J. Astrophys. Astr. (1986) **7,** 59–70

Constraints on Unstable Heavy Neutrinos from Cosmology

M. M. Vasanthi & T. Padmanabhan *Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005*

Received 1985 September 16; accepted 1985 December 11

Abstract. Cosmological scenarios with massive unstable neutrinos are discussed. Restrictions on the mass and the lifetime of the unstable neutrino are derived from (a) age and mass density of the universe and (b) the growth of primordial fluctuations. It will not be possible to accommodate unstable neutrinos with masses above ~ 1 ke V in standard cosmology unless they have exceedingly small lifetime: $\tau < 5 \times 10^8$ s.

Key words: neutrino, unstable—cosmology

1. Introduction

It is by now quite well established that there exist three species of neutrinos. The cosmological implication of massive-ν (mass ~ 10 eV) have been analyzed in the past (Gerhstein & Zel'dovich 1966; Cowsik & McClelland 1973; Doroshkevich *et al.* 1981; Bond, Efstathiou & Silk 1980; Sato & Takahara 1981). Experimental evidence indicating a small but nonzero mass for the lightest of the neutrino species, v_e (Lubimov *et al,* 1980) has generated further interest in the neutrino-dominated universe scenarios.Electron neutrino, if it is massive, can play two crucial roles: (i) it can provide the 'missing mass' in galaxies and clusters of galaxies and (ii) it may help the formation of galaxies and other structures. It is not yet clear as to whether massive neutrinos really achieve either (i) or (ii) in a consistent fashion (For difficulties with 'hot' dark matter scenario see *e.g.* White, Frank & Davies 1983; Hut & White 1984).

If the electron neutrino is massive, then there is every likelihood that the other two species are massive as well. However, a very massive neutrino cannot be stable over cosmological timescales. A stable neutrino with mass of a few ke V would provide the universe with density a few hundred times the closure density—which is clearly ruled out by observational bounds on decelaration parameter (see *e.g.* Sandage 1972). So massive heavy neutrino must be unstable. We discuss in this paper the bounds that can be imposed on the mass and lifetime of such an unstable heavy neutrino. (For previous and related work on this subject, see Davis *et al.* 1981; Turner, Steigman & Krauss 1984; Gelmini, Schramm & Vallee 1984; Olive, Seckel & Vishniac 1985; Fukugita & Yanagida 1984; Dicus,Kolb & Teplitz 1978; Turner 1985.)

It must be emphasized that the present paper is not an attempt at producing a viable cosmological scenario using unstable neutrinos. We are only concerned with the constraints that cosmology imposes on the parameters of unstable heavy neutrino. This approach is motivated by three different considerations. First one was a recent report

(Simpson 1985) claiming evidence for a heavy neutrino of about 17keV mass.So far this result has not been confirmed by other groups. The cosmological implications of this 17 keV neutrino (assuming it to be unstable) was discussed by the authors in a previous paper (Padmanabhan & Vasanthi 1985). It was shown that such a heavy neutrino makes the universe matter dominated (by the decay products of the heavy neutrino) at a redshift z $\lesssim$ 310. Virtually no growth of perturbations occurs in the radiation dominated era. Further, the decay of the heavy neutrino is likely to disrupt and smoothen out some of the past growth. We thus found that a 17 keV neutrino would be a 'burden' to cosmology.In this paper we have extended the analysis and derived bounds on the parameters of the heavy neutrino from considerations of galaxy formation.

The second motivation is related to the interactions suggested for neutrinos. It is known that the simplest weak interaction models give too large a lifetime for massive heavy neutrinos to be cosmologically acceptable. The bounds which we derive on the lifetime and mass of the unstable heavy neutrino may help to distinguish cosmologically viable particle physics models from others.

Thirdly, the study of the parameters of muon and tau neutrino can have an indirect bearing on the lightest stable neutrino. At present, models of galaxy formation distinguish between cold and hot dark matter. Cold dark matter seems to be a current favourite even though the evidence for such a choice is far from decisive. Any clearcut experimental evidence for the mass of any of the neutrinos will upset these scenarios. It seems therefore reasonable to analyze the cosmological implications of heavy neutrino to the same extent done for the light stable neutrino in the past.

Our basic results are the following: (i) It is extremely difficult to incorporate heavy neutrinos of masses greater than about 1 ke V. In order to do that (without violating cosmological constraints), the lifetime of the neutrino has to be kept as low as about 5×10^8 s. This lifetime is much shorter than the value usually obtained ($\sim 5 \times 10^{13}$ s) in flavour-changing decays, (ii) For a given mass m_{H} (< 1 keV) of the heavy neutrino, the lifetime is constrained by $\tau < 6.13 \times 10^{15}$ $(m_{\mathrm{H}}$ / 1 keV$)^{-2}$ h_0^4 s, where $H_0 = 100\ h_0$ km s^{-1} Mpc^{-1} . (iii) For a very narrow range of parameters, it is possible to have a radiation-dominated universe at the present epoch.

