Theory of Two Dimensional Mean Field Electron Magnetohydrodynamics

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Abstract

The theory of mean field electrodynamics for diffusive processes in Electron Magnetohydrodynamic (EMHD) model is presented. In contrast to Magnetohydrodynamics (MHD) the evolution of magnetic field here is governed by a nonlinear equation in the magnetic field variables. A detailed description of diffusive processes in two dimensions are presented in this paper. In particular, it has been shown analytically that the turbulent magnetic field diffusivity is suppressed from naive quasilinear estimates. It is shown that for complete whistlerization of the spectrum, the turbulent diffusivity vanishes. The question of whistlerization of the turbulent spectrum is investigated numerically, and a reasonable tendency towards whistlerization is observed. Numerical studies also show the suppression of magnetic field diffusivity in accordance with the analytical estimates.

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I. INTRODUCTION

The transport and amplification properties of a large scale magnetic field remains an area of active investigation. This is primarily due to its relevance in a variety of physical phenomena. For example, the existence of magnetic field in the universe is being understood on the basis of amplification process by some kind of dynamo mechanism. Another interesting phenomenon is the release of high energy bursts in solar flares etc. It is believed to occur as a result of the reconnection of magnetic fields, which can happen in the presence of finite diffusivity. However, there is only modest quantitative understanding of these processes. The amount of magnetic energy released by reconnection depends on the value of diffusivity, which turns out to be too small to provide an explanation of the vast energy released in these bursts. There have been attempts then to understand these phenomenon on the basis of turbulent magnetic field diffusivity, which is directly related to the question of transport of a large scale magnetic field in the presence of turbulence. Most theories put forward in these areas are cast within the Magnetohydrodynamic system. Lately, however, there has been some work which make use of models pertaining to faster time scales. It is on this regime that we are going to focus here.

In the present work we address the question of the diffusion of a long scale magnetic field in the presence of small scale turbulent magnetic fluctuation ocurring at time scales which are faster than the ion response time. For such phenomena the evolution of magnetic field is governed by the electron flow velocity. The ions being stationary, the flow velocity of the electrons determines the current and hence is thus directly related to the curl of magnetic field. Thus unlike MHD, in this approximation, heretofore referred as the Electron Magnetohydrodynamic (EMHD) approximation, the magnetic field itself evolves through an explicitly nonlinear equation. This should be contrasted to the MHD model in which the nonlinear effects creep indirectly through the lorentz force operating on the plasma flow.

The paper is organized as follows. In section II we present the salient features of the Electron Magnetohydrodynamics (EMHD) model. In section III we study the evolution

of mean magnetic field in two dimensions within the framework of EMHD description. In two dimensions there is no amplification of the large scale field, it can only diffuse. We obtain an expression for the effective diffusion coefficient and show that it is suppressed from the naive quasilinear estimates. For complete whistlerization, i.e. when the turbulence is comprised only of randomly interacting whistler waves (whistler modes being the normal modes of the EMHD model), we show that there is no turbulent contribution to diffusivity. This, then raises the pertinent question about the constituents of the turbulent state in this particular model. It becomes important to know whether the turbulent state comprises entirely of randomly interacting whistler waves or is it merely a collection of random eddies or is it that a combination of both whistlers and eddies which represent the true scenario? We address these question in section IV by numerically simulating the decaying turbulence for EMHD equations. The initial condition is chosen to be random, i.e. no whistlers to begin with. The study of final state reveals evidence of whistlerization. In section V we numerically investigate the problem of diffusion, which shows suppression of magnetic field diffusivity, essentially confirming our analytical findings of section III. Section VI contains the discussion and conclusion.

II. THE MODEL

Electron Magnetohydrodynamics (EMHD) is the theory of the motion of magnetized electron fluid in the presence of self consistent and external electric and magnetic fields. Such a theory is applicable when the time scales of interest are fast (e.g. lying between electron and ion gyrofrequencies) so that ions being massive and unmagnetized play a passive role as a neutralizing background, and the dominant role in dynamics is played by a strongly magnetized electron species. Phenomena having such time scales are often encountered in a number of plasma operated devices (e.g. switches, focusing devices, fast Z-pinches etc. [1]). Moreover, in the description of collisionless magnetic reconnection [2] as well as in certain problems related to ionosphere, the EMHD paradigm is invoked frequently. The

3

entire whistler physics is premised on the EMHD regime of dynamics.

The EMHD model is obtained by using the (i) electron momentum equation (ii) the current expressed in terms of electron velocity $\vec{J} = -n_e e v_e$ as the ions are stationary at fast time scales depicted by the model; and (iii) the Ampere's law, where displacement current is ignored under the assumption ($\omega << \omega_{pe}^2/\omega_{ce}$). The magnetic field then evolves through the following equation

$$\frac{\partial}{\partial t}(\nabla \times \vec{P}) = \nabla \times (\vec{v_e} \times (\nabla \times \vec{P})) - m_e \nu \nabla \times \vec{v_e} \tag{1}$$

Here m_e and $\vec{v_e}$ are the electron mass and the velocity respectively, $\vec{P}$ is the canonical momenta defined as $\vec{P} = m_e \vec{v_e} - e\vec{A}/c$ ($\vec{A}$ being the vector potential of the magnetic field), ν represents the electron ion collision frequency. Using the current and electron velocity relationship we obtain $\nabla \times \vec{P} = e(d_e^2 \nabla^2 \vec{B} - \vec{B})/c$; where $d_e = c/\omega_{pe}$ is the skin depth.

