

In conclusion then, recent studies of Cygnus A have revealed the following properties. First, there are compact radio structures, perhaps as small as 1 arc sec, which may indicate that radio components are interacting with gaseous matter outside the radio galaxy. Second, there is a large-scale magnetic field within the source components. Third, it is now certain that the east component has an anomalously high rotation measure. Finally, the optical nucleus is highly excited and shows evidence of continuous excitation. Mr. President, thank you.

The President. I fear that Dr. Mitton has ended his paper right on time so I'm afraid we leave questions aside. The meeting is now adjourned until 1972 October 13.

THE INCREASING RÔLE OF GENERAL RELATIVITY IN ASTRONOMY

By S. Chandrasekhar

Halley Lecture for 1972. Delivered in Oxford, May 2

MR. VICE-CHANCELLOR, LADIES AND GENTLEMEN,—

In a memorable essay, Maynard Keynes wrote

Newton was not the first of the age of reason. He was the last of the magicians, the last of the Babylonians and Sumerians, the last great mind that looked out on the visible and the intellectual world with the same eyes as those who began to build our intellectual inheritance rather less than 10,000 years ago.

If Newton was not the first of the age of reason, that place could be fairly claimed for Halley. Halley's attitude and approach to the physical sciences was in no way different from ours. In particular, he sought to find, in Nature, manifestations of the basic physical laws. Thus, having convinced himself that the existence of periodic comets is consistent with Newton's laws of gravitation, he set about to discover if there was evidence for them, with results that are well known.

In view of Halley's enthusiasm for the Newtonian theory of gravitation, it is, in the first instance, somewhat surprising that none of the earlier Halley lectures should have been devoted to the rôle which the general theory of relativity may be expected to play in astronomy. This theory which has been described by Hermann Weyl as "one of the greatest examples of the power of speculative thought" was founded on the recognition that the Newtonian theory requires modifications if it is to be compatible with other parts of physics such as electro-dynamics and optics. And in the twenties, the general theory of relativity was an intoxicating subject. Thus, Eddington described his part in the verification of Einstein's prediction of the deflection of light at the solar eclipse of 1919 as "the most exciting event" in his connection with astronomy. And the meeting of the Royal Society in London on

November 6, 1919 at which the results of this expedition were reported, has been described by Whitehead in the following terms:

The whole atmosphere of tense interest was exactly that of the Greek drama: we were the Chorus commenting on the decree of destiny as disclosed in the development of a supreme incident. There was dramatic quality in the very setting—the traditional ceremonial and in the background the picture of Newton to remind us that the greatest scientific generalization was now, after more than two centuries, to receive its first modification.

In spite of the excitement of the early twenties and in spite also of the fact that the author of general relativity was, in due time, to become the most celebrated representative of science in the twentieth century, the theory itself was not pursued with intensity, in the framework either of physics or of astronomy, during the decades that followed. I am concerned here only with astronomy. And from one point of view, the reasons for the gradual abandonment of the study of the general theory of relativity in astronomy are not far to seek. The effects of general relativity were identified with Einstein's three tests; and their manifestations in the motions of the planets and the Moon were admittedly very small. As such, general relativity did not appear too relevant in the broader contexts of astronomy.

Cosmology was, of course, an exception. And in cosmology, general relativity did indeed play a vital rôle: it directed the course of the observational researches in "the realm of the nebulae", once the expansion of the Universe had been established by Hubble. These cosmological aspects have been referred to in the earlier Halley lectures by Hubble, Sandage, and Schmidt. But cosmology, in spite of its fundamental interest for the physical sciences, is not, if I may say so, the staple of astronomy.

But what is the staple of astronomy? I am afraid that I may be starting a controversy, wholly foreign to my intentions, if I were to be dogmatic about this matter in any way. Certainly, I do not wish to arrogate to myself the wisdom or the prerogative to define the purposes or the elements of a science. So let me be pragmatic and state only that in the past questions pertaining to the continuing outpouring of energy by the stars and the other bodies constituting the astronomical Universe have always occupied a central place: they have stimulated and directed the course of astronomical development on a wide front. And it is in this sense that I envisage an increasing rôle for general relativity in astronomy. But first, I should like to describe briefly the background against which this increasing rôle for general relativity may be projected.

When one thinks of stellar energy, the question that occurs to one, almost by reflex reaction, concerns the source of the continuing luminosity of the Sun. The Sun is constantly radiating energy to the space outside; and so far as one can tell, it has done so at its present rate for at least a few thousand million years. And the principal question concerning this continual outpouring of energy is not so much its intensity as its duration. Let me be more specific.

