Sir Alfred Bray Kempe – An Amateur Kinematician

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A short biography of A B Kempe is followed by discussions on his contributions to mathematics in general and kinematics in particular. Kempe's theorem, mechanisms for generating exact straight line, for rectilinear translation, palgiograph, angle trisector and focal mechanisms are presented.

1. Introduction

In these days of ultra-professionalism, there is no place for an amateur either in athletics or in academics. Things were different earlier; the word amateur was not so uncommon outside an English dictionary. See *Box* 1 for some famous amateurs who attained great heights in various fields, outside their professions. Even Albert Einstein did his first path-breaking research as a clerk in a patent office. In this article we discuss another great amateur academic Sir Alfred Bray Kempe (1849–1922), who was a barrister by profession but made lasting contributions to mathematics and kinematics.

This article is based on an invited talk delivered in 'The Workshop on History of Mechanisms and Machines' organized by The International Federation of Theory of Mechanisms and Machines at the Indian Institute of Science, Bangalore (India) in December, 2007.

2. Short Biography

This short bio-sketch is based on reference [1]. Alfred Bray Kempe was born on July 6, 1849 in London. He attended St. Paul's school, which was situated at that time within St. Paul's cathedral in the city of London. After finishing school he was educated at Trinity College, Cambridge. He was taught mathematics by Cayley and graduated in 1872 with distinction in mathematics. The same year he published his first paper 'A general method of solving equations of the n-th degree by mechanical means'.

Keywords
Kinematics, straight-line mechanism, plagiograph, amateur, angle-trisector.

The very next year he chose law as his profession and

Box 1. Distinguished Amateurs

Pierre de Fermat* (1601–1665): In academics, probably the most famous amateur is the great French mathematician Pierre de Fermat (1601–1665), who was a lawyer and government official by profession. He was a true amateur academic, whom E T Bell[1] called the "Prince of Amateurs". But Julian Coolidge, while writing `Mathematics of Great Amateurs' excluded Fermat on the grounds that he was "so really great that he should count as professional"[2]. He made contributions of the highest order in various fields of mathematics, that include analytic geometry, theory of maxima and minima, number theory and probability theory[3]. His explanation, contradicting that of Descartes, of Snell's law for refraction of light ultimately proved to be correct. Though he was in correspondence with the great mathematicians of his era, he never published any of his results. This duty fell on his son and friends after his death.

Sophie Germain (1776–1831): Following the usual social tradition of Europe in that era, initially her parents discouraged her unusual interest in mathematics. To prevent her working in the night, even her candles and clothes were confiscated and heating was removed[2]. Ultimately the parents relented. But Ecole Polytechnique, where Lagrange was teaching mathematics, was reserved only for men. So she had to register under a false identity as Monsieur Le Blanc. She made important breakthroughs in Fermat's last theorem and her mathematical ability impressed even the "Prince of Mathematicians" – Gauss. Besides number theory, she also made lasting contributions in applied mathematics through her 'Memoir on the vibrations of elastic plates'. Finally the Institut de France could not neglect her any more and Gauss also convinced the University of Goettingen to award her an honorary degree. But she died before this could be awarded. In her death certificate, the state official described her as a single woman with no profession.

Bankim Chandra Chattopadhyay (1838–1894): The first, and still one of the best, novelist, essayist and poet in Bengali literature was a Deputy Magistrate during the British Raj. He was one of the first two graduates from Calcutta University. He wrote seventeen novels, four religious commentaries, a couple of volumes of essays and a poetry collection. He established 'Banga Darshan' (Mirror of Bengal) – the most respected literary magazine of that era in Bengal. He is most widely known as the composer of the verse "Vande Mataram" (included in one of his novels *Anandamath* – Temple of Bliss) – which became a national song with tune set by Rabindranath Tagore.

* See *Resonance*, Vol.1, January 1996.

[1] E T Bell, *Men of Mathematics*, Simon & Schuster, 1937.

[2] Simon Singh, *Fermat's Enigma*, Anchor Books, Doubleday, 1997.

[3] Michael Sean Mahoney, *The Mathematical Career of Pierre de Fermat* 1601–1665 (2nd Edition), Princeton University Press, 1994.

Box 1. Continued...

Box 1. Continued...

W G Grace (1848–1915): He was a medical practitioner but is widely acknowledged as one of the most significant cricketers of all time. He played first class cricket for a record 44 seasons even at the age of sixty. He played his last test cricket at the age of fifty. He captained England, Gloucestershire county. He is credited as the inventor of most of the batting strokes in display today.

