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Nature and Distribution of Dark Matter: 1. Dwarf Spheroidals and Milky Way

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Abstract. We argue that observations on Milky Way and dwarf spheroidals imply existence of individual haloes around dwarf spheroidals. If neutrinos (or any other 'hot' particle) provide the dark matter then we show that: (i) Embedding of visible matter inside large (~ few Mpc) dark matter islands is observationally untenable. (ii) Dwarf spheroidals possess dark matter haloes of about 10 kpc radius around them, and have an (*M/L*) ratio of about 10^4. (iii) The haloes of spiral galaxies (*e.g.* Milky Way) extend to about 100 kpc in radius. If 'cold' dark matter makes up the haloes, then no significant constraints are obtained. We discuss briefly the effect of these constraints on larger scales.

Key words: dark matter—dwarf spheroidals—neutrinos—Milky Way

1. Introduction and summary: Is dark matter hot or cold?

It is likely that most of the matter in the universe is invisible; that is, it emits little or no electromagnetic radiation. The dark matter makes its presence known through gravitational effects. The flat rotation curves of spiral galaxies (Rubin 1979; Rubin *et al.* 1982; Rubin, Thonnard & Ford 1982) and the mass to light ratios of large virialized clusters (see *e.g.* Rood 1981; Faber & Gallagher 1979), are most easily interpreted in terms of dark matter haloes. (Alternative interpretations, involving modification of dynamical laws will not be considered in this paper; (see Milgrom 1983; Bekenstein & Milgrom 1984).

What does the invisible halo consist of? Since most of the visible matter is made of baryons, one may attempt to build the haloes from baryonic matter. However, a variety of observational constraints make baryonic dark matter an unattractive alternative, if not an impossibility, (Hegyi 1984).

Leptonic dark matter could consist of any of the host of particles postulated to exist by the particle physicists. Among leptons, massive neutrinos were one of the earliest candidates (Gerhstein & Zeldovich 1966; Cowsik & McClelland 1972; Marx & Szalay 1972). An experimental claim (as yet unconfirmed by other teams) that electron neutrinos are massive gave impetus to this idea (Lubimov *et al.* 1980). Considerable amount of work was done in recent years regarding the kinematics and dynamics of neutrino dominated universe (Sato & Takahara 1980; Bond, Efstathiou & Silk 1980; Doroshkevich *et al.* 1981; Klinkhamer & Norman 1981; Wasserman 1981; Peebles

1982). Two disturbing features emerged from this analysis: (i) If massive neutrino haloes exist around dwarf spheroidals, with scale lengths comparable to that of visible matter, then the neutrino mass should be greater than about 530 eV which is completely ruled out (Aaronson 1983; Lin & Faber 1983). (ii) The numerical experiments suggest that galaxy formation in a neutrino dominated universe would have taken place at redshifts $z < 2$, which is in contradiction with the existence of high redshift objects conventionally interpreted as the nuclei of galaxies (Frenk, White & Davies 1983; Dekel & Aarseth 1984; Hut & White 1984; Kaiser 1983; Mellot 1983; Faber 1984). These two features make neutrinos rather unattractive. (For attempts to reconcile these features with the hypothesis of neutrino dominance, see Cowsik & Ghosh 1986; Mellot 1985.)

Motivated by these considerations, many people have attempted to model the dark matter by supersymmetric fermions ('sparticles') and axions. Since these particles are heavier than neutrinos, their 'thermal' velocities will be lower, earning them the name 'cold dark matter'. (Neutrino, on the other hand, is an example of 'hot dark matter'.) Cold dark matter can be made to avoid the two difficulties mentioned in the previous paragraph with relative ease (Blumenthal *et al.* 1984; Primack 1984). On the other hand they seem to face some trouble in explaining the largest scale structures *viz.* superclusters and voids (Primack & Blumenthal 1984). Besides, the experimental evidence for the existence of many of the cold dark matter candidates is weaker than that for the nonzero mass of the neutrino. (Theoretical ideas have to be hastily reshaped if the mass of any species of neutrinos is proved to be definitely non-zero!).

Taking an unprejudiced viewpoint, one may ask: Do observations give a clear cut 'yes' or 'no' answer to the existence of 'hot' or 'cold' dark matter?

We attempt to discuss this question in a series of three papers. In the present paper we analyse the constraints on dark matter distribution which arise from observations on our Galaxy and the dwarf satellites. The second paper will discuss groups and clusters of galaxies; the third paper will consider various dynamical aspects of clustering.

Within the scope of existing observations, we have *not* been able to provide a clear cut 'yes-no' answer to the question we have raised. However, rather stringent constraints can be imposed on the scale length, shape and densities of dark matter haloes. We find that neutrino ('hot' dark matter) distribution is much more severely constrained than any cold dark matter scenario. If neutrinos constitute the dark matter, then, we show:

(i) Scenarios in which galaxies are embedded in large (~ Mpc) neutrino 'islands' are ruled out by observations.
(ii) The halo around our Galaxy cannot extend significantly beyond ~ 60 kpc.
(iii) Dwarf spheroidals must have, a halo which extends upto about 10 kpc from their centre. This will give dwarf spheroidals a mass to light ratio of 10^4(!) making them very peculiar objects.

Cold dark matter, on the other hand does not lead to such stringent conditions. We leave the reader to judge for himself whether these constraints effectively rule out neutrino dominance.

The paper is organized as follows: In Section 2, we review and analyse the existing observational data about Milky Way and dwarf spheroidals. Section 3 compares standard theoretical modelling with the observations and determines the constraints. Section 4 discusses various offshoots, arguments and counterarguments based on the previous sections.

2. Dark matter observations

2.1 *Milky Way*

The visible matter in our Galaxy does not extend significantly beyond l0 kpc. Presumably the effects of dark matter haloes would be most pronounced at distances > 10 kpc. The rotation curve derived from 21 cm observations is reasonably flat right from about 8 kpc. Using CO observations of molecular complexes related to H II regions, the flat rotation curve can be extended out to 16 kpc. At larger distances, globular clusters can be used as tracers of dark matter. The data from globular clusters (as well as 21 cm and CO observations) are summarized in Fig. 1. (The data are taken from the conclusions of Innanen, Harris & Webbink 1983; Faber & Gallagher 1979; Hartwick & Sargent 1978; Peebles 1979; Mihalas & Binney 1981; Gunn, Knapp & Tremaine 1979; Similar data are also presented in Lynden-Bell 1983.)