In the nineteenth century, the only known physical process that could release energy from a self-gravitating mass such as a star is by a slow secular

contraction. By such contraction, gravitational potential energy is released; and while a fraction of this released energy goes towards raising the average temperature of the star, the remaining fraction is available for radiation to space outside. It was in terms of such a contraction hypothesis that Kelvin and Helmholtz sought to account for the radiation of the Sun and the stars. As a physical process that could play a rôle in astronomy, it is an eminently reasonable one. Indeed, as we now know, it does play a part in current schemes of stellar evolution. But when applied to the Sun to account for its radiation, the hypothesis failed because it provided a continuing source of energy for a period of only a few million years, contrary to the evidence from many directions that the age of Earth's crust must already be measured in thousands of million years.

Even though the contraction hypothesis of Kelvin and Helmholtz failed with reference to the Sun, it played an important rôle in many important astronomical and related developments: it provided, for example, the principal impetus for calibrating and in some cases replacing the qualitative methods of geology for estimating the ages of rocks by the quantitative methods of dating by the content of radioactive minerals. And as I said, it continues to play a part in current schemes of stellar evolution.

As I stated, the contraction hypothesis failed the test of duration. One was, therefore, forced to seek sources of energy that could last longer: in particular, provide for the Sun a life of at least 10^{10} years. And the possibility of such a source became evident when it was noted that the mass of the helium nucleus was less than four times the mass of the proton by 0.8 per cent. Consequently, if some way could be found for synthesizing a helium nucleus out of four protons, then the energy equivalent of 0.8 per cent of the mass of the proton would be freed as available energy. And it was apparent, already in the twenties, that if this energy derived from the binding of the helium nucleus could be released, then we would be assured for the Sun an energy source that could pass the test of duration. Whether the contemplated nuclear transformation could take place under the conditions of temperature and pressure that had been deduced by Eddington and others during the twenties, for the interiors of the stars, could not be confirmed before the advent of quantum mechanics and nuclear physics. The understanding of how protons of even relatively low energy can combine to form deuterium nuclei, and also penetrate the nuclei of carbon and nitrogen with sufficient probabilities to result in the effective burning of hydrogen into helium, came only during the late thirties. And much needed information on the cross-sections for the various nuclear reactions was obtained during the fifties. In any event, it is a fact that the detailed understanding of the source of the energy of the stars provided the central inspiration for much of the astronomy of the fifties and the sixties. It made possible the development of a detailed theory of stellar evolution; and these theoretical developments in turn stimulated most of the observational photometric studies of star clusters and galaxies; and by directing the studies towards the determination of their ages, they naturally led to the revised cosmological distance-scale and a re-evaluation of the Hubble constant.

It appears that the increasing rôle of general relativity in astronomy that one may witness during the seventies and the eighties will not be unlike the rôle of nuclear physics during the fifties and the sixties. To state this same view somewhat differently, it appears that physical processes, understandable

only in terms of the general theory of relativity, can satisfy the astronomical requirements that seem to be beyond the scope of nuclear physics. To mention quasi-stellar sources, radio galaxies, the violent events that seem to occur in the centres of galaxies, and the events reported by Weber and attributed by him to bursts of energy in the form of gravitational waves coming from the centre of our Galaxy, to mention all these, is to conjure up a list that demands processes that will release much larger fractions of the rest mass as energy than the paltry one per cent provided by the binding energies of nuclei. Such processes do seem possible in the framework of general relativity; and the kind of phenomena that must be antecedent to such processes also seem necessary in the larger astronomical contexts. I shall consider first these larger astronomical contexts and then return to the particular processes of energy release that general relativity suggests.

In considering the problem of solar energy, I emphasized that in seeking sources of energy for astronomical bodies duration is an essential desideratum. In the case of the Sun, nuclear processes involving light nuclei meet the test: they provide for the Sun a life of some ten thousand million years at its present rate of radiation. By the same token, they cannot provide comparable lives for stars that are substantially more massive than the Sun; for massive stars are proportionately far more luminous. Thus a star that is ten times more massive than the Sun has a luminosity that is ten thousand times greater. These stars will accordingly burn up their available nuclear fuel in a relatively much shorter time: indeed, they cannot endure for more than ten to twenty million years. Since our Galaxy, in somewhat its present form, must have lasted for a period at least a thousand times longer, the conclusion is inescapable that these stars are young and that they must have been formed within the last ten to twenty million years. An immediate corollary that follows from this last conclusion is that the process of star formation is a continuing process in the Galaxy. On this account the question of the eventual fate of these short-lived massive stars becomes one of central importance for astronomy. And the relevance of this question in the more general context of stellar evolution was recognized long before the problem of stellar energy was clarified. Indeed, the question occurs to one almost inevitably: for no matter what the source of energy is, it must be exhausted sooner or later; and sooner or later the question must be confronted. The question was in fact formulated by Eddington in one of his famous aphorisms: "*a star will need energy in order to cool*". And in the spirit of this aphorism we can ask: will the massive stars have the necessary energy to cool?

Let me rephrase the statement and the question in a less oracular fashion. The stellar material in the interiors of the normal stars is mainly in the form of atomic nuclei and electrons: and except in a purely ionized hydrogen-gas, the electrons outnumber the nuclei by a factor exceeding two; and in general they contribute by far the larger share to the total gas pressure. The question we ask is: can a star of assigned mass, composed of such matter, attain a state of zero-point energy at a high density? Or, to state the question as R. H. Fowler framed it, can the star attain a state in which it can be described "as a gigantic molecule in its lowest quantum state?"

