Crisis in Cosmology

Observational Constraints on Ω and H_o

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Two decades ago, in an article in Nature, Gunn and Tinsley[1] had reviewed the then available data in cosmology to conclude: " New Data on the Hubble diagram, combined with constraints on the density of the universe and the ages of galaxies, suggest that the most plausible cosmological models have a positive cosmological constant, are closed, too dense to make deuterium in the big bang, and will expand for ever...". Thanks to new technology of observations and fresh inputs from particle physics, cosmology has since advanced on both observational and theoretical fronts. The standard hot big bang model has, if at all, become more deeply rooted in cosmology today than in 1975. It is therefore opportune that we take fresh stock of the cosmological situation today and examine the observational and theoretical constraints as they are now. Not surprisingly, some of the issues discussed by Gunn and Tinsley [op. cit.] continue to be relevant today whereas fresh ones have replaced the rest. The purpose of this article is to carry out a similar exercise in the modern cosmological framework. The bottom line in this review is that despite the availability of the cosmological constant as an extra parameter for flat Friedmann models, the allowed parameter space for such models has shrunk drastically. The observations that we will consider here include the ages of globular clusters, measurement of Hubble's constant, abundance of rich clusters of galaxies, fraction of mass contributed by baryons in rich clusters and abundance of high redshift objects. We begin with a brief description of the theoretical models in standard cosmology. For the notation the reader may refer to standard textbooks[2,3].

The Standard Model The standard scenario in big bang cosmology assumes that at any given time the universe is homogenous and isotropic when averaged over a sufficiently large scale. The expansion of the universe is described by the scale factor "a" that satisfies the following equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi}{3}G\rho + \frac{\Lambda}{3} \tag{1}$$

here $k = 0, \pm 1$ represents curvature of the universe [$k = +1$ represents a closed universe, $k = -1$ an open universe and $k = 0$ is the transition or the flat universe.], Λ is the cosmological constant and ρ is the density of matter. We can rewrite this equation for the present epoch as

$$H_0^2 + \frac{kc^2}{a_0^2} = \Omega_0 H_0^2 + \Omega_\Lambda H_0^2 \tag{2}$$

where $H_0 = (\dot{a}/a)_{today}$ is the Hubble's constant, Ω_0 is the density parameter for matter and Ω_Λ is the density parameter contributed by the cosmological constant.

Radiation contributes very little to the energy density of the universe at present, though it was the dominant contituent in the early universe. For standard values of the parameters, matter decoupled from radiation at a characteristic temperature of about $3000K$. The important relic from that epoch is believed to be the currently observed microwave background with a precise blackbody distribution with a temperature of about $2.7K$.

It is also believed in the standard scenario that structure like galaxies, clusters, etc. have formed out of growth of small scale inhomogeneities via gravitational instability. Observations also suggest that luminous (baryonic) matter forms only a small fraction of the total matter density, a much larger contribution coming from "dark" matter, which is likely to be nonbaryonic, noninteracting and collisionless.

The standard model described above has no clear mechanism for generating small inhomogenities in the early universe. It is, however, possible to come up with such a mechanism if one invokes the hypothesis that the universe went through an inflationary phase at very high redshifts. The models involving inflation generically lead to two predictions: (i) The total density parameter $\Omega_0 + \Omega_\Lambda = 1$ and (ii) The initial power power spectrum of inhomogenities has the form $P_{in}(K) \propto K^n$ with $n \simeq 1$. Over the years, the idea of inflation has undergone several modifications to meet observational challenges, and it is now possible to find a model in the market that will provide almost any value for Ω_{total} and any form for $P_{in}(K)$. For the sake of definiteness we will only work with $n = 1$ models. Observations of microwave background radiation are consistent with the index n being equal to unity. As the fluctuations grow the power spectrum gets modified at small scales by different physical processes and this change is described by a transfer function. We shall

work with the transfer function suggested by Efstathiou, Bond & White[4], parametrised by $\Gamma \equiv \Omega_0 h$. The power spectrum is normalised with the COBE DMR observations[5] that give $Q_{rms-ps} = 20 \pm 3\mu K$. Here Q_{rms-ps} is the amplitude of fluctuations in the quadrapole inferred from fluctuations in the higher moments.