The plan of the paper is as follows: In Section 2 we discuss the basic scenario and work out the kinematical constraints. In Section 3 we look at the growth of fluctuations in the above scenario and consider various situations in which perturbations can grow effectively. The conclusions are discussed in Section 4.

2. Basic scenario and kinematical constraints

2.1 *Basic Scenario*

In a standard big-bang model, a neutrino with mass ranging from a few tens of eV up to a few MeV decouples at $t = t_{\mathrm{DC}}$ when the temperature of the universe was about 3.4×10^{10} K (the decoupling temperature of the universe is fairly insensitive to the mass; see for *e.g.* Dicus, Kolb & Teplitz 1978). As the universe cools, the heavy neutrino—ν_{H}—becomes non-relativistic when the temperature is

$$T_{\mathrm{NR}} = m_{\mathrm{H}}c^2/k = 1.16 \times 10^7 \mathrm{K}(m_{\mathrm{H}} / 1\,\mathrm{keV}) \tag{1}$$

where the mass of the heavy neutrino is scaled to 1 keV for convenience. Henceforth we shall not display the scaling explicitly with the understanding that the heavy neutrino m_H is in units of keV. A little later, at $t = t_{eq}$, ν_H starts dominating the energy density of the universe. The temperature T_{eq} and time t_{eq} (since big bang) for this epoch is given by,

$$T_{eq} = 1.43 \times 10^6 \, m_H \, \mathrm{K}, \tag{2}$$

$$t_{eq} = 9.39 \times 10^7 \, m_H^{-2} \mathrm{s}. \tag{3}$$

This is the epoch at which the energy density of the heavy neutrino is equal to that of the rest of the (relativistic) matter. As we shall see below the decay products of ν_H are most likely to be highly relativistic. If the lifetime of ν_H is extremely small ($\tau = t_{eq}$)most of the ν_H will decay before it becomes non-relativistic and starts dominating. If this is the case, the decay products of ν_H as well as rest of the matter will merely continue to evolve as a relativistic soup. *No essential new feature due to the unstable heavy neutrino will remain.* The condition for such (an uninteresting) scenario is $t \lesssim t_{eq}$, or in other words (using Equation 3),

$$m_H^2 (\tau/10^{10}\,\mathrm{s}) \lesssim 9.39 \times 10^{-3}. \tag{4}$$

This also implies an extremely short lifetime for the massive neutrino and cannot be incorporated in Standard particle physics models. Nevertheless, if (4) is satisfied, no further constraints can be imposed on the unstable neutrino. We shall now proceed assuming (4) is violated. That is, we assume

$$m_H^2 (\tau/10^{10}\,\mathrm{s}) > 9.39 \times 10^{-3}. \tag{5}$$

Our constraints in the rest of the paper are for this particular case. In conditions (4) and (5) we have scaled the lifetime τ in units of 10^{10} s. Hereafter we shall not display the scaling in τ explicitly. It will be assumed that τ is in units of 10^{10} s unless otherwise stated. For timescales $t > \tau$ the decay of ν_H becomes important. We can take the decay to be virtually complete at a time $t = t_D$ when $p(\nu_H) < \rho$ (others). The energy density due to ν_H at t is given by

$$\rho_H(t) \simeq m_H c^2 n_H (t_{eq}) [S_{eq}/S(t)]^3 \exp[-(t - t_{eq})/\tau] \tag{6}$$

where $n_H(t_{eq})$ is the number density of ν_H at t_{eq} and S is the expansion factor. The energy density of other forms at this time will be

$$\rho_{\mathrm{others}}(t) \simeq 2.7\, n_\gamma(t_{eq}) k T_{eq} [S_{eq}/S(t)]^4 \tag{7}$$

where $n_y(t_{eq})$is the number density of photons at t_{eq}. Comparing (6) and (7) it is easy to estimate t_D to be

$$t_D \simeq 10.34\, \tau. \tag{8}$$

In other words, ν_H dominates the dynamics up to $t < t_D \simeq 10.34\tau$.

How does the universe behave for $t > t_D$? The decay cannot be radiative because this channel ($\nu_H \to \nu_L$ +photons) is severely constrained by the photon emissivity of the galaxy (Dicus, Kolb & Teplitz 1977; Cowsik 1977; Gunn *et al.* 1978; Silk & Stebbins, 1983). Neither can the decay be due to Standard weak interactions because the lifetime for such decays would be exceedingly large (see *e.g.* de Rujula and Glashow 1980). A remaining possibility is the decay of the heavy neutrino into a light neutrino, ν_L and another light particle, *e.g.* to a Goldstone boson, that couples weakly to a flavour-changing neutrino current (see Wilczek 1982; Gelmini, Schramm & Valle 1984;

Fukugita & Yanagida 1984). Note that the heavy neutrino cannot decay into any cosmologically stable particle of masses greater than tens of electron volts. In virtually all the particle physics models which have been proposed, the companion particle of ν_L is extremely light (< 1 eV) (see references cited above). In that case the energy m_H of the heavy neutrino is shared equally between the decay products. For the sake of definiteness we shall assume the light neutrino (ν_L) produced by the ν_H decay to carry an energy $\frac{1}{2} m_H$. Such a light neutrino would be extremely relativistic (since $\frac{1}{2} m_H \gg m_L$) at the time of decay and the universe again becomes radiation dominated.