A pioneering investigation by R. H. Fowler in 1926 seemed to suggest that such a state was possible in terms of the equation of state that must govern the electron gas as its concentration is increased at a fixed temperature. This

limiting form of the equation of state of an electron gas can be derived from the following picture. We describe the states of an electron gas by quantum numbers, even as we describe the electrons in an atom by quantum numbers. In the limit of high enough concentrations, all the states for the electron with momenta less than a certain threshold value p_0 are occupied consistently with Pauli's principle, namely, that no more than one electron can occupy a state of assigned quantum numbers. While states below p_0 are all occupied, the states above p_0 are all empty. This is the completely degenerate state for an electron gas. Under these conditions it can be shown that the relation between the pressure (p) and the electron concentration (n_e) is of the form $p = k_1 (n_e)^{5/3}$, where k_1 is an atomic constant.

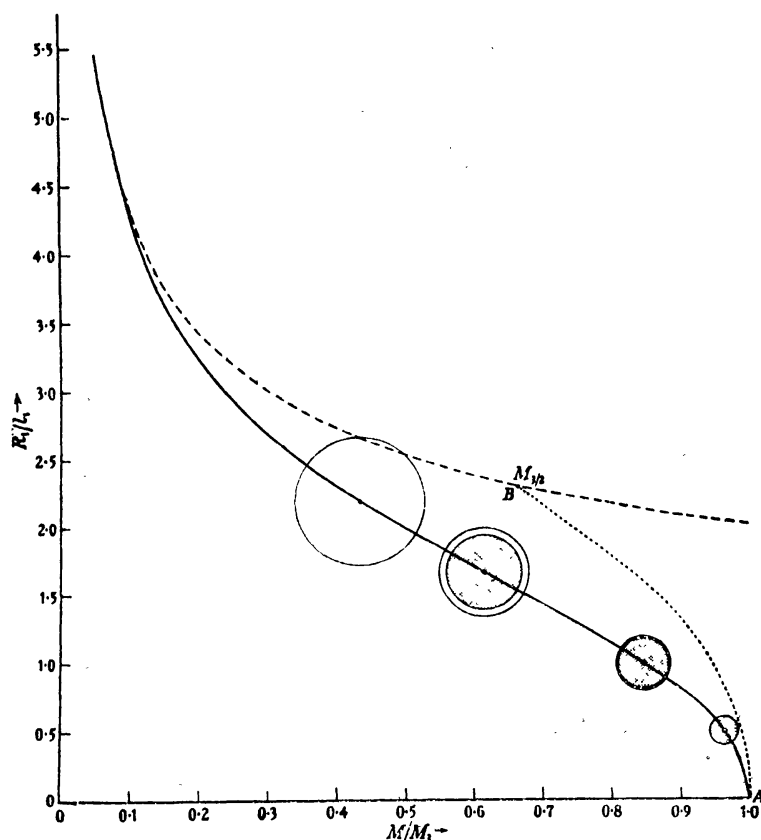


FIG. 1

The full-line curve represents the exact (mass—radius) relationship for completely degenerate configurations. The mass, along the abscissa, is measured in units of the limiting mass (denoted by M_1) and the radius, along the ordinate, is measured in the unit $l_1 = 7.72 \mu e^{-1} \times 10^8$ cm. The dashed curve represents the relation $M = \text{constant} \times R^{-3}$ that follows from the equation of state $p = k_1 (n_e)^{5/3}$; at the point B along this curve, the threshold momentum p_0 of the electrons at the centre of the configuration is exactly equal to mc . Along the exact curve, at the point where a full circle (with no shaded part) is drawn, p_0 (at the centre) is again equal to mc ; the shaded parts of the other circles represent the regions in these configurations where the electrons may be considered to be relativistic ($p_0 \geq mc$). (This illustration is reproduced from S. Chandrasekhar, *M.N.*, 95, p. 219, 1935.)