Roger Bannister (1929–): On May 6, 1954 he ran the first sub-four-minute mile in recorded history. He is a distinguished neurologist by profession. Maybe from his knowledge of the human body he knew "how to take more out of you than you have got". He was Master of Pembroke College, Oxford. He was Director of the National Hospital for Nervous Diseases in London. He is the chairman of the editorial board of the journal *Clinical Autonomic Research* and is the editor of *Autonomic Failure*.

became a barrister in November 1873. In 1909 he became a bencher of the inner circle. However, he devoted a lot of time to his hobbies, i.e., music, mathematics and mountaineering.

Kempe became an authority in ecclesiastical law and he held many Chancellorships, i.e., legal adviser to Anglican diocese. He was legal advisor to the diocese of Newcastle, Southwell, Chichester, Chelmsford, etc., finally culminating in the Chancellorship of the diocese of London in 1912. He was an admirable and courteous lawyer. He possessed a very logically clear mind. On top of that he had a sound training in rigorous mathematics. So much so, that as a lawyer he had difficulty in arguing a rotten point.

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Proposed by Cayley and Sylvester as "distinguished for his knowledge of and discoveries in kinematics" he was elected to the fellowship of the Royal Society in 1881 and in 1897 he was elected to the council of the Royal Society. He took a leading share in the management of the affairs of the society for 21 years. In 1919, he resigned from the position of the Treasurer of the Royal Society on health grounds. He was also a member of the Royal Institution for fifty years and served on its Board of Management five times. For his remarkable contributions to various government departments he was knighted in the year 1912. Besides his love for music and mathematics, his interest in the mountains of Switzerland, which he visited nearly fifty times, deserves a special mention.

He was married twice. His first wife died in 1893 and he married a second time in 1897. From his second marriage he had two sons and a daughter. During the last decade of his life, he started winding down his extraordinarily busy life and in 1922 he developed pneumonia, which ultimately led to his death.

3. Contribution to Mathematics

Kempe's early contributions to mathematics were on linkages, involving application of geometry. These are elaborated in the next section. His other contributions to mathematics, though not large in quantity, were first rate in quality. He wrote several papers mostly on algebra with special laws, which all bear the stamp of his impressive ability. His legal training always led him to make very lucid and precise statements. He was President of The London Mathematical Society during 1892–94 and he will always be remembered for his noteworthy contribution to the philosophy of mathematics. His presidential address was titled "What is mathematics?"

Some experts consider his 'Memoir on the theory of mathematical form' published in *The Philosophical Transactions of The Royal Society* in 1896 to be his most important work. It used graph theory to visualize mathematical questions and provided a framework for finding new mathematical knowledge.

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Kempe published a "proof" of the four colour theorem in 1879. This had a flaw, which was pointed out by Heawood in 1890. However, in 1976, two basic ideas of 'unavoidability' and 'reducibility' introduced by Kempe were utilized (with the aid of a computer) by Appel and

Haken to complete the proof. This required consideration of 1936 distinct cases to show that each of these is reducible.

4. Contribution to Kinematics

4.1 *Kempe's Theorem*

[1] See article by Anupam Saxena, in this issue of *Resonance* on pp.220–237.

In 1876 Kempe proved a theorem[1] on the possibility of reproducing any plane curve of degree n by means of an articulated planar mechanism consisting of only revolute (hinge joint) and prismatic (sliding joints) pairs. In 1926 Gersgorin, basing his work on Kempe's considerations, used the complex variable method to prove a more general theorem on the possibility of constructing similar mechanisms for an arbitrary algebraic function. Yet neither Kempe nor anyone else has established a method to design the simplest linkage for generating a particular curve [2]. Artobolevskii [3] gives many examples of linkages and other mechanisms for tracing curves up to the fourth degree, and even some transcendental curves.

In 1897 Konigs presented a spatial equivalence of Kempe's theorem, stating that it is possible to devise spatial linkages consisting of only revolute and prismatic pairs to guide a point along any algebraic surface or a spatial curve. It may be mentioned that except for the screw pair all other lower pairs can be thought of as combinations of revolute and prismatic pairs.

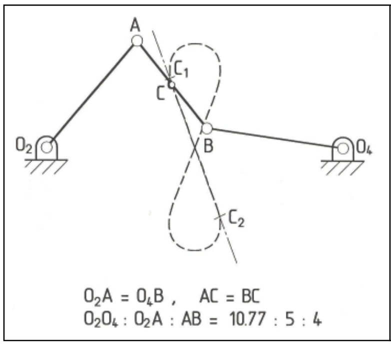
4.2 *How to Draw a Straight Line*

In 1877 Kempe published a monograph entitled 'How to draw a straight line – a lecture on linkages' [4]. These lectures were delivered to science teachers. This monograph has been reprinted after exactly hundred years [5]. The problem of generation of a straight line by an articulated mechanism is as old as James Watt's steam engine. Newton pointed out that "the description of right lines and circles, upon which the geometry is founded, belongs to mechanics. Geometry does not teach us to draw these lines, but requires them to be drawn". In geometrical construction we just copy an existing straight line. Around 1784 James Watt[2] used his famous straight-line mechanism (*Figure* 1) to guide the piston of a steam engine along an approximate straight line. This Watt linkage was popular in the 1950s to locate rear axles in racing cars [6]. James Watt himself, in a letter to his son, wrote that he was most proud of his invention of this mechanism.

