tr>
<tr>
<td>vii) Rathacakracit</td>
<td>Circle</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 15.1-6; Åśl. 12.9-12</td>
</tr>
<tr>
<td>viii) Dronacit</td>
<td>Trough</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 17.1-12; 18.1-15 Åśl. 13.4-13</td>
</tr>
<tr>
<td>ix) Śmaśānacit</td>
<td>Isoceles trap.</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 19.1-11 Åśl. 14.7-14</td>
</tr>
<tr>
<td>x) Kūrmacit</td>
<td>Tortoise</td>
<td>7½ sq. puruṣa</td>
<td>Bṣ. 20.1-1; 21.1-13</td>
</tr>
</tbody>
</table>

While giving the details of these altars the Śulbasūtras use the word vijñāyate (known as per tradition), vedevijñāyate (known as per vedic traditions) etc. very often.
3. Bricks

The altar makers used to manufacture bricks of various sizes for optional fires to get the desired shape with 200 bricks not more nor less in the first construction, fixed for each, covering an area of 7½ sq. puruṣa. For nitya agni of course, small number of bricks were required. It was indeed a tremendous exercise, sometimes the area of 7½ sq. puruṣa was covered with 125 bricks and the instruction was given to replace original bricks by its half, quarter or one-eighth bricks to fulfill the desired number of 200 bricks in each layer. The bricks were also used as units for measuring, calculating and verifying the accuracy of measured area. The square bricks caturthī (one-fourth), pañcamī (one-fifth), saṣṭī (one-sixth), and their sub-divided bricks were manufactured and each was named separately and details of dimension both in rational and irrational numbers were given. These bricks were made by a special class of specialists. No burnt bricks or bricks with cleavage were used for the purpose, only sun-dried bricks were used for the purpose. The caturthī and pañcamī bricks and dimensions of the sub-divisions are listed below:

I. Caturthī (one-fourth, square brick, size 30 ang. × ang. Five types were available:
   a) caturthī (square quarter) = 30 × 30 (ang.)
   b) ardhhā (triangular half) = 30 × 30 × 30/2 (ang.)
   c) trasra pādyā (triangular quarter) = 30 × 15/2 × 15/2 (ang)
   d) caturasra pādyā (four sided quarter) = 22½ × 15 × 15/2 × 15/2 (ang)
   e) haṃsamākhi (pentagonal half brick) = 15/2 × 7½ × 15 × 7½ × 15/2 (ang.)

II. Pañcamī (one-fifth square brick, 1/5 pu. × 1/5 pu.; 24 ang. × ang.)
   a) pañcamī (one-fifth sq. brick) = 24 × 24 (ang)
   b) adhyardhā-pañcamī (rectangular brick, side longer by one-half) = 24 × 36 (ang.)
   c) pañcamī-sapāddā (rectangular brick, side longer by one-quarter) = 24 × 30 (ang.)
   d) pañcamī-ardhhā (triangular half bricks) = 24 × 24 × 24/2 (ang.)
   e) pañcamī-pādyā (triangular quarter bricks) = 24 × 12/2 × 12/2 (ang.)
   f) adhyardhārdhā (triangular half brick of adhyardhā) = 36 × 24 × 12/13 (ang.)
   g) dirghapādyā (triangular quarter bricks of adhyardhā with larger base) = 36 × 6/13 × 6/13 (ang.)
   h) sulpādyā (triangular quarter brick of adhyardhā with shorter base) = 24 × 6/13 × 6/13 (ang.)
   i) ubhāyī (triangular brick when half brick of no. (g) & (h) are attached i.e. 30 × 12/2 × 6/13 (ang.)
   j) pañcamī-aṣṭamī (one-eighth triangular brick of pañcamī) = 12 × 12 × 12/2 (ang.)

III. Similarly there were various other bricks which were prepared to meet the
requirements of covering the agni of specified size. Baudhāyana12A says, for falcon shaped fire altar 187½ pañcamī bricks cover 7½ sq. puruṣa i.e. 187½ x (1/5 x 1/5) = 7½ sq. puruṣa, out of these bricks 3½ pañcamī bricks were recommended for-head, 52 for body, 117 for two wings and 15 for tail. It was further suggested that for filling quota of 200 bricks, a few bricks were replaced by its half and quarter without breaking their symmetry. There were various other similar example. Further their bricks were considered as small square units for verifying the area specified for each figure. The lay out of altar will give some idea about the shape, type of bricks and the diagrams of the specified size.