Within the limits of observational error, the data are very well fit by the mass radius curve,

$$M(r) = (1.4 \pm 0.2) \times 10^{10}\, M_{\odot}\, (r/1\ \text{kpc}). \tag{1}$$

The error-bar in the coefficient arises from the spread in the data points. The mean curve (solid line) and the spread (broken lines) are shown in Fig. 1.

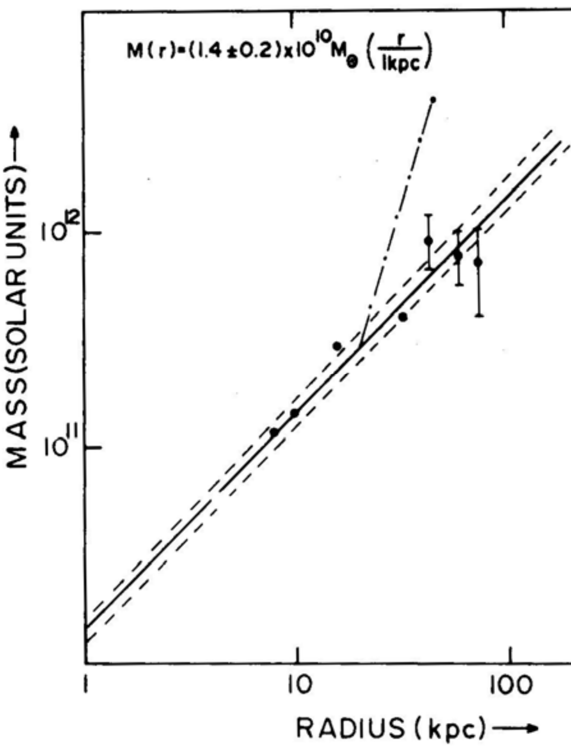

Figure 1. Total mass *M* within a radius *r* plotted against the radius *r* of the Milky Way galaxy. The solid line is the best fit curve for *M*(*r*) equation. The broken lines are the best fit lines with upper and lower limits on error bar. The dot-dash line represents the *M* – *r* curve corresponding to Equation (13) in the text.

For judging the goodness of fit, the data were tested with a power law $M \propto r^n$. The best fit value for n turns out to be 0.9 with a σ^2 (mean square deviation) of 0.0095. On the other hand, the theoretical curve $M \propto r$, shown in the figure has a σ^2 of 0.01 which is comparable to 0.0095, indicating a good fit. For future reference, we may note that $M \propto r^3$ curve leads to a σ^2 of 0.58, nearly sixty times higher.

We note that, in the range 8 kpc < r < 75 kpc, the mass distribution (1) is equivalent to the density fall off

$$\rho(\mathrm{r}) = 8.1 \times 10^{-23} \mathrm{g\ cm}^{-3}\ (r/1\ \mathrm{kpc})^{-2} \tag{2}$$

2.2 *Dwarf Spheroidals*

The seven dwarf spheroidals Fornax, Sculptor, Leo I, Leo II, Draco, Ursa Minor and Carina are usually considered to be the satellite galaxies of Milky Way. The gravitational mass of these objects were determined recently (Faber & Lin 1983; Aaronson 1983). In Fig. 2, we have plotted the gravitational mass of the dwarf spheroidals against their radii, in a log-log plot. The 'best fitting' curve is,

$$M\,(r) = 3.6 \times 10^6\, M\ \ (r/1\ \mathrm{kpc})^{2.4} \tag{3}$$

However, the σ^2 for this fit is about 0.24. For comparison we tried $M \propto R$ curve and $M \propto R^3$ curve which give σ^2 values of 0.31 and 0.26 respectively. Clearly the data are too scattered for being fitted into any single power law curve with significantly small σ^2.

For Draco and Ursa Minor mass estimates are available from velocity dispersion (Aaronson 1983) while for others the masses are estimated from tidal non-disruption. One may plot the mean density of dwarf spheroidals against their distance from Milky Way. (The distances are heliocentric distances.) This plot is shown in Fig. 3. The solid line in the figure corresponds to the relation,

$$\rho_{\mathrm{DS}}(r) = 9.2 \times 10^{-22}\ \mathrm{g\ cm}^{-3}(r/1\ \mathrm{kpc})^{-2} \tag{4}$$

The σ^2 for this fit is 0.009; for comparison, the best fit curve for $\log \rho - \log r$ has a slope of (– 2.2) and σ^2 of 0.006. Thus the mean densities of dwarf spheroidals falloff as the inverse square of the distance from Milky Way. This is to be expected because tidally limited mass estimates are used for most dwarfs. A comparison of (4) and (2) shows that ρ_{DS} exceeds the expected dark matter halo density of Milky Way at the same location by about a factor of 10. We shall discuss this point more fully in the next section.

2.3 *Spirals*

The flat rotation curve of Milky Way signals the relation $M \propto r$. It is well known that this feature is exhibited by a large number of spiral galaxies. If we denote the mass-radius relation of n^{th} spiral galaxy in a sample by

$$M\,(r) = c_n r \tag{5}$$

then we may ask the question: How different are the numbers in the set $\{c_n\}$? If the dark matter halo around the galaxies are more fundamental units than the visible galaxies, then we would expect the $\{c_n\}$ to be 'reasonably' close to each other. One main source of