Here we study constraints on two models, namely those with (i) $\Omega_0 + \Omega_\Lambda = 1$; $k = 0$ and (ii) $\Omega_0 < 1$; $\Omega_\Lambda = 0$; $k = -1$. The first one is consistent with the inflationary models though it requires an extreme fine tuning of the cosmological constant which is contrary to the spirit of inflationary scenario. [We shall comment more on this later.] The second model may be thought of as an "observer's model" in the sense that it tries to use what is known observationally. The amplitude of fluctuations for open models is obtained by rescaling the $\Omega_0 = 1$ model. Curvature effects are not important as we are interested only in scales much smaller than the curvature scale.

We next list the constraints arising from theory as well as observations, giving a brief description of methods that are used to obtain these and possible sources of errors for each constraint. Then we merge constraints together to study the allowed regions in the parameter space defined by the density parameter for matter (Ω_0) and Hubble's constant (H_0).

Ages of Globular Clusters : It is axiomatic that the age of the universe must be larger than the ages of all its constituents. Therefore ages of the oldest objects provide lower bounds for the age of the universe. Stars in the globular clusters are the oldest known objects in the universe. Ages of these stars are computed by determining their mass, and by observing metallicity and the position off the main sequence turnoff point in the HR diagram. The uncertainities associated with these determinations are now believed to be reasonably small and a fairly accurate estimate for ages of stars can be obtained by this method. Bolte and Hogan[6] compute the ages of stars in M 92 to be 15.8 ± 2.1Gyr.

The theoretical age of the universe can be readily computed given the values of Ω_0, Ω_Λ and H_0. In figure 1 we have plotted curves for $t_0 = 12, 15$ and 18Gyr (dashed lines); the allowed region for any age lies below the corresponding curve. Top frame shows these curves for flat models ($k = 0$) and lower frame shows the same curves for open models ($k = -1$; $\Omega_\Lambda = 0$).

Hubble's Constant : We will use the parametrization $h = H_0/100$ km s^{-1} Mpc^{-1}. To

measure h, we must measure distance and recession velocity of a galaxy, or a group of galaxies. Uncertainity in measurement of recession velocity of galaxies comes mainly from their peculier motions. These can be reduced by going to large recession velocities where the fractional error arising from peculier velocities is small. Error in the distance estimate depends upon the method that is used, and in general it increases with distance. Distance indicators can be divided into three classes, primary indicators like cepheid variables that can be calibrated within our galaxy and therefore the uncertainity associated with these is small. Secondary indicators like Tully-Fisher relation are based on properties of galaxies as a whole and these have to be calibrated with the help of galaxies to which distance has been measured using primary indicators. This extra step involved tends to increase errors in the computed distance. There is a class of "physics" based indicators which are independent of the entire distance ladder, such as those using supernovae, the Sunyaev-Zeldovich effect, etc.

Recent measurement[7] of distance to M 100 (a galaxy in the Virgo cluster) by the Hubble space telescope, with the use of the cepheid period luminosity relation, gives the value $h = 0.80 \pm 0.17$. This is the "local" value of Hubble constant which may differ somewhat from its global value. Turner, Cen & Ostriker[8] and Nakamura & Suto[9] have computed the probability distribution for the Hubble constant given a local value. They show that values smaller than $h = 0.5$ are ruled out at 94% confidence level. Global value of Hubble's constant can be measured with methods like Sunyaev-Zeldovich effect. The value determined by this method[10] for Abell 2218 is $h = 0.65 \pm 0.25$. Sandage and Tamman, on the other hand consistently obtain values of h in the range $0.5 - 0.6$ from a variety of methods [See, for example ref. 11].

In figure 1 we have plotted dotted lines bounding the region allowed by the value obtained for M 100 ($0.63 < h < 0.97$) and also for $h = 0.5$ as the lower limit for the global value of the Hubble constant. If we assume that $h = 0.63$ [The lowest value allowed by HST observations], then $\Omega_0 = 1$ will require the age of globular clusters to be as low as 10.6Gyr. If $h = 0.5$, we get $t_0 = 13.3$Gyr. If the age is greater than 15Gyr then we need $\Omega_\Lambda > 0.3$ for $h = 0.5$ and $\Omega_0 + \Omega_\Lambda = 1$. Thus a nonzero cosmological constant is needed to allow for globular clusters as old as 15Gyr.