When the universe expands further by a factor $f = (m_H/2m_L)$, these ν_L will become non-relativistic. The radiation temperature at the time of decay, $T(\tau)$ is given by (note that for $t_{eq} < t < \tau$, the universe is matter dominated with $S \sim t^{2/3}$)

$$\begin{aligned} T(\tau) &= T_{eq}\,(t_{eq}/\tau)^{2/3} \\ &= (6.36\times 10^4\,\mathrm{K})\,(m_H\tau^2)^{-1/3}. \end{aligned} \tag{9}$$

Similarly, the radiation temperature at the time (t_{nr}) when the decay product ν_L becomes non-relativistic is given by

$$\begin{aligned} T_{nr} &\simeq T(\tau)\,[m_L/(m_H/2)] \\ &= (1.27\times 10^3\,\mathrm{K})\,(m_H^2\tau)^{-2/3}\,(m_L/10\,\mathrm{eV}). \end{aligned} \tag{10}$$

The scenario depends on whether $T_{nr} > T_0$ or $T_{nr} < T_0$ where T_0 is the present day cosmic microwave background radiation (CMBR) temperature. If $T_{nr} < T_0$ we are still in a radiation dominated era, dominated by the relativistic decay products of ν_H. On the other hand, if $T_{nr} > T_0$, the decay products would have become non-relativistic by now. In this case we are in the familiar matter dominated universe. We shall consider these two cases separately.

Case (a): $T_{nr} > T_0$, *matter domination today*:
We shall first compute the age of the universe (t_u) in the scenario. In the radiation phase, $\tau < t < t_{nr}$ $S \sim t^{1/2}$ so that

$$T(\tau)/T_{nr} = S_{nr}/S(\tau) = (t_{nr}/\tau)^{1/2}, \tag{11}$$

while for $t_{nr} < t < t_u$, $S \sim t^{2/3}$, giving

$$T_{nr}/T_0 = S_0/S_{nr} = (t_u/t_{nr})^{2/3}. \tag{12}$$

Together,

$$\begin{aligned} t_u &= t_{nr}\,(T_{nr}/T_0)^{3/2} \\ &= \tau[T(\tau)/T_{nr}]^2\,[T_{nr}/T_0]^{3/2} \\ &= \tau T^2(\tau)/T_{nr}^{1/2}\,T_0^{-3/2}. \end{aligned} \tag{13}$$

Using Equations (9) and (10) in (13), we get

$$t_u = (1.35\times 10^{10}\,\mathrm{yr})(T_0/1.9\,\mathrm{K})^{-3/2}\,(m_L/10\,\mathrm{eV})^{-1/2}. \tag{14}$$

For scaling calculations it is proper to take the CMBR temperature to be 1.9 K. The enhancement of CMBR temperature to the observed 2.7 K is due to the $e^+ e^-$ annihilations. It is not necessary to take into account this completely extraneous effect.

If the minimum age of globular clusters is taken to be 15 billion years, then Equation (14) underestimates the age for m_L greater than 8 eV. Unfortunately, such a marginal contradiction is not of much value. Note that t_u was estimated in Equation (13) by scaling with respect to t_{eq} and by assuming an instantaneous decay of all ν_H at a fixed time τ. Because of these assumptions one cannot entirely rule out a 50 per cent change in t_u. Equation (14) also implies that $h_0 < 0.6$.

It is also easy to verify that our overall picture is consistent in the present case. We assumed in deriving Equation (13) that $t_{nr} < t_u$ or equivalently, $T_{nr} > T_0$. Using (10) we see that this is same as demanding,

$$(1.27 \times 10^3\,\mathrm{K})(m_H^2 \tau)^{-2/3}(m_L/10\,\mathrm{eV}) > 1.9\,(T_0/1.9\,\mathrm{K}). \tag{15}$$

In other words,

$$m_H^2 \tau < 1.73 \times 10^4\,(m_L/10\,\mathrm{eV})^{3/2}(T_0/1.9\,\mathrm{K})^{-3/2}. \tag{16}$$

which, as we shall see, will be identically satisfied (see Equation 22 below)

Case (b): $T_{nr} < T_0$, radiation-domination today: This situation turns out to be somewhat more tricky. To begin with, note that the constraint $T_{nr} < T_0$ is equivalent to the reverse of case (a), *i.e.*,

$$m_H^2 > 1.73 \times 10^4\,(m_L/10\,\mathrm{eV})^{3/2}\,(T_0/1.9\,\mathrm{K})^{-3/2}. \tag{17}$$

Let us now compute the age in the scenario. Calculations similar to the one above give,

$$\begin{aligned} t_u &= \tau\,[T(\tau)/T_0]^2 \\ &= (3.6 \times 10^{11}\,\mathrm{yr})\,(T_0/1.9\,\mathrm{K})^{-2}\,(m_H^2 \tau)^{-1/3}. \end{aligned} \tag{18}$$

The constraint $t_U \quad 10^{10}$ yr now implies that

$$(m_H^2 \tau) < 4.67 \times 10^4\,(T_0/1.9\,\mathrm{K})^{-6}. \tag{19}$$

Thus this scenario can operate only if τ simultaneously satisfies the inequalities,

$$(1.73 \times 10^4/m_H^2)(m_L/10\,\mathrm{eV})^{-3/2} < \tau < (4.67 \times 10^4/m_H^2)(T_0/1.9\,\mathrm{K})^{-6}.$$

We shall continuously refer to cases (a) and (b) in what follows.