Later on, a large number of approximate straight-line mechanisms like that of Roberts, Evans, Chebyshev, etc. were developed. Incidentally, the now famous Chebyshev's polynomial was developed from a concern with the Watt Linkage. Papers have been published on straight-line mechanisms as late as in 1993 [7].

Incidentally, the now famous Chebyshev's polynomial was developed from a concern with the Watt Linkage.

In 1864, eight years after Watt's discovery, M Peaucellier, a French army engineer, first came up with an eight-link mechanism, a point (Q) of which generates an exact straight line as the mechanism moves (*Figure* 2). This mechanism was rediscovered by a Russian student named Lipkin who got a substantial reward from the Russian government for his supposed originality [4]. Fortunately, thereafter Peaucellier's merit was recognized and he was awarded the great mechanical prize of the Institute of France, the *Prix Montyon*. In 1874, Sylvester introduced Peaucellier's discovery in England. In August of the same year, Hart came up with his six-link



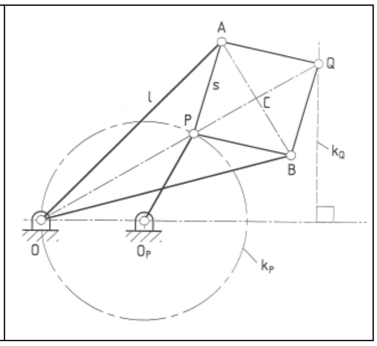


Figure 1. (left)
Figure 2. (right)

[2] See K V Gopalakrishnan, *Resonance*, Vol.14, No.6, pp.522–529, 2009; S R Deepak and G K Ananthasuresh, *Resonance*, Vol.14, No.6, pp.530–543, 2009.

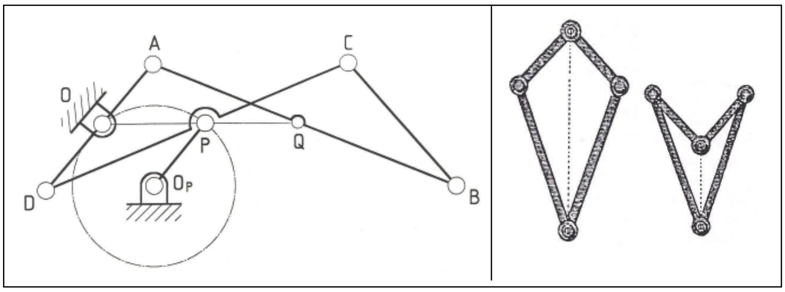


Figure 3. (left)
Figure 4. (right)

exact straight-line mechanism (*Figure* 3). Here also the point Q moves along a straight line as the mechanism is moved. Kempe [4] discussed these mechanisms in terms of two basic cells, called "kite" and "spearhead"[3], which are hinged quadrilaterals with adjacent pairs of equal lengths (*Figure* 4).

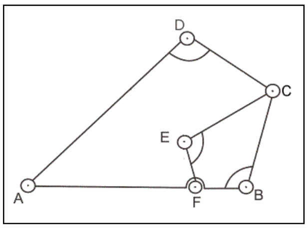
[3] The kite and spearhead shown in *Figure* 4 are also known as kite and delta seen in Penrose's aperiodic tiling.

Both these exact straight-line mechanisms are based on the principle of geometric inversion. If two points P and Q move on a plane in such a manner that with O as a fixed point, O, P and Q always remain collinear and the product OP. OQ is unchanged, then the curves traced out by P and Q are termed inverses of each other. It is known that if P moves along a circle, then Q also moves on another circle, i.e., these two circles are inverses of each other. It can be shown that the radii of these two circles are related as

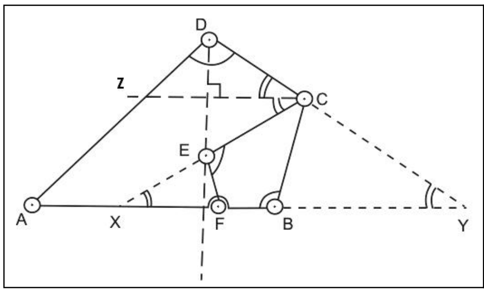
$$\frac{r_Q}{r_P} = \left| \frac{\mathrm{OP} \times \mathrm{OQ}}{\mathrm{OO}_P^2 - r_P^2} \right| , \qquad (1)$$

where O_P is the centre of the circle traced by P. If this circle passes through O, then $\mathrm{OO}_P = r_P$ and $r_Q \to \infty$, i.e., the circle traced by Q degenerates into a straight line.

