# ON THE RADIATIVE EQUILIBRIUM OF A STELLAR ATMOSPHERE. XIX

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*Received June 20, 1947*

ABSTRACT

In this paper $H$-functions which govern the law of diffuse reflection by a semi-infinite plane-parallel atmosphere scattering radiation isotropically with an albedo $\varpi_0 \leqslant 1$ are tabulated. The functions were determined numerically by solving the exact functional equations which they satisfy.

The values of the albedo for which solutions are tabulated are: $\varpi_0 = 1$, 0.975, 0.950, 0.925, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, and 0.1.

In continuation of Paper XVI,[1] in which we began the tabulation of exact solutions of transfer problems, we provide in this paper $H$-functions which govern the law of diffuse reflection by a semi-infinite isotropically scattering atmosphere for various values of the albedo, $\varpi_0 \leqslant 1$.

### DIFFUSE REFLECTION UNDER CONDITIONS OF ISOTROPIC SCATTERING WITH AN ALBEDO, $\varpi_0 \leqslant 1$

*The law of diffuse reflection from a semi-infinite plane-parallel atmosphere.*—

$$I(\mu;\mu_0) = \tfrac{1}{4}\varpi_0 F \frac{\mu_0}{\mu_0+\mu} H(\mu)\, H(\mu_0)\,.$$

The characteristic function in terms of which $H(\mu)$ is defined is

$$\Psi(\mu) = \tfrac{1}{2}\varpi_0 = \text{constant}\,.$$

TABLE 1

THE $H$-FUNCTIONS OBTAINED AS SOLUTIONS OF THE EXACT FUNCTIONAL EQUATIONS WHICH THEY SATISFY

| $\mu$ | $\varpi_0=0.1$ | $\varpi_0=0.2$ | $\varpi_0=0.3$ | $\varpi_0=0.4$ | $\varpi_0=0.5$ | $\varpi_0=0.6$ | $\varpi_0=0.7$ |
|---|---|---|---|---|---|---|---|
| 0...... | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 0.05...... | 1.00783 | 1.01608 | 1.02484 | 1.03422 | 1.04439 | 1.05544 | 1.06780 |
| 0.10...... | 1.01238 | 1.02562 | 1.03989 | 1.05535 | 1.07241 | 1.09137 | 1.11306 |
| 0.15...... | 1.01584 | 1.03295 | 1.05155 | 1.07196 | 1.09474 | 1.12045 | 1.15036 |
| 0.20...... | 1.01864 | 1.03892 | 1.06115 | 1.08577 | 1.11349 | 1.14517 | 1.18253 |
| 0.25...... | 1.02099 | 1.04396 | 1.06930 | 1.09758 | 1.12968 | 1.16674 | 1.21095 |
| 0.30...... | 1.02300 | 1.04829 | 1.07637 | 1.10789 | 1.14391 | 1.18587 | 1.23643 |
| 0.35...... | 1.02475 | 1.05209 | 1.08259 | 1.11700 | 1.15659 | 1.20304 | 1.25951 |
| 0.40...... | 1.02630 | 1.05546 | 1.08811 | 1.12516 | 1.16800 | 1.21861 | 1.28063 |
| 0.45...... | 1.02768 | 1.05847 | 1.09308 | 1.13251 | 1.17833 | 1.23280 | 1.30003 |
| 0.50...... | 1.02892 | 1.06117 | 1.09756 | 1.13918 | 1.18776 | 1.24581 | 1.31796 |
| 0.55...... | 1.03004 | 1.06363 | 1.10164 | 1.14528 | 1.19640 | 1.25781 | 1.33459 |
| 0.60...... | 1.03106 | 1.06587 | 1.10538 | 1.15087 | 1.20436 | 1.26893 | 1.35009 |
| 0.65...... | 1.03199 | 1.06793 | 1.10881 | 1.15602 | 1.21173 | 1.27925 | 1.36457 |
| 0.70...... | 1.03284 | 1.06982 | 1.11198 | 1.16080 | 1.21858 | 1.28888 | 1.37815 |
| 0.75...... | 1.03363 | 1.07157 | 1.11491 | 1.16523 | 1.22495 | 1.29788 | 1.39090 |
| 0.80...... | 1.03436 | 1.07319 | 1.11763 | 1.16935 | 1.23091 | 1.30631 | 1.40291 |
| 0.85...... | 1.03504 | 1.07469 | 1.12017 | 1.17320 | 1.23648 | 1.31424 | 1.41425 |
| 0.90...... | 1.03567 | 1.07610 | 1.12254 | 1.17681 | 1.24171 | 1.32171 | 1.42497 |
| 0.95...... | 1.03626 | 1.07741 | 1.12476 | 1.18019 | 1.24664 | 1.32875 | 1.43512 |
| 1.00...... | 1.03682 | 1.07864 | 1.12685 | 1.18337 | 1.25128 | 1.33541 | 1.44476 |

[1] *Ap. J.*, **105**, 435, 1947.

