ON THE CONTINUOUS ABSORPTION COEFFICIENT OF THE NEGATIVE HYDROGEN ION. V

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ABSTRACT

The photoionization cross-sections of the negative hydrogen ion derived in Paper IV, using Hart and Herzberg's 20-parameter wave function, are further improved by replacing the plane-wave approximation for the free electron by the Hartree approximation

I. INTRODUCTION

In Paper IV (Chandrasekhar 1958) of this series, the photoionization cross-sections of the negative hydrogen ion were revised by making use of Hart and Herzberg's (1957) 20-parameter wave function for the ground state of the ion. However, the free electron was represented by a plane wave. In this paper we shall replace this representation by a Hartree approximation and consider the extent of the improvements effected.

II. THE BASIC FORMULA AND RESULTS

By following the procedure outlined in Paper II (Chandrasekhar 1945), we find that, with Hart and Herzberg's wave function for the ground state and the Hartree wave function for the free state, the basic formula for the dipole velocity matrix element becomes

$$\int \Psi_d \left(\frac{\partial}{\partial z_1}+\frac{\partial}{\partial z_2}\right) \Psi_c d\tau = -i\,(2048\pi^3)^{1/2}\,\frac{\mathfrak{N} q^3}{k}\int_0^\infty W_2(r)\,\chi_1(r)\,dr\,, \qquad (1)$$

where $\chi_1(r)$ is the radial part of the p-spherical wave (as defined in Paper II, eqs. [6] and [7]) and

$$W_2(r) = \left(\sum_{j=-1}^{7} L_j r^j\right) e^{-ar} + \left(\sum_{j=-1}^{2} S_j r^j\right) e^{-(1+2a)r}\,. \qquad (2)$$

In equation (2) the L_j's and the S_j's have the same meanings (and values) as in Paper IV, equation (14). The quantities k, $\mathfrak{N}$, q, and a also have the same meanings as in Paper IV.

The "weight function," $W_2(r)$, computed with the values for the constants given in Paper IV is listed in Table 1; this table should be contrasted with the corresponding table (Table 1) in Paper II derived from Henrich's 11-parameter wave function.

The corresponding formula for the absorption cross-section is given by

$$\kappa_\lambda = \frac{4.02024}{k\,(k^2+0.055289)}\left|\int_0^\infty W_2(r)\,\chi_1(r)\,dr\right|^2 \times 10^{-18}\ \text{cm}^2\,; \qquad (3)$$

this cross-section refers to a wave length (cf. Paper IV, eq. [3])

$$\lambda = \frac{911.2671}{k^2+0.055289}\ \text{A}\,. \qquad (4)$$

The values of κ_λ have been evaluated in accordance with equation (3) by making use of Chandrasekhar and Breen's (1946) tabulation of the functions $\chi_1(r)$. The results are given in Table 2. For comparison, the results of the earlier calculations are also included in this table. From this comparison it would appear that no substantial improvements in the deduced cross-sections are to be expected by going to more accurate wave functions

