A primer on problems and prospects of dark energy

M. Sami

Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi 110 025, India

This review on dark energy is intended for a wider audience, beginners as well as experts. It contains important notes on various aspects of dark energy and its alternatives. The section on Newtonian cosmology followed by heuristic arguments to capture the pressure effects allows us to discuss the basic features of physics of cosmic acceleration without actually resorting to the framework of the general theory of relativity. The brief discussion on observational aspects of dark energy is followed by a detailed exposition of underlying features of scalar field dynamics relevant to cosmology. The review includes pedagogical presentation of generic features of models of dark energy and its possible alternatives.

Keywords: Dark energy, modified theories of gravity, Newtonian cosmology, pressure corrections, scalar field dynamics.

Introduction

THE 20th century has witnessed remarkable developments in the field of cosmology. The observation of redshift of light emitted by distant objects and the discovery of microwave background in 1965 have revolutionized our thinking about the universe. The Hot Big Bang model then received the status of the standard model of the universe. However, in spite of the theoretical and observational successes, cosmology remained confined to a rather narrow class of scientists; others considered it as the part of a respectable philosophy of science. Cosmology witnessed the first revolution in 1980 with the invent of cosmological inflation, making it acceptable to the larger community of physicists. Since then, it has been going hand-in-hand with high-energy physics. The scenario envisages that the universe has gone through a phase of fast accelerated expansion at early epochs. Inflation is a paradigm which can resolve some of the in-built inconsistencies of the Hot Big Bang model and provides a mechanism for generation of primordial fluctuations needed to seed the structure we see in the universe today. In the past two decades, observations have repeatedly confirmed the predictions of inflation. However, its implementation is ad hoc and requires support from a fundamental theory of high-energy physics. As inflation takes place around the Planck epoch, the needle of hope points towards string theory – a consistent theory of quantum gravity.

The second revolution cosmology witnessed in 1998, is related to late time cosmic acceleration[1,2]. The observations of high redshift supernovae reveal that the universe is accelerating at present. The phenomenon is indirectly supported by data of complimentary nature such as cosmic microwave background (CMB), large-scale structure, baryon acoustic oscillation and weak lensing. It is interesting that the thermal history of our universe is sandwiched between two phases of accelerated expansion. In the Newtonian language, cosmic repulsion can be realized by supplementing the Newtonian force by a repulsive term on phenomenological grounds. The rigorous justification of the phenomenon can only be provided in the framework of general theory of relativity (see ref. 3 for early attempts in this direction). Late time acceleration can be fuelled either by an exotic fluid with large negative pressure, dubbed the dark energy[4–10] or by modifying the gravity itself[11]. The simplest candidate of dark energy is provided by the cosmological constant Λ, though there are difficult theoretical issues associated with it[12–14]. Its small numerical value leads to a fine tuning problem and we do not understand why it becomes important today a la coincidence problem.

Scalar fields provide an interesting alternative to cosmological constant[16,17]. To this effect, cosmological dynamics of a variety of scalar fields has been investigated in the literature (see Copeland *et al.*[6] for details). They can mimic cosmological constant-like behaviour at late times and can provide a viable cosmological dynamics at early epochs. Scalar-field models with generic features are capable of alleviating the fine-tuning and coincidence problems. As for the observation, at present, it is absolutely consistent with Λ, but at the same time, a large number of scalar-field models are also permitted. Future data should allow to narrow down the class of permissible models of dark energy.

As an alternative to dark energy, the large-scale modifications of gravity could account for the current acceleration of the universe. We know that gravity is modified at short distance and there is no guarantee that it would not suffer any correction at large scales, where it is never verified directly. Large-scale modifications might arise from extra-dimensional effects or can be inspired by fundamental theories. They can also be motivated by phe-

e-mail: sami@iucaa.ernet.in

nomenological considerations such as $f(R)$ theories of gravity. However, any large-scale modification of gravity should reconcile with local physics constraints and should have potential of being distinguished from the cosmological constant. To the best of our knowledge, all the schemes of large-scale modification, at present, are plagued with some other problems.

The review is organized as follows: After introduction and a brief background, I present cosmology in Newtonian framework titled 'The homogeneous and isotropic Newtonian cosmology' and mention efforts to put it on the rigorous foundations in the domain of its validity. Next, I titled 'Beyond Newtonian physics: pressure corrections' have put forward heuristic arguments to incorporate Λ, in particular and pressure corrections, in general, in the evolution equations and describe the broad features of cosmological dynamics in presence of cosmological constant. Then a short introduction to relativistic cosmology is provided and issues associated with cosmological constant are discussed. After a brief subsection on observational aspects of cosmic acceleration, I will proceed to highlight the generic features of scalar field dynamics relevant to cosmology and mention the current observational status of dynamics of dark energy. In the last section before summary, I present a discussion on the current problems of alternatives to dark energy.

Last but not least, a suggestion for the follow-up of this review is in order. At present, there exist, a number of excellent reviews on dark energy[4–10] and cosmological constant[12–14], which focus on different aspects of the subject. Four recent and interesting reviews[7–10] which try to address the theoretical and observational aspects of late-time cosmic acceleration are highly recommended. Humility does not allow me to mention that Copeland *et al.*[6] is the most comprehensive theoretical review on dark energy with pedagogical exposition.

The smooth expanding universe

The universe is clumpy at small scales and consists of a rich structure of galaxies, local groups of galaxies, clusters of galaxies, super-clusters and voids. These structures typically range from kiloparsecs to 100 megaparsecs. The study of large-scale structures in the universe shows no evidence of new structures at scales larger than 100 megaparsecs. The universe appears smooth at such scales, which leads to the conclusion that it is homogeneous and isotropic at large scales, which serves as one of the fundamental assumptions in cosmology known as cosmological principle[18]. Homogeneity tells us that the universe looks the same when observed from any point, whereas isotropy indicates that it looks the same in any direction. In general, these are two independent requirements. However, isotropy at each point is a stronger assumption which implies homogeneity also. The cosmological principle presents an idealized picture of the universe which allows us to understand the background evolution. The departure from smoothness can be taken into account through perturbations around the smooth background. Observations confirm the presence of tiny fluctuations from smoothness in the early universe. According to modern cosmology, these small perturbations via gravitational instability are believed to have grown into the structures we see today in the universe[19–27].

One of the most remarkable discoveries in cosmology includes the expansion of the universe and its beginning from the Big Bang. The analysis of radiation spectrum emitted from distant galaxies shows that wavelengths of spectral lines are larger than the actually emitted ones; the phenomenon is known as redshift of light. Redshift is quantified by the symbol z defined as $z = (\lambda_{\text{ob}} - \lambda_{\text{em}})/\lambda_{\text{em}}$. According to the Doppler effect, the wavelength of light emitted by a source receding from the observer appears shifted towards the red end of the spectrum and the redshift is related to the velocity of recession v as $z \simeq v/c$ for $v \ll c$. In the beginning of the last century, astronomers could measure the distances to a number of distant galaxies. Hubble carried out investigations of recession velocities and plotted them against the distances to galaxies. He concluded in 1929 that there is a linear relation between recession velocity of the galaxies and the distance to them – the so called Hubble law.

The observational conclusion that the universe expands is based upon the redshift of radiation emitted by distant galaxies. Can we have another explanation for the redshift? It might look surprising that photons from larger distances emitted from galaxies reach us redshifted due to the recession of galaxies and nothing else happens to them. They travel through the intergalactic medium and could be absorbed by matter present there and then emitted, losing part of their energy in this process and thereby leading to their redshift without resorting to expansion of the universe. This apprehension can be refuted by a simple argument. As for the absorption, the underlying process is related to the scattering of photons by the particles of the intergalactic medium. If this is true, the source should have appeared blurred, which is never observed. Other efforts assuming the exotic interactions of photons could not account for the observed redshift. Thus the only viable explanation of the phenomenon is provided by the expansion of universe[22].

If we imagine moving backward in time, the universe was smaller in size, the temperature was higher and there was an epoch when the universe was vanishingly small with infinitely large energy density and temperature – the beginning of the universe dubbed as the Big Bang. Matter was thrown away with tremendous velocity; since then, the universe is expanding and cooling. At early times, it was extremely hot and consisted of a hot plasma of elementary particles. There were no atoms and no nuclei. Roughly speaking, at temperatures higher than the binding energy of hydrogen atom, the photons were freely scattering on electrons and atoms could not form. As the universe cooled below the temperature characterized by the binding energy of hydrogen atom, the electrons combined with the protons to form hydrogen atoms leading to the decoupling of radiation from matter. This was an important epoch in the history of the universe, known as recombination. The decoupled radiation since then is just expanding with the expanding universe and cooling. The discovery of microwave background, the relic of the Big Bang in 1965 confirms the hypothesis of Hot Big Bang.

The homogeneous and isotropic Newtonian cosmology

Newtonian theory of gravitation allows us to understand the expansion of a homogeneous, isotropic universe in a simple way. The Newtonian description is valid provided the matter filling the universe is non-relativistic and scales associated with the problem are much smaller than the Hubble radius. For instance, at early epochs, the universe was hot, dominated by radiation. Hence the early universe, strictly speaking, should be treated using relativistic theory. The general theory effects are also crucial at super Hubble scales. Despite its limitations, Newtonian cosmology provides a simple and elegant way of understanding the expansion of the universe[18,22,27,28].

Hubble law as a consequence of homogeneity and isotropy

Using the Newtonian notions of physics, let us show that the Hubble law is a natural consequence of homogeneity and isotropy. Let us choose a coordinate system with origin O, such that matter is at rest there and let us observe the motion of matter around us from this coordinate system. The velocity field, i.e. the velocity of matter at each point p around us at an arbitrary time, depends upon the radius vector $\mathbf{r}$ and time t. We should now look for the most general velocity field in a homogeneous and isotropic universe. Let us assume another observer located at point O' with radius vector $\mathbf{r}_{O'}$ and moving with velocity $\mathbf{v}(\mathbf{r}_{O'})$ with respect to the observer O. If we denote the velocity of point p relative to O and O' at time t by $\mathbf{v}(\mathbf{r}_p)$ and $\mathbf{v}'(\mathbf{r}'_p)$, we have,

$$\mathbf{r}'_p = \mathbf{r}_p - \mathbf{r}_{O'}, \tag{1}$$

$$\mathbf{v}'(\mathbf{r}'_p) = \mathbf{v}(\mathbf{r}_p) - \mathbf{v}(\mathbf{r}_{O'}), \tag{2}$$

where $\mathbf{r}_p$ and $\mathbf{r}'_p$ denote the radius vectors of point p with respect to O and O' respectively. The cosmological principle tells us that the velocity field should have the same functional form at any point,

$$\mathbf{v}(\mathbf{r}'_p) = \mathbf{v}(\mathbf{r}_p) - \mathbf{v}(\mathbf{r}_{O'}), \tag{3}$$

which clearly implies that the velocity field is a linear function of its argument $\mathbf{r}$,

$$v(\mathbf{r}, t) = T(t)\mathbf{r}, \tag{4}$$

where T is a 3×3 matrix. The matrix can always be diagonalized by choosing a suitable coordinate system. Isotropy then reduces it to Kronecker symbol ($T_{i,j} = H(t)\delta_{i,j}$) leading to

$$\mathbf{v}(\mathbf{r}, \mathbf{t}) = H(t)\mathbf{r}, \tag{5}$$

where H is known as the Hubble parameter. In general, a velocity field can always be decomposed into a rotational part, inhomogeneous part and isotropic part at each point. It is not surprising then that the homogeneous and isotropic velocity field has the form given in eq. (5), known as the Hubble law.

It can easily be verified that the Hubble law holds at any point. If we move from O to O', we can write

$$\mathbf{v}'(\mathbf{r}'_p) = H\mathbf{r}_p - H\mathbf{r}_{O'} = H(t)\mathbf{r}'_p. \tag{6}$$

The Hubble law gives the most general form of velocity field permissible by the homogeneity and isotropy of space.

Hubble law tells us how the distance between any two points in space changes with time provided we know the expansion rate given by $H(t)$,

$$\mathbf{r}(t) = \mathbf{x}e^{\int_0^t H(t)\mathrm{d}t}, \quad \mathbf{x} \equiv \mathbf{r}(t = 0). \tag{7}$$

The law of expansion depends upon how the Hubble parameter H varies with time. Equation (7) shows how distances in a homogeneous and isotropic universe scale with the scale factor $a(t)$,

$$a(t) \equiv e^{\int_0^t H(t)\mathrm{d}t} \quad \text{or} \quad H(t) = \frac{\dot{a}}{a}, \tag{8}$$

$$r(t) = a(t)x. \tag{9}$$

Complete information about the dynamics of a homogeneous and isotropic universe is contained in the scale factor; we thus need the evolution equation to determine $a(t)$. In case H is independent of time, we have an exponentially expanding universe dubbed de-Sitter space. In what follows I shall confirm that constant Hubble rate is allowed in relativistic cosmology provided the energy density of matter in the universe is constant. It is believed that the universe has passed through an exponentially expanding phase known as inflation at early times.

According to the Hubble law, in a homogeneous and isotropic universe, all the material particles move away radially from the observer located at any point in the universe. This motion is refereed to as Hubble flow. Indeed, any freely moving particle in such a background would ultimately follow the Hubble flow. Motion over and above the Hubble flow is called peculiar motion, which can only arise in a perturbed universe. It often proves convenient to change a coordinate system dubbed comoving, which expands with the expanding universe. Matter which follows the Hubble flow will be at rest in the comoving coordinate system, i.e. matter filling a homogeneous, isotropic universe is at rest with respect to the comoving observer. Both the frames are physically equivalent. Let us clarify that the universe does not appear homogeneous and isotropic to any observer; for instance, if an observer is moving with a large velocity, say, towards a particular galaxy, the universe looks different to him/her. A physical coordinate system is a system in which matter is at rest at the origin and moves away radially at other points. The radius vector **r** of any point in this system called physical, changes with time, whereas its counterpart **x** in the comoving system is constant. This means that physical distance between any two points in the expanding universe is given by the comoving distance multiplied by a factor that depends upon time, which is precisely expressed by eq. (7) or equivalently by eq. (9).