Abundance of Rich Clusters : Mass per unit volume contained in rich clusters can

be estimated from the observed number density of such clusters and their average mass. Clusters are identified from x-ray observations by requiring the central temperature to exceed 7keV. Mass of these clusters can be estimated by a variety of methods like assuming virial equilibrium and using the velocity dispersion of galaxies, gravitational lensing etc. One way of representing the observed number is to state the contribution of mass in these clusters to the density parameter, $\Omega^{obs}_{clusters}$. This number can be computed for any theoretical model using the Press-Schechter method[12] and successful models should satisfy the equality $\Omega(> M_{clusters}) = \Omega^{obs}_{clusters}$, within the errors of observations.

A comparison of observations with theory can also be carried out in a more involved manner by converting the number density of clusters into amplitude of density fluctuations at their mass scale. This amplitude is then scaled to $8h^{-1}Mpc$ assuming power law form for σ, the *rms* fluctuations in density perturbations. The index is chosen to match that expected in the model being considered[13]. The result is expressed as a constraint on σ_8, the rms fluctuations at $8h^{-1}$Mpc.

Errors in determination of $\Omega^{obs}_{clusters}$ are related mostly to determination of mass for clusters. Various considerations show that masses higher than those used for calculation are completely ruled out, and in fact we may be overestimating the mass of these objects. A change towards lower masses will tend to lower the allowed values of σ_8. This will shift the allowed region towards lower values of $\Gamma = h\Omega_0$.

We have used observational constraints given by Viana & Liddle[14]. For flat model the constraints are similar to those given by White, Efstathiou & Frenk[13]. We have [in figure 1] plotted thick lines showing region within one sigma of the mean. Thin lines show the bounds if the uncertainity in COBE normalisation is taken into account. Top frame shows these curves for the flat model while the corresponding curves for open models are shown in the lower frame. These figures leave very little room in the parameter space for open models. One may like to relax (i.e., lower) the globular cluster ages and/or (lower) the Hubble constant value somewhat to widen the allowed region, but all values have to be pushed to their extreme limits for this purpose. The allowed region for flat models is somewhat larger than in the $k = -1$; $\Omega_\Lambda = 0$ case. Only three contraints have been used so far and all of these are fairly robust.

Baryon content of galaxy clusters : Rich clusters of galaxies like the Coma cluster have

been studied in great detail. It is possible to determine the fraction of mass contributed by baryons to rich clusters by assuming the Coma cluster to be a prototype. It is found that[15]

$$\frac{M_B}{M_{tot}} = \frac{\Omega_B}{\Omega_0} \geq 0.009 + 0.050h^{-3/2} \tag{3}$$

with 25% uncertainity in the right hand side. This can be combined with the value of Ω_B determined from primordial nucleosynthesis to further constrain Ω_0.

Light nuclei form in the early universe as it cools from a very dense high temperature phase. Relative abundance of different elements is a function of $\Omega_B h^2$. The observed relative abundance of elements can be used to put limits on this parameter [See ref.16]. We use the values[17] $0.01 \leq \Omega_B h^2 \leq 0.02$. [There is no consensus on the allowed range; therefore we are using a conservative set of values.] By combining this value with the fraction of mass contributed by baryons in clusters we can constrain Ω_0. Plotted in figure 2 are the lowest and the highest bounds on matter density after the uncertainity in the observations of fraction of mass contributed by baryons has been taken into account. The permitted region lies to the left of the curve. Reduction in uncertainity associated with these observations can restrict the allowed region in parameter space very effectively. Generalising to inhomogenous primordial nucleosynthesis does not help as that tends to reduce the value of the baryon density[18], leading to a tighter bound on Ω_0.

Comparing figure 2 with figure 1, we notice that this constraint supplements that given by the abundance of rich clusters for high values of h. For small h it is a stronger constraint and rules out more region from the parameter space. A reduction in uncertainity in Ω_B/Ω_0 or a lowering of the upper bound on $\Omega_B h^2$ can further reduce the allowed region. For example, if we use the mean values of observations that combine to give this constraint, we will rule out about one half of region that survives in the parameter space in figure 1.

Abundance of High Redshift Objects : Existence of high redshift objects like radio galaxies and damped lyman alpha systems (DLAS) allows us to conclude that the amplitude of density perturbations is of order unity at $M \simeq 10^{11} M_\odot$ at redshift $z = 2$. We have plotted this lower bound in the top frame of figure 2 for flat models and in the lower frame for open models. For flat models, the curve runs almost parallel to lines of constant age and thus provides *an upper bound for the age of the universe*. If this constraint becomes

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stronger or we discover globular clusters with age greater than 18Gyr, very little region will be left in the parameter space we are considering. Similar results follow for open models.