It is clear from Eq.1 that the $(d_e^2 \nabla^2 \vec{B} - \vec{B})$ is frozen in the electron fluid flow. In the limit when the electron inertia can be ignored, it is simply the magnetic field which is carried along with the electron fluid. Since $v_e \sim -\nabla \times \vec{B}$; the evolution equation for magnetic field is nonlinear in $\vec{B}$. This can be contrasted with the MHD model where the magnetic field evolution is governed by an equation which is intrinsically linear in $\vec{B}$. In MHD, the nonlinear effects then arise as a result of back reaction on the fluid flow through the Lorentz force terms. Basically, in EMHD $\vec{v_e} \sim -\nabla \times \vec{B}$, and so the flow is directly related to the instantaneous magnetic field; whereas in MHD the evolution of flow velocity $\vec{v}$ depends on magnetic field through the Lorentz force term and hence $\vec{v}$ has a memory of the past magnetic field configuration. The MHD model is applicable for scale lengths which are longer than the ion skin depth. EMHD on the other hand depicts phenomenon having scale lengths shorter than the ion skin depth. Another distinction from MHD arises from the presence of intrinsic scale, viz. the electron skin depth $d_e = c/\omega_{pe}$ in the EMHD model, which separates the two regimes one in which electron inertia is important and the other where the electron inertia plays no role. The character of the EMHD equation changes in these two disparate regimes of scale lengths.

In two dimensions (i.e. when the variations are confined in $x - y$ plane) Eq.1 can be simplified and cast in terms of two scalar variables ψ and b which define the total magnetic field by the expression $\vec{B} = \hat{z} \times \nabla \psi + b\hat{z}$. The following coupled set then represents the evolution of these scalar variables

$$\frac{\partial}{\partial t}(\psi - \nabla^2 \psi) + \hat{z} \times \nabla b \cdot \nabla(\psi - \nabla^2 \psi) = \eta \nabla^2 \psi \tag{2}$$

$$\frac{\partial}{\partial t}(b - \nabla^2 b) - \hat{z} \times \nabla b \cdot \nabla \nabla^2 b + \hat{z} \times \nabla \psi \cdot \nabla \nabla^2 \psi = \eta \nabla^2 b \tag{3}$$

Here we have chosen to normalize length by electron skin depth $d_e = c/\omega_{pe}$, magnetic field by a typical amplitude B_0 and time by the corresponding electron gyrofrequency. In the nonresistive limit the above coupled equations support the following quadratic invariants

$$E = \frac{1}{2} \int [(\nabla \psi)^2 + b^2 + (\nabla^2 \psi)^2 + (\nabla b)^2] dx dy$$

which represents the total energy (sum of the magnetic and the kinetic energy),

$$H = \int (\psi - \nabla^2 \psi)^2 dx dy$$

the mean square magnetic potential and

$$K = \int (\psi - \nabla^2 \psi)(b - \nabla^2 b) dx dy$$

the cross helicity. The fields b and ψ are chosen to be uncorrelated initially in our numerical simulations. On the basis of the existence of these quadratic invariants it can be infered that the mean square magnetic potential cascades towards longer scale. We will be making use of this later in our derivation for turbulent diffusivity.

Linearizing the evolution equations in the presence of uniform magnetic field B_0 pointing in the y direction leads to the following dispersion relation

$$\omega = \pm \frac{k k_y d_e^2 \omega_{ci}}{(1 + k^2 d_e^2)}$$

for whistlers, the normal mode of oscillations in the EMHD regime. It is clear form the dispersion relation that the propagation of these waves is preferentially parallel to the magnetic

field. Furthermore, the whistler wave excitation leads to the coupling of the form $b_k = \pm k\psi_k$ between the two perturbed fields. This relation between the perturbed fields then leads to an equipartition between the energy associated with the poloidal and the axial fields. An initial unequal distribution of energy in the poloidal and axial fields ultimately has a tendency towards redistribution and achieving equipartition as a result of the whistlerization of the spectrum. It is observed that time asymptotically the turbulent state in EMHD consists of a gas of whistlers interspersed with a collection of random eddies.

There has been considerable interest lately to understand features of EMHD turbulence both in two and three dimensions in terms of power spectra and the cascade properties of the square invariants supported by the model [3]. Our attempt here, however, is to understand the role of EMHD turbulence in determining the diffusion of long scale magnetic field.

III. SUPPRESSION OF TURBULENT MAGNETIC DIFFUSIVITY IN 2D

In this section we concentrate on the transport of magnetic field in two dimension. In 2D the magnetic field can only diffuse, thus our endeavour here is to estimate the effective magnetic diffusivity in the presence of turbulence.