On the basis of this equation of state, one can readily determine the structure which a configuration of an assigned mass M will assume when in equilibrium under its own gravity. One finds that equilibrium states are possible for any assigned mass: one finds in fact a mass-radius relation of the form $M = \text{constant} \times R^{-3}$. Accordingly, the larger the mass, the smaller is its radius. Also the mean densities of these configurations are found to be in the range of 10^5 – 10^6 grams per c.c. when the mass is of solar magnitude. These masses and densities are of the order one meets in the so-called white-dwarf stars. And it seemed for a time that the white-dwarf stage (or rather the “black-dwarf” stage as Fowler described it) represented the last stage of stellar evolution for all stars. Since a finite state seemed possible for any assigned mass, one could rest with the comfortable assurance that all stars will have the “necessary energy to cool”. But this assurance was soon broken when it was realized that the electrons in the centres of degenerate masses begin to have momenta comparable to mc where m is the mass of the electron. Accordingly, one must allow for the effects of special relativity. These effects can be readily allowed for and look harmless enough in the first instance: the correct equation of state, while it approximates to that given before for low enough electron concentrations, tends to $p = k_2 (n_e)^{4/3}$ as the electron concentration increases indefinitely. (k_2 is another atomic constant.) This limiting form of the equation of state has a dramatic effect on the predicted mass-radius relation: instead of predicting a finite radius for all masses, the theory now predicts that the radius must tend to zero as a certain limiting mass is reached. The value of this limiting mass is $5.76 \mu_e^{-2}$ solar masses where μ_e denotes the mean molecular weight per electron. For the expected value $\mu_e = 2$, the limit is 1.44 solar masses.

The existence of this limiting mass means that a white-dwarf state does not exist for stars that are more massive. In other words “*the massive stars do not have sufficient energy to cool*”.

Fig. 1 exhibits the mass-radius relation that was deduced in 1935 on the basis of the exact equation of state (of which the equations given before are the appropriate limiting forms).

The conclusion that was reached at that time was stated in the following terms:

The life-history of a star of small mass must be essentially different from the life-history of a star of large mass. For a star of small mass the natural white-dwarf stage is an initial step towards complete extinction. A star of large mass cannot pass into the white-dwarf stage and one is left speculating on other possibilities.

Statements very similar to the one I have just quoted from a paper written 38 years ago frequently occur in current literature. But why, it may be asked, were these conclusions not accepted forty years ago? The answer is that they did not meet with the approval of the stalwarts of the day. Thus Eddington, commenting on the foregoing conclusion, stated:

Chandrasekhar shows that a star of mass greater than a certain limit remains a perfect gas and can never cool down. The star has to go on radiating and radiating and contracting and contracting until, I suppose, it gets down to a few kilometres' radius when gravity becomes strong enough to hold the radiation and the star can at last find peace.

If Eddington had stopped at that point, we should now be giving him credit

for having been the first to predict the occurrence of black holes—a topic to which I shall return presently. But alas! he continued to say:

I felt driven to the conclusion that this was almost a *reductio ad absurdum* of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think that there should be a law of Nature to prevent the star from behaving in this absurd way.

And similarly E. A. Milne (who was a professor here in Oxford and was a great personal friend of mine) wrote:

To me it is clear that matter cannot behave the way you predict.

In spite of the then prevalent opposition, it seemed to me likely that a massive star, once it had exhausted its nuclear sources of energy, will contract and in the process eject a large fraction of its mass; and further that if by this process, it reduced its mass sufficiently, it could find a state in which to settle.

A theoretical advance in a different direction suggested another possibility. It is that as we approach the limiting mass along the white-dwarf sequence, we must reach a point where the protons at the centre of the configuration become unstable with respect to electron capture. The situation is this. Under normal conditions, the neutron is β -active and unstable while the proton is a stable nucleon. But if in the environment in which the neutron finds itself (as it will in the centre of degenerate configurations near the limiting mass), all the electron states with energies less than or equal to the maximum energy of the β -ray spectrum of the neutron are occupied, then Pauli's principle will prevent the decay of the neutron. In these circumstances the proton will be unstable and the neutron will be stable. At these high densities, the equilibrium that will obtain will be one in which, consistent with charge neutrality, there will be just exactly the right number of electrons, protons, and neutrons with appropriate threshold energies that none of the existing protons or neutrons decays. At these densities the neutrons will begin to outnumber the protons and electrons by large factors. In any event it is clear that once neutrons begin to form, the configuration essentially collapses to such small dimensions that the mean density will approach that of nuclear matter and in the range 10^{13} – 10^{15} grams per c.c. These are the neutron stars that were first studied by Oppenheimer and Volkoff in 1939, though their possible occurrence had been suggested by Zwicky some five years earlier.

From the work of Oppenheimer and Volkoff it appeared likely that a massive star, during the course of its evolution, could collapse to form a neutron star if during the process of contracting it had reduced its mass sufficiently. The process would clearly be cataclysmic, and it seemed likely that the result would be a supernova phenomenon. But the formation of a neutron star, as the result of the collapse, will depend on whether a star, initially more massive than the limiting mass for the white-dwarf stars, ejects just the right amount of mass in order that what remains is in the permissible range of masses for stable neutron stars.