Evolution equations

I now turn to the evolution equation for the scale factor. Thanks to isotropy, we can employ spherical symmetry to derive the evolution equation. At a given time t called the cosmic time, let us consider a sphere centred at O with radius $r(t)$. Let $\rho_b(t)$ be the density of matter in the homogeneous, isotropic space referred to as background space hereafter. We assume that the net gravitational force on a particle of mass m situated on the surface of the sphere due to matter outside the sphere is zero, which means that matter inside the sphere alone can influence the motion of the particle. The total energy of the particle on the surface of the sphere (Figure 1) at any time is constant given by the expression[29],

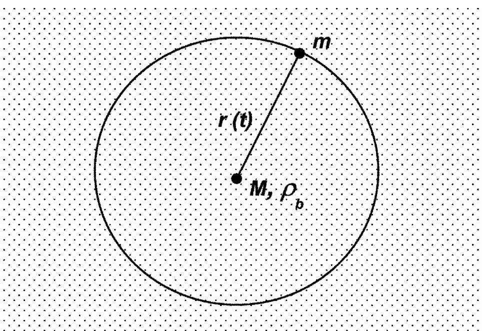

Figure 1. Particle of mass m on the surface of a sphere of radius $r(t)$ in an expanding universe with uniform matter density.

$$E_{\text{Tot}} = \frac{1}{2} m\dot{r}^2 - \frac{4\pi}{3} mG\rho_b r^2. \tag{10}$$

This equation can be cast in the following convenient form,

$$H^2 \equiv \left(\frac{\dot{r}(t)}{r(t)}\right)^2 = \frac{8\pi}{3} G\rho_b(t) + \frac{2E_{\text{Tot}}}{mr^2(t)}, \tag{11}$$

which readily translates into an evolution equation for $a(t)$ (see eq. (9)) known as the Friedmann equation,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_b(t) - \frac{K}{a^2}, \quad K = \frac{2E_{\text{Tot}}}{x^2 m}, \tag{12}$$

where K can be zero, negative or positive depending on how kinetic energy compares with the potential energy.

In order to solve the evolution equation for $a(t)$, we need to know how matter density $\rho_b(t)$ changes with time, i.e. we need the conservation equation in the expanding universe. For a non-relativistic fluid, the continuity equation that gives us the evolution of matter density of the fluid is,

$$\frac{\partial \rho_b(t)}{\partial t} + (\nabla \cdot \rho_b \mathbf{v}) = 0. \tag{13}$$

Remembering that the matter density of the background fluid is independent of the coordinates and fluid velocity is given by the Hubble law (i.e. eq. (6)), we transform the continuity equation to have the usual form,

$$\frac{\partial \rho_b(t)}{\partial t} + 3H\rho_b = 0, \tag{14}$$

which formally integrates to,

$$\rho_b(t) = \rho_b^{(0)} \left(\frac{a_0}{a}\right)^3, \tag{15}$$

where the subscript '0' denotes the quantities at the present epoch. The evolution of matter density of nonrelativistic fluid has a simple meaning that the mass of the fluid in a comoving volume remains constant.

Though eq. (12) formally resembles the evolution equation of relativistic cosmology, its derivation presented above is defective. The expression for the potential energy is written with an assumption that gravitational potential can be chosen as zero at infinity, which is not true in an infinite universe. Since the mass density ρ_b is constant in space, the total mass of universe diverges as r^3. As a result, the potential $-4\pi G\rho_b r^2/3$ cannot be normalized to zero at $r = \infty$. One could try to circumvent the problem by assuming that ρ_b vanishes for a given large value of r, but it would conflict with the underlying assumption of homogeneity. Therefore, conservation of energy is difficult to understand in an infinite universe with uniform matter density.

We can also derive the evolution equations using the Newtonian force law[18]. The force on the unit mass situated on the surface of a homogeneous sphere with radius r is given by

$$\mathbf{F} = -\frac{4\pi G}{3}\rho_b \mathbf{r}. \tag{16}$$

The Euler's equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{\nabla P_b}{\rho_b} + \mathbf{F} \tag{17}$$

in a homogeneous isotropic background simplifies to

$$\mathbf{F} = (\dot{H} + H^2)\mathbf{r}, \tag{18}$$

where $\mathbf{F}$ is the force per unit mass on the fluid element given by eq. (16). We have used the fact that pressure gradients are absent in a homogeneous, isotropic background and the velocity field is given by the Hubble law. It should also be noted that the pressure $P_b = 0$ for the non-relativistic background fluid under consideration. Using eqs (16) and (18), we obtain the equation for acceleration,

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\rho_b(t), \tag{19}$$

which could also be obtained directly from eq. (16). Equation (19) can easily be integrated to give the Friedmann equation. Indeed, by multiplying the above equation by $\dot{a}$ and using the evolution of mass density allows us to write

$$H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2}, \tag{20}$$

$$K \equiv a_0^2\left(\frac{8\pi G\rho_b^{(0)}}{3} - H_0^2\right). \tag{21}$$

The above derivation is also problematic as it assumes that the mass outside the sphere, used while writing eq. (16), can be neglected, which is not true for an infinite universe with constant mass density.

The problem can be circumvented using the geometric reformulation of Newtonian gravity in the language of Cartan. According to Cartan's formulation, orbits of particles are assumed to be the geodesics of an affine space and gravity is then described by the curvature of the affine connection (see Tipler[30] and references therein). According to Tipler[30], no pathology in cosmology associated with Newton's force law then occurs and the evolution equations of Newtonian cosmology,

$$H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2}, \tag{22}$$

$$K \equiv a_0^2\left(\frac{8\pi G\rho_b^{(0)}}{3} - H_0^2\right), \tag{23}$$

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\rho_b(t), \tag{24}$$

$$\frac{\partial \rho_b(t)}{\partial t} + 3H\rho_b(t) = 0, \tag{25}$$

can be put on rigorous foundations. Equations (22), (24), and (25) are identical to the evolution equations of Friedmann cosmology for non-relativistic fluid filling the universe. Whether or not one adopts the formulation presented in Tipler[30], Newtonian cosmology is nevertheless elegant and simple.

Let us point out an important feature of Newtonian cosmology. We note that the expression of K/a^2 remains unchanged under the scale transformation $a(t) \to Ca(t)$, C being constant. As a result, the evolution equations (i.e. eqs (22) and (24)) also respect the scale-invariance. This invariance is a characteristic of specially flat Friedmann cosmology. The Newtonian cosmology can mimic all three topologies of relativistic cosmology corresponding to $K = 0, \pm 1$, in spite of the fact that the underlying geometry in Newtonian cosmology is Euclidean. Let us note that the scale factor in Newtonian cosmology can always be normalized to a convenient value at the present epoch. This is related to a simple fact that the Friedmann equation (eq. (22)) does not change if we re-scale the scale factor, which leaves the normalization of a arbitrary. The often used normalization fixes the scale factor $a(t) = 1$ at the present epoch, i.e. $a_0 = 1$. In case of relativistic cosmology, the latter can only be done in the case of $K = 0$, whereas in the case of $K = \pm 1$, the numerical value of the scale factor a_0 depends upon the matter content of the universe.

The second important feature of Newtonian cosmology is that it leads to an evolving universe. Indeed, we could ask for a static solution given by $\dot{a}(t)$ and $\ddot{a}(t) = 0$, which is permitted by the Friedmann equation (eq. (22)) but not allowed by the equation for acceleration (eq. (24)). It is remarkable that Newtonian cosmology gives rise to an evolving universe. It is an irony that the discovery of expansion of the universe had to wait for the general theory of relativity. This is related to the commonly held perception of static universe, which was prevalent before Friedmann discovered the non-static cosmological solution of Einstein equations. So much so that Einstein himself did not believe in the Friedmann solution in the beginning and tried to reconcile his theory with the static universe by introducing cosmological constant, which he later withdrew.

The past, future and how old are we?

The general features of solutions of the evolution equations can be understood without actually solving them. What can we say about the past and the fate of universe? The equation for acceleration tells us that $\ddot{a} < 0$ for standard form of matter. This means that $a(t)$ as a function of time is concave downward. We need input regarding $\dot{a}$ at present to make important conclusions about the past. Observation tells us that $\dot{a}(t) > 0$ at present. Thus $a(t)$ monotonously decreases as t runs backward. It is therefore clear that there was an epoch in the history of the universe when $a(t)$ vanishes identically. Without the loss of generality, we can take $t = 0$ corresponding to $a(t) = 0$.

As for the fate of the universe, the problem is similar to that of escape velocity, namely if $K > 0$, the kinetic energy is less than the potential energy. In this case $a(t)$ would increase to a maximum value where $\dot{a}(t) = 0$, it would start decreasing thereafter till it vanishes and the universe ends itself in a big crunch. In case $K < 0$, the scale factor would go on increasing forever; $K = 0$ represents the critical case. Three different possibilities, $K = 0$, $K > 0$ or $K < 0$ correspond to critical, closed and open universe respectively. We should emphasize that the fate of the universe also crucially depends upon the nature of matter filling the universe. In some case, the universe may end itself in a singular state or the cosmic doomsday.

Which of the three possibilities is realized in nature? To answer this question, let us rewrite eq. (12) in a convenient form,

$$\Omega_b(t) - 1 = \frac{K}{(aH)^2}, \Omega_b(t) = \frac{\rho_b(t)}{\rho_c(t)}, \tag{26}$$

where the critical density is defined as, $\rho_c(t) = 3H^2(t)/8\pi G$. Specializing eq. (26) to the present epoch, we find that,

$$\Omega_b^0 > 1\,(\rho_b^{(0)} > \rho_c^{(0)}) \Rightarrow K > 0 \rightarrow \text{ closed universe,}$$

$$\Omega_b^0 = 1\,(\rho_b^{(0)} = \rho_c^{(0)}) \Rightarrow K = 0 \rightarrow \text{critical universe,}$$

$$\Omega_b^0 < 1\,(\rho_b^{(0)} < \rho_c^{(0)}) \Rightarrow K < 0 \rightarrow \text{open universe.}$$

where the superscript '0' designates the corresponding physical quantities at the present epoch. Since we know the observed value of $\rho_c^{(0)}$, one of the three types of universe we live in, depends upon how matter density in the universe compares with $\rho_c^{(0)}$. Observations on CMB indicate that the universe is critical to a good accuracy or $K \simeq 0$, which is consistent with the inflationary paradigm.

Let us come to the solution of the Newtonian cosmology in case of $K = 0$. Substituting $\rho_b(t)$ from eq. (15) in eq. (22), we find that, $\dot{a}^2 \sim a^{-1}$, which easily integrates giving rise to

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}, \tag{27}$$

$$\rho_b(t) = \rho_b^{(0)}\left(\frac{t_0}{t}\right)^2, \tag{28}$$

$$H(t) = \frac{2}{3}\frac{1}{t}. \tag{29}$$

The above solution is known as Einstein–de-Sitter solution. We can estimate the age of universe using eq. (29),

$$t_0 = \frac{2}{3}\frac{1}{H_0}. \tag{30}$$

Interestingly, if gravity were absent, the universe would expand with constant rate given by H_0. Using the Hubble law we would then find,

$$t_0 = \frac{1}{H_0}, \tag{31}$$

which is the maximum limit for the age of the universe in the Hot Big Bang model ($2H_0^{-1}/3 \le t_0 < H^{-1}$). The presence of standard matter always leads to deceleration, thereby leading to smaller time taken to reach the present Hubble rate of expansion. The presence of cosmological constant or any other exotic form of matter can crucially alter this conclusion.

Cosmological constant a la Hooke's law

We have seen that Newtonian cosmology gives rise to the evolving universe but for the historical reasons, cosmology had to wait for the general theory of relativity to discover it. The fact that Newtonian cosmology leads to non-stationary solution was known before the general theory was discovered, but it could receive attention as it conflicted with the perception of the static universe. Attempts were then made to modify Newtonian gravity to reconcile it with the static universe. Clearly, the modification should be such that it becomes effective at large scales, leaving the local physics unchanged. Looking at the Newton's force law (eq. (16)), it is not difficult to guess that a static solution is possible provided that we add a repulsive part proportional to the radius vector **r** in eq. (16). Newton's law of gravitation should therefore be supplemented by linear force law[18,22,31,32]

$$\mathbf{F} = -\frac{4\pi G}{3}\rho_{\mathrm{b}}\mathbf{r} + \frac{1}{3}\Lambda\mathbf{r}, \tag{32}$$

where Λ is known as the cosmological constant which is positive in the present context. It is interesting to note that there are only two central forces, namely the inverse square force and the linear force, which give rise to stable circular orbits.

Our discussion of cosmological constant is heuristic and the motivation here is to incorporate the repulsive effect in the evolution equations. We rewrite the modified force law (eq. (32)) as an equation of acceleration using the comoving coordinates,

$$\frac{1}{a}\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} = -\frac{4\pi G}{3}\rho_{\mathrm{b}}(t) + \frac{\Lambda}{3}, \tag{33}$$

which shows that a positive Λ term contributes to acceleration, as it should. The integrated form of eq. (33) is given by,

$$H^2 = \frac{8\pi G}{3}\rho_{\mathrm{b}}(t) - \frac{K}{a^2} + \frac{\Lambda}{3}, \tag{34}$$

where the integration constant K can be formally written again through physical quantities defined at the present epoch. The modified force law (eq. (32)) was proposed much before Einstein's general theory of relativity by Neumann[31] and Seeliger[32] in 1895–96.

Let us note that adding the cosmological constant to Newtonian force is equivalent to adding a constant matter density $\rho_\Lambda = \Lambda/8\pi G$ to the background matter density ρ_{b} which does not to go well with the continuity equation (eq. (14)). Since the acceleration equation also gets modified in the presence of Λ, we should check whether the modified evolution equations allow this possibility. If we differentiate eq. (34) with respect to time and respect the modified acceleration equation, we find that constant matter density is permissible in the expanding universe. As for the continuity eq. (14), it is valid for a perfect non-relativistic fluid. The cosmological constant does not belong to this category; the pressure corresponding to constant energy density is not zero. The continuity equation should take the note of pressure and get appropriately modified. As pointed out earlier, the present discussion of cosmological constant here is qualitative. Rigorously speaking, we are trying to get the right thing in the wrong place! I shall come back to this point after I incorporating the pressure corrections in the evolution equations.

The evolution equations, i.e. eqs (34) and (33) admit a static solution ($a = \text{const} = a_0$) in case of $K > 0$. Static Einstein universe ($\dot{a} = 0$ and $\ddot{a} = 0$) is possible provided that Λ has definite numerical value

$$\Lambda = \Lambda_{\mathrm{c}} = 4\pi G\rho_{\mathrm{b}}^{(0)}. \tag{35}$$

We shall observe after a short while that the static Einstein universe is unstable under small fluctuations.

