AN ATTEMPT TO INTERPRET THE RELATIVE ABUNDANCES OF THE ELEMENTS AND THEIR ISOTOPES

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ABSTRACT

In this paper an attempt is made to derive some information concerning the prestellar stage at which the elements are supposed to have been formed. By using first the relative abundances of the isotopes of a single element (e.g., *O, Ne, Mg, Si*, and *S*), it is shown that a temperature of the order of a few billion degrees is indicated. The equilibrium between the fundamental nuclear particles (protons, neutrons, α-particles, electrons, and positrons) at temperatures ranging from 5 to 10 billion degrees is then studied to establish the relative concentrations of protons and neutrons as a function of the temperature. This relation is then used to compute theoretical mass-abundance-curves under different physical conditions. From such calculations it is concluded that under the physical conditions specified by $T = 8 \times 10^9$ degrees and $\rho = 10^7$ gm/cm³ the theoretical mass-abundance-curve from oxygen to sulphur agrees fairly satisfactorily with the known abundance-curve according to V. M. Goldschmidt (Fig. 2). An important feature of the nuclear mixture considered is that hydrogen and helium are the two most abundant constituents, which is in agreement with known facts. However, the conditions indicated are seen to be quite insufficient to account for the existence of the heavy nuclei to any appreciable extent. It is, therefore, suggested that we should distinguish at least two epochs in the development of the prestellar stage. We imagine that at the earliest stages conditions of extreme temperatures and densities prevailed at which the heavier nuclei could have been formed. As the matter cooled to lower temperatures and densities, appreciable amounts (1 part in 10^6) of the heavy elements must have been "frozen" into the mixture. At temperatures of the order of from 5×10^9 to 8×10^9 degrees and densities of the order of from 10^4 to 10^7 gm/cm³ the present known relative abundances of the elements from oxygen to sulphur may have been established.

1. *Introduction.*—It is now generally agreed that the chemical elements cannot be synthesized under conditions now believed to exist in stellar interiors. Consequently, the question of the origin of the elements is left open. On the other hand, the striking regularities which the relative abundances of the elements and their isotopes reveal (e.g., Harkins' rule) require some explanation. It has therefore been suggested that the elements were formed at an earlier, *prestellar*, stage of the universe. If this is accepted, we then have a tentative basis for deriving some information concerning the physical conditions which would have prevailed during this hypothetical prestellar stage. More particularly, we may attempt to interpret the relative abundances of the elements and their isotopes in terms of the physical conditions under which these observed abundances can be realized as a consequence of thermal equilibrium between nuclei of all sorts, neutrons, electrons, and positrons. We do not know a priori whether such an interpretation is possible in terms of a single density and temperature. Indeed, a preliminary discussion of this problem by von Weizsäcker[1] has indicated that we should distinguish at least two distinct epochs in the prestellar state: an initial epoch of extreme density and temperature, when the heaviest elements, like gold and lead, were formed; and a later epoch of relatively "moderate" conditions, during which the present relative abundances of the lighter elements beyond oxygen (to at least sulphur, as we shall see in § 4) came to be established. However, von Weizsäcker's discussion was largely qualitative and was based on very few comparisons with experimental data. Since that discussion our knowledge, both of relative abundances and masses, has advanced sufficiently to justify a more detailed examination of the problem. We therefore propose to rediscuss the problem of the relative abundances of the elements in the light of the increased information which is now available.

[1] *Phys. Zs.*, **39**, 633, 1938 (see particularly pp. 641–645).

2. *Determination of the neutron concentration and the temperature from the relative abundances of the isotopes of a single element.*—In some ways the simplest method of obtaining some information concerning the prestellar stage is from the relative abundances of the isotopes of a single element which has three (or more) isotopes.[2] An accurate knowledge of the masses and the relative abundances of the isotopes of a single such element will lead to a direct determination of the neutron concentration and the temperature.