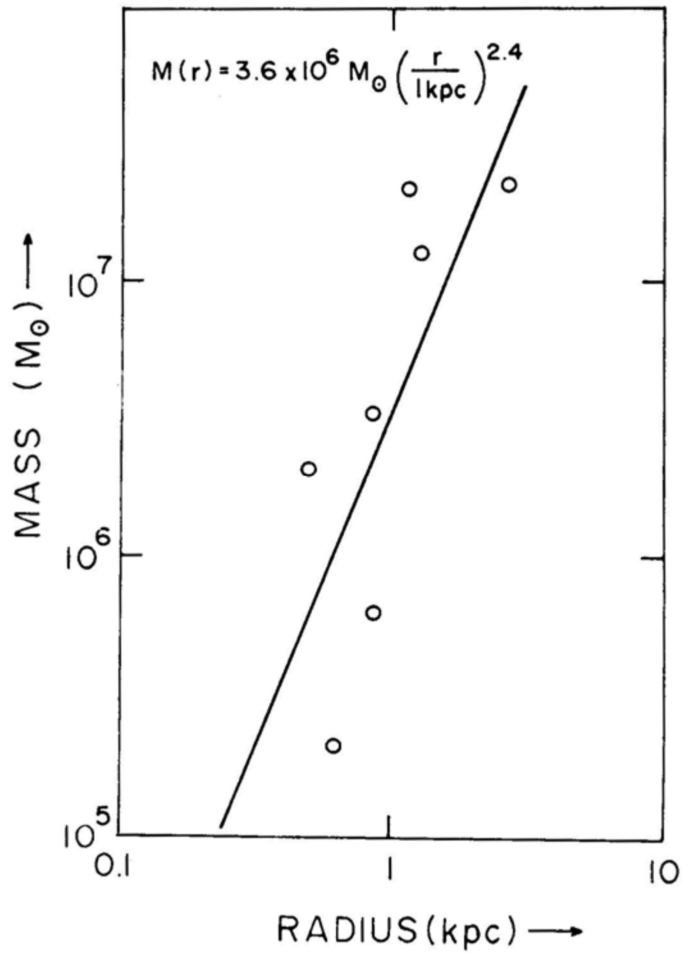

Figure 2. The mass M of the dwarf spheroidals plotted against the radius r of the dwarf spheroidals. The solid line is the best fitting curve,

$$M(r) = 3.6 \times 10^6\, M_\odot (r/1\ \mathrm{kpc})^{2.4}.$$

scatter in the $\{c_n\}$ is the mass variation in the visible part of the galaxies themselves. (Since M in (5) is the total mass, the variation in visible part will affect c_n). This scatter can be minimized by studying the relation (5) for various types of spirals individually.

In Figs 4(a, b) we have plotted the mass–radius relation for SAB and SA types spirals. The data are taken from the table given by Faber & Gallagher (1979). The gravitational mass estimated from rotation velocity at the Holmberg radius is plotted against the corresponding Holmberg radius. In a further attempt to minimize the effect of visible part of the galaxy we have used only those spirals with (Holmberg) radius greater than 15 kpc.

In the case of SAB galaxies all the points lie within a strip indicated by dotted lines in Fig. 4(a) corresponding to (with solid line showing the best fit),

$$M(r) = (3\text{–}9) \times 10^{10}\, M_\odot (r/1\ \mathrm{kpc}). \tag{6}$$

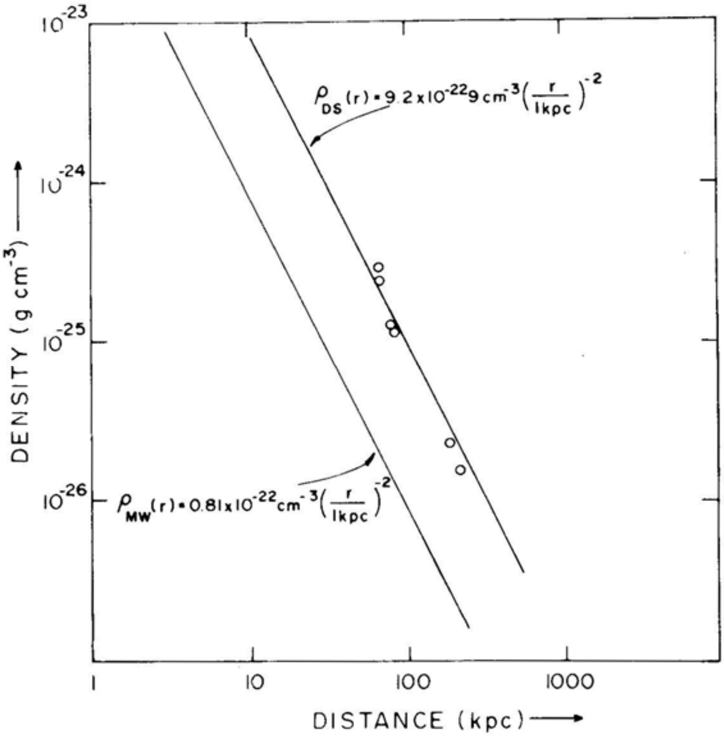

Figure 3. The solid line shows mean density of dwarf spheroidals plotted against the distance of dwarf spheroidals from the centre of the Milky Way. The thin line shows the density falloff of MW as a function of its distance from the centre of MW.

On the other hand all SA galaxies are bound within the mass–radius curves

$$M(r) = (0.5\text{–}2.5) \times 10^{11}\, M_\odot\, (r/1\ \mathrm{kpc}). \tag{7}$$

Considering the scales and uncertainties involved one may reasonably assume that c_n's do not differ from each other drastically.

In other words, all spirals are embedded in individual dark matter haloes with r^{-2} density profile. We shall attempt later (in Section 4 as well as in subsequent papers) to treat galaxies with dark haloes as basic units. Spiral galaxy observations add credibility to this assumption.

We shall now consider the constraints on the theoretical models which arise from the above observations.

3. Theoretical models

The constraints on theoretical models naturally depend on some basic assumptions as well as on the nature of the dark matter: (a) neutrino (hot) or (b) cold dark matter. We shall discuss (a) and (b) separately below.

