Fast Zonal Field Dynamo in Collisionless Kinetic Alfven Wave Turbulence

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The possibility of fast dynamo action by collisionless kinetic Alfven Wave turbulence is demonstrated. The irreversibility necessary to lock in the generated field is provided by electron Landau damping, so the induced electric field does not vanish with resistivity. Mechanisms for self-regulation of the system and the relation of these results to the theory of alpha quenching are discussed. The dynamo-generated fields have symmetry like that of zonal flows, and thus are termed zonal fields.

\[ \alpha = \frac{\alpha_K}{[1 + R_m v_{A0}^2/(\bar{v}^2)]]}. \]  
\[ (1) \]

Here \( v_{A0} = \langle B \rangle /\sqrt{\pi \rho_0} \) is the Alfven speed in the mean field, \( \langle \bar{v}^2 \rangle \) is the mean square turbulence velocity, \( R_m \) is the magnetic Reynolds number and \( \alpha_K = \sum_k (\bar{\omega} \cdot \bar{\omega})/\Delta_k \). \( \tau_e \) is an eddy self-correlation time. Eqn. (1) states that the alpha effect is quenched for \( v_{A0} > (1/\sqrt{R_m}) \bar{v}_{rms} \), i.e. for very small \( \langle B \rangle \), since \( R_m >> 1 \) in most relevant applications. Note that the above expression for \( \alpha \) may be re-written as \( \alpha = \alpha_K /\eta /[\eta + \tau_c v_{A0}^2] \), where \( \tau_c \) is the integral scale turbulence correlation time. This expression suggests that one way of interpreting the alpha 'quench' is that despite naive expectations of turbulent mixing, etc., the collisional resistivity ultimately controls the rate at which 'stretched and twisted' fields are folded to increase \( \langle B \rangle \). This constraint necessarily precludes the possibility of a fast dynamo, i.e. one which operates on time scales independent of \( \eta \). Another possible interpretation of the quench is that of 'Alfvenization', whereby the growing \( \langle B \rangle \) converts eddys into Alfven waves (which are intrinsically non-kinematic), thus initiating the quench. Indeed, with this in mind, it is interesting to note that the EMF for an individual, circularly polarized visco-resistive MHD Alfven wave is \( (\eta/2B_0)(1-v/\eta)k^2z_0 \bar{A}^2 \). Apart from the 'cross-helicity' factor \( (1-v/\eta) \) and a somewhat different expression for the helicity spectrum, this formula is identical to that given by Eqn. (1), thus reinforcing the notion that Alfvenization is at work in the alpha quench.

In this paper, we report on the theory of a fast, collisionless dynamo. In view of the above discussion of Alfvenization, we proceed to directly consider a dynamo driven by Kinetic Shear Alfven Wave (KSAW) turbulence. Here, the dynamo instability mechanism is that of a modulational instability of the KSAW spectrum. In this case, the dynamically generated field is orthogonal to an externally prescribed mean field \( B_i = B_{i0} \hat{z} \), along which the KSAW's propagate. Collisionless dissipation, via Landau damping, locks in the dynamo-generated field at a rate independent of \( \eta \), thus facilitating fast dynamo action. Since, assuming radially inhomogeneous turbulence, the dynamo-generated fields have azimuthal symmetry and are thus analogous to zonal flows, we hereafter refer to them as zonal fields.

As the current is carried by the electrons in KSAW turbulence, we start from the drift-kinetic equation

\[ \frac{\partial f}{\partial t} + v || \nabla || f - \frac{e}{B_0} \nabla \phi \times \hat{z} \cdot \nabla f - \frac{|e|}{m_e} E || \partial f}{\partial v ||} = c(f). \]  
\[ (2) \]
Here \( E_\parallel = -\nabla_\parallel \phi - (1/c)\partial A/\partial t \), where \( \phi \) is the electrostatic potential, \( A \) is the \( \hat{z} \) component of the vector potential, and \( \nabla_\parallel = \partial/\partial z + \nabla(A/B_\parallel) \times \hat{z} \cdot \nabla \). Neglecting electron inertia, averaging Eqn. (2), and using Ampere’s Law then yields the mean field Ohm’s Law for collisionless plasma:

\[
\frac{1}{c} \frac{\partial (A)}{\partial t} - \langle \vec{E}_z \vec{n}/n_o \rangle + \frac{\partial \Gamma}{\partial \Gamma} = \frac{\eta}{c} \nabla_{\perp}^2 \langle A \rangle .
\]  