While the question of the ultimate fate of massive stars with all its implications was not faced till recently, the theory of the white-dwarf stars, based on the relativistic equation of state for degenerate matter, gained gradual acceptance during the forties and fifties. The principal astronomical reasons

for this acceptance were twofold. First, the number of the known white-dwarf stars had, in the meantime, increased very substantially, largely through the efforts of Luyten; and the study of their spectra, particularly by Greenstein, confirmed the adequacy and in some cases even the necessity of the theoretically deduced mass-radius relation exhibited in Fig. 1. Secondly, since a time-scale of the order of ten million years for the exhaustion of the nuclear sources of energy of the massive stars requires the continual formation of these stars, one should be able to distinguish a population of young stars from a population of old stars. Spectroscopic studies provided evidence that the chemical composition of the young stars differs systematically from the chemical composition of the old stars; and in fact the difference is in the sense that the young stars appear to have been formed from matter that has been cycled through nuclear reactions. This last fact is consistent with the picture that during the course of the evolution of the massive stars a large fraction of their masses is returned to interstellar space. It also seemed likely that this returning of processed matter to the interstellar space was via the supernova phenomenon.

While all these ideas became a part of common belief, it remained only as belief. Their full implications were not seriously explored before the discovery of the pulsars. The discovery, in particular, of a pulsar (with the shortest known period) at the centre of the Crab nebula added much credence to the views that I have described, since the Crab nebula is itself the remnant of a supernova explosion that was observed by the Chinese and the Japanese astronomers in the year A.D. 1054. The discovery of the association of further pulsars (of longer periods) with what are believed to be the remnants of more ancient supernova explosions strengthens one's conviction. The story of the pulsars and their identification with neutron stars are matters of such common knowledge that I shall not spend any more time on them.

The principal conclusions that follow from these theoretical and observational studies can be summarized very simply.

Massive stars in the course of their evolution must collapse to dimensions of the order of ten to twenty kilometres once they have exhausted their nuclear source of energy. In this process of collapse, a substantial fraction of the mass will be returned (as processed matter) to the interstellar space. If the mass ejected is such that what remains is in the permissible range of masses for stable neutron stars, then a pulsar will be formed. The exact specification of the permissible range of masses for stable neutron stars is subject to uncertainties because of uncertainties in the equation of state for neutron matter; but it is definite that the range is narrow: the current estimate is between 0.3 to 1.0 solar mass. While the formation of a stable neutron star could be expected in some cases, it is clear that their formation is subject to vicissitudes. It is not in fact an *a priori* likely event that a star initially having a mass of, say, ten solar masses ejects, during an explosion, subject to violent fluctuations, an amount of mass just sufficient to leave behind a residue in a specified narrow range of masses. It is more likely that the star ejects an amount of mass that is either too large or too little. In such cases the residue will not be able to settle into a finite state; and the process of collapse must continue indefinitely till the gravitational force becomes so strong that what Eddington concluded is a *reductio ad absurdum* must in fact happen: "the gravity becomes strong enough to hold the radiation". In other words, a black-hole must form; and it is to the subject of black-holes that I now turn.

Let me be more precise as to what one means by a black-hole. One says that a black-hole is formed when the gravitational forces on the surface become so strong that light cannot escape from it. That such a contingency can arise was surmised already by Laplace in 1798. Laplace argued as follows. For a particle to escape from the surface of a spherical body of mass M and radius R , it must be projected with a velocity v such that $\frac{1}{2}v^2 > GM/R$; and it cannot escape if $v^2 < 2GM/R$. On the basis of this last inequality, Laplace concluded that if $R < 2GM/c^2 = R_s$ (say) where c denotes the velocity of light, then light will not be able to escape from such a body and we should not be able to see it!

By a curious coincidence, the limit R_s discovered by Laplace is *exactly* the same that general relativity gives for the occurrence of a *trapped surface* around a spherical mass. (A trapped surface is one from which light cannot escape to infinity.) While the formula for R looks the same, the radial coordinate r (in general relativity) is so defined that $4\pi r^2$ is the area of the 3-surface of constant r ; it is not the proper radial distance from the centre.

That, for a radial coordinate $r = R_s$, the character of space-time changes is manifest from the standard form Schwarzschild's metric that describes the geometry of space-time external to a spherical distribution of mass located at the centre. For a mass equal to the solar mass, the Schwarzschild radius R_s has the value 2.5 kilometres. At one time, the thought that a mass as large as that of the Sun could be compressed to a radius as small as 2.5 km would have seemed absurd. One no longer thinks so: neutron stars have comparable masses and radii.

The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge.

Let us first consider in the framework of the Newtonian theory what would happen to an incoherent mass, with no internal pressure, distributed with exact spherical symmetry about a centre. It would simply collapse to the centre in a finite time since in the absence of pressure there is nothing to restrain the action of gravity. In the Newtonian theory this result of matter collapsing to an infinite density may be considered as a *reductio ad absurdum* of the initial premises: a distribution of matter that is exactly spherically symmetric and the absence of any sustaining pressure are both untenable in practice. If either of these two premises is not exactly fulfilled, then the collapse to an infinite density will not happen.