The equilibrium between the successive isotopes of an element differing by one mass number is maintained according to the scheme

$$ {}^{A}_{Z}X + \nu \rightleftarrows {}^{A+1}_{Z}X \,. \tag{1}$$

Accordingly, we have

$$\frac{n^{A}_{Z} n_{\nu}}{n^{A+1}_{Z}} = 2\,\frac{G^{A}_{Z}}{G^{A+1}_{Z}}\left(\frac{A}{A+1}\right)^{3/2}\frac{(2\pi M k T)^{3/2}}{h^{3}}\,e^{-E_A/kT}\,, \tag{2}$$

where n^{A}_{Z}, n^{A+1}_{Z}, and n_{ν} denote, respectively, the number of nuclei of species (A, Z) and $(A + 1, Z)$ and the number of neutrons per unit volume; further G^{A}_{Z} and G^{A+1}_{Z} are the statistical weights of the ground states of the respective nuclei, M the unit of atomic mass,[3] T the temperature,

$$E_A = c^2(M^{A}_{Z} + M_{\nu} - M^{A+1}_{Z}) \,; \tag{3}$$

where M^{A}_{Z} is the weight of the atom of nuclear charge Z containing A heavy particles and M_{ν} is the mass of the neutron, while the rest of the symbols have their usual meanings.

Similarly, the equilibrium equation

$$ {}^{A+1}_{Z}X + \nu \rightleftarrows {}^{A+2}_{Z}X \tag{4}$$

will provide another equation between n_{ν} and T. From these two equations we readily obtain the relation

$$kT = \frac{E_{A+1} - E_A}{ln\left[\dfrac{n^{A}_{Z} n^{A+2}_{Z}}{(n^{A+1}_{Z})^2}\,\dfrac{(G^{A+1}_{Z})^2}{G^{A}_{Z} G^{A+2}_{Z}}\left(\dfrac{[A+1]^2}{A[A+2]}\right)^{3/2}\right]}\,, \tag{5}$$

where *ln* represents the logarithm to the base *e*. From the foregoing equation we can obtain T if we know the relative abundances of the three isotopes, their masses (*very* accurately), and their statistical weights. Once T has been determined, equation (2) will suffice to specify log n_{ν}.

We have used the foregoing method to make five independent determinations of log n_{ν} and T from the data available for the isotopes of oxygen, neon, magnesium, silicon, and sulphur. The results are summarized in Table 1. An examination of this table shows that, while an average temperature of several billion degrees is indicated, the neutron concentration varies between limits too wide to draw any safe conclusion.[4] It

[2] This method was first suggested by von Weizsäcker (*ibid.*).

[3] The mass of the proton, the mass of the hydrogen atom, or the mass of the neutron may be used at this point in the equation without any noticeable difference in the result. Actually, the mass of the hydrogen atom was used in the computation.

[4] It should not, however, be concluded that these variations in the neutron concentration correspond to equally wide variations in the physically more important quantity, namely, the density (cf. §§ 3, 4, and 5).

therefore appears that a more detailed discussion of the known mass-abundance-curve for the elements is necessary before any trustworthy estimate of the density can be made. But we already have the suggestion that the principal uncertainty in the discussion will be the density.

3. *The equilibrium between protons, neutrons, α-particles, electrons, and positrons.*—Consider an assembly of nuclei of all sorts, protons, free neutrons, electrons, and positrons at some given temperature. Let n_p, n_ν, n^-, and n^+ denote the equilibrium concentrations of protons, neutrons, electrons, and positrons, respectively. Further, let n_Z^A

TABLE 1

DETERMINATION OF LOG n_ν AND T FROM THE RELATIVE ABUNDANCES OF THE ISOTOPES OF A SINGLE ELEMENT

Element	Mass (Atomic)*	Relative Abundance†	T	log n_ν
O { ^{16}O	16	99.76	4.2×10^9	26.5
^{17}O	17.00450	0.04		
^{18}O	18.00490	0.20		
Ne { ^{20}Ne	19.99881	90.00	2.9×10^9	19.7
^{21}Ne	21.00018	0.27		
^{22}Ne	21.99864	9.73		
Mg { ^{24}Mg	23.99189	77.4	10.0×10^9	30.7
^{25}Mg	24.99277	11.5		
^{26}Mg	25.99062	11.1		
Si { ^{28}Si	27.98639	89.6	12.9×10^9	31.2
^{29}Si	28.98685	6.2		
^{30}Si	29.98294	4.2		
S { ^{32}S	31.98306	95.0	3.3×10^9	19.1
^{33}S	32.98260	0.74		
^{34}S	33.97974	4.2		

* Masses of the oxygen isotopes as listed by O. Hahn, S. Flügge, and J. Mattauch in *Phys. Z.*, **41**, 1, 1940. Other masses as listed by E. Pollard in *Phys. Rev.*, **57**, 1186, 1940.