A more rigorous calculation can be done along the same lines as that described for abundance of clusters. However in the case of DLAS, theoretically computed value of density parameter $\Omega(> M, z)$ should be greater than or equal to the observed value as not all systems in that mass range host a DLAS. Observations of DLAS give us the mean column density ($\langle \bar{N} \rangle$) of neutral hydrogen and the number of DLAS per unit redshift (dN/dz). Using these and the estimated neutral fraction for gas ($f_N \sim 0.5$) we can estimate the density parameter contributed by DLAS (for more details, see e.g. ref.19). It is also possible to compute the density contributed by collapsed objects at a given redshift using the Press-Schechter formalism. It is important to ensure that collapsed objects of the relevant mass scale, in a given model, are produced with the required abundance. It turns out that this constrant is satisfied if DLAS are associated with masses less than $10^{12} M_{\odot}$.

Discussion : These constraints rule out large regions and the surviving region shrinks further or may even disappear if observational uncertainity is reduced. In figure 3 we have shaded regions that are allowed after taking all the constraints into account. We have assumed that globular clusters are not older than 12Gyr and assumed $h > 0.5$. A somewhat less conservative interpretation of observations will lead to a much smaller allowed region, shown here as cross hatched area.

This figure clearly shows that present observations rule out large regions in the space of cosmological parameters. Flat models with cosmological constant have a better chance of surviving as compared to open models. We have not used other constraints coming from detailed structure formation models like velocity fields, shape of the correlation function etc. One reason for not considering them is the large uncertainty associated with values derived from these. Another is that we are able to rule out large regions in the parameter space with only a handful of fairly robust constraints. Lastly, all the constraints we use can be scaled trivially if the observational uncertainities or values of some input parameters change.

Allowed region in the parameter space can be widened if we allow tilted spectra, i.e. spectra with the index of power spectrum $n \neq 1$. This does not allow considerably greater freedom and we should keep in mind that this puts strong limits on another parameter,

namely the index of the power spectrum. We have not specifically discussed any mixed dark matter models as $\Omega_0 = 1$ models are ruled out by constraints discussed above.

Conclusions This brief review highlights the new developments in cosmology since the review of Gunn and Tinsley[1]. Although the constraints of "age" have been with the big bang cosmology for several decades, only now are they coming into focus, thanks to the greater precision in the measurements of Hubble's constant and an improved understanding of stellar evolution. Even allowing for errors on both fronts,the conclusion today is inescapable that the standard big bang models *without* the cosmological constant are effectively ruled out.

The constraints from structure formation, abundances of clusters, primordial nucleosynthesis and high redshift objects are all relatively recent ; but they additionally constrain the models *even with the cosmological constant.* Indeed, with the present understanding of extragalactic astronomy very little parameter space is now left for the standard model with or without the cosmological constant.

Finally, we would like to comment on the issue of "fine-tuning". If we take absence of fine-tuning to imply the dictum "all dimensionless parameters should be of order unity" then one would consider $\Omega_{total} = 1$ models as natural. [Any other model would require fine-tuning of this parameter in the early universe, a difficulty usually called "flatness problem"]. By the same token one would have insisted that $\Omega_\Lambda = 0$. Such a model is clearly reuled out by the observations. It is indeed hard to understand why the left over cosmological constant is such as to exactly conform to the flatness condition. As pointed out by Weinberg[20] this requires fine tuning to one part in 10^{108}. There have been attempts in the past to invoke a dynamically evolving cosmological constant to circumvent this difficulty ; however, none of these models have any compelling features about them. At present, we must conclude that there is indeed a crisis in cosmology.