4. Antiquity of the Tradition

The earliest enumeration of the three nitya-agni in the Rgveda13 imply the gārhapatya, the āhavaniya and the dakṣināgni. The Rgveda14 also refers to optional fire altar in the form of sūrya. It also refers,15 ‘Let the priest decorate altar (vedi), let them kindle the fire to the east’, ‘May the measure-lengths of the sacrificial post be to our felicity, may the sacred grass be (strewn) for our happiness, may altar be (raised for) our happiness’. Let the gārhapatyaciti be constructed with 21 bricks according to Taittirīya Samhitā.16 Similar passages are found in the Maitrāyani Samhitā, Kātha Samhitā18 and Kapishṭha Samhitā.19 The Saumīki vedi has been described with face 24 prakramas, base 30 prakramas and height 36 prakramas in the Taittirīya Samhitā and in other texts.20 Regarding optional fire, the Taitt. Samhitā gives various details. It says ‘He should pile a śyanacaci who desires the heaven, the śyena was the best flyer among the birds; verily becoming śyena, he flies to the world of heaven..... He should pile in the form of alaja bird with four furrows who desires support, there are four quarters, verily he finds support in the quarters. He should pile in the form of triangle (praujaciti) who has rival, verily he repels his rival. He should pile in the form of a rhombus (ubhayata prauja) who desires, ‘May I annihilate the rivals I have and those I shall have’, verily he annihilates the rivals, he has and those he will have. He should compile in the form of chariot wheel (rathacakraciti), who has foes, the chariot is a thunderbolt, verily he hurls the thunderbolt at his rival. He should pile in the form of a wooden trough (dronacaci) who desires food; in a wooden trough food is kept, verily he gains food together with its place of birth. He should pile one that has to be collected together, who desires cattle, verily he becomes rich in cattle. He should pile one in a circle (paricāyya) who desires a village, verily he gets possession of the village. He should pile in the form of smaśānacaci who desires the place where his forefathers have gone; verily he attains the place where his forefathers have gone’. It further says22, ‘He should pile (the fire) of a 1000 bricks upto knee-deep when first piling saptavidha agni (7½ puruṣa), this world is commensurate with 1000, verily he conquered this world. He should pile it of 2000 upto naval deep, when piling a second time. He should pile it of 3000 upto neck-deep when piling further”. The expert was also called agniciti (construction of the agni) appearing in the Taitt. Sam. 23 According to Sāthapatha Brāhmaṇa, the kāmyaciti (7⅔ sq. puruṣa) was to be constructed first as saptavidha, then by the increment of 1 sq. puruṣa in succession ekasatidha agni (101½ sq. puruṣa would be constructed). Baudhāyana, Āpastamba follow the same tradition. It accepted
‘heavens’ for builders of śyenacit, the world of the supreme spirit (Brāhma-loka for kārma-cit, the destroyer of enemies for rathacakra-cit and so on25. Baudhāyana and Āpastamba refer to ekavidha (1½ sq. puruṣas); dvividha (2½ sq. puruṣas) and so on. The ekavidha, dvividha do not have wings and tails, only saptavidha (7½ sq. puruṣas) agni is the first to be constructed. Thereafter altars of bigger size are obtained by increasing the area by 1 sq. puruṣa26.

Many a conjecture have been made by scholars27 like Buhler (1879), Macdonell (1904), Thibaut (1899), Keith (1914), Kane (1968), Rāmagopal (1959) placing the time of Śulbasūtras varying from 800 B.C. to 200 B.C. The sequence of Baudhāyana, Āpastamba, Pāṇini, Kātyāṇa, Patañjali were considered by Kane and Rāmagopal carefully who concluded that the principal sūtras were composed between c. 800 B.C. and 500 B.C. Whatever be the date of principal sūtras like Baudhāyana and Āpastamba Śulbasūtras, the details show the evidence of altar construction occurring in all strata of Vedic literature and the earliest reference goes back to the time of the Rgveda. The Rgveda has been dated 2000-1500 B.C. by Whitney and even earlier by Jacob28. Further, the study of chronology becomes irrelevant when we think that there was a long tradition and the tradition may be old by 200 years or 2000 years. It would be difficult to interpret anything unless we are definite. The construction of geometrical figures on the ground with the help of peg and cord, verification of area and obtaining desired shapes, the nature of treatment is very fundamental and is primary in nature.