Now if the same problem of pressure-free collapse is considered in the framework of general relativity—as it first was by Oppenheimer and Snyder—one finds, as in the Newtonian theory, that the matter collapses to the centre in a finite time (as measured by a co-moving clock). But in contrast to the Newtonian theory, the inclusion of pressure or the allowance of departures from exact spherical symmetry do not seem to make any difference to the final result. The reasons are the following. In general relativity, pressure contributes to the inertial mass; and this contribution becomes comparable to the contribution by the mass density when the radius of the object approaches the Schwarzschild limit. On this account, after a certain stage, the allowance for pressure actually facilitates, rather than hinders, the collapse. It is also clear that small departures from spherical symmetry cannot matter. For unlike in the Newtonian theory, the aim to the centre need not be perfect: it will suffice to aim within the Schwarzschild radius. More generally,

theorems by Penrose and Hawking show that, in the framework of general relativity, allowing for factors that derive from pressure and lack of spherical or other symmetry will not prevent the matter from collapsing to a singularity in space-time if a certain well-defined point of no-return—an event horizon—is passed.

If we consider then the gravitational collapse of a massive star and allow for all factors that derive from pressure but permit only small departures from spherical symmetry, the end result is the same as in a strictly spherically-symmetric pressure-free collapse. There is no alternative to the matter collapsing to infinite density at a singularity once a point of no-return is passed. The reason is that once the event horizon is passed, all time-like trajectories must necessarily get to the singularity: “all the King’s horses and all the King’s men” cannot prevent it. And as far as the external observer is concerned the energy associated with the departures from spherical symmetry will be radiated away as gravitational waves; and the event horizon will eventually settle down to a smooth spherical surface with an exterior Schwarzschild metric for a certain M —the mass of the black-hole.

It is important to notice that the phenomenon of spherical collapse will be described differently by an observer moving with the surface of the collapsing star and by an observer stationed at infinity. This difference is illustrated in Fig. 2.

Imagine that the observer on the surface of the collapsing star transmits time signals at equal intervals (by his clock) at some prescribed wavelength (by his standard). So long as the surface of the collapsing star has a radius that is large compared to the Schwarzschild radius, these signals will be received by the distant observer at intervals that he will judge as (very nearly) equally spaced. But as the collapse proceeds, the distant observer will judge that the signals are arriving at intervals that are gradually lengthening and that the wavelength of his reception is also lengthening. As the stellar surface approaches the Schwarzschild limit the lengthening of the intervals as well as the lengthening of the wavelength of his reception will become exponential by his time. The distant observer will receive no signal after the collapsing surface has crossed the Schwarzschild surface; and there is no way for him to learn what happens to the collapsing star after it has receded inside the Schwarzschild surface. For the distant observer, the collapse to the Schwarzschild radius takes, strictly, an infinite time (by his clock) though the time scale in which he loses contact in the end is of the order of milliseconds.

The story is quite different for the observer on the surface of the collapsing star. For him nothing unusual happens as he crosses the Schwarzschild surface: he will cross it smoothly and at a finite time by his clock. But once he is inside the Schwarzschild surface, he will be propelled inexorably towards the singularity: there is no way in which he can avoid being crushed to zero volume at the singularity and no way at all to retrace his steps.

From what I have said, collapse of the kind I have described must be of frequent occurrence in the Galaxy; and black-holes must be present in numbers comparable to, if not exceeding, those of the pulsars. While the black-holes will not be visible to external observers, they can nevertheless interact with one another and with the outside world through their external fields. But one important generalization is necessary and essential.

It is known that most stars rotate. And during the collapse of such rotating stars, we may expect the angular momentum to be retained except for that part which may be radiated away in gravitational waves. The question now arises as to the end result of the collapse of such rotating stars.