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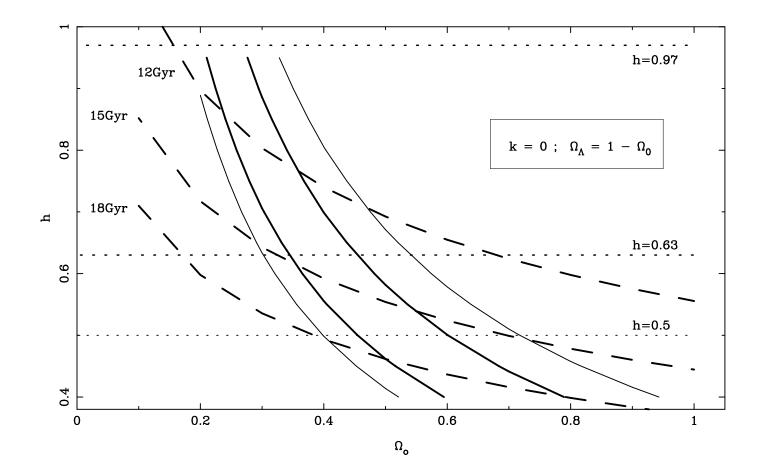
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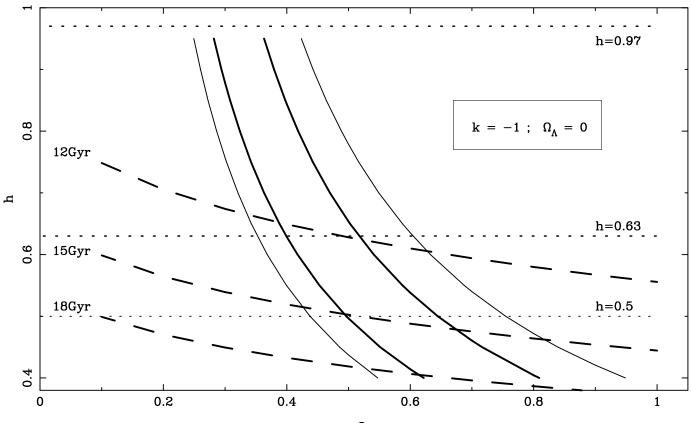
Figure Captions

