SYMBOL FOR ZERO IN MATHEMATICAL NOTATION
IN INDIA

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SYNOPSIS

The necessity of expressing any number in mathematical calculation by means of word-numerals led to the introduction of a word implying "nothingness" or "void". At a later date, when the word-numerals were substituted by symbol, the latter was represented by a dot or circular symbol. This gave rise to the symbol for zero. The Babylonians (14th Century B.C.) and the Chinese (8th Century A.D.) left a blank space or gap between the two numeral figures to represent what we understand by zero. The Greeks (700 B.C. to 150 A.D.) used the circular symbol to serve as a separation mark or empty space for purposes other than mathematical calculation and not as a notation of symbol for zero. The Indians for the first time used the word sunya and its various synonyms (200 A.D. as well as circular dot (400 A.D.) on decimal place-value scale. The nine numerals along with zero-symbol began to be used among the Arabian and European scholars from the 10th Century A.D. onwards.

SINOPSIS

La necesidad de expresar cualquier número en cálculos matemáticos por medio de palabras-cifras, condujo a la introducción de una palabra que implicara "ninguno" o "vacío". En una fecha posterior, cuando se sustituyeron las palabras-cifras por símbolos, este último fue representado por un punto o símbolo circular. Esto dio lugar al símbolo cero. Los babylonios (siglo 14 a. C.) y los chinos (siglo 8 d. C.) dejaban un espacio en blanco o un claro entre los dos dígitos numéricos a fin de representar lo que actualmente entendemos por cero. Los griegos (700 a. C. a 150 d. C.) utilizaban el símbolo circular como signo de separación o espacio en blanco para otros propósitos que involucraban cálculos matemáticos y no como símbolo cero. Los indios utilizaron por primera vez la palabra sunya y sus diversos sinónimos (200 d. C.) como así también el punto circular (400 d. C.) en la escala de valor decimal. Los nueve dígitos y el símbolo cero comenzaron a ser utilizados entre los eruditos árabes y europeos desde el siglo 10 d. C. en adelante.
The concept of 'nothingness' or 'void' which is inherent and instinctive to man of all ages found its expression in the philosophical and literary works of all ancient civilised nations. The feeling of necessity to give a symbolical expression of this concept in mathematical calculation which would permit the representation of a number of only high order without any limit led ultimately to discovery of a symbol for what is now known as zero.

The concept of 'nothingness' or 'void' in mathematical system was first introduced by the Babylonians (14th Century B.C.) who left an empty space in between two other numeral symbols along with their sexagesimal place-value system\(^1\). But this gave rise to much confusion for giving any quantitative expression of the numerical value involved as there can be no fixed standard for an empty space to allow for its unambiguous repetitions. It, therefore, helped very little in the symbolical expression of mathematical figures.

In certain Greek astronomical Papyri Mss. (100 B.C.) as well as Ptolemy's table of chords (150 A.D.), symbols resembling that of modern zero were used to indicate the ending of a sentence or a marginal gap\(^2\). According to some scholars, this served as the origin of the introduction of the symbol for zero in mathematical notation. There is no justification of such a view as their application in mathematical calculation could not be traced in any of the ancient contemporary Greek mathematical works or thereafter.

In India, the application of zero as a part of the numeral system, is characterised by its two types of uses as A) word-numerals, B) symbolic-use.

A) Word-numerals: The word-numerals were based on a decimal scale of notation namely \textit{ekā, daśa, śatā, sahasra} \ldots\ldots upto 16 places from the time of \textit{Sapītās} (1500 B.C.)\(^3\). The scale which is undoubtedly better than the Babylonian sexagesimal scale, has a slight approach to the place-value as it could express a number above hundred by using the decimal scale of higher value to be followed by one of lesser value and the number below hundred by word-numerals from one to ten and by its nine multiples\(^4\). The \textit{Ṣatapatha Brahmaṇa}, \textit{Taittirīya Brahmaṇa}, \textit{Chāndogya Upaniṣad}, \textit{Vedāṅga Jyotiṣa} etc. had similar uses of numerals.
This idea developed when the decimal scale was associated with the value of the places in a right to left order in the scale with the introduction of symbol for zero. That means, the symbols or word-numerals took values $x\, 1, \times 10, \times 100 \ldots \ldots$ when placed in 1st, 2nd, 3rd places \ldots \ldots respectively. From the beginning of the Christian era, the word-numerals with their various synonyms began to be used to avoid repetition of the same word and to keep rhythm of ālokas.

