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Inverse Compton Scattering – Revisited

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Abstract. The inverse Compton scattering of high energy electrons by photons is discussed and a simple derivation of the total power radiated is presented. The derivation is completely classical and exhibits clearly why similar formulas are applicable in the case of inverse compton scattering and synchroton radiation.

Key words: Radiation mechanisms.

An important radiative process, used in several astrophysical contexts to generate high energy photons, is the inverse compton scattering. In this process, relativistic electrons transfer part of their kinetic energy to low energy photons thereby creating high energy photons. Conventional text book derivation (Rybicki & Lightman 1979) of this process uses the photon picture, kinematics of electron-photon scattering and a judicious choice of Lorentz transformations to arrive at the final result: the net energy transferred per second from electron to photons is

$$P = \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{4}{3}\sigma_T c\gamma^2 \left(\frac{v}{c}\right)^2 U_{\mathrm{rad}}, \tag{1}$$

where $\sigma_T = (8\pi/3)\ (e^2/mc^2)^2$ is the Thompson scattering cross section, U_{rad} is the energy density of the radiation field, v is the speed of the electron and $\gamma = (1 - v^2/c^2)^{-1/2}$

The conventional derivation raises some interesting questions regarding the nature of this process. To begin with, the final answer has no $\hbar$ dependence, suggesting that the result has nothing to do with the quantum nature of the radiation. In other words, there must exist a purely classical derivation of the total power radiated in inverse compton process. The second interesting feature regarding (1) is the striking similarity between this formula and the one describing the power radiated by an isotropic distribution of electrons in a synchroton process. Several textbooks have emphasized the fact that the net power radiated by relativistic electrons, moving in a constant magnetic field B, can be obtained by replacing U_{rad} by $U_B = (B^2/8\pi)$. In the case of synchroton emission, it is virtually impossible to provide an interpretation in terms of photons and the derivation is completely classical.

The above two observations suggest that there must exist an alternative derivation of (1) based on simple classical considerations. In this note, we shall provide such a derivation.

Consider an electron moving with a four velocity u^i through a region containing electromagnetic radiation. Let the stress tensor corresponding to the radiation be

T_{ab}. The electron, accelerated by the electromagnetic field of the radiation, will radiate energy and – consequently – will feel a drag (four) force g^i. If we can find g^i then the rate of emission of energy can be determined from the component g^0. It turns out that g^i can be determined quite easily from the following considerations.

In the rest frame of the electron, the spatial components of the drag force can be expressed as $\sigma_T \bar{T}^{\mu}{}_0$ where $\bar{T}^{\mu}{}_0$ is the momentum flux of the radiation in the rest frame of the electron with μ = 1, 2, 3 (Landau & Lifshitz 1979). The spatial components of the four vector $f^i = \sigma_T T^i_k u^k$ will have this form in the rest frame of the electron. We can, of course, add to f^i any vector of the form $\sigma_T A u^i$ without altering this conclusion, since the spatial components of the latter vector will vanish in the rest frame. Thus, we expect g^i to have the form $g^i = \sigma_T\,(T^i_k u^k + A u^i)$ where A is yet to be determined. We can fix A by using the requirement that, for any four force, $g^i u_i = 0$; this gives $A = -T_{ab}u^a u^b$. Hence we find that the drag four force acting on an electron, moving through a region containing electromagnetic radiation stress tensor T_{ab}' must have the form

$$g^i = \sigma_T[T^i_k u^k - u^i(T_{ab}u^a u^b)]. \tag{2}$$

As far as the author knows, this result has not been stated explicitly in the literature. A direct derivation of the above formula from the expression for radiation reaction is given in the appendix. This result remains valid whenever F^{ik} does not change rapidly in the region at which the particle is moving. If that is not the case, one can still derive an expression for g^i but it will involve derivatives of F^{ik}.

We shall now use the above formula to obtain the rate of energy emission in the inverse Compton process and some related cases. In the case of an electron moving through a radiation bath, we have $T_{ab} = U_{\rm rad}$ dia(l, 1/3,1/3,1/3) and $u^i = (\gamma\ ,\gamma\ \mathbf{v})$. Hence

$$T_{ab}u^a u^b = U_{\rm rad}\gamma^2(1 + v^2/3); \quad T^a_b u^b = U_{\rm rad}\gamma(1, -\mathbf{v}/3). \tag{3}$$

From these results, we immediately find that g^i— $(\gamma \mathbf{f}\cdot\mathbf{v}, \gamma\mathbf{f})$ with

$$\mathbf{f} = -\tfrac{4}{3}\sigma_T U_{\rm rad}\gamma^2\mathbf{v}; \quad -\mathbf{f}\cdot\mathbf{v} = \tfrac{4}{3}\sigma_T U_{\rm rad}\gamma^2 v^2. \tag{4}$$

Since the rate of energy emission by inverse Compton scattering is $-\mathbf{f}\cdot\mathbf{v}$ we immediately obtain the result (1). The simplicity of the above derivation, compared to the conventional analysis, is noteworthy.