The qualitative features of solutions of the evolution equations can be understood without actually solving them. Equation (33) can be thought of as an equation of a point particle in one dimension[4,33],

$$\ddot{a} = -\frac{\partial V}{\partial a}, \tag{36}$$

moving in potential field

$$V(a) = -\left(\frac{4\pi G\rho_{\mathrm{b}}a^2}{3} + \frac{\Lambda a^2}{6}\right), \tag{37}$$

where I have used the fact that $\rho_{\mathrm{b}} \sim a^{-3}$. The Hubble equation acquires the form of the total energy of the mechanical particle

$$E = \frac{\dot{a}^2}{2} + V(a), \tag{38}$$

where $E = -K/2$. In order to make the mechanical analogy transparent, let us compute the minimum of the kinetic energy. If the minimum exists, it should obviously correspond to the numerical value of the scale factor that gives rise to the maximum of the effective potential $V(a)$. It is easy to see that the kinetic energy is minimum if $a = a_{\mathrm{m}}$,

$$a_m = (A/\Lambda)^{1/3}, \tag{39}$$

$$\left(\frac{\dot{a}^2}{2}\right)_m = \frac{1}{2}(A^{2/3}\Lambda^{1/3} - K), \tag{40}$$

where $A = 4\pi G\rho_{\mathrm{b}}^{(0)}a_0^3$. Note that $V(a)$ is maximum at $a = a_{\mathrm{m}}$. From eq. (40), we infer that the kinetic energy of the system at the top of the potential is,

$$\left(\frac{\dot{a}^2}{2}\right)_m \geq 0 \quad \text{if} \quad \Lambda \geq \Lambda_{\mathrm{c}} \equiv \frac{K^3}{A^2}. \tag{41}$$

In case $\Lambda = \Lambda_{\mathrm{c}}$, the system barely makes to the hump of the potential $(\dot{a} = 0)$ corresponding to $a_m = a_0$, where

$(\ddot{a} = 0)$, as it should be (see eq. (36)); this is nothing but the Einstein's static solution. We can now provide a qualitative description of the solutions of the evolution equations. For $\Lambda < \Lambda_c$, the kinetic energy is formally negative for $a = a_m$, which means that it vanishes before the particle reaches the maximum of the potential. In Figure 2 we have displayed the plot of $V(a)$ vs the scale factor a. We show three possible configurations of interest: (A) corresponds to motion starting from the left of the barrier with $a = 0$. (B) depicts the situation in which the potential barrier is approached beginning from the right with a large value of the scale factor. (C) represents the possibility of static solution.

We first analyse the case of $K > 0$ or $E < 0$, which gives rise to a variety of interesting possibilities.

(1) $\Lambda < \Lambda_c$: In this case, the kinetic energy is insufficient to overcome the potential barrier giving rise to the following interesting solutions.

(a) Oscillating solution: In this case, motion starts from $a = 0$ with insufficient kinetic energy to reach the hump of the potential. In this situation, the scale factor increases up to a maximum value where $\dot{a} = 0$ for $a < a_m$, marking the turning point followed by the contraction to $a = 0$.

(b) Bouncing universe: If the potential barrier is approached from the right side with $a = \infty$, the scale factor first decreases and reaches a minimum value and then bounces to expanding phase as the kinetic energy is not enough to overcome the barrier.

(c) Einstein static universe: This configuration corresponds to the maximum of the potential with $\dot{a} = 0$ and $\ddot{a} = 0$, possible for a particular value of Λ obtained earlier. Clearly, the static universe corresponding to a point particle sitting on the hump of the potential, is not stable. Small perturbations would derive it to either contracting $(a \to 0)$ or expanding $(a \to \infty)$ universe.

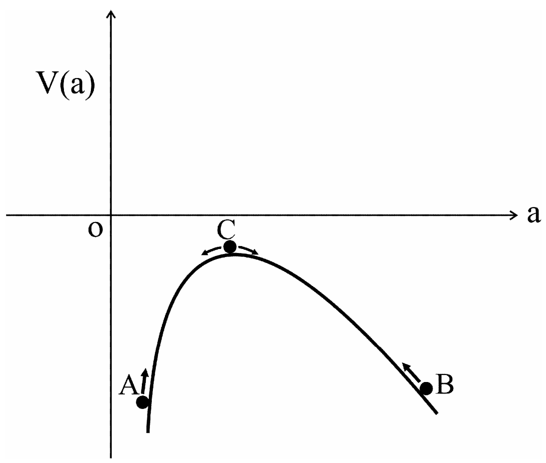

Figure 2. Plot of the effective potential $V(a)$ vs the scale factor a. Configurations (A) and (B) correspond to motion of system beginning from $a = 0$ and $a = 1$ respectively. (C) corresponds to static solution unstable under small fluctuations.

(2) $\Lambda < \Lambda_c$: The kinetic energy is sufficient to overcome the barrier for this choice of Λ. As a result, motion first decelerates till the system reaches the top of the potential and then slides down the hill with acceleration. The scale factor exhibits the point of inflection at $a(t) = a_m < a_0$. If Λ slightly exceeds its critical value, an interesting possibility dubbed loitering universe can be realized. The scale first increases as it should, approaches a_0 and remains nearly frozen for a substantial period before entering the phase of acceleration. Such a scenario has important implications for structure formation.

For $K \leq 0$ or $E \geq 0$, the system always has enough kinetic energy to surmount the barrier allowing the scale factor to increase from $a = 0$ to large values as time increases. This case is similar to the one with $K > 0$ and $\Lambda > \Lambda_c$.

For any given value of Λ, the scale factor exhibits the point of inflection at $a = a_m = (4\pi G\rho_b^{(0)} a_0^3 / \Lambda)^{1/3}$. This is also clear from eqs (32) and (33) in which the first term is the attractive character and dominates in the beginning leading to deceleration. However, as the scale factor increases and reaches a particular value, the repulsive term takes over; the scale factor exhibits the point of inflection and the expansion becomes accelerating thereafter.

Observations should tell us when deceleration changed into acceleration. This crucially depends upon how $4\pi G\rho_b^{(0)}$ compares with Λ or how ρ_Λ compares with $\rho_M^{(0)}/2$. The transition from deceleration to acceleration should have taken place around the present epoch. Had it happened much earlier it would have obstructed structure formation[34]. We shall come back to this point to confirm that cosmic acceleration is indeed a recent phenomenon.

Beyond Newtonian physics: pressure corrections

The formalism of Newtonian cosmology is not applicable to relativistic fluids. Relativistic fluids essentially have non-zero pressure. For instance, radiation is a relativistic fluid with pressure $P_b = \rho_b c^2/3$. The cosmological constant also belongs to the category of relativistic systems. In the general theory of relativity, pressure appears on the same footing as energy density. Here we present heuristic arguments to capture the pressure corrections in the evolution equations (see Zel'dovich and Novikov[22]).

Let us consider a unit comoving volume in the expanding universe and assume the expansion to be adiabatic. The first law of thermodynamics states that

$$dE + P_b dV = 0, \qquad (42)$$

where $P_b(t)$ is the pressure of the background fluid. The first law of thermodynamics applies to any system, be it relativistic or non-relativistic, classical or quantum – thermodynamics is a great science.

The energy density of the fluid can always be expressed through the mass density,

$$E = \frac{4\pi}{3} a^3 \rho_b c^2. \quad (43)$$

Substituting eq. (43) into eq. (42), we obtain the continuity equation in the expanding universe,

$$\dot{\rho}_b + 3H\left(\rho_b + \frac{P_b}{c^2}\right) = 0. \quad (44)$$

Thus the continuity equation responds to pressure corrections: $\rho_b \to \rho_b + P_b/c^2$. For a non-relativistic fluid, rest energy density dominates over pressure and the second term in the parenthesis can be neglected. For instance, for dust, $P_b \simeq 0$. At early times, the universe was hot and dominated by radiation. Hence the early universe should be treated by relativistic theory; Newtonian description becomes valid at late times when matter dominates. For the sake of convenience, we shall use the unit $c = 1$. With this choice, relativistic mass density and energy density are the same.

We can now present the cosmological constant as a perfect fluid with constant energy density. The continuity equation (eq. (44)) then implies that $\rho_\Lambda = -P_\Lambda$. Next we claim that the correct equation of acceleration in the case of background fluid with energy density ρ_b and pressure P_b is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_b + 3P_b) + \frac{\Lambda}{3}. \quad (45)$$

To verify, let us multiply eq. (45) by $\dot{a}$:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\dot{a}^2) = -\frac{4\pi G}{3} a(\rho_b \dot{a} + 3P_b \dot{a}) + \frac{\Lambda}{3} a\dot{a}. \quad (46)$$

Using the continuity equation, we can express the term containing pressure P_b in eq. (46) through ρ_b, $\dot{\rho}_b$ and $\dot{a}$:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\dot{a}^2) = \frac{4\pi G}{3}\frac{\mathrm{d}}{\mathrm{d}t}\left[\rho_b a^2 + \frac{\Lambda}{6} a^2\right]. \quad (47)$$

which can be put in the form of the Friedmann equation in the presence of matter with non-zero pressure.

$$H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2} + \frac{\Lambda}{3}. \quad (48)$$

We again observe that pressure corrects the energy density. Positive pressure adds to deceleration, whereas negative pressure contributes towards acceleration (see eq. (45)). It seems completely opposite to our intuition that a highly compressed substance explodes out with a tremendous impact, whereas in our case the pressure acts in the opposite direction. It is important to understand that our day-to-day intuition with pressure is related to pressure force or pressure gradient. In a homogeneous universe pressure gradients cannot exist. Pressure is a relativistic effect and can only be understood within the framework of general theory of relativity. Pressure gradient might appear in Newtonian framework in the inhomogeneous universe, but pressure can only be induced by relativistic effects. Strictly speaking, it should not appear in Newtonian cosmology. This applies to Λ also with negative pressure which we introduced in Newtonian cosmology by hand. Equations (44), (45) and (48) coincide with the evolution equations of relativistic cosmology. Their derivation presented here is heuristic. The rigorous treatment can only be given in the framework of the general theory of relativity, where cosmological constant appears naturally.

In order to solve the evolution equations, we need a relation between the energy density and pressure known as equation of state. In case of barotropic fluid, the equation of state is given by $w_b = P_b/\rho_b$. Dust and radiation correspond to $w_b = 0$, 1/3 respectively. Assuming that the universe is filled with perfect fluid with constant equation-of-state parameter w_b, we find from eqs (44) and (34) in case of $K = 0$,

$$\rho_b \propto a^{-3(1+w)}, \quad (49)$$

$$a(t) \propto t^{\frac{2}{3(1+w)}} \; (w > -1), \quad (50)$$

$$a(t) \propto \mathrm{e}^{\sqrt{\frac{\Lambda}{3}}t} \quad (w = -1). \quad (51)$$

In the case of radiation, $w_b = 1/3$ and as a result $\rho_b \equiv \rho_r \propto a^{-4}$. In contrast to the case of the dust-dominated universe, the radiation energy density decreases faster with the expansion of the universe. The positive radiation pressure adds to the energy density, making the gravitational attraction stronger. Consequently, the Hubble damping in the conservation equation increases, allowing the energy density to decrease faster than dust in the expanding universe. This can also be understood in a slightly different way, if we assume that radiation consists of photons. As the universe expands, the number density of the photons scales as a^{-3}, as usual. But since any length scale in the expanding universe grows proportional to the scale factor, the energy of a photon, hc/λ decreases as $1/a$, leading to $\rho_r \sim a^{-4}$ and $a(t) \propto t^{1/2}$. It is clear that radiation dominated at early epochs as $\rho_M \sim a^{-3}$ for dust.

Let us make an important remark on the dynamics in the early universe which was dominated by radiation (for simplicity, we ignore here other relativistic degrees of freedom). As $\rho_r \sim a^{-4}$, the first term on the RHS of evolution of the Hubble equation dominates over the curvature term K/a^2; obviously, cosmological constant plays no role in the present case. We therefore conclude that all the models effectively behave as the $K = 0$ model at early times,

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_{\rm r}^{(0)}\frac{a_0^4}{a^4} \to \frac{a(t)}{a_0}\left(\frac{32\pi G\rho_{\rm r}^{(0)}}{3}\right)^{1/4} t^{1/2}. \quad (52)$$

We next assume that radiation was in thermal equilibrium characterized by the blackbody distribution,

$$\rho_{\rm r} = bT^4, \quad (53)$$

where b is the radiation constant. From eqs (52) and (53), we find how temperature scales with the expansion of the universe,

$$T = \left(\frac{\rho_{\rm r}^{(0)}}{b}\right)^{1/4}\frac{a_0}{a}, \quad (54)$$

which on using eq. (52) tells us how early universe cooled with time,

$$T = \left(\frac{32\pi G}{3b}\right)^{1/4} t^{-1/2}. \quad (55)$$

At $t = 0$, both the radiation density and temperature become infinitely large; all the physical quantities diverged at that time referred to as the Big Bang. The Big Bang singularity is not the artefact of homogeneity and isotropy. It is a generic feature of any cosmological model based upon classical general theory of relativity. Classical physics breaks down as the Big Bang is approached. In the framework of classical general relativity, the Big Bang is taken to be the beginning of our universe. The universe was thus born in a violent explosion-like event, throwing away cosmic matter and giving rise to expansion of the universe. Since gravity is attractive (provided the universe is filled with matter of non-negative pressure), its roll is to decelerate the expansion. What caused Big Bang has no satisfactory answer. The Big Bang is a physical singularity which should be treated by quantum gravity. The inflationary paradigm can mimic Big Bang without singularity, but in that case, we do not know what caused inflation. In the cosmic history, there was an epoch when matter took over, leading to matter-dominated era. It turns out that it took around 10^5 years for radiation energy density to equalize with energy density of matter. The age of the universe, i.e., the time elapsed since the Big Bang till the present epoch given by eq. (30) changes insignificantly, if we consider the universe filled with both radiation and dust. This is because the time taken from the Big Bang till radiation matter equality is negligibly small compared to the actual age of universe which is around 14 Gyr. Thus the age given by eq. (30) is a reliable theoretical estimate. Unfortunately, the age given by eq. (30) falls short of the age of some old objects found in the universe. This is one of the old problems of Hot Big Bang model. We shall discuss its possible remedy in the dark energy-dominated universe.

Dark energy

Equations (48) and (46) tell us that the positive cosmological constant Λ contributes positively to the background energy density and negatively to pressure. It can be thought of as a perfect barotropic fluid with,

$$\rho_\Lambda = -\frac{\Lambda}{8\pi G},\ P_\Lambda = \frac{\Lambda}{8\pi G}, \quad (56)$$

which corresponds to $w_\Lambda = -1$. In general, we find from eq. (45) that expansion has the character of acceleration for large negative pressure,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_{\rm b} + 3P_{\rm b}), \quad (57)$$

$$\ddot{a} > 0 \Rightarrow P_{\rm b} < -\frac{\rho_{\rm b}}{3}: \text{ Dark energy,}$$

where we have included Λ in the background fluid. Thus, we need an exotic fluid dubbed dark energy to fuel the accelerated expansion of the universe. The various datasets of complimentary support the late-time acceleration of the universe. The simplest candidate of dark energy is provided by the cosmological constant with $w_\Lambda = -1$. Observations at present do not rule out the phantom dark energy with $w < -1$ corresponding to super-acceleration. In this case the expanding solution takes the form,

$$a(t) = (t_{\rm s} - t)^n,\ (n = 2/3(1+w) < 0), \quad (58)$$

$$H = \frac{n}{t_{\rm s} - t}, \quad (59)$$

where $t_{\rm s}$ is an integration constant. It is easy to see that the phantom-dominated universe will end itself in a singularity in future, known as big rip or cosmic doomsday, as $t \to t_{\rm s}$. Clearly, as $t \to t_{\rm s}$, both the Hubble parameter and the background energy density diverge[35].