Terms \( \langle \vec{E}_z \vec{n}/n_o \rangle \) and \( \partial \Gamma/\partial \Gamma \) refer to parallel electron acceleration and the spatial transport of parallel current, respectively. The flux \( \Gamma \) is given by \( (1/\Omega_e) \int d\nu_\parallel \nu_\parallel \langle (\nabla_\parallel (\phi - (v_\parallel/c)A_\parallel)) \rangle \hat{f} \). Proceeding as in quasilinear theory, we can use the coherent response \( \vec{f}_k \):

\[
\vec{f}_k = \frac{e}{T_e} (\phi - \psi) \nu_\parallel - \frac{e\omega_k}{T_e} (\nu_\parallel - k_\parallel v_\parallel) ,
\]  

(4a)

\( \Gamma = \frac{\pi T_e c}{\Omega_e} \sum_k 2 k_\parallel \hat{z} \omega_k^2 \left| \frac{\nu_\parallel}{k_\parallel} \right| \left| \frac{e(\phi - \psi)_k}{T_e} \right|^2 f_{\nu_\parallel/k_\parallel} \).  

(4b)

Here \( \nu_\parallel = \nu_\parallel \hat{A}_\parallel/c_k \) and \( \phi \) and \( \psi \) are related by \( \psi_k = (1 + k_\parallel^2 \rho_e^2) \phi_k \). Only the non-adiabatic piece of \( \vec{f} \) contributes to \( \langle \vec{E}_z \vec{n}/n_o \rangle \) and \( \Gamma \). Note that the flux of parallel current is proportional to a spectrally averaged factor of \( (k_\perp x \hat{z})/k_\parallel \), which requires that the turbulence have a net spectral chirality, in order for \( \Gamma \) to be non-zero. This property is the manifestation of helicity in the KSAW dynamo problem. Note also that \( \Gamma \) is independent of resistivity \( \eta \), since KSAW Landau damping now provides the irreversibility which permits ‘fast’ transport of current!

The stability of the KSAW spectrum to a zonal magnetic field perturbation \( \delta B_\theta = \delta B_\theta (r) \hat{\theta} \) may be determined by modulating Eqn. (3), i.e.

\[
\frac{1}{c} \frac{\partial (\delta \langle A \rangle)}{\partial t} = \frac{\delta (\vec{E}_z \vec{n}/n_o)}{\delta \langle A \rangle} \delta (\langle A \rangle) + \frac{\delta \Gamma_r}{\delta \langle A \rangle} \delta (\langle A \rangle) = -\frac{\eta q^2}{c} \delta (\langle A \rangle) q,
\]  

(5a)

where \( \delta (\langle A \rangle) \) is the associated modulation in the vector potential, i.e. \( \delta B_\theta (r) = -\partial \delta (\langle A \rangle)/\partial r \). Anticipating the use of methods from adiabatic theory, \( \langle \vec{E}_z \vec{n}/n_o \rangle \) and \( \Gamma_r \) are conveniently re-expressed in terms of the KSAW action density \( N_k \) as

\[
\langle \vec{E}_\parallel \vec{n}/n_o \rangle = -\frac{\pi}{c} T_e \sum_k \frac{2 k_\parallel^2 k_\perp^2}{2 + \rho_e^2 k_\perp^2} \frac{\omega_k^2}{k_\parallel |k_\parallel|} \left| f_{\nu_\parallel/k_\parallel} \right| N_k ,
\]  

(5b)

\[
\Gamma_r = \frac{\pi}{c \Omega_e} T_e \sum_k \frac{2 k_\parallel^2 k_\perp^2}{2 + \rho_e^2 k_\perp^2} k_\parallel |k_\parallel| \left| f_{\nu_\parallel/k_\parallel} \right| N_k ,
\]  

(5c)

where \( N_k = W_k/\omega_k \) and the KSAW energy density (normalized to the thermal energy density \( n_o T_e \)) is

\[
W_k = \frac{1}{8 \pi n_o T_e} (2 |A_{\parallel k}|^2 + c_e^2 k_\perp^2 |\phi_{\parallel k}|^2).
\]  

(5d)