2.2 *Constraints from Energy Density of Decay Products*

In both the above cases—(a) and (b)—a further constraint on m_H and τ can be obtained based on the energy density contributed by the decay products of ν_H. This energy density is easily computed to be (see *e.g.* Dicus, Kolb & Teplitz 1978),

$$\begin{aligned} \rho &= m_H\, n_H(t_{DC})(1.9\,\mathrm{K}/T_{DC})^3 \int_{t_0}^{t_u} \mathrm{d}t/\tau\,(t/t_u)^{1/2} \exp[-(t-t_0)/\tau] \\ &\simeq n_H(t_{DC})(1.9\,\mathrm{K}/T_{DC})^3\,(\sqrt{\pi}/2)\,m_H\,(\tau/t_u)^{1/2} \\ &= (1.66 \times 10^{-28}\,\mathrm{g\,cm^{-3}})\,m_H(\tau/t_u)^{1/2}. \end{aligned} \tag{20}$$

The decay time t is not scaled in this expression; also it is assumed that $t_{DC} < T < t_u$.

Demanding $\rho < \Omega_m p_c$, where Ω_m is the maximum density parameter contributed by the decay product (if Ω_b is the baryonic contribution and W_L is the contribution from the primordial light neutrino, then $\Omega_m = [1 — (\Omega_L + \Omega_b)]$, we get (with $H_0 = 100\,h_0$ kms $^{-1}$ Mpc^{-1})

$$(\tau/t_u) < 1.44 \times 10^{-2}\,\Omega_m^2 h_0^4 m_H^{-2}. \tag{21}$$

Translation of this constraint in terms of m_{H} and τ is different for the two cases (a) and (b). In case (a) t_u is given by Equation (14) and combining (14) and (21) we get

$$m_{\mathrm{H}}^2 \tau \leqslant 6.13 \times 10^5 \Omega_{\mathrm{m}}^2 h_0^4 (T_0/1.9\,\mathrm{K})^{-3/2} (m_{\mathrm{L}}/10\,\mathrm{eV})^{-1/2}. \tag{22}$$

Similarly for case (b) we get [combining (18) and (21)],

$$m_{\mathrm{H}}^2 \tau < 2.51 \times 10^5 \Omega_{\mathrm{m}}^{3/2} h_0^3 (T_0/1.9\,\mathrm{K})^{-3/2}. \tag{23}$$

which is only marginally different from (19). Since (19) is anyway satisfied in case (b) we do not get any new constraint from (23).

All the kinematic constraints are summarized in Fig. 1 where lifetime is plotted against the mass m_{H}. The vertical line at m_H = 10 eV and the horizontal line at $\tau = 4.26 \times 10^{17}$ seconds limit the region we are considering. Physically this corresponds to assuming the ν_{H} to be more massive than ν_{L} (~ 10 eV) and unstable within the age of the universe(i.e. $\tau < t_u$). The broken line above this line ($\tau = 1.73 \times 10^4\, m_{\mathrm{H}}^{-2}$) separates case (a) and case(b). The region above this corresponds to universes which are radiation dominated now. In that case, Equation (19) gives the upper bound based on the age of the universe. In order for the universe to be radiation dominated today, ν_{H} has to decay fairly late (i.e. in the recent past). On the other hand, this would lead to a rather youngish universe. Hence only a narrow region marked in the figure is allowed for case (b).

We do not get such constraints in case (a). Energy density requirement represented by Equation (22) does not lead to any new constraint because case (a) already satisfies the stronger constraint *via* Equation (16). In other words, the constraints are fairly weak for case (a).