Figure 5.



Kempe devised a particularly ingenious exact straight-line mechanism, which is *not* based on the principle of geometric inversion [8]. First let us consider the six-link chain shown in *Figure* 5. Here AB = AD, BC = CD = CE, BF = FE and (AB/BC) = (BC/BF).



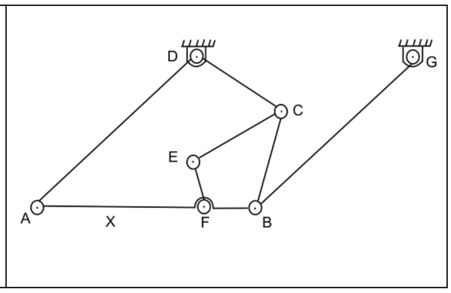


Figure 6 . (left)
Figure 7. (right)

It is obvious that ∠ABC = ∠ADC = ∠CEF. Now we can show that as this linkage (with any of the links in the chain fixed) moves, the line DE always remains perpendicular to AB. Towards this end, consider *Figure* 6, where CE and DC are extended to meet AB at X and Y respectively. It is easy to see that ∠EXF = ∠CYB. Now draw CZ parallel to AB through C, when ∠DCZ = ∠CYB = ∠EXF = ∠ZCE. Thus, CZ is the bisector of ∠DCE in the isosceles triangle DCE. Therefore, DE and CZ are at right angles, i.e., DE is perpendicular to AB which is parallel to CZ.

Next we add two more links DG and GB, with DG = GB = AB and hold DG fixed (*Figure* 7). In this eight-link mechanism, AB moves parallel to itself and DE remains perpendicular to AB, so that E moves along a straight line perpendicular to AB (and DG).

4.3 *Rectilinear Translation of a Rigid Body by Articulated Mechanism*

The mechanism discussed in Section 4.2 was extended by Kempe to generate rectilinear translation of a rigid body by using a ten-link mechanism shown in *Figure* 8.

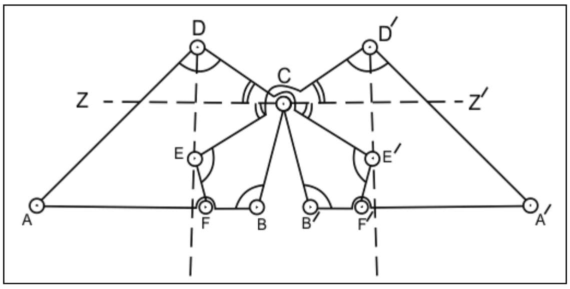


Figure 8.

Here, *Figure* 5 is reflected about a line passing through C and perpendicular to AB. Thereafter, DC and CE′ are converted into one rigid link. So are EC and CD′. Thus four links are connected at hinge C. In this chain, if the link AB is held fixed, then the link A′B′ undergoes rectilinear translation along the direction AB. To justify this statement, one needs to only prove that point C always remains equidistant from lines AB and A′B′.

First, we note that the bisectors CZ of ∠DCE and CZ′ of ∠D′CE′ are parallel to AB and A′B′, respectively. Since ZCZ′ is one line, AB and A′B′ always remain parallel. Moreover, line A′B′ always lies on the extension of AB, because the two identical rhomboids ABCD and A′B′C′D′ are always congruent for all configurations of the linkage. It should be noted that a rhomboid is completely defined by the lengths of its sides and the angle between a pair of opposite sides. Kempe again discussed all these mechanisms in terms of the basic cells of kite and spearhead.

Starting from the antiparallelogram cell (*Figure* 9a) of Hart's exact straight-line mechanism, Kempe devised a cell by bending all the straight links of *Figure* 9a through 90° in the middle as shown in *Figure* 9b, which is reproduced from [4]. In this cell, the product OC.OP remains constant. Sylvester named the points O, C, O′ and P as the four foci of the quadriplane.