Using the relation between potentials \( \phi_k \) and \( A_{\parallel k} \), this can be re-expressed as:

\[
N_k = \frac{2 + \rho_e^2 k_\perp^2}{2 \omega_k} \frac{2 k_\parallel^2 k_\perp^2}{|e \phi_k|} \frac{1}{T_e} \left| \frac{e \phi_k}{T_e} \right|^2 .
\]  

(5e)
Here $\rho_s = c_s/\Omega_i$ and $c_s^2 = T_e/m_e$. In this formulation, then, the modulations $\delta(E_z/\nu_0)$ and $\delta \Gamma$ can now be calculated simply by computing the modulational response $\delta N_k/\delta (A)$. The modulation in the number of KSAWs induced by the dynamo generated zonal field $\delta (A)$, i.e. $\delta N_k/\delta (A)$, may be obtained by using the wave kinetic equation to determine the response $\delta N_k$ to $\delta (A)$. The action density of a wave packet evolves according to the wave kinetic equation

$$\frac{\partial N_k}{\partial t} + \nu_g \cdot \nabla N_k - \Delta \omega_k \cdot \frac{\partial N_k}{\partial k} = -\gamma_k N_k,$$

where $\gamma_k$ is the wave damping decrement. The dynamo generated field refracts the underlying KSAWs and thus induces spectral modulations. In the presence of dynamo-generated zonal fields $\omega_k \to \omega_k + \delta \omega_k$. Here $\omega_k = k^2_\parallel v_0^2 (1 + k^2_\perp \rho^2_e)$ is the linear wave frequency and

$$\delta \omega_k = \frac{\omega_k}{k^2_\perp} k_\parallel B_0 \delta B_0,$$

is the perturbation in the wave frequency induced by the zonal dynamo field. Linearizing Eqn. (6) then gives the action density modulation

$$\delta N_{k,q} = \left[ \frac{i \omega_k}{\Omega_q - qv_{g,r} + i \gamma_k} \right] k_\theta q^2 \delta (A)_q \frac{\partial \langle N_k \rangle}{\partial k_r},$$

where $q$ is the radial wave number of the modulation $\delta (A)$ and $\Omega_q$ is its frequency. $\langle N_k \rangle$ is the mean KSAW action distribution. Eqn. (6) may then be substituted into Eqns. (8a,8b) to obtain the general expression for $\Omega_q$, which is (neglecting resistive dissipation)

$$\Omega_q = \pi \rho^2_e a^2 \nu_{th} q^2 \sum \frac{(1 + \rho^2_e k^2_\parallel)^{3/2}}{2 + \rho^2_e k^2_\parallel} \Omega_q k^2_\parallel + i \omega_k k_\theta q \frac{k^2_\parallel k_\theta}{k^2_{\parallel}} \frac{\partial}{\partial k_r} \left( \frac{< W_k >}{(1 + \rho^2_e k^2_\parallel)^{1/2}} \right) \mathcal{F}_o \left( \omega, k_{\parallel} \right).$$

Here $d_e = c/\omega_{pe}, \rho_e$ is the electron gyroradius and $f_o = \mathcal{F}_o/\nu_{th}$. Now, noting that the radial group velocity is $v_{g,r} = \partial \omega_k/\partial k_r = (k^2_\parallel \omega_A/\omega_k) k_\parallel \rho^2_e$ and considering the limit $qv_{g,r} > \gamma$, $\Omega_q$ (note here $\Omega_q$ is small, i.e. of $O(E^2_{||})$, and $qv_{g,r} > \gamma$ is consistent with the assumption of weak wave damping), we ultimately obtain the zonal field eigenfrequency $\Omega_q$, i.e.

$$\Omega_q = \pi \rho^2_e a^2 \nu_{th} q^2 \sum \frac{(1 + \rho^2_e k^2_\parallel)^{2}}{2 + \rho^2_e k^2_\parallel} \frac{k_\theta}{k_{\parallel}} \frac{\partial}{\partial k_r} \left( \frac{< W_k >}{(1 + \rho^2_e k^2_\parallel)^{1/2}} \right) \mathcal{F}_o \left( \omega, k_{\parallel} \right)$$

$$-i4\pi(c^2_e/\nu_{th}) a^2 \nu_{th} q^2 \sum \frac{(1 + \rho^2_e k^2_\parallel)^{5/2}}{2 + \rho^2_e k^2_\parallel} \frac{k_\theta}{k_{\parallel}} \frac{\partial}{\partial k_r} \left( \frac{< W_k >}{(1 + \rho^2_e k^2_\parallel)^{1/2}} \right) \mathcal{F}_o \left( \omega, k_{\parallel} \right).$$