† The relative abundances of the isotopes were taken from J. J. Livingood and G. T. Seaborg in *Rev. of Modern Phys.*, **12**, 30, 1940. Also, for nuclei of known spins the statistical weights were taken to be $(2S+1)$; when such information was lacking, a weight of 1 was adopted for nuclei of even mass numbers and a weight of 2 for the odd ones.

denote the concentration of a typical nucleus of charge Z and mass-number A. The fundamental reactions maintaining equilibrium are:

$$p+e^- \rightleftarrows \nu\ , \tag{6}$$

$$e^-+e^+ \rightleftarrows \gamma\text{-rays}\ , \tag{7}$$

and

$$^A_ZX + p\,\Delta p + \nu\,\Delta\nu \rightleftarrows {}^{A'}_{Z'}X\ , \tag{8}$$

where

$$A' = A+\Delta p + \Delta\nu\ ; \qquad Z' = Z + \Delta p\ . \tag{9}$$

In equations (8) and (9) Δp and $\Delta\nu$ are arbitrary integers. (It should be remarked that the product nucleus ${}^{A'}_{Z'}X$ need not, in general, be a stable nucleus.) Finally, we have the condition that the whole assembly is electrically neutral:

$$n^- = \Sigma Z n_Z^A + n^+ , \tag{10}$$

where the summation on the right-hand side is extended over nuclei of all charge and mass numbers.

Strictly speaking, the rigorous solution of the equations (6)–(10) under given physical conditions of density and temperature involves the consideration of a simultaneous system of equations of a very high order. In practice, however, the system of equations is effectively reduced very considerably, for, under prescribed conditions only a few of the nuclear particles occur with sufficient abundance to affect appreciably equation (10). As we shall see later, in § 4, under conditions of greatest interest in the present connection, the most abundant particles are protons, neutrons, α-particles, electrons, and positrons. We shall therefore begin our discussion of the theoretical mass-abundance-curves under given conditions by considering the equilibrium between these particles.

First, the equation governing the concentrations of free neutrons, protons, and electrons is *either*

$$\frac{n_p n^-}{n_\nu} = 2\,\frac{(2\pi m_e kT)^{3/2}}{h^3}\,e^{-c^2(M_1^1 - M_\nu)/kT} \tag{11}$$

or

$$\frac{n_p n^-}{n_\nu} = 16\pi\left(\frac{kT}{hc}\right)^3 e^{-c^2(M_1^1 - M_\nu)/kT} , \tag{12}$$

depending on whether the electrons are assumed to obey the nonrelativistic or the relativistic equations of statistical mechanics.[5] In the foregoing equations m_e, M_ν, and M_1^1 denote the mass of the electron, neutron, and hydrogen atom, respectively. A comparison of equations (11) and (12) shows that in a first approximation we may use equation (11) for $T \leqslant 4 \times 10^9$ degrees, while equation (12) should be used for higher temperatures.

Second, the equation governing the concentrations of electrons and positrons at a given temperature is *either*

$$n^- n^+ = 4\,\frac{(2\pi m_e kT)^3}{h^6}\,e^{-2m_e c^2/kT} \tag{13}$$

or

$$n^- n^+ = 256\pi^2\left(\frac{kT}{hc}\right)^6 e^{-2m_e c^2/kT} , \tag{14}$$

again depending on whether T is less than or greater than 4×10^9 degrees.[6]

Third, the equation giving the concentration of α-particles in terms of the proton and neutron concentrations is

$$\frac{(n_p)^2(n_\nu)^2}{n_2^4} = 2\left(\frac{1}{4}\right)^{3/2}\left[2\,\frac{(2\pi M kT)^{3/2}}{h^3}\right]^3 e^{-D_2^4/kT} , \tag{15}$$

[5] See, e.g., S. Chandrasekhar, *M.N.*, **91**, 446, 1931.