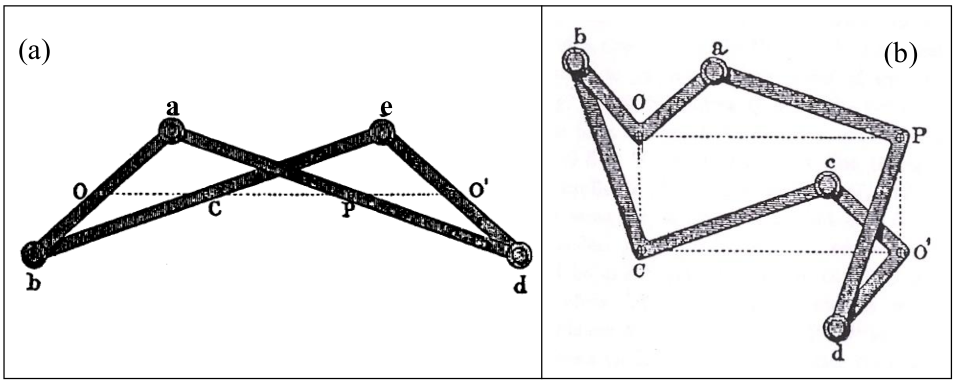
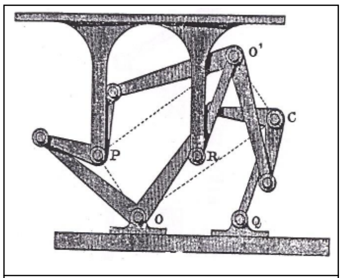


Figure 9.

Starting from the basic cell shown as OPO′C in *Figure* 9b but with a different orientation, we first make QC = OQ in *Figure* 10 [3]. Then following the principle of palgiograph (discussed in the next section) to this basic cell we add one more point R (*Figure* 10), with PR = O′R = OQ. In this eight-link mechanism, the so-called Kempe–Sylvester parallel motion generator, the worktable moves with exact horizontal motion.



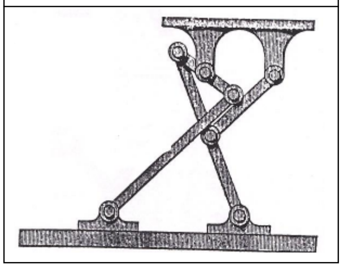


Figure 10. (top)
Figure 11. (bottom)

A six-link mechanism, starting from Chebyshev's approximate straight-line mechanism, can be designed to produce approximate horizontal motion of the worktable. Such a mechanism is shown in *Figure* 11, which is reproduced from [4]. Kempe noted that whether these mechanisms devised by mathematicians are of any use should be left to the mechanicians to decide. But he suggested that these could be used for the slide rests in lathes, traversing tables, punches, drills, drawbridges, etc.

4.4 *Kempe–Sylvester Plagiograph*

Kempe first reported to Sylvester that the motion of a coupler point E (*Figure* 12) on the coupler AB of a four-bar linkage OABD can be reproduced with a constant scale factor (= AB/AE) by point F of the six-bar linkage shown in *Figure* 12, where OABC is a parallelogram.

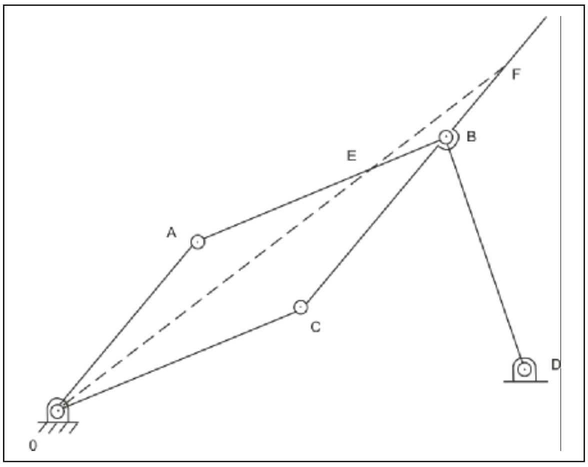


Figure 12.

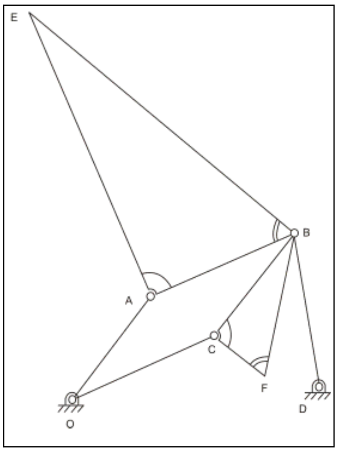


Figure 13.

Sylvester immediately extended it to general coupler point E not necessarily lying on AB (*Figure* 13). Here triangles ABE and CBF are similar and OABC is a parallelogram. In this mechanism the curve traced out by F is a scale drawing (with a constant factor = AB/AE) of that produced by E and oriented at a fixed angle α (*Figure* 14). In both these six-link mechanisms, the link DB only decided the coupler path. Removing this link, one gets a five-link mechanism (*Figure* 14) with two degrees of freedom. So, point E can now be moved to trace any arbitrary planar curve, which will be reproduced to a scale and oriented at a constant angle as proved below. This copying mechanism is called 'Plagiograph'.