5. Nature of Knowledge

One can guess the nature of geometrical and mathematical knowledge which could originate from such altar constructions. However, the Śulbasūtras have summarised some of these knowledges. The order in which these have been given is summarized below as per Baudhāyana Śulbasūtra. Almost the same order has been maintained by the Āpastamba Śulbasūtra though it belongs to a different school. How this is possible? The scholars were ordered to maintain strict secrecy of the occurrence of important results from other schools. It indicates that they follow from the same older source. The older sources of Samhitās and Brāhmaṇas, the way these appear in the Śulbasūtras. There is no doubt that most of the result had followed from the earlier tradition.

5.1 Give various units of linear measurement used for the purpose. Units29: 1 prādeśa = 12 aṅgulas, 1 pada = 15 aṅg., 1 iṣā = 188 aṅg., 1 aṅka = 104 aṅg., 1 yuga = 86 aṅg., 1 jānu = 32 aṅg., 1 saṃyā = 36 aṅg., 1 bāhu = 36 aṅg., 1 prakrama = 2 pada, 1 aratni = 2 prādeśa, 1 puruṣa = 5 aratni, 1 vyāyāma = 4 aratni, 1 aṅg. = 14 anus = 34 tilas = 3/4 inch approx. Āpastamba30 gives the same units. The units like aṅg., pada, prakrama, prādeśa, bāhu, aratni carry long tradition and have been used earlier in Samhitā and Brāhmaṇic literature in the same sense as these have been used in the Śulbasūtras.

5.2. Knowledge of rational numbers like 1/2, 1/3, 1/4, 1/8, 1/16, 3/2, 5/12, 7/₄,
5.3. Describes construction of square, rectangle, isosceles trapezium with use of cord and peg.

Baudhāyana gives two methods of construction of square,31 Āpastamba32 gives four methods of construction of square, two being different and so on.

5.4. Gives ideas of irrational numbers33 e.g., diagonal of a square of unit length is \( \sqrt{2} \) (i.e. 1.141421356237...), diagonal of a rectangle of sides \( \sqrt{2} \), and 1 is \( \sqrt{3} \) (i.e. 1.7320508075688772935274...), followed by general statement of the theorem of the square on the diagonal. For easy verification Baudhāyana also suggested triples expressed in rational number, (3, 4, 5), (12, 5, 13), (15, 8, 17) (7, 24, 25), (12, 35, 37), and (15, 36, 39). The same triples appear in the Āpastamba Śulbasūtra34. The various other triples also appeared in the Kātyāyana and Mānava Śulbasūtra. A few examples are: (1, 3, \( \sqrt{10} \)), (2, 6, \( \sqrt{40} \)), (1, \( \sqrt{10} \), 11), (188, 78\( \sqrt{3} \)), (203\( \sqrt{3} \)), (6, 2\( \sqrt{5} \), 5\( \sqrt{12} \)), (10, 4\( \sqrt{6} \), 10\( \sqrt{6} \)) and so on. There are 25 triples given in the Śulbasūtras.

5.5. Construction of a square as the sum or difference of two squares.35

5.6. Transformation36 of square into a rectangle, isosceles trapezium, and rectangle into a square, triangle, rhombus.

5.7. Calculation of the value of \( \sqrt{2} = 1 + 1/3 + 1/3.4 - 1/3.4.34 \) (approx.).37

In decimal notations \( \sqrt{2} = 1.41, 42, 56 \), the modern value being 1.41421356237... This shows that the value is correct to four places of decimal.