Age crisis and its possible resolution

Apart from the cosmic acceleration, dark energy has important implications, in particular, in relation to the age problem. In any cosmological model with normal form of matter, the age of the universe falls short compared to the age of some known objects in the universe. Since the age of the universe crucially depends upon the expansion history, it can serve as an important check on building in cosmology. In order to appreciate the problem, let us first consider the case of flat dust-dominated universe ($\Omega_M = 1$) in which case, as shown earlier,

$$t_0 = \frac{2}{3}\frac{1}{H_0}. \tag{60}$$

The observational uncertainty of H_0 gives rise to the following estimate,

$$H_0^{-1} = 9.8h^{-1}\ \text{Gyr}, \tag{61}$$

$$0.64 \lesssim h \lesssim 0.8 \rightarrow t_0 = (8 - 10)\ \text{Gyr}. \tag{62}$$

This model is certainly in trouble as its prediction for age of the universe fails to meet the constraint following from the study of ages of old stars in globular clusters[36]: 12 Gyr $\lesssim t_0 \lesssim$ 15 Gyr. One could try to address the problem by invoking the open model with $\Omega_M^{(0)} < 1$. In this case the age of universe is expected to be larger than the flat dust-dominated universe – for less amount of matter, it would take longer for gravitational attraction to slow down the expansion rate to its present value. Looking at eq. (22), it is not difficult to guess that in this case, $H_0 t_0 \rightarrow 1$ for $\Omega_M^{(0)} \rightarrow 0$, which is a substantial improvement. However, this model is not viable for the several reasons. In particular, the study of large scale structure and its dynamics constrain the matter density: $0.2 < \Omega_M^{(0)} < 0.3$ and observations on CMB un-isotropy reveal that the universe is critical to a good accuracy.

The age problem can be resolved in a flat universe dominated by dark energy. Let us rewrite the Friedmann equation in a convenient form,

$$\left(\frac{\dot a}{a}\right)^2 = H_0^2\left[\Omega_M^{(0)}\left(\frac{a_0}{a}\right)^3 + \Omega_{DE}^{(0)}\left(\frac{a_0}{a}\right)^{3(1+w)}\right], \tag{63}$$

which allows us to write the expression of t_0 in the closed form

$$t_0 = \frac{1}{H_0}\int_0^\infty \frac{\mathrm{d}z}{(1+z)[\Omega_M^{(0)}(1+z)^3 + \Omega_{DE}^{(0)}(1+z)^{3(1+w)}]^{1/2}}, \tag{64}$$

where $\Omega_M^{(0)}$ is the contribution of dark matter and $(1 + z) \equiv a_0/a$, z being the redshift parameter. The dominant contribution to the age of the universe comes from the matter-dominated era and we, therefore, have omitted Ω_r in eq. (63). In case dark energy is the cosmological constant ($w_\Lambda = -1$), we get the analytical expression for the age of the universe,

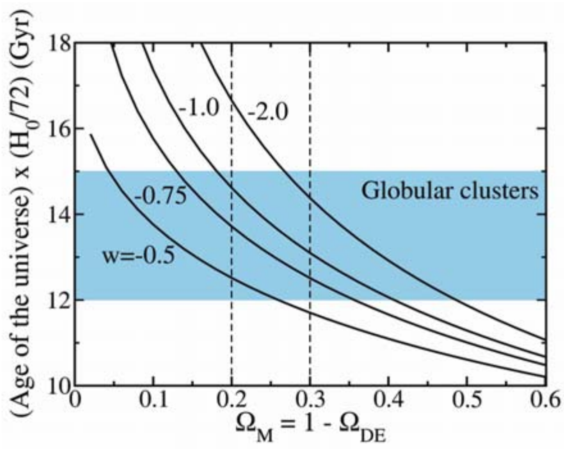

Figure 3. Plot of age of universe vs Ω_M (at present epoch) for a flat universe with matter and dark energy with constant equation-of-state parameter w (from Frieman[10]).

$$t_0 = \frac{2}{3}\frac{H_0^{-1}}{\Omega_\Lambda^{1/2}}\ln\left(\frac{1+\Omega_\Lambda^{1/2}}{\Omega_M^{(0)^{1/2}}}\right). \tag{65}$$

For dark energy other than the cosmological constant, the integral in eq. (64) should be computed numerically. In Figure 3, we have plotted the age of universe vs $\Omega_M^{(0)}$ for various possibilities of dark energy, including the phantom case. The age constraint can be met by flat, dark energy models provided that $-2 \lesssim w \lesssim -0.5$ for $\Omega_M^{(0)}$ lying between 0.2 and 0.3 (see Frieman[10]). It is remarkable that the Hot Big Bang model can be rescued by introducing the dark energy component. Interestingly, the cosmological constant was invoked to address the age problem before the invention of cosmic acceleration. The observation of cosmic acceleration in 1998 was a blessing in disguise for the cosmological constant.

The discovery of cosmic acceleration and its confirmation

The direct evidence of current acceleration of the universe is related to the observation of luminosity distance by high redshift supernovae in 1998 by two groups, independently. The luminosity distance for critical universe dominated by non-relativistic fluid and cosmological constant is given by

$$d_{\mathrm{L}} = \frac{(1+z)}{H_0}\int_0^z \frac{\mathrm{d}z'}{\sqrt{\Omega_M^{(0)}(1+z')^3 + \Omega_{DE}^{(0)}(1+z')^{3(1+w)}}}. \tag{66}$$

Equation (66) is the expanding universe generalization of absolute luminosity L_s of a source and its flux $\mathcal{F}$ at a distance d given by $\mathcal{F} = L_s/(4\pi d^2)$. It follows from eq. (66) that $D_L \simeq z/H_0$ for small z and that

$$d_L = 2(1+z-(1+z)^{1/2})H_0^{-1}, \quad \Omega_M^{(0)} = 1, \tag{67}$$

$$d_L = z(1+z)H_0^{-1}, \quad \Omega_{DE}^{(0)} = \Omega_\Lambda = 1, \tag{68}$$

which means that luminosity distance at high redshift is larger in the universe dominated by cosmological constant, which also holds true in general for an arbitrary equation of state w corresponding to dark energy. Therefore, supernovae would appear fainter in case the universe is dominated by dark energy. The luminosity distance can be used to estimate the apparent magnitude m of the source given its absolute magnitude M

$$m - M = 5\log\left(\frac{d_L}{Mpc}\right) + 25. \tag{69}$$

Let us consider two supernovae 1997*ap* at redshift $z = 0.83$ with $m = 24.3$ and 1992*p* at $z = 0.026$ with $M = 16.08$ respectively. Since the supernovae are assumed to be the standard candles, they have the same absolute magnitude. Equation (69) then gives the following estimate

$$H_0 d_L \simeq 1.16. \tag{70}$$

Then theoretical estimate for the luminosity distance is given by

$$d_L \simeq 0.95 H_0^{-1}, \quad \Omega_M^{(0)} = 1, \tag{71}$$

$$d_L \simeq 1.23 H_0^{-1}, \quad \Omega_M^{(0)} = 0.3, \Omega_\Lambda = 0.7, \tag{72}$$

where I have used the fact that, $d_L \simeq z/H_0$ for small z. The above estimate lends a strong support to the hypothesis that late-time universe is dominated by dark energy (see Figure 4)[37,38].

The observations related to CMB and large-scale structure (LSS) provide an independent confirmation of the dark energy scenario. The acoustic peaks of angular power spectrum of the CMB temperature anisotropies contains important information. The location of the major peak tells us that the universe is critical to a good accuracy, which fixes for us the cosmic energy budget. Specializing the Friedmann equation, i.e. eq. (63) to the present epoch ($a = a_0$), we have

$$\Omega_b^{(0)} = \Omega_M^{(0)} + \Omega_{DE}^{(0)}. \tag{73}$$

The contribution of radiation to total fractional energy density $\Omega_b^{(0)}$ is negligible at present. The study of LSS and its evolution indicates that nearly 30% of the total energy content is contributed by nonluminous component of non-barionic nature with dust-like equation-of-state popularly known as dark matter. The missing component which is about 70% is dark energy. The recent data on baryon acoustic oscillation are yet another independent probe of dark energy. The combined analysis of data of complimentary nature demonstrates that $\Omega_{DE}^{(0)} \simeq 0.7$ and $\Omega_M^{(0)} \simeq 0.3$ (see Figure 5)[39]. The constraint on the equation-of-state parameter w and $\Omega_M^{(0)}$ shows that w is restricted to a narrow strip around $w_\Lambda = -1$ (Figure 6). It is clear from Figure 6 that the combined analysis allows super-negative values of w corresponding to phantom energy. Let us now confirm that the transition from deceleration to cosmic acceleration took place in the recent past. Indeed, observations allow us to estimate the time of transition from deceleration to acceleration. Let us rewrite eq. (38) through dimensionless density parameters,

$$\frac{\dot{a}^2}{2} = \frac{H_0^2}{2}\left(\frac{\Omega_M^{(0)} a_0^3}{a} + \Omega_\Lambda a^2\right). \tag{74}$$

Using eq. (74), we can find out the numerical value of (a/a_0) corresponding to the minimum of kinetic energy $(\dot{a}^2/2)$, which precisely gives the transition from deceleration to acceleration,

$$\left(\frac{a}{a_0}\right)_{\rm tr} = \left(\frac{\Omega_M^{(0)}}{2\Omega_\Lambda}\right)^{1/3} \Rightarrow z_{\rm tr} = \left(\frac{2\Omega_\Lambda}{\Omega_M^{(0)}}\right)^{1/3} - 1 \simeq 0.67, \tag{75}$$

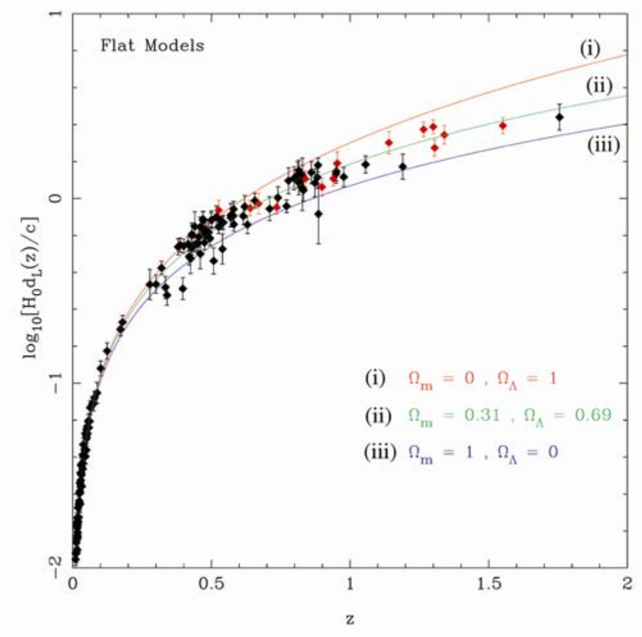

Figure 4. Plot of luminosity distance $H_0 d_L$ vs redshift z for a flat cosmological model. The black points come from the 'Gold' datasets by Riess *et al.*[37], whereas the red points show the recent data from HST. Three curves show the theoretical values of $H_0 d_L$ for (i) $\Omega_M^{(0)} = 0$, $\Omega_\Lambda = 1$, (ii) $\Omega_M^{(0)} = 0.31$, $\Omega_\Lambda = 0.69$ and (iii) $\Omega_M^{(0)} = 1$, $\Omega_\Lambda = 0$ (from Choudhury and Padmanabhan[38]).

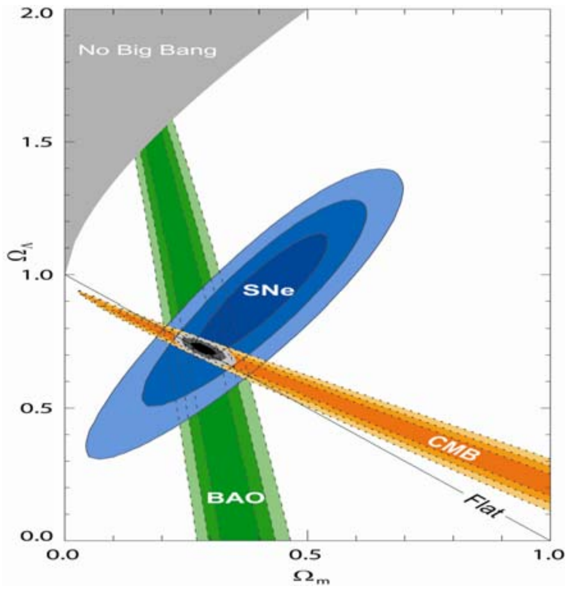

Figure 5. Best-fit regions in the (Ω_Λ, Ω_M) plane obtained using the CMB, Baryon Acoustic Oscillations (BAO) and supernovae data (from Kowalski *et al.*[39]).

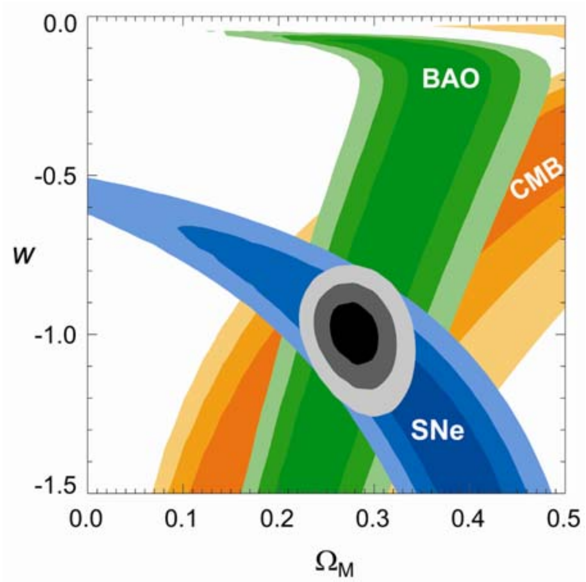

Figure 6. Constraints on the dark energy equation of w and Ω_M obtained from CMB, BAO and supernovae observations (from Kowalski *et al.*[39]).

for the observed values of density parameters ($\Omega_M^{(0)} \simeq 0.3$; $\Omega_\Lambda \simeq 0.7$). This confirms that the contribution of Λ to cosmic dynamics became important at late times, such that the cosmic acceleration is indeed a recent phenomenon.

Relativistic cosmology

In the last section I have presented heuristic arguments to capture the pressure effects in the evolution equations. Pressure in cosmology is a relativistic effect which can be consistently understood in the framework of general theory of relativity. Einstein equations are complicated, nonlinear equations which do admit analytical solutions in the presence of symmetries. Homogeneity and isotropy of the universe is an example of a generic symmetry of space–time. The assumption of homogeneity and isotropy forces the metric to assume the FRW form

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$
$$K = 0, \pm 1, \tag{76}$$

where $a(t)$ is scale factor. Coordinates (r, θ, ϕ) are the comoving coordinates. A freely moving particle comes to rest in these coordinates.

Equation (76) is purely a kinemetic statement. The information about dynamics is contained in the scale factor $a(t)$. The Einstein equations allow to determine the scale factor provided the matter content of the universe is specified. Constant K in the metric (eq. (76)) describes the geometry of the spatial section of space-time. $K = 0$, ± 1 corresponds to spatially flat, sphere-like and hyperbolic geometry respectively.