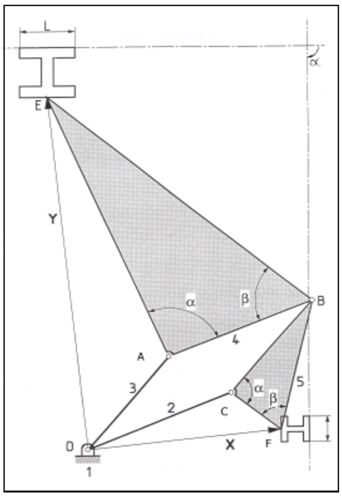


Figure 14.

Referring to *Figure* 14, the position vector of point F can be written as

$$\mathbf{X} = \mathbf{OC} + \mathbf{CF}$$

$$= \mathbf{AB} + (\mathrm{CF/CB})\ \mathbf{CB}\mathrm{e}^{-i\alpha}$$

$$= (\mathrm{AB/AE})\mathbf{AE}\mathrm{e}^{-i\alpha} + (\mathrm{CF/CB})\mathbf{CB}\mathrm{e}^{-i\alpha}$$

$$= (\mathrm{AB/AE})\mathbf{AE}\ \mathrm{e}^{-i\alpha} + (\mathrm{CF/CB})\mathbf{OA}\mathrm{e}^{-i\alpha}$$

$$= (\mathrm{AB/AE})\mathrm{e}^{-i\alpha}(\mathbf{OA} + \mathbf{AE}) \quad \text{since } (\mathrm{CF/CB}) = (\mathrm{AB/AE})$$

$$= (\mathrm{AB/AE})\mathrm{e}^{-i\alpha}\ \mathbf{Y}\ ,$$

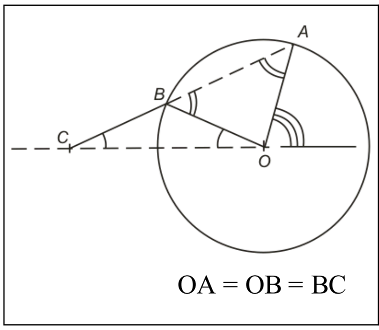
where $\mathbf{Y}$ is the position vector of the coupler point E. The movements of points F and E are thus related as follows:

$$\mathrm{d}\mathbf{X} = (\mathrm{AB/AE})\ \mathrm{e}^{-i\alpha}\ \mathrm{d}\mathbf{Y}\,.$$

Thus, point F reproduces the figure drawn by point E to a scale (AB/AE) and at an angle α in the clockwise direction.

4.5 *Angle Trisector*

Trisection of an arbitrary angle geometrically, i.e., by



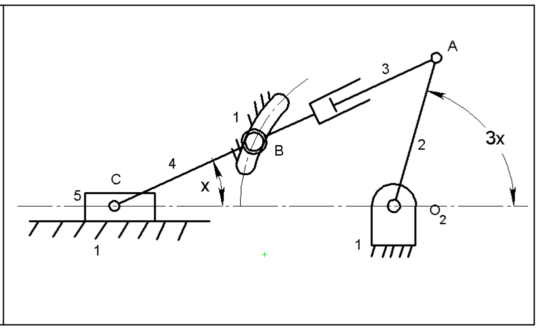


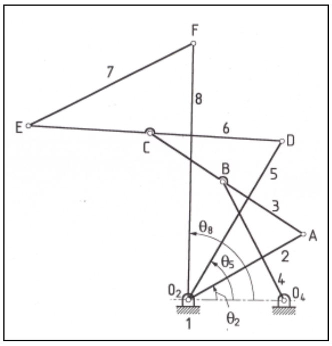
Figure 15. (left)
Figure 16. (right)

using only a compass and a straight edge, is as old as Euclidean geometry. The problem was attempted for more than 2000 years until the impossibility of the construction was proved by algebraic means. A large number of approximate solutions using geometric methods are available. Some of these methods can be used to design mechanisms (not restricted to compass and straight edge) for trisecting an arbitrary angle. One such method and the corresponding mechanism are explained in *Figures* 15 and 16. This is a five-link mechanism, which has two prismatic pairs and a higher pair.

Trisection of an arbitrary angle geometrically, i.e., by using only a compass and a straight edge, is as old as Euclidean geometry. The problem was attempted for more than 2000 years until the impossibility of the construction was proved by algebraic means.