5.8. i) Transformation of square (side = s) into a circle38 (diameter = d) i.e.
\[
\frac{d}{s} = \frac{1}{\sqrt{3}} \times \left(2 + \sqrt{2}\right)
\]

ii) Circle into square:39 First method: \( \frac{s}{d} = 1 - 1/8 + 1/8.29 - 1/8.29.6.8 \)
Second method: \( s/d = 1 - 2/15 \)

5.9. Area of square = a.a = a\(^2\); area of the rectangle = a.b; area of the triangle = 1/2 of the rectangle = 1/2 a.b, where a and b are two consecutive sides. Area of the isosceles trapezium = 1/2 c (a+b) where a and b are the parallel sides and c is the distance between the parallel sides. Though the formulae have not been mentioned but the result are evident from the construction of different agnis having the same area and from the transformation of the one figure into another. For example (Fig. 1) for caturasra syenacit, the body = 4 square puruṣas, two wings = 2 \( \times (1 + 1/5) \times 1 = 12\( \frac{1}{5} \) sq. puruṣas, 1 tail = (1 + 1/10) \times 1 = 11/10 sq. puruṣas, total = 4 + 12\( \frac{1}{5} \) + 11/10 = 7\( \frac{1}{2} \) sq. puruṣas.
5.10. Gives details of bricks and construction of compulsory and optional sacrificial fires and altars.

6. **KNOWLEDGE OF IRRATIONAL (OR SURD) NUMBERS AND THEOREM OF SQUARE ON THE DIAGONAL**

6.1. For construction of square, rectangle, circle, isosceles trapezium, various technical terms práci (east-west line), párśvamáni (line parallel to east-west line), tiryabháni (line parallel to north-south line i.e. line perpendicular to east-west line), askṣṇayá (diagonal of rectangle), karaṇi (producer), dvikaraṇi (√2), tṛkaṇi (√3), dvitiyākaraṇī (√2/2 = 1/√2), tṛtāyākaraṇi (√3/3 = 1/√3) etc. were used. It is said by Baudháyana, Āpastamba and other śulbakárás that the diagonal of the square of unit length is √2, diagonal of the rectangle of breadth √2 and height 1 is √3, the diagonal of a rectangle of breadth √10 and height 1 is √11. They have also expressed the diagonal of sides 3 and 1 as √10 and so on (vide Fig. 2.).

In the figures, OA = √2, OB = √3, OC = √4,... OE = √10 and of the numbers of like 1, √2, √3, √10, 4 etc. are represented on the same number line and they had also clear idea that the √2 is the greater than 1 and the √3 is the greater than √2. They had also attempted to approximate the value of √2 = 1 + 1/3 + 1/3.4 − 1/3.4.34 by geometrical method (vide item no. 5.7.).
As regards statement on the square of the diagonal, question has been raised whether Baudhāyana and other śulbakāra understood the truth of the general statement made by them. Many a conjecture have been made by Heath, Burk, Muller, Thibaut, Datta and others about the possible proofs of the theorem. I have not discussed any of these proofs, rather I have tried to re-examine systematically the statements made by Baudhāyana which is to my mind gives enough support for a general statement of the theorem.

6.2. Baudhāyana, sūtra 1.9 states that the diagonal of square produces double its area and it is \( \sqrt{2} \). This is clear from the Paîtrikivedi in which the vedi is oriented towards cardinal directions E, F, N, and S. Baudhāyana was a great professional and an expert in laying bricks and verifying their areas. The following diagrams drawn on the basis of the hints available in the Śulbasutras justify possibly the truth of the statement both geometrically and arithmetically.

**Geometrical explanation:**

Fig. 3 (a) shows \( AB^2 = I + II + III + IV = AC^2 + BC^2 \)
Arithmetical explanation:

Fig. 3 (b) shows that the diagonal $\sqrt{2}$ has been divided into two parts each being $\sqrt{2}/2 = 1/\sqrt{2}$. Obviously the square on the diagonal breaks into four square units, each having an area of $1/\sqrt{2} \times 1/\sqrt{2} = 1/2$ and the two squares together becomes equal to one unit. Obviously $AB^2 = (I+II) + (III+IV) = 1 + 1 = AC^2 + BC^2$. Since the sulbakāras could construct a square as the sum of two squares. They had no problem in comparing and verifying that both geometrical and arithmetical attempts lend to the same result, $AB^2 = AC^2 + BC^2$