The differential equation for the scale factor follows from the Einstein equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{77}$$

where $G_{\mu\nu}$ is the Einstein tensor and $R_{\mu\nu}$ is the Ricci tensor. The energy momentum tensor $T_{\mu\nu}$ takes a simple form reminiscent of ideal perfect fluid in FRW cosmology

$$T_\mu^\nu = \mathrm{Diag}(-\rho_b, P_b, P_b, P_b). \tag{78}$$

Note that pressure in the general theory of relativity appears on the same footing as energy density. In the FRW background, the components of $G_{\mu\nu}$ can easily be computed:

$$G_0^0 = -\frac{3}{a^2}(\dot{a}^2 + K), \quad G_i^j = \frac{1}{a^2}(2a\ddot{a} + \dot{a}^2 + K). \tag{79}$$

Other components of $G_{\mu\nu}$ are identically zero. The Einstein equations then give rise to the following two independent equations:

$$H^2 = \frac{8\pi G}{3}\rho_b - \frac{K}{a^2}, \tag{80}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_b + 3P_b). \tag{81}$$

Let me remind that ρ_b designates the total energy density of all the fluid components present in the universe. The continuity equation $\dot{\rho}_b + 3H(\rho_b + P_b) = 0$ can be obtained by using eqs (80) and (81), which also follows naturally from the Bianchi identity. As mentioned earlier, we can normalize the scale factor to a convenient value at the present epoch in case of specially flat geometry. In other cases, it should be determined from the relation $a_0H_0 = (|\,\Omega_b^{(0)} - 1\,|)$, where $\Omega_b^{(0)}$ defines the total energy content of universe at the present epoch.

Let us note that the Einstein equations (eq. (77)) with the energy momentum tensor of standard fluid with positive pressure cannot lead to accelerated expansion. The repulsive effect can be captured either by supplementing the energy momentum tensor (on the right-hand side of the Einstein equations) with large negative pressure or by modifying the geometry itself, i.e. the left-hand side of the Einstein equations. We can ask for a consistent modification of the Einstein equations (equation of motion should be of second order with the highest derivative occurring linearly so that the Cauchy problem is well posed) in four space–time dimensions within the classical framework. Under the said conditions, the only admissible modification is provided by the cosmological constant. Thus we can add a term $\Lambda g_{\mu\nu}$ on the left-hand side of eq. (77), which we can formally carry to the right-hand side and interpret it as part of energy momentum tensor of a perfect fluid[12] (see also ref. 40 for a different approach to cosmological constant),

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}. \tag{82}$$

Such a modification is allowed by virtue of the Bianchi identity. It is remarkable that the cosmological constant does not need ad hoc assumption for its introduction; it is always present in the Einstein equations. It could be considered as a fundamental constant of the classical general theory of relativity at par with Newton's constant G. It is also interesting to note that the model based upon cosmological constant is consistent with all the observational findings in cosmology at present. However, there are deep theoretical problems related to the cosmological constant.

Theoretical issues associated with Λ

There are important theoretical issues related to the cosmological constant. The cosmological constant can be associated with vacuum fluctuations in the quantum field theoretical context[12–14]. Though the arguments are still at the level of numerology, they may have far-reaching consequences. Unlike the classical theory, the cosmological constant in this scheme is no longer a free parameter of the theory. Broadly, the line of thinking takes the following route. The ground-state energy, dubbed zero-point energy or vacuum energy ρ_{vac} of a free quantum field with spin j given by

$$\rho_{\text{vac}} = \frac{1}{2}(-1)^{2j}(2j+1)\int_0^\infty \frac{d^3\mathbf{k}}{2\pi^3}\sqrt{k^2+m^2} \tag{83}$$

$$= \frac{(-1)^{2j}(2j+1)}{4\pi^2}\int_0^\infty dk\, k^2\sqrt{k^2+m^2}, \tag{84}$$

is ultraviolet divergent. This contribution is related to the ordering ambiguity of fields in the classical Lagrangian and disappears when normal ordering is adopted. Since this procedure of throwing out the vacuum energy is ad hoc, one might try to cancel it by introducing the counter-terms. The latter, however, require fine-tuning and may be regarded as unsatisfactory. The divergence is related to the modes of very small wavelength. As we are ignorant of the physics around the Planck scale, we might be tempted to introduce a cut-off around the Planck length L_p, and associate with this a fundamental scale. Thus we arrive at an estimate of vacuum energy $\rho_{\text{vac}} \sim M_p^4$ (corresponding mass scale, $M_{\text{vac}} \sim \rho_{\text{vac}}^{1/4}$), which is away by 120 orders of magnitude from the observed value of this quantity, which is of the order of 10^{-48} (GeV)^4. The vacuum energy may not be felt in the laboratory, but plays an important role in general theory of relativity through its contribution to the energy momentum tensor as

$$\langle T_{\mu\nu}\rangle_0 = -\rho_{\text{vac}} g_{\mu\nu}, \quad \rho_{\text{vac}} = \Lambda/8\pi G, \tag{85}$$

and appears on the right-hand side of the Einstein equations.

The problem of zero-point energy is naturally resolved by invoking supersymmetry, which has many other remarkable features. In the supersymmetric description, every bosonic degree of freedom has its Fermi counterpart which contributes zero-point energy with opposite sign compared to the bosonic degree of freedom thereby doing away with the vacuum energy. It is in this sense that the supersymmetric theories do not admit a non-zero cosmological constant. However, we know that we do not live in supersymmetric vacuum state and hence it should be broken. For a viable supersymmetric scenario, for instance, if it is to be relevant to hierarchy problem, the supersymmetry breaking scale should be around $M_{\text{susy}} \simeq 10^3$ GeV. We still remain away from the observed value by many orders of magnitude. We do not know how Planck scale or SUSY breaking scales are related to the observed vacuum scale!

At present there is no satisfactory solution to cosmological constant problem. One might assume that there is some way to cancel the vacuum energy. One can then treat Λ as a free parameter of classical gravity similar to Newton constant G. However, the small value of cosmological constant leads to several puzzles including the fine tuning and coincidence problems. The energy density in radiation at the Planck scale is of the order of Plank energy density $\rho_P \simeq 10^{72}$ GeV^4 and the observed value of the dark energy density, $\rho_\Lambda \simeq 0.7 \times \rho_c^{(0)} \simeq 10^{-48}$ GeV^4 which implies that $\rho_\Lambda/\rho_P \sim 10^{-120}$. Thus ρ_Λ needs to be fine tuned at the level of one part in 10^{-120} around the Planck epoch, in order to match the current universe. Such an extreme fine tuning is absolutely unacceptable at theoretical grounds. Secondly, the energy density in cosmological constant is of the same order as matter energy density at the present epoch. The question what causes this coincidence has no satisfactory answer.

Efforts have recently been made to understand Λ within the framework of string theory using flux compactification. String theory predicts a very complicated landscape of about 10^{500} de-Sitter vacua[14]. Using Anthropic principal, we are led to believe that we live in one of these vacua.

A novel approach to cosmological constant problem is provided in ref. 15. The line of thinking takes following route: in the conventional framework, the equations of motion for matter fields are invariant under the shift of the matter Lagrangian by a constant while gravity breaks this symmetry. Thus, one cannot obtain a satisfactory solution to the cosmological constant problem until the gravity is made to respect the same symmetry. An effective action suggested by Padmanbhan in ref. 15 is explicitly invariant under the 'shift symmetry'. In his approach, the observed value of the cosmological constant should arise from the energy fluctuations of degrees of freedom located in the boundary of a spacetime region.

Scalar field dynamics relevant to cosmology

The fine tuning problem associated with cosmological constant led to the investigation of cosmological dynamics of a variety of scalar field systems such as quintessence, phantoms, tachyons and Kessence[16,17,41–43] (see ref. 6 for details). Scalar fields can easily mimic dark energy at late times and posses rich dynamics in the past. We should note that scalar fields models do not address the cosmological constant problem, they rather provide an alternative way to describe dark energy. The underlying dynamics of these systems has been studied in great detail in the literature. Scalar fields naturally arise in models of high energy physics and string theory. It is worthwhile to bring out the broad features of their dynamics that make these system viable to cosmology.

Quintessence

A standard scalar field (minimally coupled to gravity) capable of accounting for the late time cosmic acceleration is termed as quintessence. Its action is given by

$$S = \int \mathcal{L}\sqrt{-g}\,\mathrm{d}^4x = -\int\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + V(\phi)\right)\sqrt{-g}\,\mathrm{d}^4x. \tag{86}$$

The energy momentum tensor corresponding to this action is given by

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right], \tag{87}$$

which gives rise the following expression for energy density and pressure in FRW background

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi),\ P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{88}$$

The Euler-Lagrangian equation

$$\partial^\alpha \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\partial^\alpha\phi} - \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta\phi} = 0, \tag{89}$$

$$\sqrt{-g} = a^3(t), \tag{90}$$

for the action (eq. (86)) in FRW background acquires the form

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0, \tag{91}$$

which is formally equivalent to the continuity equation and can put in the form

$$\rho_\phi = \rho_\phi^0 \exp\left(-\int 3(1+w(\phi))\frac{\mathrm{d}a}{a}\right), \tag{92}$$

where $w(\phi) = P_\phi/\rho_\phi$. Equation (88) tells us that for a steep potential $\dot{\phi}^2 >> V(\phi)$, the equation of state parameter approaches the stiff matter limit, $w(\phi) \to 1$ whereas $w(\phi) \to -1$ in case of a flat potential, $\dot{\phi}^2 << V(\phi)$. Hence the energy density scales as $\rho_\phi \sim a^{-n}$, $0 \le n \le 6$. Let us note that while the field rolls along the steep part of the potential, its energy density ρ_ϕ scales faster than ρ_r.

From eq. (81) we find that

$$\ddot{a} > 0 \to \rho_b + 3P_b < 0 \Rightarrow \dot{\phi}^2 < V(\phi), \tag{93}$$

which means that we need nearly flat potential to account for accelerated expansion of universe such that

$$\frac{1}{V}\left(\frac{V_{,\phi}}{V}\right)^2 << 1,\quad \frac{V_{,\phi\phi}}{V^2} << 1. \tag{94}$$

In case of field domination regime, the two conditions in eq. (94) define the slow roll parameters which allow to neglect the $\ddot{\phi}$ term in equation of motion for ϕ. In the present context, unlike the case of inflation, the evolution of field begins in the matter dominated regime and even today, the contribution of matter is not negligible. The traditional slow roll parameters cannot be connected to the conditions on slope and curvature of potential which essentially requires that Hubble expansion is determined by the field energy density alone. Thus the slow roll parameters are not that useful in case of late time acceleration, though, eq. (94) can still be helpful.

The scalar field model aiming to describe dark energy should possess important properties allowing it to alleviate the fine tuning and coincidence problems without interfering with the thermal history of universe. The nucleosynthesis puts an stringent constraint on any relativistic degree of freedom over and above that of the standard model of particle physics. Thus, a scalar field has to satisfy several important constraints if it is to be relevant to cosmology. Let us now spell out some of these features in detail, see refs 6 and 16 for details. In case the scalar field energy density ρ_ϕ dominates the background (radiation/matter) energy ρ_b, the former should redshift faster than the latter allowing radiation domination to commence which in turn requires a steep potential. In this case, the field energy density overshoots the background and becomes subdominant to it. This leads to the locking regime for the scalar field which unlocks the moment the ρ_ϕ is comparable to ρ_b. The further course of evolution crucially depends upon the form of the scalar potential. For the non-interference with thermal history, we require that the scalar field remains unimportant during radiation and matter dominated eras and emerges out from the hiding at late times to account for late time acceleration. To address the issues related to fine tuning, it is important to investigate the cosmological scenarios in which the energy density of the scalar field mimics the background energy density. The cosmological solution which satisfy this condition are known as scaling solutions,

$$\frac{\rho_\phi}{\rho_b} = \text{const.} \tag{95}$$

The steep exponential potential $V(\phi) \sim \exp(\lambda\phi/M_P)$ with $\lambda^2 > 3(1 + w_b)$ in the framework of standard GR gives rise to scaling solutions whereas the shallow exponential potential with $\lambda \leq \sqrt{2}$ leads to a field dominated solution ($\Omega_\phi = 1$). Nucleosynthesis further constraints λ. The introduction of a new relativistic degree of freedom at a given temperature changes the Hubble rate which crucially effects the neutron to proton ratio at temperature of the order of one MeV when weak interactions freeze out. This results into a bound on λ, namely[6],

$$\Omega_\phi \equiv 3(1 + w_b)/\lambda^2 \lesssim 0.13 \Rightarrow \lambda \gtrsim 4.5. \tag{96}$$

In this case, for generic initial conditions, the field ultimately enters into the scaling regime, the attractor of the dynamics, and this allows to alleviate the fine tuning problem to a considerable extent. The same holds for the case of undershoot (Figure 7).

Scaling solutions, however, are not accelerating as they mimic the background (radiation/matter). One therefore needs some late time feature in the potential. There are several ways of achieving this: (1) The potential that mimics a steep exponential at early epochs and reduces to power law type $V \sim \phi^{2p}$ at late times gives rise to accelerated expansion for $p < 1/2$ as the average equation of state $\langle w(\phi)\rangle = (p - 1)/(p + 1) < -1/3$ in this case[44,45]. (ii) The steep inverse power law type of potential which becomes shallow at large values of the field can support late time acceleration and can mimic the background at early time[46].

The solutions which exhibit the aforesaid features are referred to as tracker solutions. For a viable cosmic evolution we need a tracker like solution. However, on the basis of observations, we cannot rule out the non-tracker models at present.

In the second class of models where trackers are absent, there are two possibilities. First, if ρ_ϕ scales faster than ρ_b in the beginning, it then overshoots the background and enters the locking regime. In case of the undershoot, the field is frozen from the beginning due to large Hubble damping. In both the cases, for a viable cosmic evolution, models parameters are chosen such that $\rho_\phi \sim \rho_\Lambda$ during the locking regime. Hence at early times, the field gets locked ($w(\phi) = -1$) and waits for the matter energy density to become comparable to field energy density which is made to happen at late times. The field then begins to evolve towards larger values of $w(\phi)$ starting from

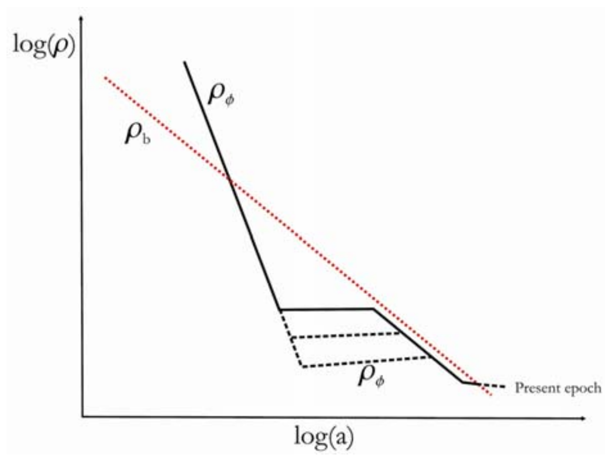

Figure 7. Cosmologically viable evolution of field energy density vs the scale factor. The dotted line shows the evolution of background (matter/radiation) energy density. The field energy density ρ_ϕ (with different initial conditions) joins the scaling regime and mimics the background. At late times it exits the scaling matter regime to become the dominant component and to account for the late time acceleration.