Several aspects of this result are of interest. First, note that zonal field growth requires a normal population profile, i.e. $\partial \langle N_k \rangle/\partial k^2_{\parallel} < 0$, similarly to the case of zonal flow generation by drift waves. This condition is virtually always satisfied for Alfvénic MHD turbulence. Second, note that zonal field growth $Im \Omega_q$ is independent of $\eta$, since electron Landau damping provides the requisite dissipation. However, concomitant with this is the fact that the dynamo drive is ultimately proportional to $|E_k|^2/k^2_\parallel = (|\phi - \psi|)_k^2$, and thus depends directly upon field-fluid decoupling due to finite gyro-radius (and possibly finite ion inertial layer width, i.e. finite $k^2_\parallel c^2/\omega_{pi}$) effects. This is a direct consequence of the fact that finite $E_k$ is required for Landau damping. Since, in turn, Landau damping provides the necessary mechanism for irreversibility, the KSAW dynamo necessarily then requires non-zero $E_k$ on KSAW scales. Note also that while a mean $\Gamma$ (and thus a mean field KSAW dynamo) requires a net chirality or helicity in the KSAW turbulence, *zonal field growth does not*. This is because zonal field formation is a spontaneous symmetry breaking phenomenon, whereby the system acts to reinforce an initial $\delta B_0$ of either sign, but does not amplify the total flux. In this respect, the zonal field amplification process is more like the small-scale dynamo than the mean field dynamo. However, we hasten to add that a broad spectrum of growing zonal fields, with $(\rho_i/L_\perp) < q \rho_i < 1$, can be expected (N.B. Here $L_\perp$ is the scale of the Alfvén wave spectrum inhomogeneity in the direction perpendicular to $B_0$). Thus, the zonal field dynamo field is not restricted to hyper-fine scales such as $q \rho_i \sim 1$, etc. A further observation is that the zonal field dynamo mode are predicted to oscillate as well as grow, i.e. $\Omega_q$ is complex. A net chirality (i.e. finite spectrum averaged $k_\parallel B_0 k_\perp$) is required for $Re \Omega_q \neq 0$. 

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Since KSAWs (with finite $k_r$) will be refracted by zonal fields, the KSAW spectrum will necessarily be modified by the growth of the dynamo-generated fields. Thus, a self-consisted KSAW dynamo theory, which treats both the zonal field generation as well as their back-reaction on the KSAW’s, is called for. The effects of random refraction of KSAW’s by the generated fields can be described by the quasilinear wave kinetic equation

$$\frac{\partial \langle N_k \rangle}{\partial t} - \frac{\partial}{\partial k_r} \left( \frac{\partial \tilde{N}_k}{\partial x} \right) = - \gamma_k \langle N_k \rangle, \quad (10a)$$

which, using $\tilde{\omega}_k$ and $\tilde{N}_k$ as given by Eqns. (9a,9b), can be simplified to

$$\frac{\partial \langle N_k \rangle}{\partial t} - \frac{\partial}{\partial k_r} D_k \frac{\partial \langle N_k \rangle}{\partial k_r} = - \gamma_k \langle N_k \rangle, \quad (10b)$$

where:

$$D_k = \left( \frac{\omega_k k \theta}{k_z B_o} \right)^2 \sum q \sigma^2 |\delta B_{\theta, q}|^2 R(\Omega_q - q v_{gr}), \quad (10c)$$

Here, the irreversibility intrinsic to $D_k$ is provided by KSAW electron Landau damping via $\gamma_k$. Since zonal field frequencies are low (i.e. $\Omega_q \sim (\delta B / B_o)^2$), $D_k$ is effectively non-resonant in character. Note the effect of random refraction is to 'diffuse' $k_r$ to higher values. This refraction occurs as KSAW packets traverse the layered structures of zonal fields. Since wave dissipation is likely to be stronger at high $k_r$, this constitutes a route for feedback of the fields on the KSAW intensity, as well as its spectral distribution.

