ZENITH ANGLE RESPONSE FOR INCLINED MESON TELESCOPES

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DIRECTIONAL measurements of cosmic ray intensity are mostly carried out by Geiger Counter Telescopes. In order to correlate the daily variation of μ meson intensity measured by such telescopes with anisotropy of primary cosmic radiation, it is essential to know their response characteristics.

Response characteristics of a telescope for any direction can be calculated by knowing the sensitive area of the telescope for the particles coming from that direction and the intensity of radiation in that direction. The sensitive area of a telescope is maximum for particles coming in a direction perpendicular to the plane of the counter tray. A vertical meson telescope, for example, has a maximum sensitive area for particles incident from the vertical direction. A simple geometrical consideration shows that the solid angle available for particles coming in inclined directions is greater than the one available for those coming in vertical direction. Taking into account both these factors one can calculate the geometrical sensitivity G. S. (θ) of any telescope arrangement for different values of the inclination θ which the incoming cosmic ray trajectories make with the vertical. An expression for the same for meson telescopes of cubical geometry was obtained by Parsons.[1] Parsons' method was later extended by the present authors[2] to obtain an expression for the geometrical sensitivity of vertical counter telescopes having rectangular dimensions. Radiation sensitivity and Cumulative sensitivity were calculated assuming a zenith angle attenuation of the form $I_\theta = I_0 \cos^2\theta$ where I_0 and I_θ are cosmic ray intensities in the vertical direction and in a direction inclined to the vertical at an angle θ respectively.

In the case of inclined telescopes, the calculation of response characteristics is rather complicated. No annular ring around the axis of the telescope corresponding to a particular value of θ, the angle of inclination of the incoming particle with respect to the zenith, will have uniform intensity all round the ring. However, considering the problem only in the plane in which the angular opening of the telescopes is narrower (usually E-W plane), we have determined the response of inclined telescopes for different values of θ.

Consider a geometrical arrangement in which the top and bottom trays are represented by XY and AB respectively. Let the breadth $AB = XY = d$ and the length of the telescope be "l". Let the separation between the two trays be $BY = AX = a$. The axis of the telescope is inclined to the vertical at an angle α so that the breadth d is inclined at an angle $(90 - \alpha)$ to the vertical while the length l is horizontal. Depending on the value of θ with respect to α, θ can be grouped into two ranges for each of which the formula to be used for deriving the geometrical sensitivity will be different.