[6] See R. H. Fowler, *Statistical Mechanics*, 2d ed., p. 654, Cambridge, England, 1936.

where, quite generally D_Z^A is given by

$$D_Z^A = c^2[(A - Z)M_\nu + ZM_1^1 - M_Z^A] . \qquad (16)$$

Finally, we have the equation insuring the electrical neutrality of the whole assembly:

$$n^- = n^+ + n_p + 2n_2^4 + \ldots . \qquad (17)$$

For any assigned neutron concentration and temperature the foregoing equations can be solved (by trial and error), and Table 2 summarizes the results of such computations

TABLE 2

THE ELECTRON—POSITRON—NEUTRON—PROTON—α-PARTICLE EQUILIBRIUM

LOG n_ν	$T=5\times10^9$ DEGREES				$T=6.5\times10^9$ DEGREES				$T=8\times10^9$ DEGREES			
	log n_p	log $2n_2^4$	log n^-	log n^+	log n_p	log $2n_2^4$	log n^-	log n^+	log n_p	log $2n_2^4$	log n^-	log n^+
23.0	24.28	18.42	29.81	29.81								
23.5	24.78	20.42	29.81	29.81								
24.0	25.28	22.42	29.81	29.81								
24.5	25.78	24.42	29.81	29.81								
25.0	26.28	26.42	29.81	29.81								
25.5	26.78	28.42	29.81	29.81								
26.0	27.02	29.90	30.07	29.55	26.99	22.78	30.27	30.27				
26.5	26.91	30.68	30.68	28.94	27.49	24.77	30.27	30.27				
27.0	26.74	31.34	31.35	28.27	27.98	26.76	30.27	30.27	27.79	21.91	30.61	30.61
27.5					28.47	28.75	30.28	30.26	28.29	23.91	30.61	30.61
28.0					28.76	30.31	30.49	30.04	28.79	25.90	30.61	30.61
28.5					28.65	31.11	31.11	29.43	29.28	27.87	30.62	30.60
29.0					28.49	31.78	31.78	28.77	29.73	29.78	30.67	30.55
29.5									29.83	30.98	31.08	30.15
30.0									29.71	31.72	31.71	29.52
31.0									29.38	33.05	33.04	28.19

for three different temperatures. The resulting variations are illustrated graphically in Figure 1.

An examination of Figure 1 reveals that for low neutron concentrations the material density is almost entirely due to the electron-positron pairs which occur in a "vacuum" in thermal equilibrium. At higher neutron concentrations the protons and α-particles increase in numbers. Eventually the proton abundance reaches a maximum and decreases (asymptotically) according to the law

$$n_p \propto n_\nu^{-1/3} . \qquad (18)$$

At the same time,

$$n_2^4 \propto n_\nu^{4/3} . \qquad (19)$$

It should, however, be remarked that this increase of the α-particle concentration cannot continue indefinitely, for, as the neutron concentration increases appreciably beyond the maximum proton concentration, some of the heavier nuclei become abundant and the corresponding nuclear equilibrium must be included in setting up the primary neutron-proton equilibrium and satisfying the condition for electrical neutrality. However, as we have already remarked, in the following sections we shall not be interested in conditions when the abundant nuclei are different from protons or α-particles.

4. *The theoretical mass-abundance-curves for elements beyond oxygen and comparison with Goldschmidt's empirical curve.*—For the range of physical conditions considered in § 3, the relative concentrations of protons and neutrons being known as a function of temperature, it is now possible to compute the complete nuclear abundance-curve to be expected under equilibrium conditions at given temperatures and neutron concentra-