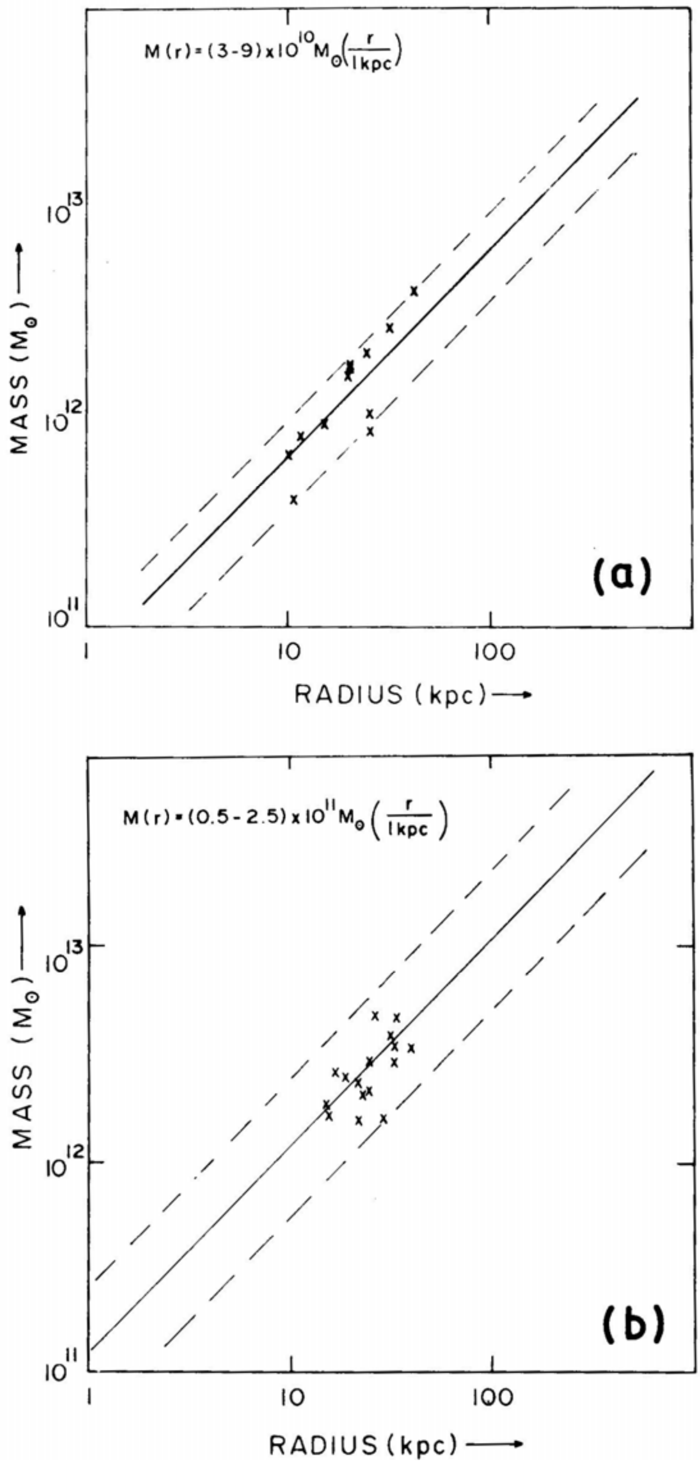

Figure 4. Mass-radius relationship for a) SAB spirals b) SA spirals. The solid line shows the best fit curve.

3.1 *Neutrino* (*Hot*) *Dark Matter*

Assuming neutrinos constitute the dark matter in the universe, we have the well-known constraint from cosmology (Gerhstein & Zeldovich 1966; Cowsik & McClelland 1972)

$$\sum_i m_i \leqslant 100\,\mathrm{eV}\cdot\Omega h^2. \tag{8}$$

Here the sum is over all species of neutrinos, h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1} and Ω is the ratio between mass density of the universe and critical density. Assuming a single species of neutrino and that $\Omega h^2 \sim 1/2$, we shall take m_ν to be less than 50 eV. The only existing experimental support for massive neutrinos suggests a bound 14 eV $< m_\nu <$ 40 eV. Thus $m_\nu \sim$ 30 eV will be used for scaling the expressions in what follows.

Tremaine & Gunn (1979) have shown that m_ν must satisfy the constraint,

$$m_\nu^4 > \frac{9\,h^3}{4(2\pi)^{5/2} g_\nu G \sigma_\nu r_{cv}^2} \tag{9}$$

where σ_ν is the velocity dispersion of neutrinos bound in a gravitational potential well of core radius r_{cv}. The constraint (9) arises from the theorem (see for example, Lynden-Bell 1967), which states that maximum coarse grained phase space density can only decrease with time. Equation (9) can be rewritten as (note a crucial printing error in the original paper of Tremaine & Gunn 1979).

$$m_\nu > (101\,\mathrm{eV})\,(\sigma_\nu/100\,\mathrm{km\,s^{-1}})^{-1/4}(r_{cv}/1\,\mathrm{kpc})^{-1/2}\,g_\nu^{-1/4}. \tag{10}$$

If we make the following two crucial assumptions: (i) the σ_ν for neutrinos in dwarf spheroidals is the same as that of baryonic matter ~ 10 km s^{-1} (Aaronson 1983) and (ii) the r_{cv} for neutrinos in dwarf spheroidals are of the same order as that of tidal radii of baryonic matter ~ 1 kpc, then we get $m_\nu >$ 170 eV, in violent contradiction with (8). Clearly hot dark matter picture is in trouble.

The only way to escape this situation is to relax the assumptions (i) and (ii) in the above paragraph. If one assumes that the velocity dispersion of neutrinos is much higher than that of baryonic matter, making the r_{cv} much larger than $r_{c\ \mathrm{baryons}}$ ($= r_{cb}$). Clearly this will bring down the right hand side of (10) allowing one to escape the constraint. The question that faces us is the following: How much can one push up r_{cv} and σ_ν?

There are three essentially different approaches which one may consider at this stage: (i) Neutrinos form large (few Mpc) dark matter islands in which our local group and dwarf spheroidals are embedded (Cowsik & Ghosh 1986). There is no dark matter bound to dwarf spheroidals individually, (ii) Dwarf spheroidals are imbedded in the haloes of our Galaxy. The neutrino halo extends for ~ 250 kpc around Milky Way. No significant amount of dark matter is attached to dwarf spheroidals. (iii) Dwarf spheroidals do have a neutrino halo around them, but this halo has $r_c \gg r_{cb}$.

We shall now show that approaches (i) and (ii) are theoretically and observationally unsound; it is essential to assume that there exists dark matter bound individually to dwarf spheroidals.

To see this consider the mass and density profiles of Milky Way shown in Figs 1 and 3. Milky Way dark matter, right up to 75 kpc, is definitely not a constant density profile.

The density enhancement between 10 kpc and 100 kpc is by a factor 100 and cannot be treated at all as a small perturbation on a constant density background. The best analysed dwarf spheroidals Draco and Ursa Minor are at about 67 kpc which is in the region of $M \propto r$ behaviour. (This conclusion is independently supported by the standard Milky Way modelling by Lin & Lynden-Bell (1982) based on Magellanic Stream and by Frenk & White (1982) based on globular cluster orbits.) Dark matter halo around our Galaxy is best represented by an isothermal sphere.