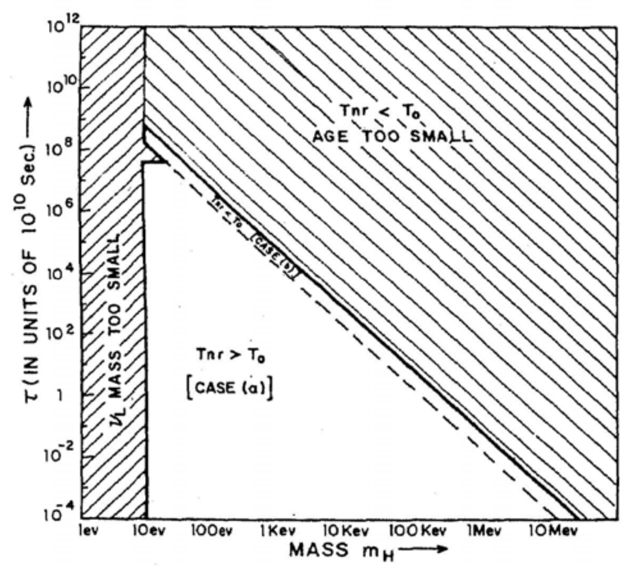

Figure 1. The lifetime τ (in units of 10^{10} s) is plotted against the mass of the heavy neutrino, m_{H}. The broken line separates the two cases (a) and (b) (see text). For the case (a), the regions above the horizontal line at $\tau \simeq 10$ı s are not allowed from the constraint that τ is less than the age of the universe. For case (b) this constraint is given by the thick line just above the broken line. In both the cases the regions which are not allowed are struck out.

We shall now study the growth of fluctuations in the above scenario, which will lead to much stronger constraints.

3. Growth of fluctuations

tudies about the growth of density fluctuations in an expanding universe (Peebles 1979) have led to the following conclusions: (a) The growth is significant only when the wavelength of fluctuation is less than the size of the horizon, (b) No fluctuations can grow in a universe dominated by relativistic particles (Meszaros 1974). (c) In the matter dominated era the density contrast $\delta = (\delta\rho/\rho)$ grows as $t^{2/3}$. In other words $\delta \propto S(t) \quad [T(t)]^{1}$.

Baryonic fluctuations cannot grow until the recombination era ($z \sim 10^3$) and hence the baryonic density perturbations cannot grow by more than a factor of 10^3 between recombination and today. This implies the existence of $\delta_B \geqslant 10^3$ at $z{\sim}10^3$. For adiabatic fluctuations, $(\delta\rho/\rho)_B$ is directly related to the fluctuations in CMBR temperature. A value of $\delta_B \sim 10^{3}$ is entirely ruled out by the observational upper limit ($\Delta T/T < 6 \times 10^{-5}$; see Fixsen, Cheng & Wilksen 1982) on the anisotropy of CMBR. One possible solution to this dilemma is to have a scenario in which non-baryonic matter can dominate before the recombination era. Their growth of fluctuations can create a potential well into which the baryons can fall immediately after they decouple from radiation.

We have seen that the non-relativistic ν_H starts dominating the energy density from T_{eq} until they decay at the temperature $T(\tau)$. It is possible for perturbations in ν_H to grow during this epoch when the universe is matter dominated. Present day studies of galaxy-galaxy correlation functions show that the scales which are entering the nonlinear regime today (*i.e.* $\delta \lesssim 1$) have sizes $L \simeq 5h_0^{-1}$ Mpc (Davis & Peebles 1983). This length scale L would have entered the horizon in the past at $t = t_L$ when the radiation temperature was, say, T_L. We shall now estimate T_L. Two scenarios arise depending on whether this scale entered before the decay of ν_H or after (*i.e.* whether $T_L > T_t$ or $T_L < T_t$). We will consider these cases separately.

If $T_L < T_T$ then fluctuations enter the horizon in the radiation dominated era. If $d_H(t_L)$ is the horizon size at time t_L, then

$$d_H \simeq c t_L = c\tau T^2(\tau)/T_L^2. \tag{24}$$

The physical wavelength λ of the fluctuations scale with expansion linearly; *i.e* $\lambda(t) \propto S(t)$. Fluctuations with a wavelength L today would have a wavelength $\lambda(t_L)$ at time $t = t_L$, where

$$\lambda(t_L) = L(T_0/T_L). \tag{25}$$

Therefore the condition $d_H = \lambda$ ('fluctuations entering horizon') implies (compare (24) and (25))

$$T_L = (c\tau/L)[T^2(\tau)/T_0]. \tag{26}$$

Our condition $T_L < T(\tau)$ is equivalent to

$$\tau < 3.33\, h_0^{-3} m_H (T_0/1.9\,\mathrm{K})^3. \tag{27}$$

The fluctuations enter the horizon after the ν_H decay, in the radiation dominated universe; however *it cannot grow in this epoch.* If $T_{nr} < T_0$ (case (b) considered in the previous section in which the universe remains radiation dominated from $t = \tau$ until today) then the *fluctuations cannot grow at all.* This completely rules out the situation corresponding to case (b) if $T_L < T(\tau)$. On the other hand if $T_{nr} > T_0$ the fluctuations can grow after (and only after) the decay products have become non-relativistic. Thus the growth occurs from $t = t_{nr}$ till today. The growth factor in this matter-dominated era is

$$\varepsilon \equiv T_{nr}/T_0$$
$$\simeq 6.68 \times 10^2 (m_H^2 \tau)^{-2/3} (m_L/10\,\mathrm{eV})\,(T_0/1.9\,\mathrm{K})^{-1}. \tag{28}$$