We will concentrate here on turbulent scale lengths longer than the electron skin depth. In this regime of scale lengths i.e. for $kd_e << 1$ the electron inertia effects are unimportant and as mentioned in earlier section the magnetic field lines are frozen in the electron fluid flow. Thus turbulence in the electron velocity leads to the diffusion of magnetic flux. This diffusion of magnetic field lines, arising as a result of turbulence and not due to resistivity, is termed as the turbulent diffusivity of the magnetic field. The effective turbulent diffusivity would thus depend on the electron fluid flow velocity. A naive quasilinear estimate would thus predict that the magnetic field diffusivity $\beta \sim \tau v_e^2 \sim \tau(\nabla b)^2$, where τ is some averaged correlation time for the electron flow velocity $v_e = \hat{z} \times \nabla b$ in the $x - y$ plane, and b is the z component of the turbulent small scale magnetic field. This suggests that the magnetic field diffusion in the $x - y$ plane is solely determined by the turbulent properties of the

z (i.e. the axial) component of the magnetic field. However, this does not represent the complete picture. We will now show that the presence of small scale turbulence in the poloidal magnetic field results in the suppression of such estimates of diffusivity. This is similar to the work carried out by Gruzinov [4], Cattaneo [5] and others in the context of MHD. In MHD the magnetic field lines are tied to the plasma flow velocity. It is observed that the magnetic field diffusivity is suppressed from the quasilinear estimates given solely in terms of plasma flow velocity. The presence of small scale turbulence in the magnetic field, which opposes the fluid motion through the $\vec{J} \times \vec{B}$ backreaction is found to be responsible for such a suppression.

We choose to represent the small scale turbulence in the fields b and ψ as

$$b(x,t) = \sum_k b_k(t) exp(i\vec{k} \cdot \vec{r})$$

$$\psi(x,t) = \sum_k \psi_k(t) exp(i\vec{k} \cdot \vec{r})$$

In addition to this we assume the existence of a large scale magnetic field pointing along y direction characterized by the magnetic stream function of the following form

$$\psi_0 = \psi_q exp(iq_x x) + c.c$$

This magnetic field has a scale length $q^{-1} >> k^{-1}$ and hence when considering averaging over the scale of turbulence this field can be essentially treated as a constant in space. We are interested in understanding the process of diffusion of this long scale field in the presence of small scale turbulence in the variables b and ψ, i.e. we seek an equation of the kind

$$\frac{\partial \psi_q}{\partial t} = -\beta q_x^2 \psi_q \tag{4}$$

and are interested in determining β in terms of the properties of small scale turbulence. The qth fourier component of Eq.2 yields

$$(1 + q_x^2)\frac{d\psi_q}{dt} + < \hat{z} \times \nabla b \cdot \nabla(\psi - \nabla^2 \psi) >_q = -\eta q_x^2 \psi_q \tag{5}$$

The second term in the equation signifies the generation of qth mode as the result of nonlinear coupling between the high k turbulent fields. The angular brackets indicate the ensemble average. The above equation can be rewritten as

$$(1 + q_x^2)\frac{d\psi_q}{dt} + i\vec{q}\cdot < \hat{z} \times \nabla b(\psi - \nabla^2\psi) >_q = -\eta q_x^2 \psi_q$$

We denote $< \hat{z} \times \nabla b(\psi - \nabla^2\psi) >_q$ by $\vec{\Gamma}$ representing the nonlinear flux. Since $q_y = 0$, $i\vec{q}\cdot\vec{\Gamma} = iq_x\Gamma_x$. The suffix x stands for the x component. Now

$$\Gamma_x =< -\frac{\partial b}{\partial y}(\psi - \nabla^2\psi) >_q = -\sum_k ik_y(1 + k_1^2) < b_k\psi_{k_1} >$$

where $k_1 = q - k$.

To estimate the correlation $< b_k\psi_{k_1} >$ we make use of the quasilinear approximation where each of these fields gets generated from the other through the interaction with the large scale field. Thus we can write

$$< b_k\psi_{k_1} >=< b_k\delta\psi_{k_1} > + < \delta b_k\psi_{k_1} >,$$

where it is understood that $\delta\psi_{k_1}$ is the magnetic perturbation in the plane arising as the result of turbulent stretching of the mean magnetic field by the electron flow velocity $\hat{z} \times \vec{k}b_k$; and δb_k is the perturbation in the elecron flow (viz. $\hat{z} \times \vec{k}\delta b_k$) arising from the Lorentz force $\hat{z}k_1^2\psi_{k_1} \times \hat{y}q_x\psi_q$. It should be noted here that the first term corresponds to that derived from a kinematic treatment, wherein the response of magnetic field on flow is not considered. The second term takes account of the back reaction of the magnetic field on the electron velocity. Thus dropping the second term would be tantamount to a purely kinematic approximation. We will now show that the second term leads to a significant suppression of the estimates of diffusivity obtained purely from the kinematic treatment. The equations for δb_k and $\delta\psi_{k_1}$ are

$$(1 + k_1^2)(-i\omega_k + \delta\omega_k)\delta\psi_{k_1} = -\eta k_1^2\delta\psi_{k_1} - ik_y b_{-k}iq_x(1 + q^2)\psi_q$$

and

$$(1 + k^2)(-i\omega_k + \delta\omega_k)\delta b_k = -\eta k^2\delta b_k - ik_{y1}(k_1^2 - q^2)\psi_{-k_1}iq_x\psi_q$$