Is it possible to assume that the visible matter in dwarf spheroidals only sample the neutrinos which are actually bound to the halo around Milky Way, but just happen to be streaming in the vicinity of dwarfs? This is impossible because of two reasons.

Firstly observations show that the density of gravitating matter in dwarf spheroidals is more than ten times higher than that of Milky Way halo density at that point. (See Equations 2 and 4.) This ten-fold local enhancement of density indicates the existence of about 10^6 $M_\odot$ of dark matter *bound* to the dwarf spheroidals. Secondly, considerations of tidal stability imply the existence of dark matter *bound* to dwarf spheroidals. Consider embedding a dwarf spheroidal in the isothermal halo of our Galaxy. The condition for the tidal stability of the dwarf spheroidals can be derived from Chandrasekhar (1942); (Section 5.5; Equations 5.601 to 5.613.) We get

$$(\bar{\rho}_\nu + \bar{\rho}_B)_{\rm bound} > 6\,\rho_{\rm bg}. \tag{11}$$

Here $\bar{\rho}_\nu$ and $\bar{\rho}_B$ on the left denote average density of neutrinos and baryons (stars) *bound* to the dwarf spheroidal, while $\rho_{\rm bg}$ is the density of background dark matter (*not* bound to dwarf spheroidal; part of Milky Way halo or still larger structure), at the vicinity of the dwarf spheroidal. Since $\rho_{\rm bg}$ in the vicinity of Draco or Ursa Minor (~ 67 kpc) is about 2 times 10^{-26} g cm^{-3} , the density of matter bound to Draco should satisfy the condition,

$$(\bar{\rho}_\nu+\bar{\rho}_B)_{\rm bound} > 1.2\times10^{-25}\ {\rm g\ cm^{-3}} \tag{12}$$

In a size of 1 kpc, this implies a gravitationally bound mass of ~ 8 ×10 $M_\odot$, which is more than one order of magnitude higher than the visible matter (both Draco and Ursa Minor have visible mass of ~ 2×10^5 $M_\odot$; see Faber & Lin 1983). In other words the amount of dark matter *bound* to these dwarf spheroidals must be quite high (~7.8 $\times10^6$ $M_\odot$) to ensure tidal stability. Thus, both observationally and theoretically, there must exist dark matter *bound* to dwarf spheroidals.

Before we proceed further we would like to emphasize three important points relevant to this discussion, (i) Equation (11) was derived assuming the satellite galaxy to be orbiting Milky Way in a low eccentricity orbit. Eccentricity of the orbit will change the numerical coefficient 6 in Equation (11) to a higher value making matters worse (King 1962). (ii) The background density $\rho_{\rm bg}(r)$ may vary very little over the *visible* extent (~1 kpc) of the dwarf spheroidal. We shall show later that even $(\rho_\nu)_{\rm bound}$ does not vary significantly over the visible extent of the dwarf spheroidals. This does not affect the tidal stability argument, (iii) On the other hand, the fact that $\rho_{\rm bg}$ is; not globally constant is crucial for the tidal stability argument. This is because a globally constant density distribution tidally *compresses* matter rather than *disrupts*.

The last point mentioned above shows that we may be able to make the tidal constraint less severe by flattening the $r_{\rm bg}(r)$ to a constant value beyond some radius $r_{\rm f}$. Since Draco is at ~ 67 kpc, $r_{\rm f}$ < 67 kpc (at least). Further Draco and Ursa Minor have ρ_ν of the order of ~2×10^{-25} gcm^{-3}. To provide this much of background density the

Milky Way halo must flatten from about $r_f \sim 20$ kpc. (From (2) we see that $\rho_{bg} \simeq 2 \times 10^{-25}$ gem^{-3} when $r \sim 20$ kpc.) In other words, one requires a density profile like,

$$\rho_{bg} = 8 \times 10^{-23} \text{ g cm}^{-3} (r/1 \text{ kpc})^{-2} \qquad r \leqslant 20 \text{ kpc}$$

$$= 2 \times 10–25 \text{ g cm–3} \qquad r \geqslant 20 \text{ kpc} \qquad (13)$$

Such a drastic assumption leads to many more problems: First of these is the direct contradiction with accepted mass-radius relation for Milky Way halo. (We have indicated modified mass profile based on (13) by a dash dot line in Fig. 1 which goes outside the error bars of known observations.) Secondly it is impossible to ensure the dynamical stability of an artificial configuration like the one in (13) where density falls by *two orders* of magnitude and then remains constant. Lastly a configuration like (13) will over-estimate the mass of local group (Milky Way–Andromeda systems) by a large factor. For example, if we assume (13) to be valid up to ~ 300 kpc (tidally limited radius of Milky Way by Andromeda) the mass contribution of Milky Way to our local group is greater than $3 \times 10^{14} M_\odot$! This is nearly two orders higher than the *upper bound* ($8.6 \times 10^{12} M_\odot$) obtained by studying the infall of local group towards Virgo cluster (Lynden-Bell 1983). In short, there is no escape from assuming the existence of dark matter bound to dwarf spheroidals.

Granted that neutrinos are bound around dwarf spheroidals what kind of constraints can we obtain regarding their distribution?