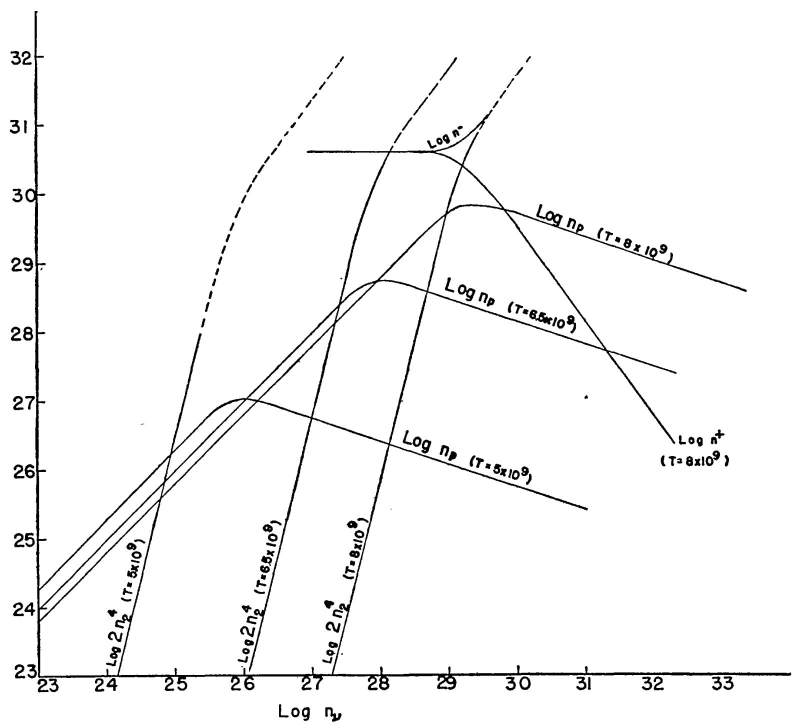

FIG. 1

tions. The relative concentration of the nuclei ${}^{A}_{Z}X$ and ${}^{A'}_{Z'}X(A' = A + \Delta p + \Delta\nu;$ $Z' = Z + \Delta p)$ is determined by the equation

$$\frac{n^{A}_{Z}(n_p)^{\Delta p}(n_\nu)^{\Delta\nu}}{n^{A'}_{Z'}} = \frac{G^{A}_{Z}}{G^{A'}_{Z'}}\left(\frac{A}{A'}\right)^{3/2}\left[2\,\frac{(2\pi MkT)^{3/2}}{h^3}\right]^{\Delta p+\Delta\nu} e^{-(D^{A'}_{Z'}-D^{A}_{Z})/kT}\,, \tag{20}$$

where D^{A}_{Z} has the same meaning as in equation (16). We may note that the foregoing equation can be re-written in the following form, for computing:

$$\left.\begin{aligned}\log n^{A'}_{Z'} = {} & \log n^{A}_{Z} - \log\,(G^{A}_{Z}/G^{A'}_{Z'}) - \tfrac{3}{2}\log\,(A/A') \\ & + \Delta p(\log n_p - 34.08 - \tfrac{3}{2}\log T) \\ & + \Delta\nu(\log n_\nu - 34.08 - \tfrac{3}{2}\log T) + \frac{4.692}{T}\,(D^{A'}_{Z'} - D^{A}_{Z})\,,\end{aligned}\right\} \tag{21}$$

where T is expressed in units of billions of degrees and D^{A}_{Z} in millimass units.

It should be noted that, according to equations (20) and (21), we can compute the equilibrium concentrations not only of the stable nuclei but also of the unstable nuclei if we have the necessary data for them. The table of atomic masses given by W. H. Barkas[7] gives theoretical masses for some of the unstable nuclei. Using these masses, we may compute theoretical abundance-curves. In doing this we can take the "net" abundance of a stable nuclei to be equal to the abundance of this nucleus *plus* the abundances of all unstable nuclei, which, after an appropriate number of β^- or β^+ disintegrations, transform into the particular nucleus under consideration. However, it was found, for the densities and temperatures for which computations were made, that the relative abundance of the unstable nuclei "bordering" on a given stable nucleus was generally quite small in the range of the periodic table considered. Consequently, the effect of these unstable nuclei was ignored.

Our object, then, is to compare the computed theoretical abundance-curves with the observed relative abundances of the stable nuclei, as given by V. M. Goldschmidt.[8] Remembering that our object is to obtain some information concerning the prestellar stage (as indicated by the known relative abundances of the nuclei), we adopted the following procedure.

For an assigned temperature the neutron concentration was so adjusted that the relative concentration of the nuclei $^{16}_{8}O$ and $^{36}_{18}A$ occurring in the equilibrium mixture was approximately the same as that known to occur in the "cosmos"—according to Goldschmidt, approximately in the ratio 15,000:1. After having adjusted the physical conditions in this manner, the complete theoretical mass-abundance-curve was computed according to equation (20) and the table of atomic masses given by Barkas.

