The Radiative Equilibrium of Extended Stellar Atmospheres.

GENTLEMEN,—

In a recent paper (*M. N.* **94.** 444, 1934, referred to as I) the standard problems in the theory of radiative equilibrium which arise in the study of stellar atmospheres have been rediscussed without neglecting the curvature of the outer layers of the star. Thus for an infinite stellar atmosphere the solution for the Schwarzschild problem was found to be (equation 51)

$$B=\tfrac{3}{4}F_0\int_0^\tau r^{-2}\,d\tau. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

There are similar solutions for the other problems of the stellar atmospheres, and in fact even for the Schwarzschild problem there exists another form for the solution derived on the basis of a different method of averaging (*cf.* (I), §§ 6, 7, 8). But the solution of the Schwarzschild problem in the form (1) has been derived independently by N. A. Kosirev in a paper immediately preceding the author's (*M. N.* **94.** 430, 1934). However, Kosirev has gone very much further than the author in working out the consequences of (1) and derives quite unexpected results regarding the colour distribution in the emergent radiation and attempts in this way to remove the well-known temperature anomalies in the P Cygni and Wolf-Rayet stars. Kosirev's results are obviously very important and are so surprising that a full discussion of the radiative equilibrium of extended atmospheres would be well worth undertaking. In this letter, however, I only wish to draw attention to one particular result in the theory of the extended stellar atmospheres, which incidentally throws some light on Kosirev's results.

Now if $\kappa\rho \propto r^{-n}$ $(n>1)$, then from equations (53), (57) and (58) of my paper one easily derives that

$$I(\lambda)=2\pi\int_0^\infty I(\lambda, p)p\,dp,$$

$$=\pi R_{\frac{1}{2}}^{1}\int_0^\infty B(\lambda, T)\Phi(\tau)\tau^{-2/(n-1)}\,d\tau, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where

$$\Phi(\tau)=2\int_0^\pi [\exp.-(n-1)\tau\,\mathrm{coesc}^{n-1}\theta\int_0^\theta \sin^{n-2}\theta\,d\theta]\,.$$
$$\sin\theta\,d\theta, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and R_1 is the radius of the configuration at the point where $\tau=1$. Further if T_1 is the temperature at this point, then one easily verifies from I (53) and (54) that

$$T=T_1\tau^{(n+1)/4(n-1)}.$$

Hence by (2) we can now rewrite for $I(\lambda)$

$$I(\lambda)=\pi R_1^2 C\lambda^{-5}E(\lambda, T_1), \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$E(\lambda, T_1)=\int_0^\infty [\exp. c_2(\lambda T_1\tau^{(n+1)/4(n-1)})^{-1}-1]^{-1}\Phi(\tau)\tau^{-2/(n-1)}d\tau \quad . \quad . \quad . \quad . \quad . \quad (5)$$

With $n=3/2$, equations (3), (4), (5) go over into Kosirev's equations (22), (23), and (24). But when $n=1$ we obtain by a similar calculation

$$I(\lambda) \propto \lambda^{-1}\int_0^\pi e^{-\alpha\int_0^\theta \operatorname{cosec}\theta\, d\theta} \sin\theta\, d\theta\equiv 0,$$

which means *that the intensity of the emergent radiation from an extended stellar atmosphere in which* $\kappa\rho$ *varies inversely as* r *is identically zero in all wave-lengths*. But at the same time in the *spectral decomposition* the variation of intensity with wave-length follows the law λ^{-1}, and hence we can have arbitrarily high colour temperatures at sufficiently small wave-lengths. The above is precisely equivalent to Kosirev's general result which he obtained by discussing the case $n=3/2$.

It should clearly be of interest to have a complete discussion of the equations (3), (4), (5) for different values of n. It would be sufficient to study the functions $E(\lambda, T_1)$ for one particular temperature as we can always deduce the results for a different temperature as λ and T_1 occur in $E(\lambda, T_1)$ only as the product λT_1. Such studies are being undertaken by the writer, but it may be of interest now to draw attention to the fact that when $n=3$

$$\Phi(\tau)=4Ei_2(\tau). \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Again, if following Kosirev we define the "effective temperature" by

$$I=\pi R_1^2\sigma T_{eff}^4, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where I is the integrated radiation of the star and R_1 as defined previously, then we find that

$$T_{eff}=\sqrt[4]{A}\,T_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where

$$A=2\int_0^{\pi} \sin^{2n-1}\theta \,.\, (\psi(\theta))^{-2}\, d\theta, \quad . \quad . \quad . \quad . \quad (9)$$

and

$$\psi(\theta)=(n-1)\int_0^{\theta} \sin^{n-2}\theta\, d\theta. \quad . \quad . \quad . \quad . \quad (10)$$

When $n=3$, we find that $A=4/3$.

I am, Gentlemen,

Yours faithfully,

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