Kempe devised an ingenious eight-link mechanism with only revolute pairs (*Figure* 17) as an angle trisector. Here again, he started from the basic cell of Hart's mechanism in the form of an anti-parallelogram O_2ABO_4, where $O_2O_4 = AB$ and O_2 $A=O_4B$ (*Figure* 17). Another anti-parallelogram O_2DCA is connected as shown in the figure. The ratio of the lengths of the longer to that of the shorter sides of these two anti-parallelogram is the same, i.e., $(O_2A/AB) = (O_2D/O_2A)$. One of the longer sides of the first anti-parallelogram O_2A becomes one of the shorter sides of the second anti-parallelogram and one of the shorter sides of the first one, AB, is extended to create one of the longer sides AC of the second one. Now, a third anti-parallelogram O_2FED having similar characteristics is added. It can be easily verified from the count of the links and joints that the resulting eight-link mechanism has a single degree of freedom. Since the

Figure 17.



the three anti-parallelograms are scale drawings of one another,

$$\angle \mathrm{AO_2O_4} = \angle \mathrm{DO_2A} = \angle \mathrm{FO_2D}.$$

Thus $\theta_5 = 2\theta_2$, and $\theta_8 = 3\theta_2$. In other words, $\mathrm{O_2A}$ is the trisector of $\angle \mathrm{FO_2O_4}$.

It may be mentioned that by continuing in the same way with n anti-parallelograms, one gets a mechanism, which can simultaneously construct all the fractions, $\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n-1}{n}$ of any arbitrary angle.

4.6 *Kempe–Burmester Focal Mechanism*

In 1878, Kempe devised another important mechanism also studied by Burmester, which is now known as Kempe –Burmester focal mechanism. This again is an eight-link mechanism (*Figure* 18). It is an over-closed mechanism as can be clearly verified from the count of the number of links and joints. In other words, the mechanism has mobility with a single degree of freedom only for special kinematic dimensions as discussed below. It consists of an arbitrary four-bar linkage $\mathrm{O_2ABO_4}$ with an inside point T where the sum of the angles subtended by the two pairs of opposite sides (i.e., $\mathrm{O_2O_4}$ and AB, $\mathrm{O_2A}$ and $\mathrm{O_4B}$) is 180°. This point T is called the focal point. The points P, Q, R and S, one on each side of the original

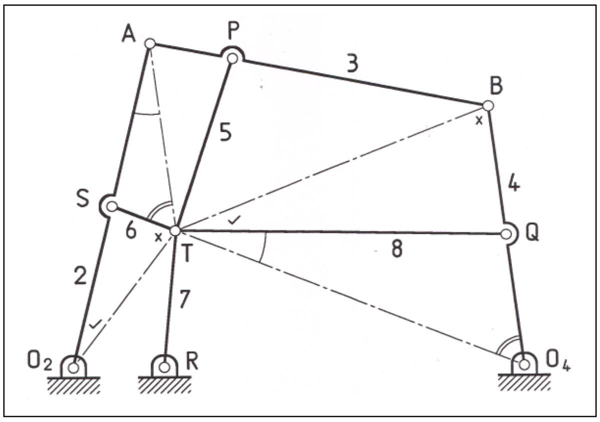


Figure 18.

four-bar linkage, are chosen so that each pair of internal four-bar linkages are reflected similar to each other as indicated in the figure. If the focal point T is outside the quadrilateral O_2ABO_4, then the angles subtended by the opposite sides should be the same. For a detailed discussion on this focal mechanism one may refer to [12]. In this focal mechanism, any one of the simple hinges is redundant and can be removed maintaining the same relative motion (with single degree of freedom) between the links.

Referring to *Figure* 18, the following possible practical applications of Kempe–Burmester focal mechanism are worth mentioning:

- Link 4 undergoes pure rotation about O_4, even when the revolute pair at O_4 is removed. Thus, if required, one can use the floating link 4 as a turntable without providing a spindle.
- Link AP of the six-link mechanism 1 – 2 – 3 – 5 – 6 – 7 or link BP of the mechanism 1 – 7 – 5 – 3 – 8 – 4 can serve as a complete coupler-cognate of the original four-bar linkage O_2ABO_4. With infinite choices for T and correspondingly for P, Q, R and S, one gets infinite choices for such cognates.
- Suppose a four-bar linkage function generator is designed to correlate the angle between the coupler (link 3) and the output link (4) with the input angle (i.e., between the frame and link 2). Then, the focal mechanism guides to infinite number of alternatives once a design has been reached.
- Points P, Q, R and S always remain on a circle as the mechanism moves. The radius of the circle changes continuously with the movement of the mechanism. Four cutters centred at these four points were used in 1970's to cut glass television tubes of varying cross-section [9].