Figure 1 : This figure shows the constraints on the density parameter contributed by matter, Ω_0, and the Hubble's constant h arising from : (i) ages of globular clusters, (ii) measurements of Hubble's constant, and (iii) abundance of rich clusters. Top frame shows the constraints for a model with $k = 0$, $\Omega_\Lambda \neq 0$ and $\Omega_0 + \Omega_\Lambda = 1$. The lower frame is for $k = -1$, $\Omega_\Lambda = 0$ and $\Omega_0 \leq 1$ model. Lines of constant age are shown as dashed lines for specified values of Ω_0 and h. Dotted lines mark the band enclosing value of local Hubble constant $(0.63 < h < 0.97)$ obtained from HST measurements. We have also shown the assumed lower limit for its global value $(h = 0.5)$. Thick unbroken lines enclose region which is permitted by the observed abundance of clusters. Thin unbroken lines show the extent to which this region can shift due to uncertainity in the COBE normalisation of power spectrum. Note that these three constraints rule out large regions in the parameter space. In particular, it is clear that the $\Omega_0 = 1$ model is ruled out.
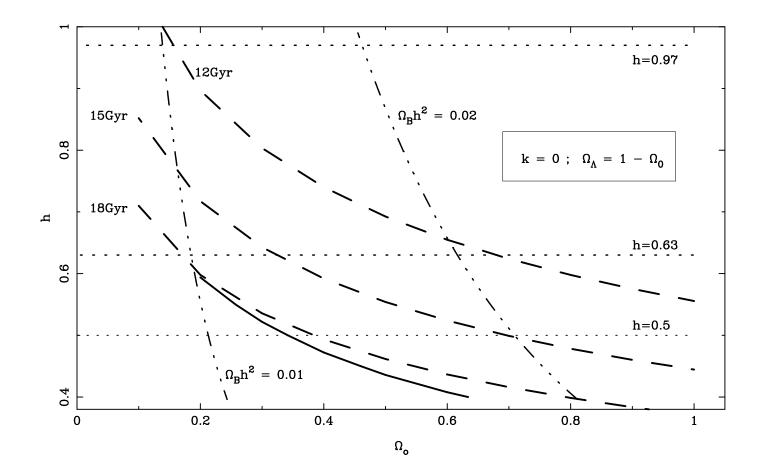
Figure 2 : This figure shows the constraints on the density parameter contributed by matter, Ω_0, and the Hubble's constant h arising from : (i) ages of globular clusters, (ii) measurements of Hubble's constant, (iii) abundance of high redshift objects, and, (iv) fraction of mass contributed by baryons in clusters and primordial nucleosynthesis. Top frame shows the constraints for a model with $k = 0$, $\Omega_\Lambda \neq 0$ and $\Omega_0 + \Omega_\Lambda = 1$. The lower frame is for $k = -1$, $\Omega_\Lambda = 0$ and $\Omega_0 \leq 1$ model. Lines of constant age are shown as dashed lines for specified values of Ω_0 and h. Dotted lines mark the band enclosing value of local Hubble constant ($0.63 < h < 0.97$) obtained from HST measurements. We have also shown the assumed lower limit for its global value ($h = 0.5$). Thick unbroken line is a lower bound on permitted values of h from abundance of high redshift objects. This line depicts $\sigma(10^{11} M_\odot, z = 2) = 1$. Note that this constraint implies that a $k = 0$ universe can not be much older than 18Gyr. Dot-dashed lines mark the extreme upper limits allowed by primordial nucleosynthesis and fraction of mass contributed by baryons in clusters. For a given Ω_0 allowed values of h lie below this curve ; conversly, for a given h, allowed values of Ω_0 lie to the left of this curve. Uncertainities in all observations have been included while plotting this curve.
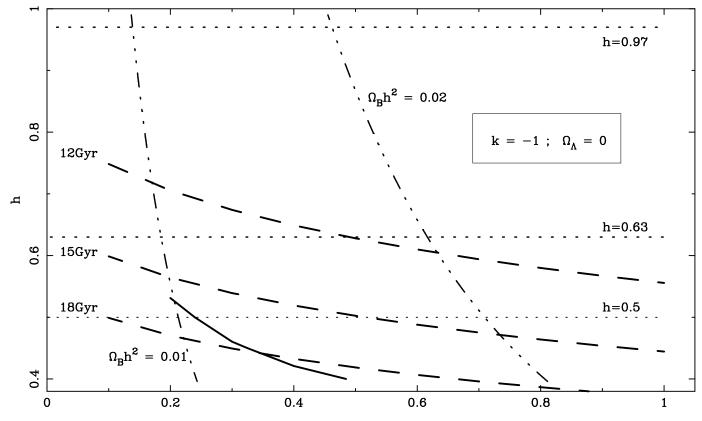
Figure 3 : This figure summarises all the constraints plotted in figures 1 and 2. Shaded region is permitted for $t_0 > 12$Gyr, $h > 0.5$ and other constraints being satisfied. Cross hatched area shows region with $t_0 > 15$Gyr and cluster abundance in the allowed region without taking uncertainity in COBE normalisation into account. If the uncertainities in the observations are pushed to the extreme limits then the allowed parameter space corresponds to the shaded region. A somewhat less conservative interpretation of observations will lead to a much smaller allow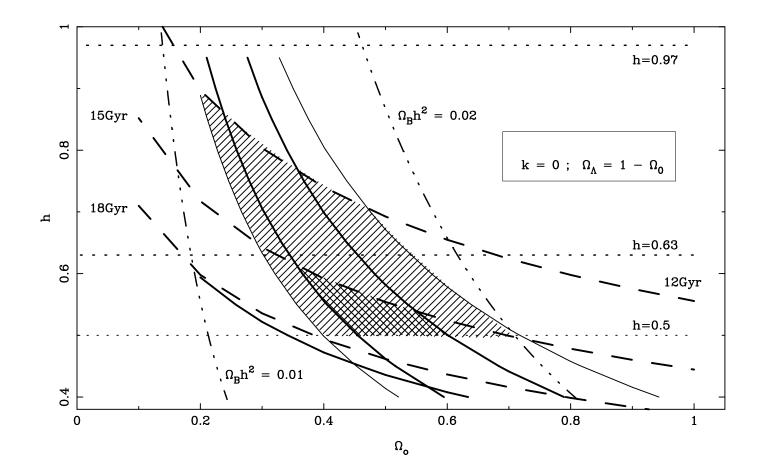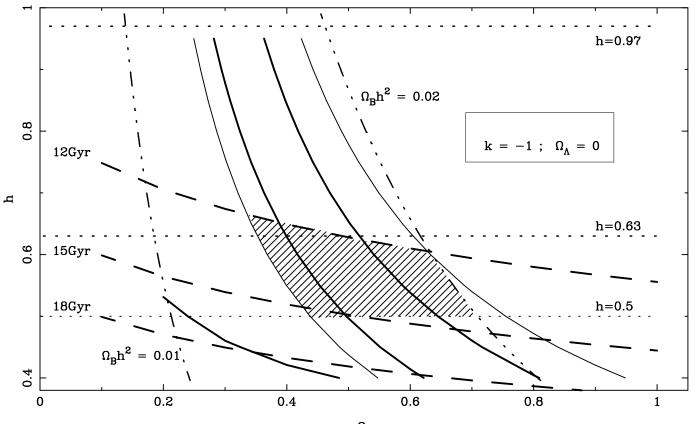ed region, shown here as cross hatched area.