From (10) it is clear that one has to increase the velocity dispersion σ_v and the core radius r_ν of the neutrino distribution. We shall now see how much latitude exists in these parameters. Suppose that the baryons are described by a truncated isothermal sphere with core radius r_b, velocity dispersion σ_b and cutoff radius r_t. (The sharp cutoff at an outer radius could be tidal truncation or because of the Gaussian falloff arising from neutrino core.) Observationally, for the four dwarfs Sculptor, Draco, Ursa Minor and Carina (which Faber & Lin 1983 cite as having $M/L > 1$), the tidal cutoff radius r_t varies between 0.5 kpc to 1.28 kpc with a mean value of 0.94 kpc. One may take the core radius r_b to be about one-tenth of cutoff radius; *i.e.* $r_b \sim 100$ pc. The σ_b for dwarfs are quite uncertain. Estimating from the visible mass, as

$$\sigma_b \sim (GM_b/r_t)^{1/2} \qquad (14)$$

it varies between 0.59 km s^{-1} and 2.63 km s^{-1} with an average of 1.4 km s^{-1} . (This value is consistent with Aaronson's (1983) measurements, though he uses a χ^2 bound of 6.4 km s^{-1} for this discussion.) As for neutrino halo we shall assume it to be a concentric isothermal sphere with core radius r_v and velocity dispersion σ_v. When concentric isothermal spheres form out of violent relaxation (or virialization) their core densities will be comparable (see *e.g.* Sato 1981). Since central densities of isothermal spheres are related to core radius and velocity dispersion as,

$$\rho_c = 9\sigma^2/(4\pi G r^2) \qquad (15)$$

the equality $\rho_{vc} \simeq \rho_{bc}$ implies, (for a discussion of this assumption, see Appendix 2),

$$\sigma_v/\sigma_v = \sigma_b/r_b. \qquad (16)$$

Using Equation (16), after some simple algebra Equation (10) can be rewritten as,

$$m_\nu > (1010\text{eV})(1 \text{ km s}^{-1}/\sigma_b)^{1/4}(100 \text{ pc}/r_b)^{1/2}(r_b/r_\nu)^{3/4} \qquad (17)$$

or equivalently,

$$(r_v/r_b) > 109(m_v/30\ \text{eV})^{-4/3}(1\ \text{km s}^{-1}/\sigma_b)^{1/3}(100\ \text{pc}/r_b)^{2/3}. \quad (18)$$

The right-hand side is scaled to 'physically reasonable' parameters. (For comparison we note that with σ_b = 10km s^{-1}, r_b = 200 pc the coefficient 109 becomes 32.) We see from (18) that the neutrino core radius must be nearly 100 times the baryonic core radius; with $r_b \sim 100$ pc, $r_v \sim 10$ kpc. (By resorting to the extreme limits of $\sigma_b \sim 10$ km s^{-1}, $r_b \sim 200$ pc, we will get $r_v \sim 6.4$ kpc.)The baryonic matter with cutoff at $r_t \sim 1$ kpc is situated well within the neutrino core.

Such a large neutrino core will give an *M/L* value *at the tidal radius* r_t to be about,

$$M/L = 3(r_t/r_b)^2 \sim 300. \quad (19)$$

However the true *M/L* associated with the dwarf spheroidal is much higher than 300, because the halo extends much further than r_t. One may estimate the tidally limited halo radius by setting,

$$(M_v/x^3) > [M_{bg}/(R-x)^3] \quad (20)$$

where $M_v(x)$ is the halo mass around dwarf within a distance x from its centre and M_{bg} is the mass in the halo of Milky Way within a distance $R - x$ from Milky Way. (R is the centre to centre distance between the spheroidal and Milky Way.) Simple calculation based on our model will give,

$$x/R < [1+ (225\ \text{km s}^{-1}/\sigma_v)]^{-1} \quad (21)$$

For Draco, with $R \sim 70$ kpc, $\sigma_v \sim 100$ km s^{-1}, x is about 20 kpc. Within 20 kpc, Draco will have a halo mass of $5 \times 10^{10}\ M_\odot$ and *M/L* ratio of about 10^4! If neutrinos constitute the dark matter, then dwarf spheroidals are the most peculiar objects in this universe.

In this picture, dwarf spheroidals of mass $\sim 5 \times 10^{10}\ M_\odot$ is moving in the halo of Milky Way galaxy. Dynamical friction will lead to an orbital relaxation timescale of the order of, (Chandrasekhar 1942)

$$t_d \sim v^3/G.^2 M\rho = 6 \times 10^{10}\ y\ (M/5 \times 10^{10}\ M_\odot)^{-1}(v/200\ \text{km s}^{-1})^3\ (\rho/10^{-26}\ \text{g cm}^{-3})^{-1}. \quad (22)$$

This estimate of t_d can easily change by a factor of 10 when finite size of haloes are taken into account. Chandrasekhar's original formula, for example has a coefficient, of the order of $(8\pi)^{-1}$ on the right-hand side making matters worse. To save the situation it is necessary to assume that Milky Way halo density is considerably depleted at the vicinity of dwarf spheroidals.

If all the above assumptions are granted, then 30 eV neutrinos may still be used to model Milky Way and dwarf spheroidals, as far as *kinematic* features are concerned. We hope to discuss the dynamical features in a subsequent paper.

3.2 *Cold Dark Matter*

The fact that cold dark matter candidates can easily account for dark matter in dwarf spheroidals has been emphasized in literature repeatedly (Blumenthal *et al*. 1984; Primack & Blumenthal 1983; Primack 1984). Since cold dark matter has significantly lower velocity dispersion than neutrinos, they can cluster easily at $10^7\ M_\odot$ scales. As the mass of cold dark matter candidates are expected to be about > 1 keV, the phase space constraint is easily satisfied. If the existence of stringent constraints is treated as a negative aspect, then cold dark matter does better than neutrinos.

4. Discussion and conclusions

Contrary to claims often made in literature, the existence of dark matter in dwarfs does not rule out the possibility of neutrino dominance. On the other hand, it does impose stringent conditions on the distributional properties of dark matter. It is important to see whether these constraints can be respected at the scales of groups and clusters of galaxies. We intend to discuss this matter in detail in a subsequent paper; we shall merely present an outline here.

To begin with, we expect the kinematic modelling of dark matter in the universe to satisfy the following two criteria:

(i) There is only one kind of dark matter in the universe

This is probably most drastic of the simplifying assumptions we are making. Unfortunately this assumption is required to obtain any reasonable constraint whatsover. (With just two components for dark matter, it turns out that dark matter parameters get completely out-of-hand.) Within the context of this paper, this assumption implies that dark matter in dwarf spheroidals is of the same kind as dark matter in our Galaxy. When larger structures are considered, this assumption works as a powerful Occam's razor.

(ii) Dark matter clustering pattern is similar all over the universe.

As stated above, this assumption is (admittedly) vague. If the dark matter dominates the dynamics at all scales, we expect baryons to be secondary perturbations in the sea of dark matter. Thus we expect dark matter to be distributed in a similar pattern all throughout the universe. For example, the clustering scale of dark matter is expected to be of the same order everywhere in the universe.