TABLE 1—*Continued*

| $\mu$ | $\varpi_0=0.8$ | $\varpi_0=0.9$ | $\varpi_0=0.925$ | $\varpi_0=0.95$ | $\varpi_0=0.975$ | $\varpi_0=1.0$ |
|---|---|---|---|---|---|---|
| 0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 0.05 | 1.0820 | 1.0999 | 1.1053 | 1.1117 | 1.1196 | 1.1368 |
| 0.10 | 1.1388 | 1.1722 | 1.1828 | 1.1952 | 1.2111 | 1.2474 |
| 0.15 | 1.1866 | 1.2349 | 1.2506 | 1.2693 | 1.2936 | 1.3508 |
| 0.20 | 1.2286 | 1.2914 | 1.3123 | 1.3373 | 1.3703 | 1.4503 |
| 0.25 | 1.2663 | 1.3433 | 1.3692 | 1.4008 | 1.4427 | 1.5473 |
| 0.30 | 1.3006 | 1.3914 | 1.4224 | 1.4604 | 1.5117 | 1.6425 |
| 0.35 | 1.3320 | 1.4363 | 1.4724 | 1.5170 | 1.5778 | 1.7364 |
| 0.40 | 1.3611 | 1.4785 | 1.5197 | 1.5709 | 1.6414 | 1.8293 |
| 0.45 | 1.3881 | 1.5183 | 1.5646 | 1.6224 | 1.7027 | 1.9213 |
| 0.50 | 1.4132 | 1.5560 | 1.6073 | 1.6718 | 1.7621 | 2.0128 |
| 0.55 | 1.4368 | 1.5918 | 1.6480 | 1.7191 | 1.8195 | 2.1037 |
| 0.60 | 1.4590 | 1.6259 | 1.6869 | 1.7647 | 1.8753 | 2.1941 |
| 0.65 | 1.4798 | 1.6583 | 1.7242 | 1.8086 | 1.9295 | 2.2842 |
| 0.70 | 1.4995 | 1.6893 | 1.7600 | 1.8509 | 1.9822 | 2.3740 |
| 0.75 | 1.5182 | 1.7190 | 1.7943 | 1.8918 | 2.0334 | 2.4635 |
| 0.80 | 1.5358 | 1.7474 | 1.8274 | 1.9313 | 2.0833 | 2.5527 |
| 0.85 | 1.5526 | 1.7746 | 1.8592 | 1.9695 | 2.1320 | 2.6417 |
| 0.90 | 1.5685 | 1.8008 | 1.8898 | 2.0065 | 2.1795 | 2.7306 |
| 0.95 | 1.5837 | 1.8259 | 1.9194 | 2.0423 | 2.2258 | 2.8193 |
| 1.00 | 1.5982 | 1.8501 | 1.9479 | 2.0771 | 2.2710 | 2.9078 |

In Paper XVI (§ 2) we outlined a procedure by which the functional equations satisfied by the $H$-functions can be solved numerically. The same methods were adopted in the computations of the present paper, though with increased experience we were able

TABLE 2

COMPARISON OF THE INTEGRALS $\int_0^1 H(\mu)d\mu$ EVALUATED WITH THE AID OF THE TABULATED FUNCTIONS WITH THEIR EXACT VALUES $2[1-(1-\varpi_0)^{1/2}]/\varpi_0$

| $\varpi_0$ | Iterated | Exact | $\varpi_0$ | Iterated | Exact |
|---|---|---|---|---|---|
| 0.1 | 1.02632 | 1.0263340 | 0.8 | 1.3819 | 1.381966 |
| .2 | 1.05572 | 1.0557281 | 0.9 | 1.5194 | 1.519494 |
| .3 | 1.08892 | 1.0889331 | 0.925 | 1.5699 | 1.570030 |
| .4 | 1.12698 | 1.1270167 | 0.950 | 1.6344 | 1.634512 |
| .5 | 1.17157 | 1.1715729 | 0.975 | 1.7269 | 1.726946 |
| .6 | 1.22512 | 1.2251482 | 1.000 | 1.9999 | 2.000000 |
| 0.7 | 1.29219 | 1.2922213 | | | |

to reduce appreciably the number of iterations and integrations which were required before a further iteration did not alter the function.

An idea of the accuracy reached in our present calculations can be obtained from Table 2, where we have made a comparison between the values of the integral

$$\int_0^1 H(\mu)\,d\mu\,,$$

evaluated numerically with the aid of the tabulated functions, and the exact values given by the formula

$$\frac{2}{\varpi_0}\left[1-(1-\varpi_0)^{\frac{1}{2}}\right].$$