6.3. Baudhāyana’s next sūtra 1.10 — 1.11 justify the same statement from the diagonal of a rectangle. It states that the diagonal of a rectangle of breadth $\sqrt{2}$ and height 1 is $\sqrt{3}$. The truth of this statement also follows from the similar diagram given below. The geometrical diagram is in the pattern of the construction of syenaciti in which wings and tail are adjusted for fulfilling the area of $7\frac{1}{2}$ sq. purusa. For arithmetical proof Baudhāyana asks us to divide the diagonal $\sqrt{3}$ into nine square units (navamastubhumerbhāgo). It was not clear originally why Baudhāyana, in connection with the statement of theorem on the square of diagonal has directed to divide the diagonal into nine square units. The picture will be cleared from the following diagrams.
Geometrical explanation:

Fig. 4 (a) shows that \( AB^2 = I + II + III + IV + \text{area } P (\text{side } = \sqrt{2}) \). From \( BC^2 \) a strip (side \( \sqrt{2} \)) is cut off. From the cut off strip, square area \( P \) (side \( \sqrt{2} \)) is kept aside and the remaining rectangular strip is transferred and joined to the area \( AC^2 \). Obviously,

\[
\begin{align*}
BC^2 &= I + II + \text{area } P + \text{area } Q \\
AC^2 &= III + IV - \text{area } Q \\
BC^2 + AC^2 &= I + II + III + IV + \text{area } P \\
\therefore AB^2 &= BC^2 + AC^2
\end{align*}
\]

Arithmetical explanation:

Fig. 4 (b) shows that the diagonal \( \sqrt{3} \) has been divided into three parts, each being \( \sqrt{3}/3 = 1/\sqrt{3} \). Obviously the square of the diagonal breaks into nine square units, each having area \( 1/\sqrt{3} \times 1/\sqrt{3} = 1/3 \). Three square units together become equal one unit in area and six square units become \( 1/3 \times 6 = 2 \) square unit area. Hence \( AC^2 = (I + II + III) + (IV + V + \ldots + IX) = 1 \text{ unit } + 2 \text{ units } = AC^2 + BC^2 \). This shows that sūlakār's directive of breaking the diagonal into nine square units is quite meaningful which establishes the theorem of the square on the diagonal of a rectangle.

6.4. The Baudhāyana sūtrasutra 1.12 gives the general statement of the theorem of the square on the diagonal which states ‘that the areas of the square produce separately by the length and breadth of a rectangle together equal the area of the square produced by the diagonal’.

Next sūtra 1.13 says that this can easily be understood for rectangle having sides (3,4,5), (12,5,13), (15,8,17), (7,24,25), (12,35,37), (15,36,39). Items 6.1 to 6.4 show the mode of treatment in the Śulbasūtras which undoubtedly hints that an attempt was made systematically to understand the theorem of square on the diagonal of a square of rectangle, having rational and irrational values of sides.

7. Triples in other Cultural Areas

Pythagorean triple (3,4,5) is found in almost in all the old cultures. The relation \( x^2 + y^2 = z^2 \), where \( x, y, z \) are the length, breadth and the hypotenuse of a rectangle was discovered by the Babylonians and Egyptians much before Pythagoras. According to Van der Waerden, the Chinese followed a more or less the same Babylonian algebraic technique in a more developed form. According to him the Chinese type is more geometrical than the Babylonian though types of treatment are the same. Let us now discuss these in a nutshell.

7.1. The Babylonian & the Chinese:

Neugebauer and Sach’s translations of Babylonian text (BM 34568) gives 19
problems involving $X^2 + Y^2 = Z^2$ and Plimpton (No. 322) gives 15 such triples in sexagesimal numbers. The script of the text is old Babylonian and according to them it is supposed to belong to the period 1900 B.C. to 1600 B.C. The problems have been divided by Van der Waerden 40 into 13 types as follows:

Type I : Given $x$ and $y$, to find $z$
Type II : Given $y$ and $z$ to find $x$
Type III : Given $x = a$, and $z - y = d$, to find $y$ and $z$
Type IV : Given $x - y = d$, $z = c$, find $x$ & $y$.
Type V : Given $z - x = a$ and $z - y = b$ to find $x$, $y$, and $z$
Type VI : Given $x = a$, and $x + z = b$, to find $x$ and $z$
Type VII : Given $x = a$, $y - z = p/q y$, to find $y$ and $z$.
Type VIII: Given $x = a$, $y = b$ to find the side $s$ of the inscribed square.
Type IX : Given $x = a$, $y = b$, to find the diameter of the inscribed circle.
Type X : Given $x + y = s$ and $xy = F$, to find $x$ & $y$.
Type XI : Given $y - z = d$, $xy = F$, to find $x$ & $y$.
Type XII : Given $x + y = s$ and $z = c$ to find $x$ & $y$.
Type XIII : Given $x + y + z = s$ and $xy = F$ to find $z$.