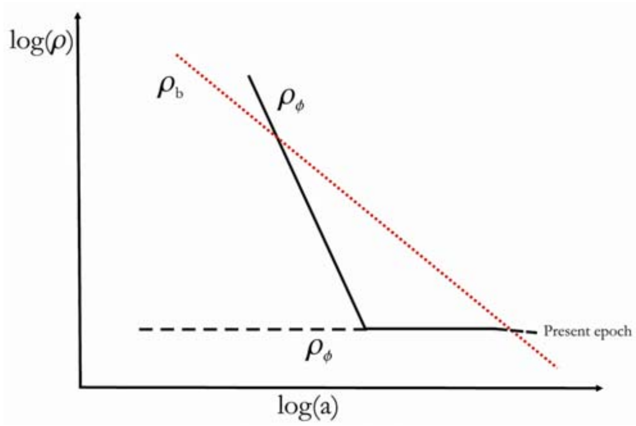

Figure 8. Evolution of ρ_ϕ and ρ_b in absence of scaling regime in case of overshoot and undershoot. The field remains trapped in the locking regime till its energy density becomes comparable to that of the background component. It then starts evolving slowly and overtakes the background to become dominant at late times.

$w(\phi) = -1$ (Figure 8). In this case one requires to tune the initial conditions of the field. The two classes of scalar fields are called freezing and thawing models[7,42]. In case of tracker (freezing) models, one needs to tune the slope of the field potential. Nevertheless, these are superior to thawing models as they are capable of addressing both the fine tuning and the coincidence problems.

Before we proceed further, we should make an honest remark about scalar field models in general. These models lack predictive power: for a given cosmic history, it is always possible to construct a field potential that would give rise to the desired evolution. Their merits should therefore be judged by the generic features which arise in them. For instance tracker models deserve attention for obvious reasons. Scalar fields inspired by a fundamental theory such as rolling tachyons are certainly of interest.

Tachyon field as source of dark energy

Next we shall be interested in the cosmological dynamics of tachyon field which is specified by the Dirac-Born-Infeld (DBI) type of action given by (see ref. 6 and references therein),

$$\mathcal{S} = \int -V(\phi)\sqrt{1-\partial^\mu\phi\,\partial_\mu\phi}\sqrt{-g}\ \mathrm{d}^4x, \qquad (97)$$

where on phenomenological grounds, we shall consider a wider class of potentials satisfying the restriction that $V(\phi) \to 0$ as $\phi \to \infty$. In FRW background, the pressure and energy density of ϕ are given by

$$P_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2}, \qquad (98)$$

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}. \qquad (99)$$

The equation of motion which follows from eq. (97) is

$$\ddot{\phi} + 3H\dot{\phi}(1-\dot{\phi}^2) + \frac{V'}{V}(1-\dot{\phi}^2) = 0, \qquad (100)$$

where H is the Hubble parameter

$$H^2 = \frac{1}{3M_\mathrm{p}^2}(\rho_\phi + \rho_\mathrm{b}). \qquad (101)$$

Tachyon dynamics is different from that of the quintessence. Irrespective of the form of its potential

$$w(\phi) = \dot{\phi}^2 - 1 \Rightarrow -1 \le w(\phi) \le 0. \qquad (102)$$

The investigations of cosmological dynamics shows that in case of the tachyon field, there exists no solution which can mimic scaling matter/radiation regime. These models necessarily belong to the class of thawing models. Tachyon models do admit scaling solution in presence of a hypothetical barotropic fluid with negative equation of state. Tachyon fields can be classified by the asymptotic behaviour of their potentials for large values of the field: (i) $V(\phi) \to 0$ faster then $1/\phi^2$ for $\phi \to \infty$. In this case dark matter like solution is a late-time attractor. Dark energy may arise in this case as a transient phenomenon. (ii) $V(\phi) \to 0$ slower then $1/\phi^2$ for $\phi \to \infty$; these models give rise to dark energy as late-time attractor. The two classes are separated by $V(\phi) \sim 1/\phi^2$ which is scaling potential with $w(\phi)$ = const. These models suffer from the fine tuning problem; dynamics in this case acquires dependence on initial conditions.

Phantom field

The scalar field models discussed above lead to $w(\phi) \ge -1$ and cannot give rise to super acceleration corresponding to phantom dark energy with $w(\phi) < -1$ permitted by observations, see Figure 6. The simplest possibility of getting phantom energy is provided by a scalar field with negative kinetic energy. Phantom field is nothing but the Hoyle-Narlikar's creation field (C-field) which was introduced in the steady-state theory to reconcile the model with the perfect cosmological principle. Though the quantum theory of phantom fields is problematic, it is nevertheless interesting to examine the cosmological consequences of these fields at classical level. Phantom field is described by the following action

$$\mathcal{S} = \int\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)\sqrt{-g}\ \mathrm{d}^4x. \qquad (103)$$

Its corresponding equation of state parameter is given by

$$w(\phi) = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 - V(\phi)}, \tag{104}$$

which tells us the $w(\phi) < -1$ for $\dot{\phi}^2/2 < V(\phi)$. An unusual equation of motion for ϕ follows from eq. (103)

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0. \tag{105}$$

It should be noted that the evolution equation of phantom field is same as that of the ordinary scalar field but with inverted potential allowing the field with zero kinetic energy to rise up the hill. As mentioned earlier, phantom energy is plagued with big rip singularity which is characterized by divergence of the Hubble parameter and curvature of space time after a finite interval of time. In such a situation, quantum effects become important and one should include higher curvature corrections to general theory of relativity which can crucially modify the structure of the singularity. To the best of our knowledge, the big rip singularity can be fully resolved in the framework of loop quantum cosmology[47]. Big rip can also be avoided at the classical level in a particular class of models in which potential has maximum. In this case, the field rises to the maximum of the potential and ultimately settles on top of the potential to give rise to de-Sitter like behaviour.

For a viable cosmic history, the phantom energy density similar to the case of rolling tachyon should be subdominant at early epochs. The field then remains frozen till late times before its energy density becomes comparable to matter energy density. Its evolution begins thereafter. Clearly, dark energy models based upon phantom fields belong to the category of thawing models.

Late time evolution of dark energy

In the preceding subsections, we have described the cosmological dynamics of quintessence, phantoms and rolling tachyon. These scalar field models fall into two broad categories: (i) Tracker or freezing models in which the field rolls fast at early stages such that it mimics the background with $w_b = 0$. At late times, $w(\phi)$ starts deviating from dust-like behaviour and becomes negative moving towards de-Sitter phase as the field rolls down its potential. (ii) Non-tracker or thawing models are those in which the field is trapped in the locking regime due to large Hubble damping such that $w(\phi) = -1$. And only at late times, as ρ_ϕ becomes comparable to the background energy density, the field begins to evolve towards larger values of $w(\phi)$. As demonstrated by Caldwell and Linder[42], these models occupy narrow regions in the ($w' \equiv \mathrm{d}w/\mathrm{d}\ln(a)$, w) plane,

Freezing models: $3w(1+w) < w' < 0.2w(1+w)$

Thawing models: $1 + w < w' < 3(1+w)$

where the upper and the lower bounds are obtained using analytical arguments and numerical analysis of generic models belonging to both the classes of models. As pointed out earlier (see Figure 6), combined analysis of different observations reveal that dark energy equation of state parameter lies in the narrow strip around $w_\Lambda = -1$. The observational resolution between the two classes of the model which is of the order of $1 + w$ is therefore a challenge to future observations.

As mentioned earlier, the phantom and the tachyon dark energy models belong to the class of thawing models. In this case, we can simplify the dynamics around the present epoch by using the approximation that $|1 + w| << 1$ and that the slope of the potential is small. The validity of the second approximation can be verified numerically in each case. In this scheme of a plausible approximation, one arrives at an amazing result: all the different dynamical systems, thawing quintessence, phantom, tachyon and phantom tachyon follow a unique evolutionary track. The distinction between the four classes of scalar field systems and the distinction between different models within each class is an effect of higher order than $|1 + w|$ (ref. 43) which certainly throws a great challenge to future generation experiments! Indeed, a recent examination of observational data including 397 Type-Ia supernovae at redshifts $0.015 \le z \le 1.55$ has shown that evolving dark energy models provide a slightly better fit to the data than the cosmological constant[48]. If future data confirms this result then it could mean that cosmic acceleration is currently slowing down which may have important consequences for dark energy model building.

Quintessential inflation on brane: a beautiful model that does not work

Quintessential inflation refers to attempts to describe inflation and dark energy with a single scalar field. The unifications of the two phases of accelerated expansion could be realized in the framework of Randall–Sundrum (RS) brane worlds[45,46]. In order to achieve this, the field potential should be flat during inflation but steep in radiation and matter dominated eras such that ρ_ϕ could mimic the background energy density at early epochs. At late times, it should become flat so as to allow the current acceleration of universe. Since the potential does not exhibit minimum, the conventional reheating mechanism does not work in this scenario. One could employ alternative mechanisms such as reheating via gravitational particle production or instant preheating. It is not realistic to have a potential which changes from flat to steep and back to flat at late times (Figure 9). However, it is generic to have a potential which is steep and allows to track the background at early epochs and gives rise to a viable late time cosmic evolution.

In case of a steep potential, the field energy density scales faster than radiation energy density leading to the commencement of radiative regime. But a steep potential cannot support inflation in FRW cosmology. This is precisely where the brane assisted inflation comes to our rescue. In *RS* brane world model, the Friedmann equation is modified to,

$$H^2 = \frac{8\pi G}{3}\rho_{\rm b}\left(1+\frac{\rho_{\rm b}}{2\lambda_B}\right), \qquad (106)$$

where λ_B is the brane tension. The presence of quadratic density term in the Friedmann equation changes the dynamics at early epochs in crucial manner. Consequently, the field experiences greater damping and rolls down its potential slower than it would during the conventional inflation. This effect is reflected in the slow-roll parameters which have the form,

$$\varepsilon = \varepsilon_{\rm FRW}\frac{1+V/\lambda_B}{(1+V/2\lambda_B)^2}, \qquad (107)$$

$$\eta = \eta_{\rm FRW}(1+V/2\lambda_B)^{-1}, \qquad (108)$$

where $\varepsilon_{\rm FRW}$ and $\eta_{\rm FRW}$ are the standard slow-roll parameters in absence of brane corrections. The influence of brane corrections becomes specially important when $V/\lambda_B >> 1$. In this case, we have,

$$\varepsilon \simeq \varepsilon_{\rm FRW}(V/\lambda_B)^{-1},\ \eta \simeq 2\eta_{\rm FRW}(V\lambda_B)^{-1}, \qquad (109)$$

which tells us that slow-roll (ε, $\eta << 1$) is possible when $V/\lambda_B >> 1$ even if the potential is steep ($\varepsilon_{\rm FRW}$, $\eta_{\rm FRW} > 1$). As the field rolls down its potential, the high-energy brane correction to Friedmann equation disappears giving rise to the natural exit from inflation.

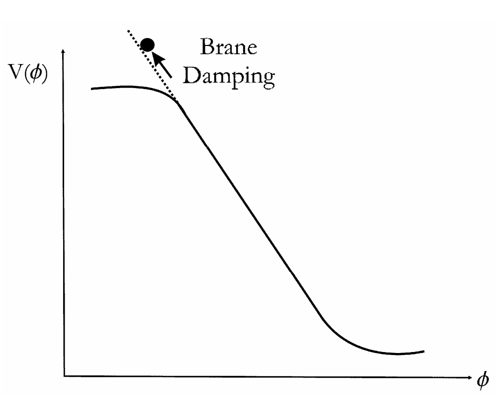

Figure 9. A desired form of potential for quintessential inflation. It is generic to have a steep potential at early times with brane corrections helping the slow-roll of the field.

It is possible to choose potentials suitable to quintessential inflation and fine tune the model parameters such that the model respects nucleosynthesis constraints and leads to the observed late time cosmic acceleration[45,46]. However, the problem occurs on the other side. Recent measurements of CMB anisotropies place fairly strong constraints on inflationary models. The tensor to scalar ratio of perturbations turns out to be larger than its observed value in case of steep brane world inflation. Clearly, the brane world unification of inflation and dark energy is ruled out by observation.

Modified theories of gravity and late time acceleration

The second approach to late time acceleration is related to the modification of left-hand side of Einstein equations or the geometry of space time. It is perfectly legitimate to investigate the possibility of late time acceleration due to modification of the Einstein–Hilbert action. In the past few years, several schemes of large scale modifications have been actively investigated. Some of these modifications are inspired by fundamental theories of high-energy physics whereas the others are based upon phenomenological considerations. In what follows, I shall briefly describe the modified theories of gravity and their relevance to cosmology.

String curvature corrections

It is interesting to investigate the string curvature corrections to Einstein gravity amongst which the Gauss–Bonnet correction enjoys special status[49–57]. These models, however, suffer from several problems. Most of these models do not include tracker like solution and those which do are heavily constrained by the thermal history of universe. For instance, the Gauss–Bonnet gravity with dynamical dilaton might cause transition from matter scaling regime to late time acceleration allowing to alleviate the fine tuning and coincidence problems. Let us consider the low energy effective action,

$$S = \int {\rm d}^4x\sqrt{-g}\begin{bmatrix}\dfrac{1}{16\pi G}R-(1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ -V(\phi)-f(\phi)R_{GB}^2\end{bmatrix}+\mathcal{S}_m, \qquad (110)$$

where R_{GB}^2 is the Gauss–Bonnet term,

$$R_{GB}^2 \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}. \qquad (111)$$

The dilaton potential $V(\phi)$ and its coupling to curvature $f(\phi)$ are given by,

$$V(\phi) \sim e^{(\alpha\phi)}, \quad f(\phi) \sim e^{-(\mu\phi)}. \tag{112}$$

The cosmological dynamics of system (eq. (110)) in FRW background was investigated in Koivisto and Mota[50], and Tsujikawa and Sami[51]. It was demonstrated that scaling solution can be obtained in this case provided that $\mu = \alpha$. In case $\mu \neq \alpha$, the de-Sitter solution is a late time attractor. Hence, the string curvature corrections under consideration can give rise to late time transition from matter scaling regime. Unfortunately, it is difficult to reconcile this model with nucleosynthesis[50,51] constraint.