It is known that the spectrum of primordial fluctuations has an amplitude $(\delta\rho/\rho) \sim 10^{-4}$ when they enter the horizon. To achieve $(\delta\rho/\rho) \sim 1$ today, the growth factor ε has to be $\geqslant 10^4$. Using (28) we get

$$m_H^2 \tau < 1.73 \times 10^{-2} (m_L/10\,\mathrm{eV})^{3/2} (T_0/1.9\,\mathrm{K})^{-3/2}. \tag{29}$$

While the above situation is a theoretical possibility it is of little practical importance. Note that the above condition is almost the same as Equation (4). This corresponds to a situation consisting of the following ordering of events: ν_H decays very early ($\tau \sim t_{eq}$) leaving a relativistic soup; fluctuations enter the horizon in the relativistic epoch; Decay products become non-relativistic at temperatures 10^4; fluctuations grow in the matter-dominated era. Clearly this is indistinguishable from the conventional single stable neutrino (with mass ~ 10 eV) scenario. We also note that the above constraint demands an extremely short lifetime for the unstable neutrino which is not compatible with standard particle physics.

On the other hand, if $T_L > T(\tau)$, fluctuations enter the horizon in the regime when the non-relativistic ν_H dominates the energy density. In this case, Equation (24) is replaced by

$$d_H = c\,\tau\,[T(\tau)/T_L]^{3/2} \tag{30}$$

and since

$$\lambda = L(T_0/T_L)$$

we get

$$T_L = T(\tau)(c\tau/L)^2 [T(\tau)/T_0]^2. \tag{31}$$

The assumption $T_L > T(\tau)$ now translates to the inequality,

$$\tau > 3.33\,(T_0/1.9\,\mathrm{K})^3\, h_0^{-3}\, m_H. \tag{32}$$

The growth of fluctuation is again different for case(a) and case(b). In case(a) fluctuations can grow during two different matter-dominated epochs: (A) $t_L < t < \tau$ and (B) $t_{nr} < t < t_u$. During the (in between) radiation-dominated phase ($\tau < t < t_{nr}$) the fluctuations oscillates as an acoustic wave with negligible growth. Let ε_A be the growth factor in the ν_H dominated era (A) and ε_B be the one that arises after the decayed products become non-relativistic (B). Then

$$\varepsilon_A = T_L/T(\tau) = (c\,\tau/L)^2 [T(\tau)/T_0]^2$$
$$= 0.45\, h_0^2\, (\tau/m_H)^{2/3} (T_0/1.9\,\mathrm{K})^{-2} \tag{33}$$

while ε_B is given by Equation (28). The total growth factor is

$$\varepsilon = \varepsilon_A \times \varepsilon_B$$
$$= 2.99 \times 10^2\, m_H^{-2}\, h_0^2\, (T_0/1.9\,\mathrm{K})^{-3} (m_L/10\,\mathrm{eV}). \tag{34}$$

The requirement $\varepsilon > 10^4$ then implies

$$m_H < 1.73 \times 10^{-1} h_0 (T_0/1.9\,\mathrm{K})^{-3/2} (m_L/10\,\mathrm{eV})^{1/2} \tag{35}$$

i.e., mass of the heavy neutrino must be less than about 0.173 keV.

We note that ε is independent of τ while dependent on m_H^{-2}. These dependences can be understood as follows: From Equation (31) note that

$$\begin{aligned} T_L &= (\tau/L)^2 [T(\tau)/T_0]^2 T(\tau) \\ &= (t_{eq}/L)^2 (T_{eq}/T_0)^2 T_{eq} \propto 1/m_H \end{aligned} \tag{36}$$

and that T_L is independent of t (of course, for computational purpose it is convenient to scale it with respect to τ). The growth factor

$$\begin{aligned} \varepsilon = \varepsilon_A \times \varepsilon_B &= [T_L/T(\tau)](T_{nr}/T_0) \\ &= (T_L/T_0)(2m_L/m_H) \propto m_L/m_H^2 \end{aligned} \tag{37}$$

which is independent of t (we have used (10) and (36)). Larger m_H leads to smaller T_L (delay in the entry of fluctuations into the horizon) and smaller T_{nr} (longer duration of relativistic epoch). Both the effects hinder the growth of fluctuations.

The final scenario that we have to consider corresponds to case(b) with $T_L > T(\tau)$. In this situation phase B does not exist because the universe is still radiation dominated. Thus, all the growth must take place during phase A. Using the expression for ε_A (see Equation 33) and imposing the condition $\varepsilon_A > 10^4$ we get

$$\tau > 3.33 \times 10^6 (T_0/1.9\,\mathrm{K})^3 h_0^{-3} m_H \tag{38}$$

The constraints from the growth of fluctuations are depicted in Fig. 2. Line marked (1) distinguishes case(a) from case(b), while the line marked (2) distinguishes $T_L < T(\tau)$ from $T_L > T(\tau)$. These two lines divide t-m_H plane into four regions. The region which is to the right of both (1) and (2) is completely ruled out. For case(a) with $T_L = T(\tau)$ the parameters are restricted by Equation (29). If $T_L > T(\tau)$ the only constraint is on the mass ($m_H < 173$ eV). We have also marked the vertical line at $m_H = 10$ eV. In case *B,* when $T_L > T(\tau)$ only regions above the line corresponding to Equation (38) are allowed.