8

Here ω represents the linear frequency and $\delta\omega$ stands for the eddy decorrelation effect arising from the coherent mode coupling. Substituting the above expression for δb_k and $\delta\psi_{k_1}$ we obtain the following expression for the nonlinear flux

$$\Gamma_x = -\sum_k \left(\tau_k(k_y^2 \mid b_k \mid^2 -k_{1y}^2 k_1^2 \mid \psi_{k_1} \mid^2) \right) iq_x \psi_q \tag{6}$$

where

$$\tau_k = \frac{1}{(1+k^2)(-i\omega_k + \delta\omega_k) + \eta k^2}$$

Here τ_k represents the spectral correlation times for the turbulent fields. We have assumed that the turbulent scales are much longer compared to the electron skin depth (i.e. $k << 1$) in the above derivation. The evolution equation for ψ_q under the approximation $q << k << 1$ can then be written as

$$\frac{d\psi_q}{dt} = -q_x^2 \left[\sum_k \tau_k k_y^2 (\mid b_k \mid^2 -k^2 \mid \psi_k \mid^2) \right] \psi_q - \eta q_x^2 \psi_q \tag{7}$$

The factor inside the square bracket in the right hand side of the above equation represents the turbulent contribution to diffusivity. It is made up of two parts. The first part, depending on $k_y^2 \mid b_k \mid^2$ represents the kinematic contribution and the second part arises as the result of small scale turbulence in the poloidal component of magnetic field. It is clear that turbulence in the poloidal component of magnetic field contributes towards suppressing the magnetic field diffusivity. It should be noted here that for complete whistlerization, the spectral components of the two fields would be related as $b_k = \pm k\psi_k$, for which the turbulent diffusivity vanishes exactly. For this extreme case, diffusion of ψ_q is determined by resistivity alone. It appears then, that the understanding of the question of whistlerization of the spectrum in the turbulent state is of paramount importance. We will study this issue in the next section.

We rewrite Eq.7 as

$$\frac{d\psi_q}{dt} = -q_x^2 \sum_k \tau_k (< v_x^2 >_k -k^2 < \tilde{B}_x^2 >_k)\psi_q - \eta q_x^2 \psi_q$$

$$= -\frac{q_x^2}{2} \sum_k \tau_k (< v^2 >_k -k^2 < \tilde{B}^2 >_k)\psi_q - \eta q_x^2 \psi_q \tag{8}$$

In the above expression $\tilde{B}_x$ is the x component of the turbulent field. In writing the second equality we have assumed that the turbulence is isotropic. Thus we can write

$$\beta = \sum_k \frac{\tau_k}{2}(<v^2>_k -k^2 <(\nabla\psi)^2>_k) + \eta$$

The kinematic diffusivity β_0 would be just $\beta_0 = \sum_k \tau_k v_k^2/2 + \eta$, dependent on the turbulent velocity alone. We can then express β in terms of the kinematic diffusivity as $\beta = \beta_0 - \sum_k \tau_k k^2 <(\nabla\psi)^2>_k /2$. Following Gruzinov et al we assume an equivalence of correlation times (i.e. assume $\tau_k = \tau$ for each mode) and write $\beta = \beta_0 - \tau <k^2><(\nabla\psi)^2>/2$. To estimate $<(\nabla\psi)^2>$ we use the stationarity of the mean square magnetic potential. This can be justified on the basis of inverse cascade property of the mean square potential. At longer scales dissipation due to resistivity is small and the assumption of stationarity of the mean square potential is reasonably good. We multiply Eq.2 by ψ and take ensemble average. This yields

$$<\psi\frac{d\psi}{dt}> = \frac{1}{2} <\frac{d\psi^2}{dt}> = 0$$

$$<\psi\hat{z}\times\nabla b\cdot\nabla\psi> = \frac{1}{2}\nabla\cdot<\hat{z}\times\nabla b\psi^2> = 0$$

we thus obtain

$$\eta <(\nabla\psi)^2> = B_0 <\psi\frac{\partial b}{\partial y}> = \beta B_0^2$$

Substituting for $<(\nabla\psi)^2>$ and writing $\tau/2$ as $\beta_0/<v^2> = \beta_0/<(\nabla b)^2>$ we obtain

$$\beta = \frac{\beta_0}{1 + \frac{<k^2>\beta_0 B_0^2}{\eta<(\nabla b)^2>}} = \frac{\beta_0}{1 + R_m \frac{<k^2>B_0^2}{<v^2>}} \tag{9}$$

Here R_m is the magnetic Reynold's number. It is clear that for $R_m >> 1$ the suppression of the magnetic field diffusivity occurs even when the turbulent velocity is larger than the effective whistler speed in the presence of B_0, the magnetic field.