Having obtained the evolution equation for $\langle N_k \rangle$, we can now self-consistently describe the coupled evolution of the dynamo driven zonal field spectrum and the KSAWs. These may be described by coupled predator-prey type equations for $|\delta B_{\theta, q}|^2$ and $\langle N_k \rangle$, which are:

$$\frac{\partial}{\partial t} |\delta B_{\theta, q}|^2 = 2 \sigma^2 (\tau_q - \eta), \quad (11a)$$

and

$$\frac{\partial \langle N_k \rangle}{\partial t} = \frac{\partial}{\partial k_r} D_k \frac{\partial \langle N \rangle}{\partial k_r} - \gamma \langle N_k \rangle, \quad (11b)$$

where

$$\tau_q = -4 \pi \left( c_s^2 d_e^2 / v_{the} \right) \sum_k \left( \frac{1 + k^2 \rho_s^2}{2 + k^2 \rho_s^2} \right) k^2 \rho_s^2 \frac{\langle W_k \rangle}{\langle 1 + k^2 \rho_s^2 \rangle^{1/2}} \frac{\partial}{\partial k_r^2} \left( \frac{\omega}{k_r} \right). \quad (11c)$$

Here $\tau_q = \tau_q \langle \langle N \rangle \rangle$ is derived directly from Eqn. (10b) for $Im(\Omega_q)$. Note that Eqn. (11c) implies that a critical KSAW intensity level $\langle N \rangle_{crit} \sim \eta$ is required for zonal flow growth via modulational instability. For weak $\eta$, this implies that only a very modest level of KSAW excitation is required for the dynamo. Moreover, in regimes of weak dissipation, it suggests that the ‘marginal point’ for the KSAW dynamo will scale with $\eta$, despite the fact that $\eta$ is otherwise completely irrelevant to the dynamics of zonal field generation. Eqn. (11b) suggests that as zonal fields grow, high $k_r$ components in the KSAW spectrum will be generated, thus ultimately quenching KSAW energy via coupling to dissipation. Finally, it should be noted that the expression for $\tau_q$ given in Eqn. (11b) assumes $q \rho_s < 1$. For $q \rho_s \sim 1$, additional FLR factors enter which force $\tau_q$ to decay for $q \rho_s \geq 1$.

A moment’s consideration of Eqns. (10a,11a,11b,11c) naturally begs the question of what happens in the limit of $\eta \to 0$ since, in this case, there is no apparent control on zonal flow growth. We speculate here that as $\eta \to 0$, zonal fields may be subject to collisionless reconnection instabilities (i.e. tearing modes), which limit zonal field growth without requiring resistive dissipation. Such instabilities are analogous to Kelvin-Helmholtz type instabilities which may limit zonal flow growth in the zero collisionality limit $\xi_0$. Alternatively, the zonal fields may regulate themselves via feedback on $\partial \langle N_k \rangle / \partial k_r^2$ (i.e. by modifying the wave spectrum $\langle \Omega \rangle$) or by trapping of KSAW packets $\langle \Omega \rangle$. The detailed dynamics of these collisionless reconnection instabilities and of the system’s behavior for $\eta \to 0$ will be discussed in detail in a future publication.
In conclusion, we have demonstrated that a zonal field dynamo can be driven by collisionless kinetic Alfven wave turbulence. This dynamo is ‘fast’ in that the rate of requisite magnetic reconnection is determined by electron Landau damping and thus is independent of the collisional resistivity. Concomitant with this, non-zero $E_{||}$ (i.e. due to FLR or finite ion inertial layer width) is required for magnetic field amplification. The zonal field dynamo self-regulates via refraction-induced diffusion of the KSAW spectrum toward high $k_r$. These predictions should be amenable to investigation in laboratory experiments. Finally, it is interesting to note that for typical interstellar medium parameters ($T_e \approx 1$ ev, $B_o \approx 10^{-6}$ G, $L \sim 1$ pc, etc.) and taking $B/B_o \approx 10^{-3}$, Eqn. (9b) predicts that the magnetic field grows at a rate $\gamma_0 \approx 10^{-17} \text{sec}^{-1}$, consistent with that needed to achieve equipartition in $10^9$ years.

We thank L. Chen, G. Falkovich, T.S. Hahm, E.-J. Kim and M. Malkov for stimulating discussions. We also thank E.-J. Kim for a careful, critical reading of the manuscript. This research was supported by Department of Energy Grant No. FG-03-88ER53275. P.H. Diamond also acknowledges partial support from the National Science Foundation under Grant No. PHY99-07949, to the Institute for Theoretical Physics at U.C.S.B., where part of this work was performed.