Before proceeding further, it is probably worthwhile to consider various assumptions upon which Fig. 2 is based. In arriving at these constraints, we have made (rather drastic) simplifications.

(i) We have neglected all the disruptive effects of the decay of ν_H on the already formed structures. The decay can effect the structure in two ways. Firstly it can violently disrupt the condensates. Secondly, free streaming of relativistic decay products can wipe out smallscale inhomogeneities. Both these effects can change the perturbation spectrum at large scales. Hopefully this will not be important at scales of few Mpc and will manifest only at supercluster scales. This effect is under investigation.

(ii) We have assumed that there is no growth during the radiation-dominated epoch. This is not strictly true because there can be some growth due to the residual velocity field of the matter after it crosses the horizon (Primack & Blümenthai 1984). This can relax the constraints slightly.

(iii) In one particular case, we have obtained the total growth factor by multiplying the growth factors at two different epochs. If too much of structure dissipation occurs following the decay, then this calculation may not be strictly correct.

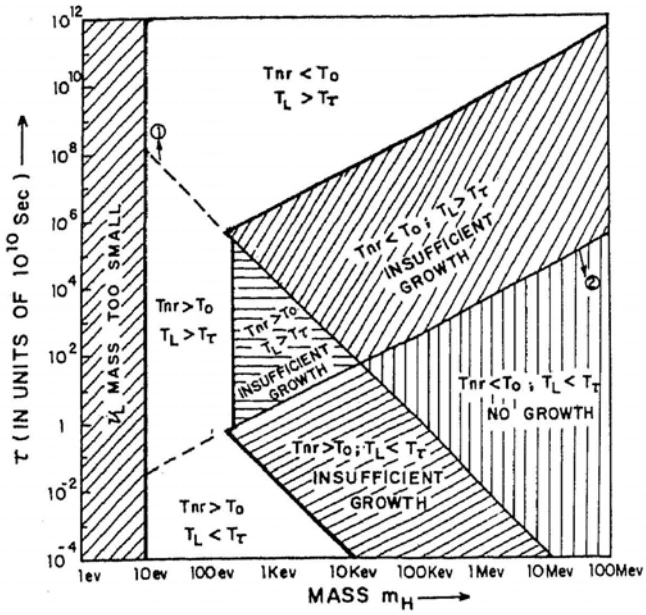

Figure 2. Lifetime τ is plotted against mass m_{H}. Line (1) distinguishes case (a) from case(b) while line (2) separates the region $T_{\mathrm{L}} > T(\tau)$ from $T_{\mathrm{L}} < T(\tau)$. The thick line separates the allowed regions (from growth of fluctuation analysis) from the 'not allowed' regions. For case (a) with $T_{\mathrm{L}} > T(\tau)$ the only constraint is that $m_{\mathrm{H}} < 173$ eV, while for $T_{\mathrm{L}} < T(\tau)$ growth is sufficient only for regions to the left of the line given by Equation (29) (see text). For case (b), $T_{\mathrm{L}} < T$ (t) has no provision for growth to occur at any time (region below line). For $T_{\mathrm{L}} > T(\tau)$ growth is sufficient only for regions above the line given by Equation (38).

We plan to investigate these dynamical effects in a future publication. At this stage we may say that the effect of (i) to (iii) above makes the boundary lines drawn in Fig. 2 (and in Fig. 3) less sharp. As long as a particular (m, τ) value falls reasonably within a region one can use these constraints with confidence.

4. Conclusions

The constraints derived in Section 2 and 3 are combined into Fig. 3. The region bordered by the thick line in Fig. 3 gives the allowed domain. Clearly the top region of Fig. 2 is severely constrained by the age of the universe as shown in Fig. 1. The allowed physics with unstable neutrinos can be understood by considering three typical points in regions marked A, B and C.

Region A probably holds the least interest. Here the perturbations enter the horizon *after* the decay of unstable neutrinos, in the radiation-dominated era. The growth can take place only after the decay products have become non-relativistic. These constraints imply that the decay must occur too early ($\tau = T_{\mathrm{eq}}$) for any interesting feature of ν_{H} to remain. Also note that the typical lifetime in this domain is $\sim 10^7$ s which is too small by conventional particle physics standards for ν_{H}.

In region B the fluctuations enter the horizon before the decay and hence can grow during the first matter-dominated era as well as after the decay products have become non-relativistic. A wide range in lifetime is available for B, but the mass of the neutrino is constrained to be less than about 173 eV.