In the case of a charged particle moving in a magnetic field (taken to be along the z-axis) we can perform a similar analysis. In this case we have $T^i_k = U_B$ dia(1, –1, –1, 1). Simple calculation gives

$$\begin{aligned} g^0 &= \sigma_T[T^0_i u^i - u^0(T_{ab}u^a u^b)] = \sigma_T\gamma U_B[1 - \gamma^2(1 + v^2 - 2v_z^2)] \\ &= -\sigma_T\gamma U_B[2\gamma^2 v^2 \sin^2\alpha], \end{aligned} \tag{5}$$

where $v_z = v\cos\alpha$ with α being the pitch angle. This is the standard formula for energy emitted in synchroton radiation by a single charge. For a system of charged particles emitting synchroton radiation it is usual to assume $\langle v_z^2\rangle = v^2/3$ or – equivalently – $\langle \sin^2\alpha\rangle = 2/3$. In this case, we find that $g^0 = -(4/3)\gamma(U_B\ \sigma_T\ \gamma^2 v^2)$.

The rate of emission of energy in the synchroton process is therefore

$$P = \tfrac{4}{3}\sigma_T U_B \gamma^2 v^2. \tag{6}$$

The correspondence between this result and (1) arises due to two facts. (i) The structure of T_{ab} for a constant magnetic field and (ii) the assumption of isotropic distribution of velocities for the electrons allowing $\langle v_z^2 \rangle = (v^2/3)$.

Finally, the equation (2) can also be used to estimate the radiative force on a charged fluid embedded in a slightly anisotropic radiation field. If the charged particles in the fluid are moving with non relativistic velocities, then equation (2) approximates to

$$g^\mu \simeq \sigma_T T_0^\mu + \sigma_T \frac{v^\alpha}{c} T_\alpha^\mu - \sigma_T \frac{v^\mu}{c} T^{00}. \tag{7}$$

We take the slightly perturbed radiation field to have an energy momentum tensor of the form

$$T_b^a = U_{\rm rad}\,{\rm dia}\left(1, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right) + \delta T_b^a, \tag{8}$$

where δT_b^a has a non zero flux $J^\mu = \delta T^\mu{}_0$ .In this case we easily find that g^μ is given by

$$g^\mu \cong \sigma_T U_{\rm rad}\left(J^\mu - \frac{4}{3} v^\mu\right). \tag{9}$$

The first term represents the "push" exerted by the radiation flux and the second term is the drag arising from the inverse Compton effect.

APPENDIX

Consider a particle with charge q and mass m moving in an electromagnetic field F^{ik} which is constant in space and time. The radiation reaction force acting on the particle is given by

$$g^i = \frac{2}{3} q^2 \left(\frac{{\rm d}a^i}{{\rm d}s} - u^i u_k \frac{{\rm d}a^k}{{\rm d}s}\right). \tag{10}$$

When F^{ik} is a constant we have

$$a^i = \left(\frac{q}{m}\right) F^i{}_k u^k; \quad \frac{{\rm d}a^i}{{\rm d}s} = \left(\frac{q}{m}\right)^2 F^i{}_k F^k{}_j u^j. \tag{11}$$

Substituting these expressions in (10) and rearranging the terms we get

$$g^i = \frac{2}{3}\left(\frac{q^2}{m}\right)^2 \left[(F^{ka}F_{kj}) u_a u^j u^i - F^{ki} F_{kj} u^j\right]. \tag{12}$$

Using the definition of T_{ab} we can write $F^{il}F_{kl}$ as

$$F^{il}F_{kl} = F^{li}F_{lk} = -(4\pi) T_k^i + \tfrac{1}{4}\delta_k^i (F_{ab}F^{ab}). \tag{13}$$

Now we can express g^i in terms of T_{ab} alone. Note that

$$\begin{aligned}(F^{ka}F_{kj})u_a u^j u^i - F^{ki}F_{kj}u^j &= u_a u^j u^i[-4\pi T^a_j + \tfrac{1}{4}\delta^a_j F^2] \\ &\quad - u^j[-4\pi T^i_j + \tfrac{1}{4}\delta^i_j F^2] \\ &= -4\pi(T^{aj}u_a u_j)u^i + 4\pi T^{ij}u_j, \end{aligned} \tag{14}$$

since the term involving $F^2 = F_{ab}F^{ab}$ cancels out. Therefore,

$$\begin{aligned} g^i &= \frac{8\pi}{3}\left(\frac{q^2}{m}\right)^2 [T^{ij}u_j - (T^{ab}u_a u_b)u^i] \\ &= \left(\frac{\sigma_T}{c}\right)[T^{ij}u_j - (T^{ab}u_a u_b)u^i], \end{aligned} \tag{15}$$

with $\sigma_T = (8\pi/3)\ (q^2/mc^2)^2$. This relation expresses the radiation reaction in terms of the energy density of electromagnetic field.

When F^{ik} is not a constant, one picks up an additional term on the right hand side of the form

$$g^i_{\text{extra}} = \frac{2q^3}{3m}\frac{\partial F^{ik}}{\partial x^l}u_k u^l. \tag{16}$$

This additional term is ignorable when F^{ik} is constant or when it is due to electromagnetic radiation with $\langle F^{ik}\rangle = 0$.

References

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