IV. WHISTLERIZATION

We have observed in the earlier section that for a turbulent state which is a collection of whistlers alone, the effective turbulent diffusivity goes to zero. Thus it is of significance

to understand the whistlerization of turbulent spectra. This is identical to studying the question of Alfvenization in the context of MHD model. It is interesting to note, however, that in the MHD model Alfvenization leads to an equipartition between the magnetic and the fluid energies. However, there can be no equipartition between the magnetic and kinetic energies as a result of the whistlerization of the spectrum. Thus, the dominance of magnetic or kinetic energies is dependent on whether the typical scale of turbulence are longer or shorter that the electron skin depth respectively. In this paper we have concentrated on the case where the turbulent scales are much longer compared to the electron skin depth. Thus the total energy is predominantly magnetic. Whistlerization of the spectrum then leads to an equipartition between the poloidal and the axial field energies.

We seek to understand the question of whistlerization by carrying out numerical simulation. We evolve the two field ψ and b by Eq.2 and Eq.3 respectively, using a fully de-aliased pseudospectral scheme. In this scheme the fields b and ψ are fourier decomposed. Each of the fourier modes are then evolved, linear part exactly, whereas the nonlinear terms are calculated in real space and then fourier transformed in k space. This requires going back and forth in real and k space at each time step. The Fast Fourier Transform (FFT) routines were used to go back and forth in the real and k space at each time integration. The time stepping is done using predictor corrector with the mid point leap frog scheme. The simulation was carried out with a resolution of $128X128$ modes as well as at a higher resolution of $256X256$ modes. The initial spectrum of the two fields b and ψ was chosen to be concentrated on a band of scales and their phases were taken to be random. The two fields were chosen to be entirely uncorrelated to begin with.

In Fig.1 we show a plot $\mid b_k \mid$ vs. $\mid k\psi_k \mid$ for the initial spectrum. It is clear from the figure that the initial spectrum is totally different from a spectrum whistler waves, which in turn would have shown up in the figure as a straight line passing through the origin with unit slope basically depicting the relationship $\mid b_k \mid = \mid k\psi_k \mid$ for whistlers. In Fig.2 and Fig.3 we plot for the evolved spectrum $\mid b_k \mid$ vs. $\mid k\psi_k \mid$ for $B_0 = 0$ and 0.5 respectively. It is clear that most of the points now cluster close to the origin. It is suggestive, when contrasted with the

initial condition of Fig.1 that the modes are trying to acquire whistler wave relationship. The scatter in the plot indicates that both eddies and whistlers constitute the final state. Thus a quantitative assessment of the turbulent state as regards whistlerization of the spectra is required. For this purpose we introduce a variable

$$w_k = \frac{abs(\mid b_k \mid^2 - \mid \psi_k \mid^2)}{(\mid b_k \mid^2 + \mid \psi_k \mid^2)} \tag{10}$$

which essentially indicates the fractional deviation of the k_{th} mode from being whistlerized. In Table I we list the fraction of modes in the spectrum for which w_k is within certain percentage.

TABLE - I

Permissible % deviation	Fraction of modes Whistlerized		
	Initial condition	Evolved state $B_0 = 0$	Evolved state $B_0 = 0.5$
2.5	0	0.028	0.031
5	0	0.053	0.054
7.5	0	0.077	0.080
10	0	0.101	0.102

It is clear from Table I that the initial state had zero fraction of modes having deviations, w_k even upto 10%, in the final state a reasonable fraction of modes acquire whistlerization within a certain percentage of deviation as measured by the parameter w_k. We also introduce an integral quantity signifying overall whislerization as $w = \int w_k dk / \int dk$. For a completely whistlerized spectrum the variable w would take a value of 0, and the maximum value that w can have is unity. For our initial spectrum $w = 0.9957$, after evolution (i)for $B_0 = 0$ (corresponding to Fig.1), $w = 0.5020$, and (ii) for $B_0 = 0.5$ (Fig.2) $w = 0.4912$. More detailed studies of this kind, addressing the evolution of whislerization with time (e.g. by studing how w evolves with time), its dependence on external magnetic field, etc. are being carried out presently and will be presented in a subsequent publication. The question of

Alfvenization of the spectrum in the context of MHD is also being pursued along similar lines and will be presented elsewhere.

It is clear from our studies that the whistlerization of the spectrum is not complete. Random eddies are also present in the evolved spectrum. This deviation from the whistler wave relationship contributes towards the residual effective turbulent diffusivity of the magnetic field lines. In the next section we will carry out a numerical study to determine the diffusivity of magnetic field in the presence of turbulence.

V. NUMERICAL RESULTS ON DIFFUSION

We saw in section III that the final expression of the effective diffusivity that we obtained was based on the fact that the effective correlation times of the interacting modes were ultimately the same for each of them. Whether this this really happens can only be verified by a fully nonlinear numerical simulation. We have carried out a set of numerical studies to investigate the question of magnetic diffusivity. We observe that the results of our investigation agrees with the expression that we have obtained earlier, thereby suggesting that the ansatz of local equivalence of correlation time is indeed correct.