TABLE 1

THE WEIGHT FUNCTION $W_2(r)$

r	$W_2(r)$	r	$W_2(r)$	r	$W_2(r)$	r	$W_2(r)$
0	0	4 4	0 18140	8 8	0 04836	16 4	0 00558
0 1	0 12665	4 5	17538	8 9	04712	16 6	00522
0 2	22763	4 6	16958	9 0	04591	16 8	00487
0 3	30707	4 7	16400	9 1	04474	17 0	00455
0 4	36851	4 8	15861	9 2	04360	17 2	00425
0 5	41502	4 9	15342	9 3	04250	17 4	00396
0 6	44920	5 0	14842	9 4	04142	17 6	00369
0 7	47326	5 1	14360	9 5	04037	17 8	00344
0 8	48907	5 2	13896	9 6	03936	18 0	00320
0 9	49819	5 3	13449	9 7	03836	18 2	00298
1 0	50193	5 4	13018	9 8	03740	18 4	00277
1 1	50140	5 5	12603	9 9	03646	18 6	00258
1 2	49748	5 6	12203	10 0	03554	18 8	00239
1 3	49094	5 7	11818	10 2	03377	19 0	00222
1 4	48239	5 8	11447	10 4	03209	19 2	00207
1 5	47232	5 9	11090	10 6	03049	19 4	00192
1 6	46116	6 0	10746	10 8	02896	19 6	00178
1 7	44922	6 1	10414	11 0	02750	19 8	00165
1 8	43678	6 2	10095	11 2	02611	20 0	00153
1 9	42428	6 3	09788	11 4	02477	20 2	00141
2 0	41117	6 4	09492	11 6	02350	20 4	00131
2 1	39831	6 5	09207	11 8	02228	20 6	00121
2 2	38556	6 6	08933	12 0	02112	20 8	00112
2 3	37299	6 7	08669	12 2	02000	21 0	00104
2 4	36066	6 8	08414	12 4	01894	21 2	00096
2 5	34862	6 9	08169	12 6	01792	21 4	00089
2 6	33689	7 0	07932	12 8	01695	21 6	00082
2 7	32549	7 1	07704	13 0	01602	21 8	00076
2 8	31443	7 2	07485	13 2	01513	22 0	00070
2 9	30373	7 3	07273	13 4	01428	22 2	00064
3 0	29338	7 4	07068	13 6	01347	22 4	00059
3 1	28337	7 5	06871	13 8	01270	22 6	00055
3 2	27371	7 6	06681	14 0	01197	22 8	00050
3 3	26439	7 7	06497	14 2	01127	23 0	00047
3 4	25540	7 8	06320	14 4	01060	23 2	00043
3 5	24672	7 9	06148	14 6	00997	23 4	00039
3 6	23836	8 0	05983	14 8	00937	23 6	00036
3 7	23029	8 1	05823	15 0	00880	23 8	00033
3 8	22252	8 2	05668	15 2	00826	24 0	00031
3 9	21502	8 3	05518	15 4	00775	24 2	00028
4 0	20779	8 4	05373	15 6	00727	24 4	00026
4 1	20083	8 5	05232	15 8	00681	24 6	00024
4 2	19411	8 6	05096	16 0	00638	24 8	00022
4 3	0 18764	8 7	0 04964	16 2	0 00597	25 0	0 00020

for the ground state of the ion. It is also unlikely that improvements will be affected by going to better representations of the free state.

TABLE 2

PHOTOIONIZATION CROSS-SECTIONS OF H^- COMPUTED WITH DIPOLE-VELOCITY FORMULA, USING HART AND HERZBERG'S 20-PARAMETER AND HENRICH'S 11-PARAMETER WAVE FUNCTIONS FOR GROUND STATE AND HARTREE WAVE FUNCTIONS FOR FREE STATE

k^2	With 11-Parameter Wave Function		With 20-Parameter Wave Function	
	λ (A)	$\kappa_\lambda \times 10^{17}$ cm^2	λ (A)	$\kappa_\lambda \times 10^{17}$ cm^2
1.75	505	0 0657	505	0 0656
0 80	1066	0 333	1065	0 342
0 50	1642	0 740	1641	0 751
0 35	2249	1 231	2248	1 226
0 25	2987	1 84	2985	1 83
0 20.	3572	2 32	3570	2 30
0 175	3960	2 62	3957	2 61
0 150	4443	2 97	4439	2 98
0 125	5059	3 39	5054	3 41
0 100.	5875	3 87	5868	3 90
0 090	6280	4 06	6272	4 09
0 070	7283	4 41	7273	4 41
0 055	8275	4 52	8263	4 49
0 050	8669	4 50	8655	4 45
0 045	9102	4 44	9086	4 37
0 035	10111	4 13	10093	4 03
0 020	12131	2 96	12104	2 83

Note added in proof: In a recent paper, Bransden, Dalgarno, John, and Seaton (1958) have published calculations for *p*-waves in the field of a neutral hydrogen atom allowing for exchange and polarization, and they have tabulated functions which are equivalent to our $\chi_1(r)$ in equation (1) for $k^2 = 0.25$. Using their functions (denoted by G_1^+ in Table 10 of their paper), together with the weight function $W_2(r)$ of the present paper, we find for κ_λ the values 1.50×10^{-17} cm^2 and 1.42×10^{-17} cm^2 for their functions with $d \neq 0$ and $d = 0$, respectively; these values should be contrasted with 1.83×10^{-17} cm^2 (see Table 1) obtained with the Hartree wave function. The difference arising from the use of a more accurate representation for the free state for $k^2 = 0.25$ (corresponding to $\lambda = 2985$ A) is therefore not inappreciable. However, the difference resulting from allowing for exchange and polarization will decrease for increasing λ, since the values of $W_2(r)$ for large values of r become increasingly important; and, as Bransden *et al.* point out, the difference between their wave functions and the Hartree wave functions (Chandrasekhar and Breen 1946) becomes negligible for $r > 4$.

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