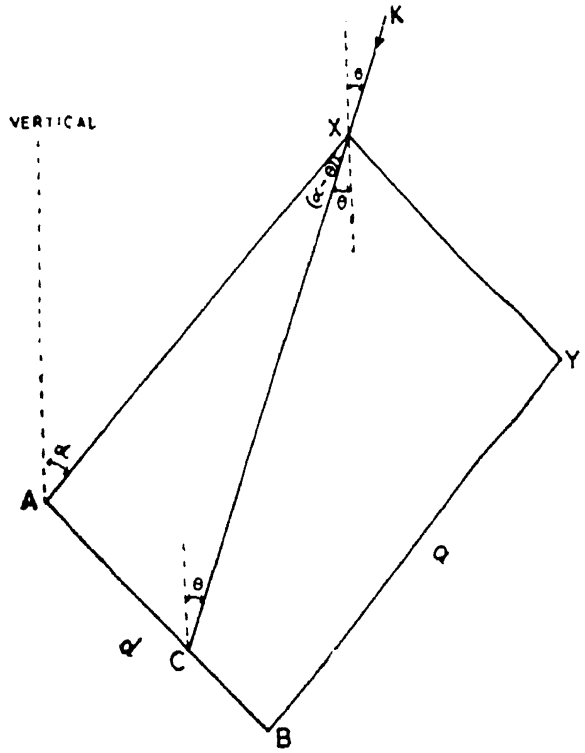

FIG. 1

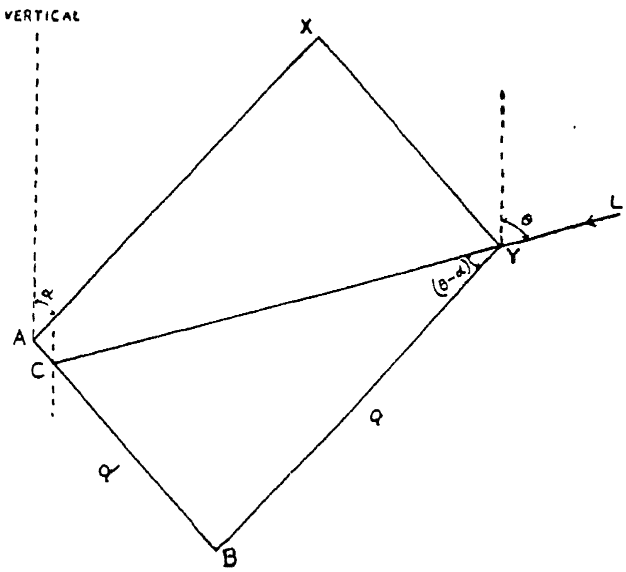

FIG. 2

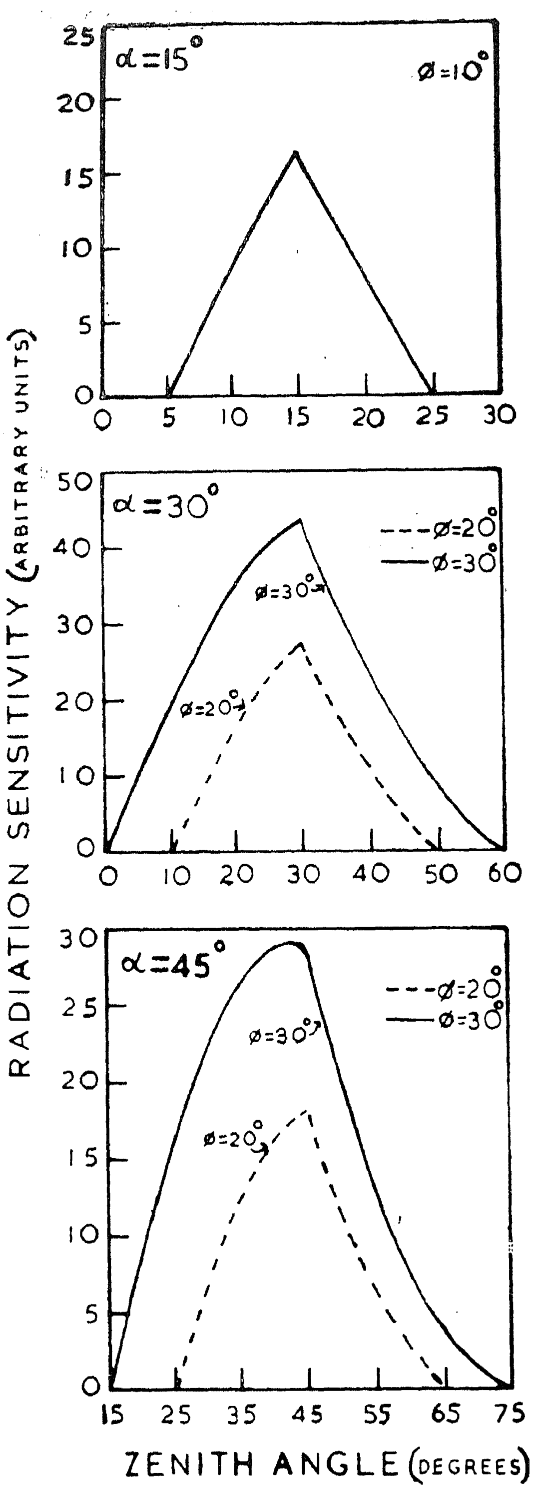

FIG. 3

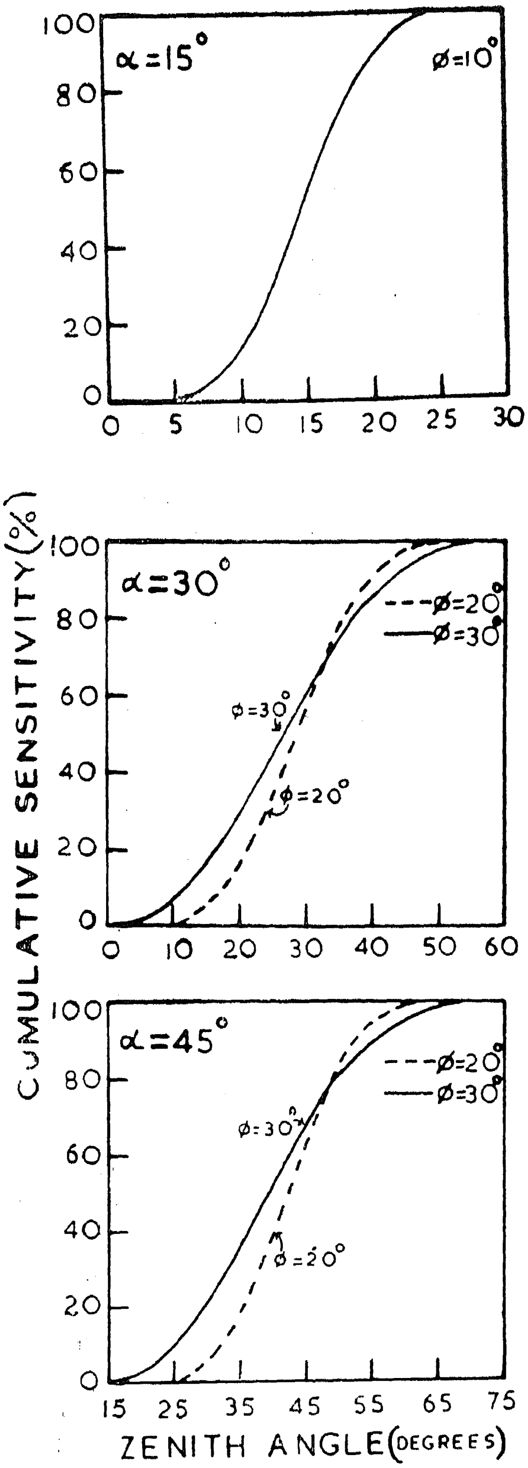

FIG. 4

Fig. 3. Zenith angle dependence of Radiation Sensitivity for meson telescopes inclined to the vertical at an angle α and having a semi-angle of opening ϕ in the narrower plane. Fig. 4. Zenith angle dependence of Percentage Cumulative Sensitivity for meson telescopes inclined to the vertical at an angle α and having a semi-angle of opening ϕ in the narrower plane.

