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Rayleigh–Taylor instability driven nonlinear vortices in dusty plasmas

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The low frequency stability of a dusty plasma with a nonuniform mass and charge distribution of the dust component is studied. It is shown that the inverse stratification of the dust mass density in a gravitational field may lead to a Rayleigh–Taylor-like instability. In the nonlinear regime this instability can produce an incompressible flow pattern of spontaneously rotating dust plasma fluid. This result can have potential applications in the interpretation of vortical patterns observed in laboratory experiments as well as in a variety of astrophysical situations where the generation and sustenance of angular momentum are important issues. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1881452]

The collective oscillations of dusty plasmas have been the object of serious study in recent years in view of their novel character and wide ranging applications.[1,2] A dusty plasma differs from an ordinary electron-ion plasma by the presence of a heavier dust species which gets dynamically charged by collisions with the electrons and ions. The collective properties of this medium are significantly influenced by a number of factors such as the presence of the third charged species (the dust component), the amount of charge on the dust, the dynamic nature of the charge on the dust, the dust mass, the shape of the dust particles, the conductivity of the dust particles, etc. In contrast to ordinary plasmas, therefore, a dusty plasma can have a wider range of collective modes and novel dissipation mechanisms which can influence the stability of these modes. Due to the large amount of charge on single dust particles the dust fluid can also exhibit strong coupling behavior which can show up as viscoelastic properties[3] of the medium and even lead to a phase transition into a "dust crystal."[4,5] For the massive dust particles gravitational effects play an important role in their dynamics which is another important distinction from the conventional electron-ion plasmas where gravity effects are normally neglected. The interplay of gravitational and electrodynamic forces has been well recognized and investigated in the astrophysical context such as in explaining the fine structure of planetary rings[6] and in understanding the dynamics of massive dust objects in galaxies.[7,8] The role of gravity is significant even for micron sized dust particles in laboratory experiments where sizable external electric fields are needed to levitate dust plasma formations. Gravity can also bring about macroscopic instabilities in many equilibrium configurations. The well known Rayleigh–Taylor instability in neutral fluids is a classic example of gravity aided instability of multilayered fluids with an unfavorable density gradient. In this paper we discuss the excitation of a similar gravitation induced Rayleigh–Taylor-like instability for a charged dust fluid arising as a result of inverse stratification of the dust mass/number density. We also obtain exact nonlinear solutions which may ultimately arise due to the saturation of such an instability. The final nonlinear state is a rotating structure which for high values of viscosity can take the form of a rigid rotator. Such rotation effects induced by self-consistent collective effects in dusty plasmas may have important astrophysical applications where the generation and sustenance of angular momentum are important issues. They may also be relevant for understanding some recent laboratory observations of vortical motion of dust clouds in the vertical plane without the presence of any magnetic field.

The linear theory of the instability can be understood quite simply from a basic fluid model for the dust component, namely,

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{V}_d) = 0, \tag{1}$$

$$\frac{\partial \vec{V}_d}{\partial t} + \vec{V}_d \cdot \nabla \vec{V}_d = \frac{\rho_c}{\rho_m} \nabla \phi - g\hat{z} + \frac{\mu}{\rho_m} \nabla^2 \vec{V}_d, \tag{2}$$

$$\nabla \cdot \vec{V}_d = 0, \tag{3}$$

where ρ_c and ρ_m stand for the dust charge and mass densities, respectively, $\vec{V}_d$ is the dust fluid velocity, ϕ is the electrostatic potential, g is the gravitational acceleration, and μ is the coefficient of viscosity for the dust flow velocity. The equilibrium condition is given by

$$\frac{\rho_{c0}}{\rho_{m0}} \nabla \phi_0 = g\hat{z}, \tag{4}$$

where $\nabla \phi_0$ is an external electric field imposed to balance the dust particles against gravity. We also assume that there are no equilibrium flows, i.e., $\vec{V}_{0d}=0$. The incompressibility condition (3) implies the neglect of sound waves and is valid for very low frequency phenomena, $\partial/\partial t \ll \omega_{da}$, where ω_{da} is the dust acoustic frequency. We now restrict our attention to two-dimensional incompressible perturbations such that all variations are in the x-z plane. Condition (3) then allows us to introduce a velocity potential ψ such that $\vec{V}_d = \hat{y} \times \nabla \psi$. Taking alternately the curl and divergence of Eq. (2) and using the definition of $\vec{V}_d$, we can rewrite Eqs. (1) and (2) in the following form:

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$$\frac{\partial \rho_m}{\partial t} + \hat{y} \times \nabla \psi \cdot \nabla \rho_m = 0, \tag{5}$$

$$\frac{\partial \vec{\Omega}}{\partial t} + \hat{y} \times \nabla \psi \cdot \nabla \vec{\Omega} = \nabla \left(\frac{\rho_c}{\rho_m} \right) \times \nabla \phi + \mu \nabla^2 \vec{\Omega}, \tag{6}$$

$$\nabla \cdot \{ (\hat{y} \times \nabla \psi \cdot \nabla)(\hat{y} \times \nabla \psi) \} = \nabla \cdot \left(\frac{\rho_c}{\rho_m} \nabla \phi \right), \tag{7}$$