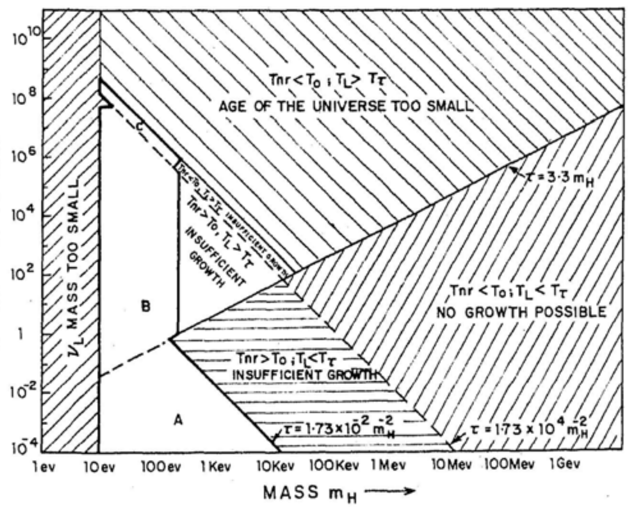

Figure 3. The constraints depicted in Figs 1 and 2 are combined in this figure. The regions allowed are within the thick lines.

Both regions A and B correspond to the situation in which the present day universe is matter dominated. In contrast, region C corresponds to a universe which is still radiation dominated. Clearly parameters in C are most severely restricted.

With no extra input from particle physics it is impossible to constrain the lower region of the figure. However, any realistic modelling of neutrino interactions will give a lower bound to the lifetime. Given such a constraint (external to cosmology) the parameter space will be restricted to a compact simply connected domain in m-τ plane. For example, if we make the conservative assumption that the lifetime is definitely greater than about 10^7 s, then the mass of the unstable neutrino cannot be greater than about 10 keV. Note that this is a reasonably powerful constraint in the modelling of neutrino interactions.

References

Bond, J. R., Efstathiou, G., Silk, J. 1980, *Phys. Rev. Lett.,* **4**5, 1980.

Cowsik, R. 1977, *Phys. Rev. Lett.,* **39, 784**.

Cowsik, R., McClelland, J. 1973, *Astrophys. J.,* **180**,

7.Davis, M., Lecar, M., Pryor, C, Witten, E. 1981, *Astrophys. J.,* **250,** 423

Davis, M., Peebles, P. J. E. 1983, *Astrophys. J.,* **267,** 465.

de Rujula, A., Glashow, S. L. 1980, *Phys. Rev. Lett.,* **45,** 942

Dicus, D. A, Kolb, E. W., Teplitz, V. L. 1977, *Phys. Rev. Lett.,* **39,** 168.

Dicus, D. A., Kolb, E. W., Teplitz, V. L. 1978, *Astrophys. J.,* **221**, 327.

Doroshkevich, A. G., Khlopov, M. Yu., Sunyaev, R. A., Szalay, A. S., Zeldovich, Ya. B. 1981, *Proc N.Y. Acad. Sci,* **375,** 32.

Fixsen, D. J., Cheng, E. S., Wilkinson, D. T. 1982, *Phys. Rev. Lett,* **50**, 620.

Fukugita, M., Yanagida, T. 1984, *Phys. Lett.,* **144B,** 386.

Gerhstein, S. S., Zeldovich, Ya. B. 1966, *JETP Lett.,* **4**, 174.

Gelmini, G, Schramm, D. N., Valle, J. W. F. 1984, *Phys. Letts.,* **146B,** 311.

Gunn, J., Lee, B. W., Lerche, L, Schramm, D. N., Steigman, G. 1978, *Astrophys. J.,* **223**, 1015.

Hut, P., White S. D. M. 1984, *Nature,* **310**, 637
Lubimov, V. A., Novikov, E. G., Nozik, V. Z., Tretyakov, E. F., Kosik, V. S. 1980, *Phys. Lett.,* **94B,** 266.
Meszaros, P. 1974, *Astr. Astrophys.,* **37**, 225.
Olive, K. A., Seckel, D., Vishniac, E. 1985, *Astrophys. J.,* **292**, 1.
Padmanabhan, T., Vasanthi, M. M. 1985, *Nature,* **317**, 335.
Primack, J. R., Blumenthal, G. R. 1984, in *Clusters and Groups of Galaxies,* Eds F. Mardirossian, G. Giuricin & M. Mezzetti, D. Reidel, Dordrecht, p. 435.
Peebles, P. J. E. 1980, *LargeScale Structure of the Universe,* Princeton Univ. Press.
Sandage, A.1972, *Astrophys. J.,* **173**, 485.
Sato, H., Takahara, P. 1981, *Prog. Theo. Phys.,* **65**, 374; **66**, 508.
Silk J., Stebbins, A. 1983, *Astrophys. J.,* **269**, 1.
Simpson, J. J. 1985, *Phys. Rev. Lett.,* **54**, 1891.
Turner, M. S. 1985, *Phys. Rev.,* **D3**1, 1212.
Turner, M. S., Steigman, G. S., Krauss, L. M. 1984, *Phys. Rev. Lett.,* **52**, 2091.
White, S. D. M., Frenk, C. S., Davis, M. 1983, *Astrophys. J.,* **274**, LI.
Wilczek, F. 1982, *Phys. Rev. Lett,* **49**, 1549.