The Plimpton text 322 contains only a list of Pythagorean triples and the text (BM 34568) has also used the triples $(3, 4, 5)$, $(5, 12, 13)$, $(8, 15, 17)$, $(20, 21, 29)$.

The Chinese used the problems which have used much more geometrical terms but the types and the mode of solutions are more elaborate and similar to those of the Babylonian. It has used the same triplets as mentioned above and also the triple $(7, 24, 25)$. It is not uncommon if we think these appeared in Chiu Chang Suan Shu (Nine chapters on the Mathematical Art, composed between 50 B.C. and 100 A.D.), about 2000 years later than the Babylonian. The Chinese might have received and reviewed the Babylonian materials and developed them in their own style.

7.2. The Egyptian:

The Egyptian had also some knowledge of such triples. The Egyptian rope stretcher played an important role in the foundation ceremony of temples. The Egyptian manuscripts of Middle Kingdom (2000 to 1800 BC.) appear to be purely arithmetical works taking geometrical materials as background for arithmetical computation. In the Berlin Papyrus (No. 6619) is published the problem as follows:

"A square and a second square whose side is one-half and one-fourth of that of the first square, have together an area of 100, show how to calculate this"

Taking two unknown $x$ & $y$, equations may be written in the form

$$y = \frac{3}{4} x \text{ and } x^2 + y^2 = 100$$
RITUAL GEOMETRY

This gives the solutions of the problem

\[ x = 8, y = 6 \text{ and } z = 10, \text{ The tirple} (6,8,10) \]
is linked with the wellknown triple (3,4,5).

7.3. The Greeks:

Seidenberg has shown that the Pythagorean legend (5th century B.C.) ‘sacrificing
\( \Delta \)X in honour of the discovery (3,4,5)’ or the Delian legend of Apollo (260-200
B.C.) “the way to fight a plague is to double an altar” or the Heron’s use (62 A.D.) of
peg and cord for the construction right angles’ have similarities with Indian traditions.
Cantor\(^4\) hold for twenty years the opinion that the geometry of the \textit{Sulbasūtras} was a
\textit{derivative} of Alexandrian knowledge and finally renounced it in 1904 when he was
announced with Schroeder’s suggestion that the Indian peg and cord construction go
back to atleasant 10th century B.C. Of course, Cantor preferred Egypt and not India as a
resource of Greek geometry, though the reason has not been explained.

8. CONCLUDING REMARKS

From the discussion that follows, it is clear that Indian tradition of altar
construction is old, original and primary in nature. It is not yet clear whether the Indian
recovered the basic knowledge of the triple of their own or got it from other sources.
The Babylonian appears to have known the relation \( x^2 + y^2 = z^2 \), because their pattern
of tackling the problem appear to have been used for teaching purposes and no history of
original development is available. The Chinese followed the Babylonian and shows
originality and simplicity in tackling some of their problems. The Egyptian, Indian &
Greek may have some links at some stages because of the use of cord and peg
method. But, Indian technique is distinct because of its concept of both rational and
irrational numbers and their application for verifying the truth of theorem of square of
the diagonal, Van der Waerden\(^4\) has proposed the idea of Pre-Babylonian tradition
playing a central role on the basis of Neolithic geometry and algebra invented
somewhere in Central Europe is just to hold all the traditions as carrier of the same
tradition without proper acknowledgement to contributions to each culture. Indian
history or irrational numbers and their attempt to use them for theorem of square on the
diagonal of square or rectangle is distinct from solutions of other cultures and deserve
recordings in world history of mathematics.