DGP model

In DGP model, gravity behaves as four dimensional at small distances but manifests its higher dimensional effects at large distances. The modified Friedmann equations on the brane lead to late time acceleration. The model has serious theoretical problems related to ghost modes and superluminal fluctuations. The combined observations on background dynamics and large angle anisotropies reveal that the model performs much worse than ΛCDM (ref. 58). However, generalized versions of DGP can be ghost free and can give rise to transient acceleration as well as a phantom phase[59].

f(R) theories of gravity

On purely phenomenological grounds, one could seek a modification of Einstein gravity by replacing the Ricci scalar in Einstein–Hilbert action by $f(R)$. The action of $f(R)$ gravity is given by[11],

$$S = \int \left[\frac{f(R)}{16\pi G} + \mathcal{L}_m\right]\sqrt{-g}\, d^4x, \tag{113}$$

The modified Einstein equations which follow from eq. (113) have the form,

$$f' R_{\mu\nu} - \nabla_\mu \nabla_\nu f' + \left(\Box f' - \frac{1}{2} f\right) g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{114}$$

which are of fourth-order for a nonlinear function $f(R)$. Here prime denotes the derivatives with respect to R. The Ricci scalar in FRW background is given by

$$R = 12H^2 + 6\dot{H}, \tag{115}$$

which tells us that the modified eq. (114) contains de-Sitter space time as a vacuum solution provided that $f(4\Lambda) = 2\Lambda f'(4\Lambda)$. The $f(R)$ theories of gravity may indeed provide an alternative to dark energy. To see this, let us write the evolution equations which follow from eq. (114) in a convenient form

$$H^2 = \frac{8\pi G}{3f'}\rho_R, \tag{116}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{f'}(\rho_R + 3P_R), \tag{117}$$

where ρ_R and P_R are energy density and pressure contributed by curvature modification

$$\rho_R = \frac{Rf' - f}{2} - 3H\dot{R}f'', \tag{118}$$

$$P_R = 2H\dot{R}f'' + \ddot{R}f'' + \frac{1}{2}(f - f'R) + f'''\dot{R}^2, \tag{119}$$

ρ_R and P_R identically vanish in case of Einstein–Hilbert action, $f(R) = R$ as it should be. As an example of $f(R)$ model let us consider, $f(R) = R - \alpha_n/R^n$, where α_n is constant for given n. In case of a power law solution $a(t) \sim t^n$, the effective equation of state parameter can be computed as

$$w_R = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}. \tag{120}$$

Choosing a particular value of n, we can produce a desired equation of state parameter for dark energy.

The functional form of $f(R)$ should satisfy certain requirements for the consistency of the modified theory of gravity. The stability of $f(R)$ theory would be ensured provided that, $f'(R) > 0$ and $f''(R) > 0$ which means that graviton is not ghost and scalar degree is not tachyon. We can understand the stability conditions heuristically without entering into their detailed investigations. From evolution equations (eqs (116) and (117), we see that the effective gravitational constant $G_{\rm eff} = G/f'$ which should be positive or $f' > 0$ in order to avoid the pathological situation. As for the second condition, V. Faraoni[60] has given an interesting interpretation: let us consider the opposite case when $f'' < 0$ which means that $G'_{\rm eff} = -f''G/f'^2 > 0$. This implies that gravitational constant increases for increasing value of R making the gravity stronger. In view of Einstein equations, it leads to yet larger value of curvature and so on which ultimately leads to a catastrophic situation. Thus we need f'' to be positive to avoid the catastrophe.

Let us note that $f(R)$ gravity theories apart from a spin two object necessarily contain a scalar degree of freedom. Taking trace of eq. (114) gives the evolution equation for the scalar degree of freedom,

$$\Box f' = \frac{1}{3}(2f' - f'R) + \frac{8\pi G}{3}T. \tag{121}$$

It should be noticed that eq. (121) reduces to an algebraic relation in case of Einstein gravity; in general f' has dynamics. It is convenient to define scalar function ϕ as,

$$\phi \equiv f' - 1, \tag{122}$$

which is expressed through Ricci scalar once $f(R)$ is specified. We can write the trace equation (eq. (121)) in the terms of V and T as

$$\Box\phi = \frac{\mathrm{d}V}{\mathrm{d}\phi} + \frac{8\pi G}{3}T. \tag{123}$$

which is a Klein–Gordon equation in presence of a deriving term. Thus ϕ is indeed a scalar degree of freedom which controls the curvature of space time.

The effective potential can be evaluated using the following relation

$$\frac{\mathrm{d}V}{\mathrm{d}R} = \frac{\mathrm{d}V}{\mathrm{d}\phi}\frac{\mathrm{d}\phi}{\mathrm{d}R} = \frac{1}{3}(2f - f'R)f''. \tag{124}$$

Models which satisfy the stability conditions belong to two categories: (i) Either they are not distinguishable from ΛCDM or are not viable cosmologically. (ii) Models with disappearing cosmological constant: in these models, $f(R) \to 0$ for $R \to 0$ and they give rise to cosmological constant in regions of high density and differ from the latter otherwise. In principal, these models can be distinguished from cosmological constant. Models belonging to the second category were proposed by Hu–Sawicki[61] and Starobinsky[62] (see also ref. 63 on the similar theme). The functional form of $f(R)$ in Starobinsky parametrization is given by,

$$f(R) = R + \lambda R_0\left[\left(1 + \frac{R^2}{R_0^2}\right)^{-n} - 1\right]. \tag{125}$$

Here n and λ are positive. And R_0 is of the order of presently observed cosmological constant, $\Lambda = 8\pi G\rho_{\mathrm{vac}}$. The model satisfies the stability conditions quoted above.

In the Starobinsky model, the scalar field ϕ, in the absence of matter, is given by

$$\phi(R) = -\frac{2n\lambda R}{R_0(1 + \frac{R^2}{R_0^2})^{n+1}}. \tag{126}$$

Notice that $R \to \infty$ for $\phi \to 0$. For a viable late time cosmology, the field should be evolving near the minimum of the effective potential. The finite time singularity inherent in the class of models under consideration severely constrains dynamics of the field.

The curvature singularity and fine tuning of parameters: The effective potential has minimum which depends upon n and λ. For generic values of the parameters, the minimum of the potential is close to $\phi = 0$ (Figure 10) corresponding to infinitely large curvature. Thus while the field is evolving towards minimum, it can easily oscillate to a singular point[64,65]. However, depending upon the values of parameters, we can choose a finite range of initial conditions for which scalar field ϕ can evolve to the minimum of the potential without hitting the singularity. We find that the range of initial conditions allowed for the evolution of ϕ to the minimum without hitting singularity shrinks as the numerical values of parameters n and λ increase. In the presence of matter, the minimum of the effective potential moves towards the origin. In case of the compact objects such as neutron stars, the minimum is extremely near the origin and the singularity problem becomes really acute[65,66].

Avoiding singularity with higher curvature corrections: We know that in case of large curvature, the quantum effects become important leading to higher curvature corrections. Keeping this in mind, let us consider the modification of Starobinsky's model[67–69],

$$f(R) = R + \frac{\alpha}{R_0}R^2 + R_0\lambda\left[-1 + \frac{1}{(1 + \frac{R^2}{R_0^2})^n}\right], \tag{127}$$

then ϕ becomes

$$\phi(R) = \frac{R}{R_0}\left[2\alpha - \frac{2n\lambda}{(1 + \frac{R^2}{R_0^2})^{n+1}}\right]. \tag{128}$$

In case $|R|$ is large, the first term which comes from αR^2 dominates. In this case, the curvature singularity, $R = \pm\infty$ corresponds to $\phi = \pm\infty$. Hence, in this modification, the minimum of the effective potential is separated from the curvature singularity by the infinite distance in the ϕ, $V(\phi)$ plane. Though the introduction of R^2 term formally allows to avoid the singularity but cannot alleviate the

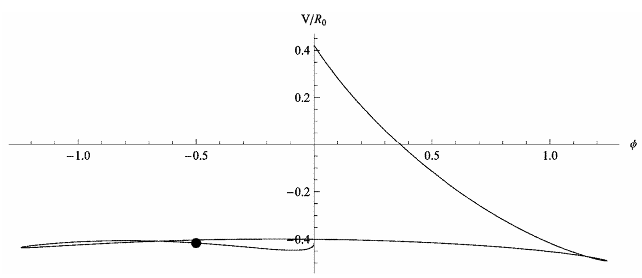

Figure 10. Plot of effective potential for $n = 2$ and $\lambda = 1.2$. The red spot marks the initial condition for evolution.

fine tuning problem as the minimum of the effective potential should be near the in generic cases. As for the compact objects, Babichev and Langlois[70] (see ref. 71 also on the similar theme) have argued that neutron stars can be rescued from singularity if a realistic equation of state for these objects is used though the numerical simulation is yet challenging for densities of the order of nuclear matter density. The problem deserves further investigation.

In scenarios of large scale modification of gravity, one should worry about the local gravity constraints. The $f(R)$ theories are related to the class of scalar tensor theories corresponding the Brans–Dicke parameter $\omega = 0$ or the PPN parameter $\gamma = (1 + \omega)/(2 + \omega) = 1/2$ unlike GR where $\gamma = 1$ consistent with observation ($|\gamma - 1| \lesssim 2.3 \times 10^{-5}$). This conclusion can be escaped by invoking the so-called chameleon mechanism[72]. In case, the scalar degree of freedom is coupled to matter, the effective mass of the field depends upon the matter density which can allow to avoid the conflict with solar physics constraints. However, the problem of singularity in these models is genuine and should be addressed.

Summary

I have given a pedagogical exposition of physics of late time cosmic acceleration. Most of that part of the review should be accessible to a graduate student. The discussion of Newtonian cosmology is comprehensive and reviews the efforts to put the formalism of Newtonian cosmology on rigorous foundations in its domain of validity. Heuristic discussion on the introduction of cosmological constant and pressure corrections in evolution equations is included. The underlying idea leading to late time cosmic acceleration is explained without the use of general theory of relativity. The basic features of cosmological dynamics in presence of cosmological constant is presented in a simple and elegant fashion making it accessible to non-experts. The review also gives the glimpses of relativistic cosmology, contains important notes on the dynamics of dark energy and discusses underlying features of cosmological dynamics of a variety of scalar fields including quintessence, rolling tachyon and phantom. Special emphasis is put on the cosmic viability of these models; the cosmological relevance of scaling solutions is briefly explained. The review ends with a discussion on modified theories of gravity as possible alternatives to dark energy. The treatment is simple but conveys the successes and problems of cosmology in the framework of modified theories of gravity. Basic features of $f(R)$ cosmology are explained avoiding the cumbersome mathematical expressions. The latest developments of $f(R)$ theories with disappearing cosmological constant are highlighted. The problem of singularities in these models and their possible resolution are discussed. I hope the review would be helpful to beginners and will also be of interest to experts.

1. Perlmutter, S. *el al*., Measurements of omega and lambda from 42 high redshift supernovae. *Astrophys. J*., 1999, **157**, 565–586.
2. Reiss, A. *et al*., Bv Ri light curves for 22 type Ia supernovae. *Astrophys. J*., 1999, **117**, 707–724.
3. Efstathiou, G. *et al*., The cosmological constant and cold dark matter. *Nature*, 1990, **348**, 705–707; Ostriker, J. P. and Steinhardt, P. J., Steinhardt, The observational case for a low-density universe with a non-zero cosmological constant. *Nature*, 1995, **377**, 600–602; Bagla, J. S., Padmanabhan, T. and Narlikar, J. V., Crisis in cosmology: observational constraints on omega and H(0). *Comments Astrophys*., 1996, **18**, 275 [astro-ph/9511102].
4. Sahni, V. and Starobinsky, A. A., The case for a positive cosmological lambda term. *Int. J. Mod. Phys*., 2000, **D9**, 373444.
5. Sahni, V. and Starobinsky, A., Reconstructing dark energy. *Int. J. Mod. Phys*., 2006, **D15**, 2105–2132 [astro-ph/0610026]; Padmanabhan, T., Dark energy: mystery of the millennium. *AIP Conf. Proc*., 2006, **861**, 179–196 [astro-ph/0603114]; Peebles, P. J. E. and Ratra, B., The cosmological constant and dark energy. *Rev. Mod. Phys*., 2003, **75**, 559–606; Perivolaropoulos, L., Accelerating universe: observational status and theoretical implications. *AIP Conf. Proc*., 2006, **848**, 698–712 [astro-ph/0601014]; Straumann, N., Dark energy. arXiv:gr-qc/0311083; Frieman, J., Lectures on dark energy and cosmic acceleration. *AIP Conf. Proc*., 2008, **1057**, 87–124 [arXiv:0904.1832]; Sami, M., Models of dark energy. *Lect. Notes Phys*., 2007, **72**, 219–256; Sami, M., Dark energy and possible alternatives. arXiv:0901.0756.
6. Copeland, E. J., Sami, M. and Tsujikawa, S., Dynamics of dark energy. *Int. J. Mod. Phys*., 2006, **D15**, 1753–1936 [hep-th/0603057].
7. Linder, E. V., Mapping the cosmological expansion. *Rep. Prog. Phys*., 2008, **71**, 056901.
8. Caldwell, R. K. and Kamionkowski, M., The physics of cosmic acceleration. arXiv:0903.0866.
9. Silvestri, A. and Mark Trodden, Approaches to understanding cosmic acceleration. arXiv:0904.0024.
10. Frieman, J., Turner, M. and Huterer, D., Dark energy and the accelerating universe. *Ann. Rev. Astron. Astrophys*., 2008, **46**, 385–432 [arXiv:0803.0982].
11. Capozziello, S., Curvature quintessence. *Int. J. Mod. Phys*., 2002, **D11**, 483–492; Capozziello, S., Carloni, S. and Troisi, A., Quintessence without scalar fields. *Recent Res. Dev. Astron. Astrophys*., 2003, **1**, 625 [astroph/0303041]; Carroll, S., Duvvuri, V., Trodden, M. and Turner, M., Is cosmic speed-up due to new gravitational physics. *Phys. Rev*., 2004, **D70**, 043528; Sotiriou, T. P. and Faraoni, V., F(R) theories of gravity. arXiv:0805.1726; Nojiri, S. and Odintsov, S., Modified gravity with Ln R terms and cosmic acceleration. *Gen. Rel. Grav*., 2004, **36**, 1765–1780 [hep-th/0308176].
12. Sahni, V., Krasinski, A. and Zeldovich, Ya. B., Republication of *The Cosmological Constant and the Theory of Elementary Particles* (eds Zeldovich, Ya. B.), *Gen. Rel. Grav*., 2008, **40**, 1557; *Sov. Phys. Usp*., 1968, **11**, 381.
13. Weinberg, S., The cosmological constant problem. *Rev. Mod. Phys*., 1989, **61**, 1–23; Carroll, S. M., The cosmological constant. *Living Rev. Rel*., 2001, **4**, 1 [arXiv:astro-ph/0004075]; Padmanabhan, T., Cosmological constant: the weight of the vacuum. *Phys. Rep*., 2003, **380**, 235–320.
14. Bousso, R., Tasi lectures on the cosmological constant. *Gen. Rel. Grav*., 2008, **40**, 607–637 [arXiv:0708.4231].
15. Padmanabhan, T., Dark energy and its implications for gravity. *Adv. Sci. Lett*., 2009, **2**, 174 [arXiv:0807.2356]; Padmanabhan, T., Dark energy and gravity. *Gen. Rel. Grav*., 2008, **40**, 529–564 [arXiv:0705.2533].