Calculations of the kind outlined in the preceding paragraph have been made for different initially assigned temperatures, and the results are summarized in Table 3.

In Figure 2 we have compared the theoretical abundances of the elements beyond oxygen, according to Table 3, with the abundances given by Goldschmidt.[9] An examination of this figure shows that the better agreement with the computed and the observed abundances is obtained under the conditions:

$$\left.\begin{aligned} T &= 8 \times 10^9 \text{ degrees} , \\ \log n_p &= 29.83 , \\ \log n_\nu &= 29.30 , \\ \log n_2^4 &= 30.3 , \end{aligned}\right\} \tag{22}$$

or

$$\rho = 10^7 \text{ gm/cm}^3 ; \qquad T = 8 \times 10^9 \text{ degrees} . \tag{23}$$

It is further seen that the theoretical abundance-curve under these conditions agrees with Goldschmidt's curve quite satisfactorily for all elements from oxygen to sulphur. It is difficult to extend the theoretical calculations beyond argon, since the mass defects are not known to sufficient accuracy.

5. *Further discussion of the physical conditions for the prestellar stage derived in section 4.*—We shall now consider in some detail the special features of the physical conditions ($T \sim 8 \times 10^9$; $\rho \sim 10^7$ gm/cm³) we have derived for the prestellar stage on the

[7] *Phys. Rev.*, **55**, 691, 1939.

[8] V. M. Goldschmidt, *Geochemische Verteilungsgesetze der Elemente*. IX. *Die Mengenverhältnisse der Elemente und der Atom-Arten*, Oslo: J. Dybwad, 1938.

[9] *Ibid.* (see particularly the table on p. 120). In the case of neon and argon, approximate values have been read from the graph, p. 123.

basis of the relative abundances of the elements from $^{16}_{8}O$ to $^{32}_{16}S$, as known from a variety of terrestrial, meteoric, and stellar sources.

TABLE 3

THE COMPUTED RELATIVE ABUNDANCES OF THE NUCLEI FOR DIFFERENT INITIAL CONDITIONS

Element	Z	A	$T=6.5\times10^9$ log $n_\nu=27.65$ log $n_p=28.60$	$T=8\times10^9$ log $n_\nu=29.30$ log $n_p=29.83$
H	1	1	28.6	29.8
H	1	2	22.8	25.2
He	2	3	20.9	23.6
He	2	4	29.0	30.3
He	2	5	....	....
Li	3	6	17.8	21.2
Li	3	7	16.3	20.1
Li	3	8	....	....
Be	4	9	16.4	20.2
B	5	10	15.9	19.5
B	5	11	16.1	19.8
C	6	12	21.5	24.0
C	6	13	17.9	21.3
N	7	14	16.6	20.0
N	7	15	17.9	21.2
O	8	16	20.5	23.1
O	8	17	16.4	20.0
O	8	18	15.2	19.1
F	9	19	14.6	18.4
Ne	10	20	17.4	20.5
Ne	10	21	15.9	19.4
Ne	10	22	15.8	19.5
Na	11	23	16.4	19.8
Mg	12	24	18.3	21.1
Mg	12	25	17.0	20.2
Mg	12	26	17.5	20.7
Al	13	27	17.1	20.3
Si	14	28	18.0	20.7
Si	14	29	17.1	20.2
Si	14	30	18.1	21.1
P	15	31	17.1	20.1
S	16	32	17.3	20.1
S	16	33	16.9	19.9
S	16	34	18.2	21.1
Cl	17	35	16.6	19.6
A	18	36	16.1	19.0
Cl	17	37	16.4	19.6
A	18	38	17.8	20.6
K	19	39	15.9	18.9
A	18	40	14.5	18.2

From one point of view the most significant characteristic of our equilibrium mixture is the enormous abundance of hydrogen and helium. In itself, this is a very satisfactory feature, for one of the most striking facts known about the abundances of the elements is precisely the extreme abundance of the two lightest elements, compared to all the others. However, according to Table 3, hydrogen and helium are, together, as much as 10^7 times more abundant than oxygen, while it is currently estimated that this ratio should be more nearly 10^4 or 10^5. In considering this discrepancy it should be remembered that in our calculations of the abundances of the different nuclear species according to equation (20) we have ignored the partition-function factor, which should, strictly, be included.