The numerical scheme is the same as outlined in the last section. However, in addition to evolving the two fields b and ψ a number of tracer particles ($N = 1600$) were placed in the two dimensional spatial $x - y$ region of integration. The particles were initially placed uniformly in the $x - y$ plane, and were then evolved using the Lagrangian electron velocity at their location (viz. $\hat{z} \times \nabla b$). Since the magnetic field lines are tied to the electron flow velocity, the behaviour of magnetic field diffusivity can be discerned from the diffusion of these particles. Thus the averaged mean square displacement of these particles is used as a measure of magnetic diffusivity (e.g. $\beta = d < (\delta x)^2 > /dt$). This method of evaluating the tracer particle diffusivity to study the diffusion of magnetic fields in two dimension has been adopted by Cattaneo in the context of the MHD model [5].

It is clear that for $\eta \neq 0$ and an initial distribution of power with random phases in the

various modes for the two fields b and ψ, Eq.2 and Eq.3 represent the case of 'decaying' EMHD turbulence. We refrain from using a random stirring force to achieve stationary state as this might lead to the particle displacement being dependent on the characteristics of the random stirrer. We will here investigate the case of decaying turbulence and we will present results in the regime where the variations can be considered as slow, i.e. we treat the problem in the quasistatic limit.

The derivation of our main Eq.9 for the suppression of magnetic field diffusivity was premised on the notion of stationarity of the mean square magnetic potential. As discussed earlier the cascade of the mean square magnetic potential towards longer scales ensures attaining such a state. This can be clearly seen in Fig.4 which shows the evolution of mean square magnetic potential with time. It is clear that the percentage variation in $\int \psi^2 dx dy$ is small after $t = 200$. For the purpose of our calculations in all our numerical runs we have restricted to the region where the percentage variations in $\int \psi^2 dx dy$ is below 3%.

In Fig.5 we show the mean square displacement of the tracer particles with time. The thick line indicated by the label 'kinematic' essentially corresponds to the displacement when the uniform magnetic field in the y direction B_0 is chosen to be zero. We will designate the slope of this curve as β_{kin}, the kinematic diffusivity. The other two lines essentially correspond to the longitudinal and the transverse displacement in the presence of a uniform magnetic field $B_0 = 1$ along the y diection. It is clear from the figure that the slope of the kinematic curve is larger than the other two curves which correspond to the displacement for finite B_0. This clearly indicates that the presence of B_0 suppresses the diffusivity; the conclusion we arrived at earlier in the last section. However, longitudinal displacements of the tracer particles are larger compared to their transverse displacement, suggesting that the assumption of isotropic turbulence in not valid in the presence of uniform magnetic field. There has been indications in earlier works both in MHD [6] as well as in EMHD [7] that the presence of strong magnetic field results in anisotropy of the spectrum. Our results showing distinct values for the longitudinal and the transverse diffusivity is another evidence for anisotropic turbulence in the presence of external magnetic field.

14

We next investigate the question whether the supression of diffusivity with increasing magnetic field is indeed given by the kind of expression (Eq.9) that we have obtained in the earlier section. For this purpose we carry out several numerical runs with varying strength of the magnetic field. The diffusivity β for each case is then given by the slope of the displacement of the tracer particles. It is clear from Fig.5 that the curve is jagged, essentially signifying that β, the diffusivity estimated from the slope of such a curve is a statistical quantity. We take a time average given by

$$\beta(t_2 - t_1) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \beta(t)dt$$

The choice of $t_2 - t_1$ is such that the in this duration the turbulence can essentially be treated as quasistationary. The averaging procedure eliminates the statistical fluctuation in the estimate of diffusity and it is observed that with varying t_2 the slope asymptotes to a constant value for each case.

In Fig.6 the y axis represents β_{kin}/β and along the x axis we vary B_0^2. It is clear from the plot that the data points nicely fit a straight line, as our analytical expression predicts.

VI. DISCUSSION

There are two important results of our present work. First, we have been able to show that the turbulent EMHD state shows tendencies towards whistlerization. The spectrum is only partially whistlerized, suggesting that both eddies and randomly interacting whistlers constitute the turbulent state. Secondly, we have carried out studies to understand the diffusion of long scale magnetic field in the context of Electron Magnetohydrodynamics. We have shown that the effective diffusivity due to turbulence in the electron flow velocity gets suppressed in the presence of small scale turbulence of the magnetic field. For complete whistlerization the turbulent diffusivity vanishes. However, since the turbulent state is only partially whistlerized the effective diffusivity does not vanish it only gets suppressed from pure kinematic estimates. We have confirmed these results numerically.