Case A. $\theta \leqslant \alpha$:—Consider cosmic rays incident at a zenith angle θ, where $\theta \leqslant \alpha$. In order to calculate the response, we have to calculate the area perpendicular to the path of the particle in each direction and multiply it by the intensity in that direction. From Fig. 1,

$$BC = AB - AC = d - a \tan(\alpha - \theta). \quad (1)$$

Projection of BC perpendicuar to the ray KC

$$= BC \cos(\alpha - \theta)$$
$$= \{d - a \tan(\alpha - \theta)\} \cos(\alpha - \theta)$$
$$= a\{\delta - \tan(\alpha - \theta)\} \cos(\alpha - \theta) \quad (2)$$

where $\delta = d/a$.

Area perpendicular to the path of the particle

$$= a.l\,\{\delta - \tan(\alpha - \theta)\} \cos(\alpha - \theta). \quad (3)$$

When $\theta = \alpha$, the expression for the area reduces to $(l \times d)$ and when $\theta = (\alpha - \tan^{-1}\delta)$, the area becomes zero, thus satisfying the boundary conditions.

Radiation sensitivity in the direction of the particle is

$$\text{R.S.}(\theta) = a.l. \cos^2\theta \{\delta - \tan(\alpha - \theta)\} \times \cos(\alpha - \theta) \quad (4)$$

if the intensity falls off with the zenith angle as $\cos^2\theta$.

Case B. $\theta \geqslant \alpha$:—Consider the case when the particles are coming in the direction LY making an angle $\theta \geqslant \alpha$ with the vertical.

From Fig. 2, it can be shown similarly that the radiation sensitivity of the telescope R.S.(θ) is given by

$$\text{R.S.}(\theta) = a.l. \cos^2\theta \{\delta - \tan(\theta - \alpha)\} \times \cos(\theta - \alpha). \quad (5)$$

In Fig. 3, are plotted the radiation sensitivity for different inclined telescopes characterised by α, the inclination of the axis of the telescope with the vertical, and $\phi = \tan^{-1}(\delta)$, the semi-angle of opening in the narrower plane.

Since the total counting rate N of a telescope is given by

$$N \propto \int_{(\alpha - \phi)}^{(\alpha + \phi)} \text{R.S.}(\theta)\, d\theta \quad (6)$$

the percentage contribution to the total counting rate of particles confined to the zenith angles between $(\alpha - \phi)$ and any value $(\alpha - \phi + \theta_0)$ is given by the cumulative sensitivity C.S. as

$$\text{C.S.} = \left[100 \int_{(\alpha - \phi)}^{(\alpha - \phi + \theta_0)} \text{R.S.}(\theta)\, d\theta\right] \div \int_{(\alpha - \phi)}^{(\alpha + \phi)} \text{R.S.}(\theta)\, d\theta. \quad (7)$$

The cumulative sensitivity for various zenith angles and for various values of α and ϕ is shown in Fig. 4.

An important result is that most of the radiation comes from a narrow cone along the axis of the telescope. Thus for example for telescopes having a semi-angle of 20° in the narrower plane and inclined to the vertical at 45°, the mean inclination of all radiation recorded is at 42·5° and 50% of the recorded radiation is incident within a range of approximately ± 5·5° of this mean value.

Since the problem has been considered here only in one plane, *viz.*, the plane in which the angular opening of the telescope is narrower, the present method is only approximate. However, the angle of maximum response calculated with the present method for a vertical telescope having semi-angles of opening 20° × 45° turns out to be 13·0° which compares favourably with the value 13·5° obtained by accurate calculations described in our earlier communication.[2] Also, the present method gives an angle of maximum response of 42·0° for Parsons'[3] telescopes of dimension 1 m. × 1 m. × 1½ m. and inclined at 45° to the vertical, which is almost the same as Parsons' calculated value, *viz.*, 42·5°. It seems, therefore, that the approximation involved in the present method does not give errors exceeding ± 0·5° even in case of wide-angle telescopes.

It may be concluded from the above results that for narrow-angle telescopes (semi-angles less than 20°) inclined to the vertical, the maximum response of the telescope is almost along the axis of the telescope. Also, about 50% of the recorded particles are confined to within ± 5° of the axis of the telescope.

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1. Parsons, N. R., *Rev. Sci. Instru.*, 1957, **28**, 265.
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3. Parsons, N. R., *Aust. J. Phys.*, 1957, **10**, 462.