One might have thought that the inclusion of angular momentum would

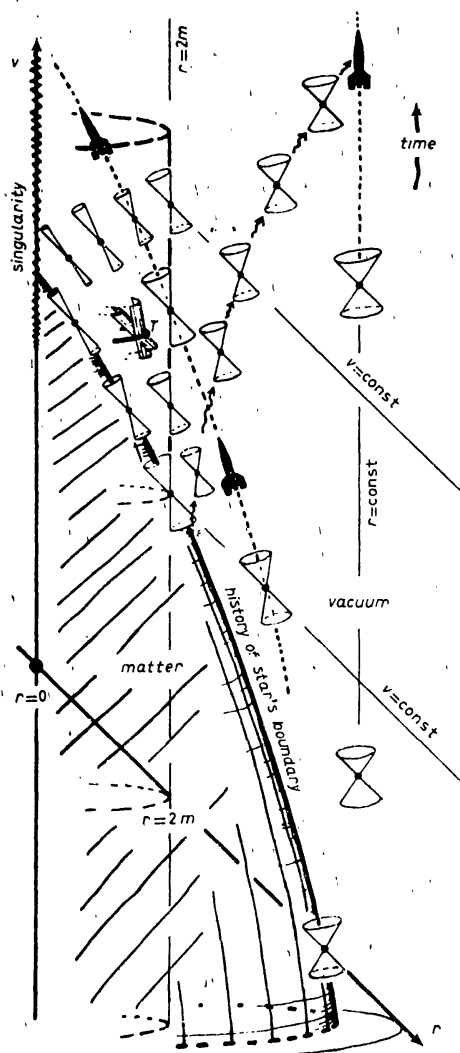


FIG. 2

Illustrating spherically symmetric collapse. At each point the future and the past light-cones are drawn; all time-like trajectories must lie within these cones. The reception by an observer, orbiting in a circular orbit at a large distance from the centre, of a light signal sent by an observer on the collapsing stellar surface is shown; it makes it clear why no signal sent after the surface passes inside the Schwarzschild surface at $r=2m$ can be received by the orbiting observer. Also, notice how all future-directed time-like paths from any point inside $r=2m$ must necessarily intersect the singularity at $r=0$. (This illustration is reproduced from R. Penrose, *Nuovo Cimento*, Serie I, Vol. I, p. 252, 1969.)

make the problem excessively complicated. But if the current ideas are confirmed, the expected end result is not only simple in all essentials, it also provides the principal justification for anticipating an increased rôle for general relativity in astronomy.

In 1963, Kerr discovered the following solution of Einstein's equations for the vacuum which has two parameters M and a and which is also asymptotically flat:

$$ds^2 = - \frac{\Delta}{\rho^2} [dt - a \sin^2 \theta d\phi]^2 + \frac{\sin^2 \theta}{\rho^2} [(r^2 + a^2) d\phi - a dt]^2 \\ + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2,$$

where $\rho^2 = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 + a^2 - 2Mr$. (The solution is written in units in which $c = G = 1$; and in a system of coordinates introduced by Boyer and Lundquist.)

Kerr's solution has *rotational symmetry* about the axis $\theta=0$: none of the metric coefficients depends on the cyclic coordinate ϕ . It is, moreover, *stationary*: none of the metric coefficients depends on the coordinate t which is time for an observer at infinity. Kerr's solution reduces to Schwarzschild's solution when $a=0$.

A test particle describing a geodesic in Kerr's metric at a large distance from the centre will describe its motion as in the gravitational field of a body having a mass M and an angular momentum $\mathcal{J} = aM$ (as deduced from the Lens-Thirring effect).

It is now believed that the end result of the collapse of a massive rotating star is a black-hole with an external metric that will eventually be Kerr's, all the asymmetries having been radiated away. I shall not attempt to explain the reasons for this belief except to say that they derive, principally, from a theorem of Carter which essentially states that *sequences of axisymmetric metrics, external to black holes, must be disjoint, i.e. have no members in common*.

The Kerr metric, like Schwarzschild's, has an event horizon; it occurs at $r = \frac{G}{c^2} [M + (M^2 - a^2)^{\frac{1}{2}}]$. In writing this formula, I have assumed that $a < M$; if this should not be the case, there will be no event horizon and we shall have a "*naked singularity*", i.e. a singularity that will be visible and communicable to the outside world. For the present, I shall restrict myself to the case $a < M$.

Trajectories, time-like or null, can cross the event horizon from the outside; but they cannot emerge from the inside. In this respect also the Kerr black-hole is like the Schwarzschild black-hole. But unlike the Schwarzschild metric, the Kerr metric defines another surface (the *stationary limit*), external

to the event horizon, whose equation is $r = \frac{G}{c^2} [M + (M^2 - a^2 \cos^2 \theta)^{\frac{1}{2}}]$. This surface touches the event horizon at the poles; and it intersects the equator ($\theta = \pi/2$) on a circle whose radius ($= 2GM/c^2$) is larger than that of the horizon. On this surface, an observer who considers himself as staying in the same place must travel with the local velocity of light: like Alice, he must run as fast as he can to stay exactly where he is! Light emitted by such an observer must accordingly appear as infinitely red-shifted to one stationed at infinity.

The occurrence of the two separate surfaces in the Kerr geometry gives rise to unexpected possibilities. These possibilities derive from the fact that in the space between the two surfaces—termed the *ergo-sphere* by Wheeler and Ruffini—the coordinate t , which is time-like external to the stationary limit, becomes space-like. Therefore, the component of the four-momentum in the t -direction, which is the conserved energy for an observer at infinity, becomes space-like in the ergo-sphere; it can accordingly assume here negative values. In view of these circumstances, we can contemplate a process in which an element of matter enters the ergo-sphere from infinity and splits here (in the ergo-sphere) into two parts in such a way that one part, as judged by an observer at infinity, has a negative energy. Conservation of energy requires that the other part acquire an energy that is in excess of that of the original element. If the part with the excess energy escapes along a geodesic to infinity while the other part crosses the event horizon and is swallowed up by the black hole, then we should have extracted some of the rotational energy of the black hole by reducing its angular momentum. The possibility that such processes can be realized was first pointed out by Penrose.

In considering the energy that could be released by interactions with black holes, a theorem of Hawking is useful. Hawking's theorem states that *in the interactions involving black holes, the total surface area of the boundaries of the black holes can never decrease*; it can at best remain unchanged (if the conditions are stationary).