15

The problem of diffusion of magnetic field lines is of great interest, as it provides a mechanism for the reconnection of magnetic field lines which is thought to underlie an understanding of the rapid release of energy in several solar and astrophysical contexts. The resistive diffusion turns out to be too small to explain the large amount of energy released. This had initiated efforts towards understanding the phenomenon of turbulent diffusivity of magnetic field lines. Earlier attempts on this were based on the Magnetohydrodynamic approximation. However, it was shown theoretically by Gruzinov et al [4] and numerically by Cattaneo [5] that the value of turbulent diffusivity gets suppressed in the presence of turbulence in small scale magnetic field. Recently, attempts to understand the reconnection phenomenon in the context of Electron Magnetohydrodynamics are being made [2]. Our work in this context becomes relevant, as we have shown here that the naive quasilinear estimates do not provide a complete picture. The effective diffusivity gets suppressed in the presence of turbulence in the magnetic field, with whistlerization of the spectrum playing an important role in this regard.

Other issue that we would like to point out in this regard is the role of whistlers in EMHD turbulence. Some recent studies on EMHD turbulence categorically rule out the presence of whistler effect in determining the energy tranfer rate on the basis of the numerically observed scaling of the power spectrum [3]. We have, on the other hand shown here that there is a tendency towards whistlerization of the turbulent spectra and that directly influences the effective diffusivity of the magnetic field lines. Invoking the Prandtl mixing length argument, which relates the transfer rate to the effective diffusivity, the question of whistler effect being present or not remains debatable. Moreover, we also have evidence of anisotropization of the turbulent spectrum in the presence of external magnetic field (this work will be presented elsewhere) which further points towards a subtle role of whistlers in governing the EMHD turbulence.

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FIGURE CAPTION

Figure 1 Plot of $\mid b_k \mid$ vs. $\mid k\psi_k \mid$ for the initial spectrum.

Figure 2 Plot of $\mid b_k \mid$ vs. $\mid k\psi_k \mid$ for the evolved spectrum when the external field $B_0 = 0$.

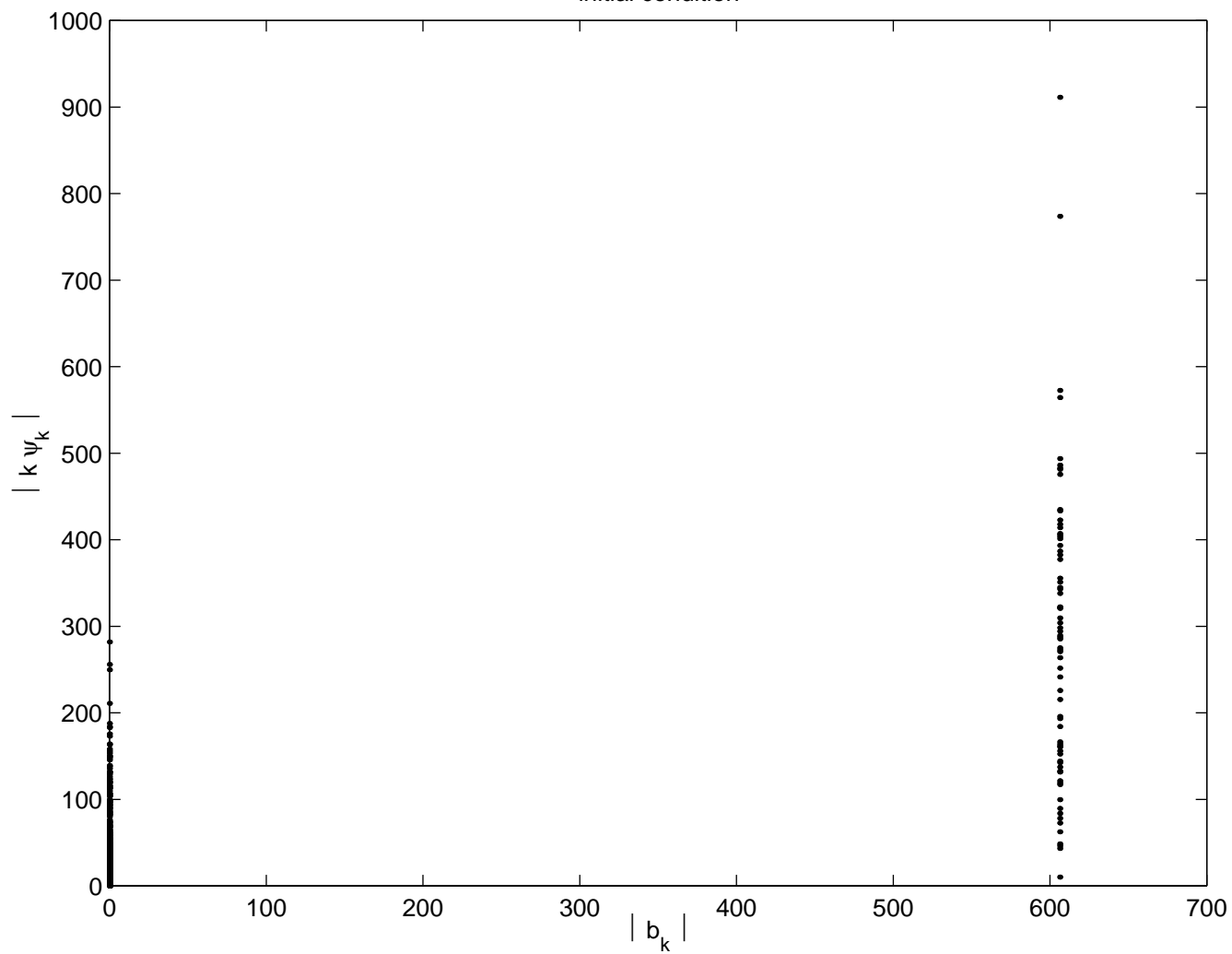Figure 3 Plot of $\mid b_k \mid$ vs. $\mid k\psi_k \mid$ for the evolved spectrum when the external field $B_0 = 0.5$.