It is known that in the corresponding formulae in the theory of ionization equilibrium these partition-function factors can, under favorable circumstances, amount to as much as 20 in favor of the species in the higher stages of excitation.[10] In the case of nuclear equilibrium, these factors are likely to be even more important, for the known increase

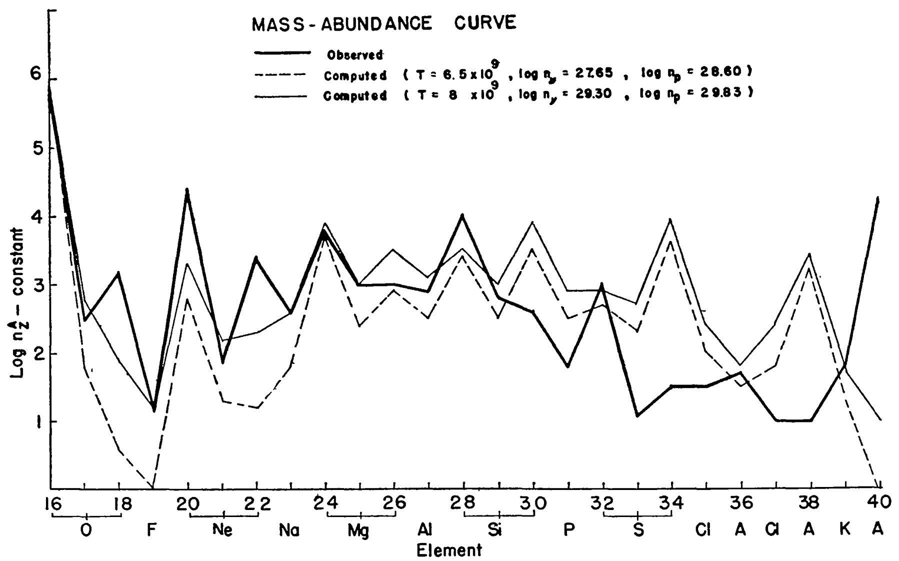

FIG. 2

in the density of excited nuclear levels with increasing mass number (particularly for the odd nuclei) would be expected to bring in large factors in the direction of increasing the oxygen-helium ratio. However, special circumstances may intervene in the cases of particular nuclei. On the whole, it appears to us that, by taking properly into account the partition-function factors in the equation of nuclear equilibrium, it should not be impossible to increase the values for the oxygen-helium ratio given in Table 3 by a factor of the order 100, while keeping the general agreement between the computed and the observed relative abundances of the nuclei from oxygen to sulphur at substantially the same level as that achieved in Figure 2 for $T = 8 \times 10^9$ degrees and $\rho = 10^7$ gm/cm^3.

The second feature to which attention should be drawn is the hydrogen-helium ratio. For the conditions derived for the pre-stellar stage in § 4 this ratio is 1:3 in favor of helium. This is contrary to the generally accepted view that hydrogen is actually the more abundant of the two. It should, however, be noted in this connection that at lower temperatures and densities the equilibrium ratio $H:He$ rapidly shifts in favor of hydrogen (see Fig. 1). It therefore seems likely that the observed high ratio between hydrogen and helium may be the result of a later stage in the prestellar development—a stage in which the relative abundances of the elements beyond oxygen had already been

[10] See, e.g., Fowler, *op. cit.*, chap. xiv, pp. 562 ff.

frozen; but the equilibrium between the fundamental nuclear particles, protons, neutrons, and α-particles still functioned sufficiently to shift the hydrogen-helium ratio in favor of the former.