Suggested Reading

[1] www-circa.mcs.st-and.ac.uk/~history/Biographies/Kempe.html

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Kempe devised the focal mechanism without any specific goal in mind. It was examined entirely from mathematical and theoretical considerations. But it found an industrial application after almost 100 years of its invention.

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5. Conclusions

The enormous contribution of Kempe in the field of mathematics and kinematics, that too as an amateur, does not need to be overemphasized. We ought to remember that all his contributions in kinematics were made in a very short span of time immediately after kinematics was established as a separate field of study. Kempe maintained the same level of devotion in all his honorary duties. He always valued the contributions of others and gave credit unhesitatingly to Sylvester, his mentor. According to Kempe's own admission, his interest in kinematics waned after Sylvester left England to move to Canada. It is a great tribute to Kempe's contribution that his works find prominent mention in texts on kinematics published more than hundred years after his time.

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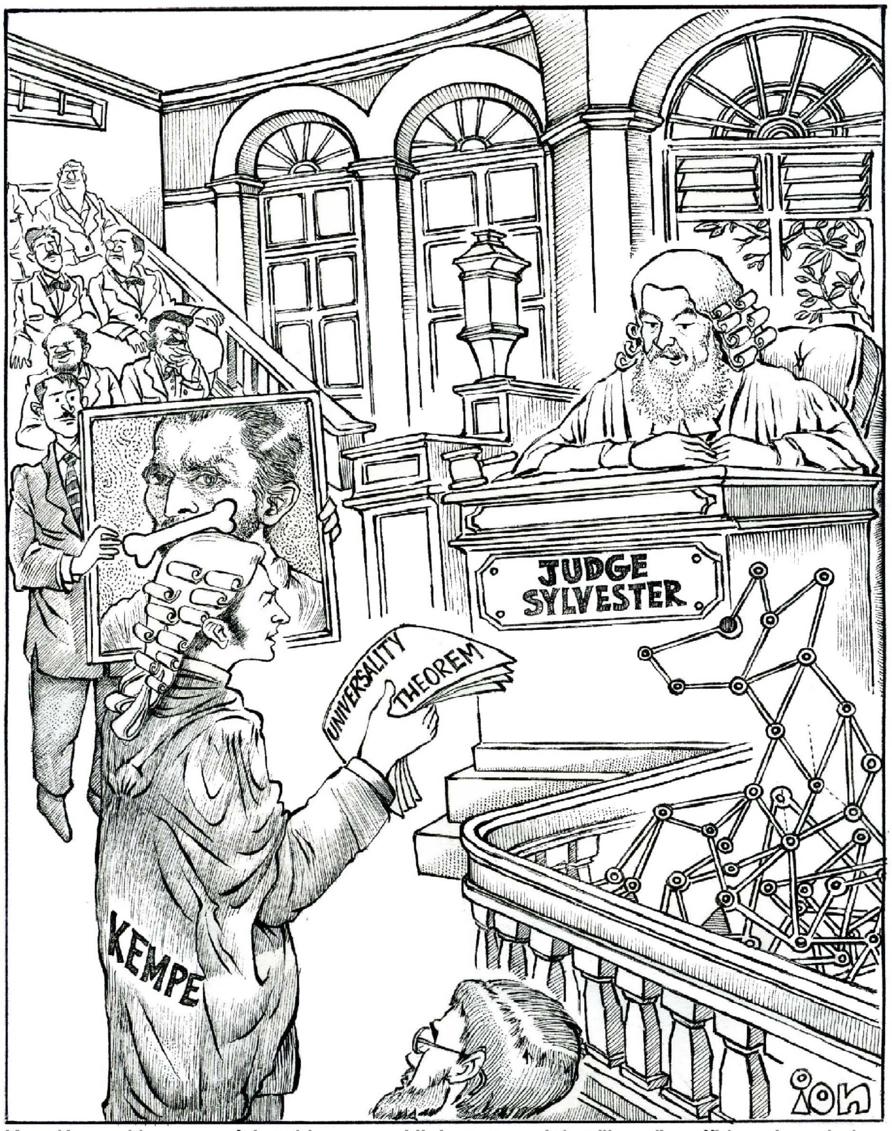
ERRATA

Resonance, Vol.15, No.12, pp. 1074–1083, 2010.
Counting subspaces of a finite vector space – 2
Amritanshu Prasad

Page 1082: In Section 6, Kathleen O'Hara's result on the unimodality of Gaussian binomial coefficients was incorrectly described. We saw in the article that the Gaussian binomial coefficients are polynomials in q with positive integer coefficients. It was the unimodality of these coefficients that Kathleen O'Hara proved: the fact that the coefficient of q^i increases with i up to some point, and then decreases.

Science Smiles

Ayan Guha



Your Honor, I have proof that this accused linkage traced that 'bone' graffiti on the painting.

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