ABBREVIATIONS

1. Āpastamba \textit{Śulbasūtra} — Āṣī
2. Baudhāyana \textit{Śulbasūtra} — Bāṣī
3. Kapisṭhala \textit{Saṃhitā} — KPS
4. Kāṭha \textit{Saṃhitā} — KS
5. Kātyāyana \textit{Śulbasūtra} — Kāṣī
6. Maitrāyaniya \textit{Saṃhitā} — MS
For edition of Āśī, Bāl., Kāl., Māl. See item no. 8 under Notes and References.

NOTES AND REFERENCES

3. Röhr, A. 'Āpastamba' Sulbasūtra', edited and published with notes and comments, Zeitschrift d. deutsch-morgenländ. Gesellschaft, 55, 543-591, 1901; 56, 327-391, 1902. Burk pointed out (55, p. 564) that the triple (8, 15, 17) satisfy the basic property of Pythagorean triple but are not amongst the triples ascribed to the Pythagoreans.
6. Seidenberg, A. 'The Ritual Origin of Geometry', Archive for History of Exact Sciences, 1, 488-527, 1962, see p. 489. Seidenberg believes in ritual origin of geometry. According to him (p. 521) temple building was a ritual and stretching cord was a ritual, so we have geometrical rituals going back to times of about 3500 B.C.
10. Saha, S. N. & Bag, A.K. The Sulbasutras (Baudhāyana, Āpastamba, Kātyāyana and Māṇava), text with English translation and commentary, Indian National Science Academy, New Delhi, 1983. The references of all sūtra texts (Bāl., Āśī, Kāl., Māl.) have been used from this edition.
13. Bāl., 10.2-10.3
14. Bāl., 11.5-11.6
15. Bāl., 11.2.1, 11.3
16. Rāg., 1.15, 16; 6.15.19, 5.11.2
17. Rāg., 1.164.22, 10.14.5, 1.8.5, 1.4.7, 2.2.4, 6.3.7
18. Rāg., 7.35.7.
19. TS., 5.2. 3.5.
20. MS., 3.2.3.
21. ER., 20.1
22. KPS., 32.3.
23. TS. 6.2.4.5; MS. 3.8.4; KS. 25.3; KPS. 3.8.6.
24. TS., 5.4. 1-2
25. TS., 5.6 8. 2-3
26. MS., 5.2.5-5.6; 5.7.6.1.
27. OS. 12.2.3. 17-18.
28. Bāl., 8.1, 20.1; Āśī., 12.9, 14.1, 14.4, 15. 1, 18.1 etc.
29. Āśī., 8.3-8.5.
30. Buhler, G. Sacred Book of the East, 14, pp. xxii. 2, pp. xxx; Macdonell, A. Brhaddevata ed. & tr. into English, 2 parts, Harvard Oriental Series, nos. 5 & 6; Cambridge. 1904, vide p. xxii; Thibaut, G.
RITUAL GEOMETRY


Thibaut, G. The Pandit n.s. 1, p. 169.

Bd. 1. 3.

Ādi. 6. 5, 15. 4.

Bd. 1. 4-1. 5, 1. 6-1. 8.

Ādi. 1. 7, 2. 1.

27 Bk. 1. 8-1. 13.

28 Ādi. 1. 5, 2. 2, 2. 3, 2. 5, 5. 5.

29 Bk. 2. 1-2. 2; Ādi. 2. 4, 2. 8.

30 Bk. 2. 3-2. 11; Ādi. 2. 7, 2. 8, 3. 3.

31 Bk. 2. 12; Ādi. 2. 9.

32 Bk. 2. 9; Ādh. 3. 2; Kūl. 3. 11.

33 Bk. 2. 19-2. 11; Ādi. 3. 3; Kūl. 3. 12.

34 Vide Notes and References No. 5.


36 van der Waerden, B. L. "On Pre-Babylonian Mathematics II" Archive for History of Exact Sciences, 23, 1980, vide p. 24; Van der Waerden believes in two way direction of Pre-Babylonian Mathematics, viz. i) For teaching purposes, collections of algebraic and geometrical problems with solutions are developed in the manner of the extant Chinese, Babylonian and Egyptian collections and ii) Ritual purposes, geometrical constructions were developed in ceremonial meeting places (henges and temples) and altars satisfying certain geometrical conditions. Of course, he remains silent about the origin.