16. Zlatev, I., Wang, L. M. and Steinhardt, P. J., Quintessence, cosmic coincidence, and the cosmological constant. *Phys. Rev. Lett.*, 1999, **82**, 896–899; Steinhardt, P. J., Wang, L. M. and Zlatev, I., Cosmological tracking solutions. *Phys. Rev.*, 1999, **D59**, 123504; Amendola, L., Coupled quintessence. *Phys. Rev.*, 2000, **D62**, 043511.
17. Armendariz-Picon, C., Mukhanov, V. and Steinhardt, P. J., A dynamical solution to the problem of a small cosmological constant and late time cosmic acceleration. *Phys. Rev. Lett.*, 2000, **85**, 4438–4441; Armendariz-Picon, C., Mukhanov, V. and Steinhardt, P. J., Essentials of K essence. *Phys. Rev.*, 2001, **D63**, 103510; Chiba, T., Okabe, T. and Yamaguchi, M., Kinetically driven quintessence. *Phys. Rev*., 2000, **D62**, 023511.
18. Bondi, H., *Cosmology*, Cambridge University Press, 1960.
19. Weinberg, S., *Gravitation and Cosmology*, John Wiley, New York, 1972.
20. Padmanabhan, T., *Large Scale Structure of the Universe*, Cambridge University Press, 1993.
21. Narlikar, J. V., *Introduction to Cosmology*, Cambridge University Press, 2004.
22. Zel'dovich, Ya. B. and Novikov, I. D., *Structure and Evolution of Universe*, Chicago University Press, 1983.
23. Kolb, E. W. and Turner, M. S., *The Early Universe*, Addison-Wesley, Publishing Company, NY, 1990.
24. Padmanabhan, T., *Theoretical Astrophysics*, Cambridge University Press, 2002, vol. III.
25. Liddle, A. R. and Lyth, D. H., *Cosmological Inflation and Large-Scale Structure*, Cambridge University Press, New York, 2000.
26. Dodelson, S., *Modern Cosmology*, Academic Press, 2003.
27. Mukhanov, V., *Physical Foundations of Cosmology*, Cambridge University Press, 2005.
28. Cahill, R. T., Unravelling the dark matter-dark energy paradigm. [arXiv:0901.4140]..
29. Milne, E., A Newtonian expanding universe. *Q. J. Math*., 1934, **5**, 64–72; McCrea, W. H., Newtonian cosmology. *Nature*, 1955, **175**, 466.
30. Tipler, F. J., *Am. J. Phys*., 1996, **64**, 1311–1315.
31. Neumann, C., Uber das newtonische Prinzip der Fernwirkung, Leipzig, 1996.
32. Seeliger, H., *Astron. Nachr*., 1895, **137**, 129.
33. Sahni, V., Feldman, H. and Stebbins, A., Loitering universe. *Astrophys. J.*, 1992, **385**, 1–8.
34. Weinberg, S., Anthropic bound on the cosmological constant. *Phys. Rev. Lett*., 1987, **59**, 2607.
35. Krauss, L. M. and Chaboyer, B., Age estimates of globular clusters in the milky way: constraints on cosmology. *Science*, 2003, **299**, 65–70.
36. Caldwell, R. R., Marc Kamionkowski and Weinberg, N. N., Phantom energy and cosmic doomsday. *Phys. Rev. Lett.*, 2003, **91**, 071301.
37. Kowalski, M. *et al*., Improved cosmological constraints from new, old and combined supernova datasets. *Astrophys. J.*, 2008, **686**, 749–778.
38. Reiess A. G. *et al.*, Type Ia supernova discoveries At Z ¿ 1 from the Hubble space telescope: evidence for past deceleration and constraints on dark energy evolution. *Astrophys. J.*, 2004, **607**, 665–687.
39. Choudhury, T. R. and Padmanabhan, T., Cosmological parameters from supernova observations: a critical comparison of three data sets. *Astron. Astrophys*., 2005, **429**, 807.
40. Stojkovic, D., Starkman, G. D. and Matsuo, R., Dark energy, the colored anti-de Sitter vacuum, and Lhc phenomenology. *Phys. Rev.*, 2008, **D77**, 063006; Batra, P., Hinterbichler, K., Hui, L. and Kabat, D., Pseudo-redundant vacuum energy. *Phys. Rev.*, 2008, **D78**, 043507 [arXiv:0801.4526].
41. Tsujikawa, S. and Sami, M., A unified approach to scaling solutions in a general cosmological background. *Phys. Lett*., 2004, **B603**, 113–123 [hep-th/0409212]; Copeland, E. J., Mohammad, R. G., Sami, M. and Shinji Tsujikawa, What is needed of a tachyon if it is to be the dark energy? *Phys. Rev.*, 2005, **D71**, 043003 [hep-th/0411192]; Sami, M., Savchenko, N. and Toporensky, A., Aspects of scalar field dynamics in Gauss–Bonnet brane worlds. *Phys. Rev.*, 2004, **D70**, 123528 [hep-th/0408140]; Sami, M., Implementing power law inflation with rolling tachyon on the brane. *Mod. Phys. Lett.*, 2003, **A18**, 691 [hep-th/0205146]; Setare, M. R., Holographic tachyon model of dark energy. *Phys. Lett*., 2007, **B653**, 116121; Setare, M. R., Interacting holographic phantom. *Eur. Phys. J*., 2007, **C50**, 991–998; Setare, M. R., The holographic dark energy in non-flat Brans–Dicke cosmology. *Phys. Lett*., 2007, **B644**, 99–103.
42. Caldwell, R. R. and Linder, E. V., The limits of quintessence. *Phys. Rev. Lett.*, 2005, **95**, 141301.
43. Sen, A. A. and Chandrachani Devi, N., Cosmological scaling solutions with tachyon: modified gravity model. *Phys. Lett.*, 2008, **B668**, 182–186; Scherrer, R. J. and Sen, A. A., Thawing quintessence with a nearly flat potential. *Phys. Rev.*, 2008, **D77**, 083515; Sen, A. A., Gupta, G. and Das, S., Non-minimal quintessence with nearly flat potential. arXiv:0901.0173; Singh, P., Sami, M. and Dadhich, N., Cosmological dynamics of phantom field. *Phys. Rev.*, 2003, **D68**, 023522 [hep-th/0305110]; Ali, A., Sami, M. and Sen, A. A., The transient and the late time attractor tachyon dark energy: can we distinguish it from quintessence? arXiv: 0904.1070.
44. Sahni, V. and Wang, L., A new cosmological model of quintessence and dark matter. *Phys. Rev.*, 2000, **D62**, 103517.
45. Sami, M., Inflation with oscillations. *Grav. Cosmol*., 2003, **8**, 309–312 [gr-qc/0106074]; Sami, M., Dadhich, N. and Shiromizu, T., Steep inflation followed by born-infeld reheating. *Phys. Lett*., 2003, **B568**, 118–126 [hep-th/0304187]; Sami, M. and Sahni, V., Quintessential inflation on the brane and the relic gravity wave background. *Phys. Rev*., 2004, **D70**, 083513 [hep-th/0402086]; Sami, M. and Padmanabhan, T., A viable cosmology with a scalar field coupled to the trace of the stress tensor. *Phys. Rev.*, 2003, **D67**, 083509.
46. Sahni, V., Sami, M. and Souradeep, T., Relic gravity waves from brane world inflation. *Phys. Rev*., 2002, **D65**, 023518 [gr-qc/01051211].
47. Sami, M., Singh, P. and Tsujikawa, S., Avoidance of future singularities in loop quantum cosmology. *Phys. Rev*., 2006, **D74**, 043514 [gr-qc/0605113].
48. Shafieloo, A., Sahni, V. and Starobinsky, A. A., Is cosmic acceleration slowing down?, arXiv:0903.5141.
49. Nojiri, S., Odintsov, S. D. and Sasaki, M., Gauss–Bonnet dark energy. *Phys. Rev.*, 2005, **D71**, 123509.
50. Koivisto, T. and Mota, D. F., Cosmology and astrophysical constraints of Gauss–Bonnet dark energy. *Phys. Lett*., 2007, **B644**, 104–108 [arXiv:astro-ph/0606078]; Koivisto, T. and Mota, D. F., Gauss–Bonnet quintessence: background evolution, large scale structure and cosmological constraints. *Phys. Rev*., 2007, **D75**, 023518 [hep-th/0609155].
51. Tsujikawa, S. and Sami, M., String-inspired cosmology: late time transition from scaling matter era to dark energy universe caused by a Gauss–Bonnet coupling. *JCAP*, 2007, **0701**, 006 [hep-th/0608178].
52. Calcagni, G., Tsujikawa, S. and Sami, M., Dark energy and cosmological solutions in second-order string gravity. *Class. Quant. Grav*., 2005, **22**, 3977–4006; Sami, M., Toporensky, A., Tretjakov, P. V. and Tsujikawa, S., The fate of (Phantom) dark energy universe with string curvature corrections. *Phys. Lett*., 2005, **B619**, 193.
53. Carter, B. M. N. and Neupane, I. P., Dynamical relaxation of dark energy: a solution to early inflation, late-time acceleration and the cosmological constant problem. *Phys. Lett*., 2006, **B638**, 94–99; Carter, B. M. N. and Neupane, I. P., Towards inflation and dark

energy cosmologies from modified Gauss–Bonnet theory. *JCAP*, 2006, **0606**, 004; Neupane, I. P., On compatibility of string effective action with an accelerating universe. *Class. Quant. Grav*., 2006, **23**, 7493–7520 [hep-th/0602097]; Leith, B. M. and Neupane, I. P., Gauss–Bonnet cosmologies: crossing the phantom divide and the transition from matter dominance to dark energy. *JCAP*, 2007, **0705**, 019 [hep-th/0702002]; Neupane, I. P., Simple cosmological de Sitter solutions on Ds(4) X Y(6) spaces. arXiv: 0901.2568; Neupane, I. P., Extra dimensions, warped Compactifications and cosmic acceleration. arXiv: 0903.4190.

54. Calcagni, G., de Carlos, B. and De Felice, A., Ghost conditions for Gauss–Bonnet cosmologies. *Nucl. Phys*., 2006, **B752**, 404–438.
55. Nojiri, S., Odintsov, S. D. and Sami, M., Dark energy cosmology from higher-order, string-inspired gravity and its reconstruction. *Phys. Rev*., 2006, **D74**, 046004; Elizalde, E., Jhingan, S., Nojiri, S., Odintsov, S. D., Sami, M. and Thongkool, I., Dark energy generated from a (super)string effective action with higher order curvature corrections and a dynamical dilaton. *Eur. Phys. J*., 2008, **C53**, 447–457 [arXiv:0705.1211].
56. Tsujikawa, S., Cosmologies from higher-order string corrections. *Ann. Phys.*, 2006, **15**, 302–315 [hep-th/0606040].
57. Tatekawa, T. and Tsujikawa, S., Second-order matter density perturbations and skewness in scalar-tensor modified gravity models. *JCAP*, 2008, **0809**, 009 [arXiv:0807.2017].
58. Durrer, R. and Maartens, R., Dark energy and modified gravity. arXiv:0811.4132.
59. Sahni, V. and Shtanov, Y., Brane world models of dark energy. *JCAP*, 2003, **0311**, 014 [astro-ph/0202346]; Shtanov, Y., Sahni, V., Shafieloo, A. and Toporensky, A., Induced cosmological constant and other features of asymmetric brane embedding. *JCAP*, 2009, **0904**, 023 [arXiv:0901.3074].
60. Faraoni, V., de Sitter space and the equivalence between F(R) and scalar-tensor gravity. *Phys. Rev.*, 2007, **D75**, 067302.
61. Hu, W. and Sawicki, I., Models of F(R) cosmic acceleration that evade solar-system tests. *Phys. Rev*., 2007, **D76**, 064004 [arXiv: 0705.1158].
62. Starobinsky, A. A., Disappearing cosmological constant in F(R) gravity. *JETP Lett*., 2007, **86**, 157.
63. Appleby, S. A. and Battye, R. A., If Gauss–Bonnet interaction plays the role of dark energy. *Phys. Lett*., 2007, **B654**, astro-ph/0608104.
64. Appleby, S. A. and Battye, R. A., Aspects of cosmological expansion in F(R) gravity models. *JCAP*, 2008, **0805**, 019 [arXiv: 0803.1081].
65. Frolov, A. V., A singularity problem with F(R) dark energy. *Phys. Rev. Lett.*, 2008, **101**, 061103 [arXiv:0803.2500].
66. Kobayashi, T. and Maeda, K., Relativistic stars in F(R) gravity, and absence thereof. *Phys. Rev.*, 2008, **D78**, 064019 [arXiv:0807.2503].
67. Abha Dev, Jain, D., Jhingan, S., Nojiri, S., Sami, M. and Thongkool, I., Delicate F(R) gravity models with disappearing cosmological constant and observational constraints on the model parameters. *Phys. Rev.*, 2008, **D78**, 083515 [arXiv:0807.3445].
68. Kobayashi, T. and Maeda, K., Can higher curvature corrections cure the singularity problem in F(R) gravity? *Phys. Rev.*, 2009, **D79**, 024009 [arXiv:0810.5664].
69. Abdalla, M., Nojiri, S. and Odintsov, S. D., Consistent modified gravity: dark energy, acceleration and the absence of cosmic doomsday. *Class. Quant. Grav.*, 2005, **22**, L35 [hep-th/0409177]; Bamba, K., Nojiri, S. and Odintsov, S. D., The universe future in modified gravity theories: approaching the finite-time future singularity. *JCAP*, 2008, **0810**, 045 [0807.2575]; Nojiri, S. and Odintsov, S. D., The future evolution and finite-time singularities in F(R)-gravity unifying the inflation and cosmic acceleration. arXiv:0804.3519[hep-th].
70. Babichev, E. and Langlois, D., Relativistic stars in F(R) gravity. arXiv:0904.1382.
71. Nojiri, S. and Odintsov, S. D., Singularity of spherically-symmetric spacetime in quintessence/phantom dark energy universe. arXiv:0903.5231.
72. Straumann, N., Problems with modified theories of gravity, as alternatives to dark energy. arXiv:0809.51; Capozziello, S. and Tsujikawa, S., Solar system and equivalence principle constraints on F(R) gravity by Chameleon approach. *Phys. Rev*., 2008, **D77**, 107501; Brax, P., Carsten van de Bruck, Anne-Christine Davis and Shaw, D. J., F(R) Gravity and Chameleon theories. *Phys. Rev.*, 2008, **D78**, 104021.

ACKNOWLEDGEMENTS. I am indebted to T. Padmanabhan for giving me an opportunity to write this review for a special issue of *Current Science*. I thank to D. Jain, V. Sahni and I. Thongkool for taking pain in going through the manuscript and making suggestions for its improvement. I also thank S. A. Abbas, N. Dadhich, S. Jhingan, K. Raza and A. A. Sen for useful comments and discussion.