A few synonyms of 0, 1, 2, 3, etc. may be given as follows:

0  
- śunya, kha, ākāśa, ambra, vyoma, nabha, pūrṇa etc.

1  
- kṛiti, dharā, prthvī, bhū, bhūmi, indu, candrā, abja, vidhū, śāśānka etc.

2  
- yama, śāvna, dāsa, ākṣi, netra, nayana, bāhu, kara etc.

3  
- rāma, guṇa, agni, bhuvana, loka etc.

The application of nine word numerals on the decimal place-value scale was found in Sanskrit works and inscriptions from 2nd Century A.D. onwards. Several examples are given below :-

(1) **Pālsasiddhānta** (200 A.D.)

- **kha-kha-aṣṭa-muṇi-rāma-śaśi-netra-aṣṭa-sara-rātripāḥ**

  0 0 8 7 3 2 2 8 5 1

  = 1582237800.

(2) **Sūryasiddhānta** (400 A.D.)

- **nava-vasu-aṣṭa-kha-nava-śaśi**

  9 8 7 8 0 9 2

  = 2908789.

- **agni-śunya-śaśi-vasu-sarpa-nava**

  3 0 2 8 8 9

  = 988203.

(3) **Pāñcasiddhāntikā** (505 A.D.)

- **śunya-dvi-paṇca-yama**

  0 2 5 2

  = 2520

- **muṇi-yama-yama-dvi**

  7 2 2 2

  = 2227

(4) **Inscription at Cambodia** (604 A.D.)

- **rasa-dāsa-saraṇ ṣakendra varṣe**

  6 2 5

  = 526 Šaka year.
(5) **Inscription at Champā**\(^{10}\) (731 A.D.)

\[\text{rāma-artha-sākhal śaka}\]

\[
\begin{array}{ccc}
3 & 5 & 6 \\
\end{array}
\]

= 653 śaka year

(6) **Inscription at Java**\(^{11}\) (732 A.D.)

\[\text{śruti-indriya-rasa śaka}\]

\[
\begin{array}{ccc}
4 & 5 & 6 \\
\end{array}
\]

= 654 śaka

Such uses of word-numerals are to be found in the Brāhmaṇaśułćasiddhānta and Khandakhādyaka of Brahmagupta (628 A.D.) and in almost all later mathematical and astronomical works. The use of the word-numeral system is also found to occur in many of the inscriptions of Cambodia, Java, Champā, Malāy etc., at about 7th Century A.D.

B) **Symbolic-use:** A good number of inscriptions using numeral symbols are now known. Some of these inscriptions used a point-symbol (śunya-bindu) as well as circular symbol for zero. The earliest evidence of the symbol for zero is found in the Bēkhsālī Ms. (400 A.D.). A chronological list of the appearance of the symbols of numerals from one to nine along with or without that of zero fitted in a decimal place-value scale, is given below:

(1) **Bēkhsālī Ms.**\(^{12}\) (400 A.D.)

Symbolic expression

| i) | \[3 3\] | (17 verso) |
| ii) | \[9 \overline{u} u\] | (22 recto) |
| iii) | \[9 \overline{u} u\] | (22 recto) |
| iv) | \[I \overline{a} \overline{a} 3\] | (56 recto) |

(2) **Gurjara Grant inscription**\(^{13}\) (895 A.D.)

\[2 \overline{a} \overline{e}\]

(3) **Khmere inscription at Sambor**\(^{14}\) (883 A.D.)

\[c \cdot e\] (śaka) (605 (śaka)

\[\text{Number}\]

\[
\begin{array}{c}
350 \\
157500 \\
947 \\
846720 \\
346 \\
605 \\
\end{array}
\]
(4) Malay inscription at Palembang\textsuperscript{15} (684 A.D.)
\[ \text{C} \text{O} \text{C} \quad (\text{saka}) \]
\[ \text{C} \text{O} \quad (\text{saka}) \]
\[ 606 \quad (\text{saka}) \]
\[ 60 \quad (\text{saka}) \]