And, finally, we should refer to the abundances of the heavy nuclei. It is found that the physical conditions under which we would predict anything like the observed relative abundances of the elements beyond oxygen and up to potassium (say) will be wholly inadequate to account for any appreciable amounts of the heavy nuclei. Thus, under the conditions indicated by our considerations in § 4 (namely, $T = 8 \times 10^9$ and $\rho = 10^7$gm/cm^3) the equilibrium oxygen-iron ratio is $1:10^{-10}$; this is in complete disagreement with the known ratio. Of course, we might ignore iron as an exceptionally abundant element. But, if we should go to elements beyond iron, the amounts predicted decrease so very rapidly that the conclusion is inescapable that to predict anything like the observed relative abundances of the heaviest nuclei (e.g., an oxygen-lead ratio of 10^{-7}) we need distinctly different conditions from those indicated by the relative abundances of the isotopes of a single element, the general run of the mass-abundance-curve beyond oxygen, and particularly the extreme abundance of hydrogen and helium.[11] We shall return to this matter in § 6.

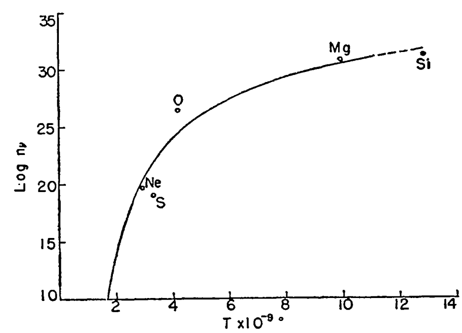

FIG. 3

6. *General remarks concerning the development of the prestellar stage.*—According to the considerations advanced in the preceding sections, we may picture the development of the prestellar stage in the following terms.

Originally, conditions of extreme temperatures ($T \sim 10^{10}$ to 10^{11} degrees) and densities (anywhere between the densities used in this paper and nuclear densities) should have prevailed. Under such conditions the heavy nuclei could presumably be formed. As the matter cooled to lower temperatures and densities, appreciable amounts of the heavy elements (about a few parts in a million) must have been "frozen" into the mixture. At temperatures of the order of from 6 to 8 billion degrees and densities ranging from 10^5 to 10^7 gm/cm^3 the present known relative abundances of the elements beyond and including oxygen must have been established; further, the extreme abundances of hydrogen and helium relative to all the other elements must also have been established during this same epoch in the development of the prestellar stage. At somewhat lower temperatures and densities the hydrogen-helium ratio must have been fixed at the known value. It may also be noted that nuclear equilibrium, as such, must have ceased completely to function at a temperature of the order of 4 billion degrees, as is evident from the discussion of § 3 and Figure 1. At still lower temperatures the abundances of the lighter nuclei—lithium, beryllium, boron, carbon, and nitrogen—must have suffered further changes from nonequilibrium processes of the character now occurring in stellar interiors. One peculiar circumstance should be mentioned at this point. In Figure 3 we have plotted the neutron concentration at which the proton maximum occurs as a function of the temperature. In the same diagram we have also plotted the neutron concentrations and temperatures derived in § 2 from the relative abundances of the isotopes of oxygen, neon, magnesium, silicon, and sulphur. It is remarkable how these points lie along the proton-maximum-curve. We are tempted to ask: "Did the original matter cool in such a way that the protons continued to exist with the maximum possible abundance?"

[11] It has already been noted by von Weizsäcker (*op. cit.*) that, to account for the relative abundances of isotopes of oxygen, conditions are needed which are very different, indeed, from that required to account, for example, for the observed oxygen-lead ratio. But our conclusions stated in the text are more general than this.

Finally, we may briefly refer to the cause of the original expansion and cooling. One suggestion is that it may be connected with the beginning of the expansion of the universe. Another (possibly related) suggestion is that it might have arisen from the loss of energy by neutrino emission in the manner contemplated by Gamow and Shoenberg in a different connection.[12]

In conclusion, it should perhaps be emphasized that the considerations of this paper should be regarded as of a purely exploratory nature and that such "agreements" as may have been obtained should not be overstressed. It should, indeed, be remembered that we are here dealing with a stage in the evolution of the universe in which conditions were utterly different from the present conditions. To emphasize this fact, it may be remarked that at an average density of the order of 10^7 gms/cm^3 the entire mass in the universe (*ca.* 10^{54} gm.) may be inclosed in a sphere of radius of the order of 10^{16} cm., i.e., somewhat less than a hundredth of a parsec.

YERKES OBSERVATORY
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[12] *Phys. Rev.*, **59**, 539, 1941.