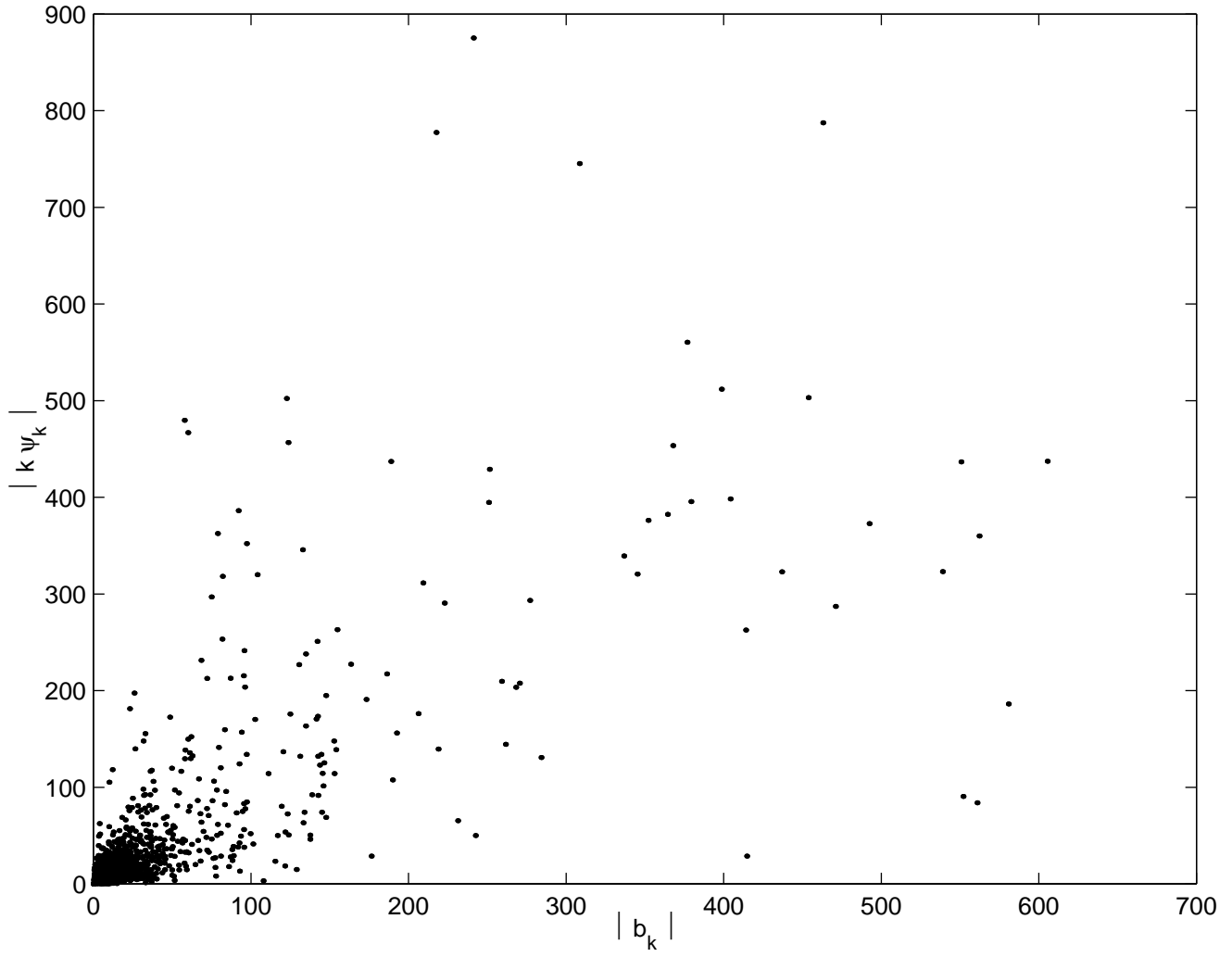Figure 4 Evolution of mean square magnetic potential.

Figure 5 Mean square displacement of the tracer particles with time is shown, thick lines (kinematic) shows the displacement in the absence of any external field. The other two lines indicated by 'longitudinal' and the 'transverse' show the mean square displacement of the tracer particles along and across the external magnetic field $B_0 = 1$.

Figure 6 A plot of β_{kin}/β vs. B_0^2.

Final State for $B_0 = 0.5$