These two assumptions imply that dark matter is predominantly distributed around the galaxies (and smaller dwarfs) as an extensive halo (~ 80 – 100 kpc). We take this pattern to be the basic unit; even though halo of individual galaxies might overlap in rich clusters if intergalactic spacing is less than about ~ 150 kpc. Consider a system of N (gravitationally bound) galaxies, each with a visible matter radius r_{v} (~ 10 kpc) and dark matter halo extension of r_{h}(~ 100 kpc), confined to a region of size R ('cluster size'). (The average intergalactic separation $D(\sim N^{-1/3} R)$ decides whether the haloes overlap or not.) We can easily estimate the $(M/L)_c$ for the cluster from the $(M/L)_g$ for individual galaxies evaluated at $r = r_{\mathrm{v}}$. The total mass in the cluster is,

$$\begin{aligned} M_{\text{total}} &= N\, M_{\text{galaxy}}(r < r_{\mathrm{h}}) \\ &= N\, M_{\text{gal}}(r < r_{\mathrm{v}}) \cdot (r_{\mathrm{h}}/r_{\mathrm{v}}) \\ &= N\, (M/L)_{\text{gal},\, r = r_{\mathrm{v}}} \cdot M_{\text{lum, gal}} \cdot (r_{\mathrm{h}}/r_{\mathrm{v}}) \end{aligned} \tag{23}$$

so that,

$$\frac{M_{\text{total}}}{N\, M_{\text{lum, gal}}} = (M/L)_{\text{cluster}} = (M/L)_{\text{galaxy},\, r = r_{\mathrm{v}}} (r_{\mathrm{h}}/r_{\mathrm{v}}). \tag{24}$$

In other words cluster (M/L) ratios will be (r_h/r_v) times the galactic (M/L) ratios measured around the visible edges of galaxies. Since we expect (r_h/r_v) to be > 10, and $(M/L)_{gal} \sim 10 - 20$ (taking ellipticals also into account) we get $(M/L)_{clusters} \sim 100 - 200$ which is not widely off the mark.

The intergalactic separation D in such a system will be of the order of $(N^{-1/3}\ R)$. If $D > 2r_h$ there is no significant merging of haloes and the dark matter resides around each individual galaxy. On the other hand, if $D < 2\ r_h$ (as it often happens), the halo material in the overlap region $1 = 2\ r_h - D$ will form a common background, of nearly uniform density. The mass in this background halo will be,

$$M_{bg} \simeq N\ M_{gal,(r<r_v)} \cdot [(2r_h - D)/r_v] \tag{25}$$

so that,

$$M_{bg}/M_{total} \simeq (2r_h - D)/r_h. \tag{26}$$

In the dense regions of the cluster, M_{bg} can be significant part of M_{total}. Such a common halo, in our picture is dynamically generated. Galaxies are primary carriers of dark matter.

The above discussion is intended to show that one may be able to describe (M/L) observations at all scales without resorting to specific clustering at various scales.

Appendix 1

Criterion for Tidal Stability

We indicate below the derivation of Equation (11) in the text, based on Chandrasekhar (1942).

Consider a coordinate system $S(x, y, z)$ with origin at the centre of Milky Way galaxy (MW). Let a dwarf spheroidal (DS) be going around MW in a circular orbit. Let the distance to the centre of DS be R. We assume both MW and DS to be spherically symmetric with relation to their respective centres, and that the linear extent of visible matter in DS (~ 1 kpc) is much smaller than the distance R (~ 70 kpc).

The gravitational potential felt by a star bound within the DS, is the sum of two terms $V(r)$ and $U(r)$: $V(r)$ is the potential due to all the matter bound to MW and is distributed in a spherically symmetric fashion about the origin (centre of MW). The background halo *attached* to MW, through which DS is moving, contributes to $V(r)$. The $U(r)$ denotes the potential due to all matter bound to DS and is spherically symmetric about the *centre of DS.* Since the DS is moving, $U(r)$ has a complicated functional dependence on the inertial coordinates. All the stars bound to the DS as well as any dark matter *bound* to DS will contribute to $U(r)$. In the rest frame of the DS, we shall take the shape of DS to be a constant density sphere, with mean density, $\bar{\rho}_{bound} = (\bar{\rho}_V + \bar{\rho}_B)_{bound}$.

Under these assumptions the tidal stability condition (Equation 5.613 of Chandrasekhar 1942) becomes,

$$[(d^2V/dr^2) - (1/r)(dV/dr)]_{r=R} + (4\pi/3)G\,\bar{\rho}_{bound} > 0 \tag{A1.1}$$

using,

$$(1/r^2)d/dr(r^2 dV/dr) = 4\pi G\,\rho_{bg}(r) \tag{A1.2}$$

where $\rho_{bg}(r)$ is the density of MW halo at r, and defining a 'mean density' by,

$$\bar{\rho}_{bg}(r) \equiv \left(\frac{4\pi}{3} r^3\right)^{-1} \int_0^r 4\pi x^2 \rho_{bg}(x)\,dx \tag{A.13}$$

we can transform (A1.1) into a more useful expression. Simple algebra gives,

$$d^2V/dr^2 - (1/r)(dV/dr) = 4\pi G \rho_{bg}(r) - (3/r)(dV/dr)$$
$$= 4\pi G[\rho_{bg}(r) - \bar{\rho}_{bg}(r)] \tag{A1.4}$$

so that (A1.l) becomes,

$$\bar{\rho}_{bound} > 3[\bar{\rho}_{bg}(R) - \rho_{bg}(R)]. \tag{A1.5}$$

This relation shows that: (i) a non-trivial criterion is esbalished only when $\bar{\rho}_{bg}(R) > \rho_{bg}(R)$. In particular, if MW halo was globally constant then $\bar{\rho}_{bg} = \rho_{bg}$ and (A1.5) is identically satisfied, (ii) the existence of MW halo matter in the vicinity of DS *does* help tidal stability (note the $(-\rho_{bg}(R))$ term on the right hand side) but not completely.