(5) Malay inscription at Kotaksapur\textsuperscript{16} (716 A.D.)
\[ \text{C} \text{O} \text{V} \quad (\text{saka}) \]
\[ 608 \quad (\text{saka}) \]

(6) Dinaya Sanskrit inscription at Java\textsuperscript{17} (760 A.D.)
\[ \text{C} \text{O} \text{C} \quad (\text{saka}) \]
\[ \text{nayana-vasu-rasa} \quad (\text{saka}) \]
\[ \frac{2}{8} \quad 6 \quad \text{saka} = \frac{682}{saka} \]

(7) Inscription at Po-nagar-Champa\textsuperscript{18} (813 A.D.)
\[ \text{I} \text{I} \text{I} \quad (\text{saka}) \]
\[ 735 \quad (\text{saka}) \]
\[ \text{I} \text{I} \text{I} \quad (\text{saka}) \]
\[ 735 \quad (\text{saka}) \]

(8) Inscription at Bakul\textsuperscript{19} (839 A.D.)
\[ \text{I} \text{E} \quad (\text{saka}) \]
\[ 751 \quad (\text{saka}) \]

The symbolical use of zero is thus found to occur in the \textit{Bakhshali} Ms. (400 A.D.) of India as well as in a number of inscriptions found at Sambor, Malay, Java used in a decimal place-value system with their dates expressed in Saka era of Indian origin. Some of these symbols including that of zero found in these inscriptions bear more or less close resemblance with those of the corresponding numbers in the \textit{Bakhshali} Ms. The \textit{Dinaya Sanskrit inscription} found at Java (760 A.D.) however contained both word-numerals and the numeral symbols. This seems to suggest strongly a common origin of all these symbols - obviously in India.

The Chinese (6th Century A.D.) left a gap or some vacant space similar to the Babylonians where a zero was required. This is found on some Thang Ms. of the Tunhuang cave-temples. One of the Ms. rolls, namely \textit{Li-Chhêng Suan Ching}\textsuperscript{20} contains the number 405 both in written form and in rod numerals like \textsuperscript{1111111}. A symbol for zero in the usual circular form appeared only in 1247 A.D. in a work \textit{Su Shu Chiu Chang} of Chhin Chiu-Shao\textsuperscript{21}. Thus it gives \textsuperscript{11} \textsuperscript{11} \textsuperscript{11} \textsuperscript{11} \textsuperscript{11} \textsuperscript{11} as 147000. Some scholars like Needham from the consideration of amalgamation
of Indian, Chinese and of the local residence of Java, Champa, Malay etc. due to
colonisation have come to this conclusion that the origin of the symbol for zero
should be attributed to the mixed culture of the Indians and the Chinese. But
such an argument is quite untenable in view of the fact that the use of the
symbol for zero fitted in a decimal place-value scale appears in China only in
Ms. of the 13th Century A.D. Furthermore, there is no evidence that the use
of word-numerals including that of zero in a decimal place-value scale was ever
made by the Chinese or even by any other nations except by the Indian prior to
their symbolic use. This has also been clearly recognised by the French scholar
Coedes.  

The earliest use of zero in Arabia by a circular symbol was found in a 10th
Century A.D. manuscript. In other Ms. during the same period, the forms such
as $\Phi$, $\infty$ and $\subset$ were used for zero. The modern Arabic symbol 0 is represented
by a symbol that looks somewhat like the modern zero and the zero is denoted by a
dot. The Indian name शून्य was taken over by the Arabs asṣ-ṣifr or şifr. This
was subsequently changed to sephirum (1202 A.D. - Fibonacci), taiphra (1340 A.D. -
Planudes) and zero, zero (C.16th Century A.D. - Italy).  

The zero in its circular symbol appeared in Europe in a 13th Century A.D.
manuscript, reproduced in Della vita e delle opere di Leonardo Pisano (Rome,
1852). Next, it is found to have been used in European Manuscripts from 14th
Century A.D. onwards.

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