Now, the surface area of a Kerr black-hole is given by

$$S = \frac{8\pi}{c^4} G^2 M [M + (M^2 - a^2)^{\frac{1}{2}}].$$

By Hawking's theorem, in a process in which energy is extracted from a Kerr black-hole, M and a must both change in such a way that S increases. By writing

$$M^2 = M_{ir}^2 + \mathcal{J}^2/4 M_{ir}^2,$$

where

$$M_{ir} = \frac{1}{2} \{ [M + (M^2 - a^2)^{\frac{1}{2}}]^2 + a^2 \}^{\frac{1}{2}}$$

and \mathcal{J} ($=aM$) is the angular momentum, Christodoulou has shown that Hawking's condition, $\delta S \geq 0$, is equivalent to $\delta M_{ir} \geq 0$. Accordingly, we may consider M_{ir} as the irreducible mass of the Kerr black-hole in the sense that by no interaction with the black-hole, effected by the injection of small amounts matter into it, can we reduce the value of M_{ir} . The contribution to M^2 by the term $\mathcal{J}^2/4 M_{ir}^2$ represents therefore the maximum rotational energy that can be extracted.

Another example illustrating Hawking's theorem (and considered by him) is the following. Imagine two spherical (Schwarzschild) black holes, each of mass $\frac{1}{2} M$, coalescing to form a single black hole; and let the black hole that is eventually left be, again, spherical and have a mass \mathfrak{M} . Then Hawking's theorem requires that

$$16 \pi \mathfrak{M}^2 \geq 16 \pi [2 (\frac{1}{2} M)^2] = 8 \pi M^2,$$

or

$$\mathfrak{M} \geq M/\sqrt{2}.$$

Hence the maximum amount of energy that can be released in such a coalescence is

$$M(1 - 1/\sqrt{2})c^2 = 0.293 Mc^2.$$

In practice, the actual amount may be much less; but it is clear that the processes of the type considered have the potentialities for releasing far larger fractions of the rest mass as energy than nuclear processes.

In connection with Weber's observations to which we have referred earlier, the possibility of a large black hole at the centre of the Galaxy and with a mass in the range 10^4 – 10^8 solar masses has often been suggested, for instance by Lynden-Bell and by Bardeen. It may be supposed that such a black hole would be continuously swallowing up stars and accreting matter. As each star is swallowed, or when matter is accreted, we may expect that a certain fraction of the mass energy is radiated as gravitational waves. And strong tidal forces that would also be operative under these circumstances may produce considerable attendant effects such as electromagnetic radiation. Various proposals in these directions are currently actively being pursued. Even if all these attempts to account for Weber's events—their frequency and their energy content—fail, there still remains the question whether naked singularities may not appear under certain circumstances with undreamt-of possibilities though the present view is that "singularities will, forever, remain concealed."

It is clear that none of the processes for energy release that I have described is anything more than a mere suggestion. The present situation is not unlike that in the twenties when the conversion of hydrogen into helium was contemplated as a source of stellar energy with no sure knowledge that it could be accomplished; only years later were well-defined chains of nuclear reactions that could accomplish it formulated. We may similarly have to wait for some years now.

In discussing the various possibilities that may arise as the result of interactions with black holes and among black holes, we are today considering seriously situations that were brushed aside as *reductio ad absurdum* not so very long ago. For my part, while considering the phenomena associated with event horizons and the impossibility of communication across them, I have often recalled a parable from Nature that I learnt in India fifty years ago.

The parable, entitled "Not lost but gone before", is about larvae of dragonflies deposited at the bottom of a pond. A constant source of mystery for these larvae was what happens to them, when on reaching the stage of chrysalis, they pass through the surface of the pond never to return. And each larva, as it approaches the chrysalis stage and feels compelled to rise to the surface of the pond, promises to return and tell those that remain behind what really happens, and confirm or deny a rumour attributed to a frog that when a larva emerges on the other side of their world it becomes a marvellous creature with a long slender body and iridescent wings. But on emerging from the surface of the pond as a fully-formed dragonfly, it is unable to penetrate the surface no matter how much it tries and how long it hovers. And the history books of the larvae do not record any instance of one of them returning to tell them what happens to it when it crosses the dome of their world. And the parable ends with the cry

... Will none of you in pity,
To those you left behind, disclose the secret?

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ON THE ABSORPTION SPECTRUM OF CALCIUM IN SOLID BENZENE

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Duley and Graham^{1,2} have recently performed interesting experiments concerning the absorption spectra of various atoms in certain hydrocarbon matrices with a view to identifying bands such as that at 4430 Å well known in the spectra of interstellar material³ and of supernovae⁴. We are concerned here with their observations of the absorption spectrum of calcium atoms in solid benzene. At low $\text{Ca}/\text{C}_6\text{H}_6$ concentrations (~ 0.01 per cent) the absorption spectrum at 55 °K in the neighbourhood of 4430 Å closely resembles the unidentified band at that wavelength in the spectra of super-