We shall assume that density of matter bound to MW, $\rho_{bg}(r)$, has the following form:

$$\rho_{bg}(r) = \rho_0 \qquad r \leqslant r_c$$
$$= \rho_0 (r_c/r)^2 \qquad r \geqslant r_c \tag{A1.6}$$

where r_c is the core radius of MW with $r_c \lesssim 8$ kpc. Using (A1.6) in (A1.3) and (A1.5), we get,

$$\bar{\rho}_{bound} > 6\rho_{bg}(R)[1 - r_c/R]. \tag{A1.7}$$

Neglecting $r_c(\lesssim 8$ kpc) compared to R ($\gtrsim 70$ kpc) we get Equation (11) of the text,

$$\bar{\rho}_{bound} > 6\rho_{bg}(R) \tag{A1.8}$$

ensuring tidal stability.

If (A1.8) is not satisfied, any normal astronomical system will get tidally disrupted. Mathematically speaking, violation of (A1.8) will make all stellar orbits, except those which satisfy very special initial conditions, to grow exponentially in time (see Chandrasekhar 1942, op. cit. Equation 5.621).

Appendix 2

Dependence of the Results on the Ratio ($\rho_{\nu c}/\rho_{bc}$) of Dwarf Spheroidals

In the text, we have assumed that the core densities of neutrinos and baryons are equal in the dwarf spheroidals (see Equations 15 and 16). We shall here examine the sensitivity of our results to this assumption. To do this, let us put, (in the text, we took $\alpha = 1$)

$$\rho_{\nu c} = \alpha^2 \cdot \rho_{bc} \tag{A2.1}$$

so that (16) is replaced by,

$$(\sigma_\nu/r_\nu) = \alpha(\sigma_b/r_b). \tag{A2.2}$$

It is straightforward to work out the α-dependence in various constraints. Equations

(17), (18) and (19) are replaced by Equations (A2.3–5):

$$m_\nu > (1010\,\mathrm{eV}/\alpha^{1/4})(\sigma_b/1\,\mathrm{km\,s^{-1}})^{-1/4}(r_b/100\,\mathrm{pc})^{-1/2}(r_b/r_\nu)^{3/4} \quad \text{(A.2.3)}$$

$$r_\nu/r_b > (109/\alpha^{1/3})(m_\nu/30\,\mathrm{eV})^{-4/3}(\sigma_b/1\,\mathrm{km\,s^{-1}})^{-1/3}(r_b/100\,\mathrm{pc})^{-2/3} \quad \text{(A2.4)}$$

$$M/L > 3\,(r_t/r_b)^2\,\alpha^2. \quad \text{(A2.5)}$$

The maximum distance up to which the DS-halo extends (due to tidal effect of our Galaxy) is determined by Equations (20), (21) in the text. This equation (21) is modified to, ($v_{MW} = 225$ km s^{-1})

$$x/R \leqslant [1 + (v_{MW}/\sigma_b)(r_b/r_\nu)(1/\alpha)]^{-1}. \quad \text{(A2.6)}$$

Given some specific model for the formation of the DS and the haloes, α can be estimated dynamically. Once α is estimated, the above equations present the constraints. In the absence of such a clearcut model, we have two possible routes open to us: (i) Use kinematic constraints to limit the range of α, and investigate results for this particular range, (ii) Use qualitative, 'guesstimates' for α based on simplified dynamics. We shall pursue both these routes here.

We know that $\rho_b > \rho_v$ (at the core) for most astronomical systems. Thus we expect α to be less than one. Observations on our Galaxy, for example, clearly indicate an increase in (*M/L*) with radial distance. Dark matter is distributed smoothly and over a larger scale, compared to visible matter, giving $\rho_b > \rho_\nu$ at the core. Dynamically, we expect $\rho_b > \rho_v$ because of the following reason: In the standard big bang scenario, baryons 'fall into' the potential well of neutrinos and quickly attain the same density contrast as neutrinos (Sato 1981). Thus, just after formation,

$$\delta_b = \frac{\rho_b - \bar{\rho}_b}{\bar{\rho}_b} = \frac{\rho_b}{\bar{\rho}_b} - 1 = \delta_\nu = \frac{\rho_\nu}{\bar{\rho}_\nu} - 1. \quad \text{(A2.7)}$$

So that in the beginning, (taking $\Omega_b \sim 0.01$ and $\Omega_\nu \sim 1$ where $\Omega = \bar{\rho}/\rho_c$)

$$\frac{\rho_\nu}{\rho_b} \approx \frac{\bar{\rho}_\nu}{\bar{\rho}_b} = \frac{\Omega_\nu}{\Omega_b} \approx 10^2. \quad \text{(A2.8)}$$

Neutrinos, being dissipationless maintain their configuration (core radius, density *etc.*) through violent relaxation and virialization. Baryons undergo dissipation and sink to the centre increasing ρ_b. Since we see baryons to be lumped at scales ten times (or more) smaller than the neutrino haloes, baryonic density would have enhanced by a factor of about 10^3. In that case (ρ_v/ρ_b) today would be $\sim 10^2 \times 10^{-3} \cdot \sim 10^{-1}$, or $\alpha \sim 0.3$. Note that a contraction of baryonic matter by a factor $\sim 10^{2/3} \sim 5$ is enough to produce an $\alpha < 1$, starting from (A2.8).

These conclusions are reinforced by consideration of the maximum value allowed for x in (A2.6). Demanding that $x_m < 20$ kpc (*i.e.* DS halo does not extend for more than 20 kpc) with,

$$x_m/R = [1 + (v_{MW}/\sigma_b)(r_b/r_\nu)(1/\alpha)]^{-1} \quad \text{(A2.9)}$$

it can be seen that $\alpha \lesssim 1$. On the other hand, too low a value of α will reduce the (*M/L*) value of DS *via* (A2.5). References cited in the text suggests an (*M/L*) value for DS galaxies to be greater than about 10.

With all these considerations in mind one may estimate the value of α to be in the range of (0.01, 1.00). (It is known from (A2.8) that $\alpha = (\rho_v/\rho_b)^{1/2}$ is definitely less than 10). For this range which varies by a factor of 100, $\alpha^{1/4}$ and $\alpha^{1/3}$ produce factors of the order of 3–5. Since only these low powers of α appear in (A2.3)and (A2.4) our results are not very sensitive to α. Note that as α varies from 10^{-2} to 1, the ratios of core densities change from 10^{-4} to 1, which is sufficiently realistic range.

These arguments (partially at least) justify not including α in our discussion in the text.

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