where $\vec{\Omega} = \hat{y} \nabla^2 \psi$ is the vorticity of dust flow. We now carry out a linear stability analysis about the equilibrium described by Eq. (4) for perturbation scale lengths that are much shorter than the equilibrium scale lengths, namely, $k \gg \rho'_{m0}/\rho_{m0}$. It is then straightforward to obtain the following dispersion relation:

$$\gamma^2 + \frac{\gamma k^2 (\mu k^2)}{(k^2 - i k_z \rho'_{m0}/\rho_{m0})} - \frac{g k_x^2 \rho'_{m0}}{\rho_{m0}(k^2 - i k_z \rho'_{m0}/\rho_{m0})} = 0, \tag{8}$$

where γ represents the growth rate and $\rho'_{m0} = d\rho_{m0}/dz$. In obtaining Eq. (8) we have ignored the fluctuations in the dust charge density and treated ρ_c to be a constant in time. We can estimate the contribution of the dust fluctuation ρ_{c1} from a simple model dust charging equation that is widely used in the literature,[9]

$$\left(\frac{\partial}{\partial t} + \eta \right) \rho_{c1} + \vec{V}_{d1} \cdot \nabla \rho_{c0} = |I_{oe}| \frac{\rho_{m0}}{m_d} \left(\frac{1}{T_e} + \frac{1}{T_i} \right) \phi_1 \tag{9}$$

and the quasineutrality condition, viz.,

$$\rho_{c1} = - n_0 e \left(\frac{e}{T_i} + \frac{n_{e0}}{n_0} \frac{e}{T_e} \right) \phi_1. \tag{10}$$

Here, η represents the inverse of the charging (discharging) rate of a dust particle, $|I_{oe}|$ is the magnitude of the equilibrium electron current falling on each dust particle, and we have assumed that the electron and ion density perturbations are given by the linearized Boltzmann law $n_{\alpha 1} = n_{\alpha 0}(-e_\alpha \phi / T_\alpha)$, with $\alpha = e, i$. For slow perturbations $\eta \gg \partial/\partial t$ we see that $\rho_{c1} \sim O(\vec{V}_{d1} \cdot \nabla \rho_{c0}/\eta)$. Thus effects due to ρ_{c1} may be neglected in comparison to those due to $\rho_{m1} \sim O[\vec{V}_{d1} \cdot \nabla \rho_{m0}/(\partial/\partial t)]$ in the linearized Eq. (2). With ρ_c as a constant in Eq. (2), the $(-\rho_c \phi)$ term can be interpreted as a sort of "pressure" term. Coming back to the linear dispersion relation, Eq. (8), an approximate condition for the onset of gravitational instability can be obtained by ignoring the imaginary terms in the denominator. We then get the condition

$$\mu k^2 \leqslant \frac{g}{L} \frac{k_x^2}{k^2} \tag{11}$$

with the characteristic growth rate $\gamma \sim (k_x/k)\sqrt{g/L}$, where L is the density scale length.

We next discuss the possibility of a saturated state of this instability due to nonlinear effects. We note that the instability is driven by the torque proportional to $\nabla \rho_m \times \nabla \phi$, which is finite because the density and potential perturbations have an x dependence (note that the growth rate $\gamma \propto k_x$). Saturation can occur if the resulting vortex motion mixes up the density layers giving a $\rho_m \equiv \rho_m(\psi)$, thereby satisfying the continuity equation. A simple solution of a uniform density ρ_m arising through mixing seems a reasonable choice. This will make the driving torque for the instability vanish. The vorticity equation, Eq. (6), would lead to a stationary state if $\{\psi, \Omega\} = \mu \nabla^2 \Omega$, where $\{\ldots, \ldots\}$ denotes a Poisson bracket. If viscosity is negligible then any function $\Omega = \Omega(\psi)$ can be a solution. However, for finite and non-negligible μ the only consistent solution is

$$\Omega \equiv \Omega(\psi) = \text{constant}. \tag{12}$$

Such a solution corresponds to a rigidly rotating dust cloud structure. An analytic solution of the nonlinear set of Eqs. (5)–(7) can be written in the form

$$\psi = C_0 r^2 + C_1 r \cos\theta, \tag{13}$$

which corresponds to a rotational velocity $V_\theta = 2C_0 r$. Typical saturated values of the rotational velocity may be estimated from the condition of importance of fluid nonlinearities for saturation, viz.,

$$\frac{\partial}{\partial t} \sim \vec{V}_d \nabla,$$

which gives

$$\frac{V_{d,\theta}}{r} \sim \gamma \sim 2 \sqrt{\frac{g}{L}} \frac{k_x}{k} - \frac{\mu k^2}{\rho_m}. \tag{14}$$

For typical laboratory plasmas with $L \sim 10$ cm, $k_x/k \sim 1$, one gets $0 \leqslant \Omega \leqslant 10\ \mathrm{s}^{-1}$. In some recent laboratory studies,[10,11] self-excitation of rotational motion of dust particles has been observed in dc glow discharge plasmas with rotational velocities in the range of a few hertz. However, dust densities in these experiments are quite low and the observed motion can be well understood in terms of single particle orbits. It would be interesting to extend the experimental investigations to higher densities such that collective physics of the dust component becomes important. Under such a circumstance it should be possible to observe the nonlinear vortex solution we have obtained in this paper. Such a study would also aid in obtaining some insight into the mechanism of angular momentum generation in astrophysical objects where charged dust is often ubiquitous.

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