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A superspace formulation of the stochastic quantisation method

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Abstract. A superspace formulation of the stochastic quantisation method of Parisi and Wu is presented for the scalar and gauge field theories.

1. Introduction

In recent years close connections have been established between Euclidean field theories and stochastic processes. It has been shown (Parisi and Wu 1981) that the n-point functions

$$\langle\varphi(x_1)\varphi(x_2)\ldots\varphi(x_n)\rangle=\int \mathscr{D}\varphi\, \varphi(x_1)\ldots\varphi(x_n)\exp(-S[\varphi]) \tag{1.1}$$

of Euclidean field theories are the same as the steady state limit of the equal time correlation functions $\langle\varphi(x_1, t)\ldots\varphi(x_n, t)\rangle_\eta$ computed from the Langevin equation

$$\partial\varphi(x, t)/\partial t=-\delta S[\varphi]/\delta\varphi(x, t)+\eta(x, t) \tag{1.2}$$

where $\eta(x, t)$ is a Gaussian stochastic process with

$$\langle\eta(x, t)\rangle=0, \qquad \langle\eta(x, t)\eta(x', t')\rangle=2\delta(x-x')\delta(t-t'). \tag{1.3}$$

This equivalence between D-dimensional Euclidean field theories and the steady state of $(D+1)$-dimensional stochastic processes has led to what is known as the stochastic quantisation method of Parisi and Wu. This method acquires particular significance in the case of gauge theories where it allows one to circumvent the problems associated with gauge fixing. It has also proved to be extremely useful for numerical simulation of field theories. The equivalence between Euclidean field theories and stochastic processes is most easily accomplished through the well established equivalence between the Langevin equation and the Fokker–Planck equation. A proof based on functional techniques has been given by Nakano (1983). Another proof using rearrangement of perturbation theory terms has been given by Nakazato *et al* (1983).

In a related development it has been shown (McClain *et al* 1983) that certain D-dimensional stochastic processes of the type

$$\delta S[\varphi]/\delta\varphi=\eta(x) \tag{1.4}$$

are completely equivalent to a $(D-2)$-dimensional Euclidean field theory. This equivalence is established first by formulating the stochastic processes in a $(D+2)$-

dimensional superspace by adding two anticommuting dimensions and then exploiting the supersymmetry of the theory thus obtained so as to effect the desired dimensional reduction. In this paper we present a similar superspace formulation of the stochastic theory described by the Langevin equation (1.2). The superspace formalism discussed below permits an easy proof of equivalence of correlation functions of the steady state theory appropriate to the Langevin equation (1.2) and the n-point function (1.1) of the Euclidean field theory.

This paper is organised as follows. In § 2 we present a superfield formulation of the Langevin equation for the scalar case. In § 3 we discuss the perturbation theory for the superfield formulation of § 2 and establish a connection with the work of Nakazato *et al.* In §§ 4 and 5 we deal with gauge theories. We conclude in § 6 with some comments and discussion of this work.

2. Superfield formulation

The steady state generating functional corresponding to the Langevin equation (1.1) may be written as

$$Z[J]=\int \mathscr{D}\eta \exp\left(\int \mathrm{d}x\, \mathrm{d}t[-\tfrac{1}{4}\eta^2(x,t)+\mathrm{i}J(x,t)\varphi_\eta(x,t)]\right). \tag{2.1}$$

Here $\varphi_\eta(x, t)$ is a solution of the Langevin equation (1.1) with initial values specified at $t=-\infty$. We may write (2.1) alternatively as

$$Z[J]=\int \mathscr{D}\eta\, \mathscr{D}\varphi\, \mathrm{Det}\left(\frac{\partial}{\partial t}+\frac{\delta^2 S}{\delta\varphi^2}\right)\delta\left(\frac{\partial\varphi}{\partial t}+\frac{\delta S}{\delta\varphi}-\eta\right)$$
$$\times\exp\left(\int \mathrm{d}x\, \mathrm{d}t[-\tfrac{1}{4}\eta^2(x,t)+\mathrm{i}J(x,t)\varphi(x,t)]\right). \tag{2.2}$$

We now introduce the following functional representations for the delta functional and the determinant which appear on the right-hand side of (2.2):

$$\delta\left(\frac{\partial\varphi}{\partial t}+\frac{\delta S}{\delta\varphi}-\eta\right)=\int \mathscr{D}\omega \exp\left[-\mathrm{i}\int \mathrm{d}x\, \mathrm{d}t\, \omega\left(\frac{\partial\varphi}{\partial t}+\frac{\delta S}{\delta\varphi}-\eta\right)\right], \tag{2.3}$$

$$\mathrm{Det}\, M=\int \mathscr{D}\bar\psi\, \mathscr{D}\psi \exp\left(-\int \mathrm{d}x\, \mathrm{d}t\, \bar\psi M\psi\right). \tag{2.4}$$

In (2.4) ψ and $\bar\psi$ are anticommuting fields. Integrations over η are now performed to give

$$Z[J]=\int \mathscr{D}\varphi\, \mathscr{D}\bar\psi\, \mathscr{D}\psi\, \mathscr{D}\omega \exp\left\{\int \mathrm{d}x\, \mathrm{d}t\left[-\omega^2+\mathrm{i}\omega\left(\frac{\partial\varphi}{\partial t}+\frac{\delta S}{\delta\varphi}\right)-\bar\psi\left(\frac{\partial}{\partial t}+\frac{\delta^2 S}{\delta\varphi^2}\right)\psi+\mathrm{i}J\varphi\right]\right\}. \tag{2.5}$$

Following Parisi and Sourlas (1979) and McClain *et al* (1983) we add to the original D dimensions two anticommuting dimensions α and $\bar\alpha$ with the properties

$$\alpha^2=\bar\alpha^2=\{\alpha,\bar\alpha\}=0, \tag{2.6}$$

$$\int \mathrm{d}\bar{\alpha}\, \mathrm{d}\alpha \begin{pmatrix} 1 \\ \alpha \\ \bar{\alpha} \\ \alpha\bar{\alpha} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \tag{2.7}$$

and introduce a superfield

$$\Phi = \varphi + \psi\bar{\alpha} + \bar{\psi}\alpha + \alpha\bar{\alpha}\mathrm{i}\omega. \tag{2.8}$$

It is easily checked that $Z(J)$ can be written in terms of the superfield Φ as

$$Z[J] = \int \mathscr{D}\Phi \exp\left\{\int \mathrm{d}\boldsymbol{x}\, \mathrm{d}t\, \mathrm{d}\bar{\alpha}\, \mathrm{d}\alpha \left[\Phi\left(\frac{\partial^2}{\partial\bar{\alpha}\,\partial\alpha} + \alpha\frac{\partial}{\partial\alpha}\frac{\partial}{\partial t}\right)\Phi + \mathrm{i}\mathscr{J}\Phi\right] - S[\Phi]\right\} \tag{2.9}$$

where

$$\mathscr{J} = J(\boldsymbol{x}, t)\delta(\alpha)\delta(\bar{\alpha}). \tag{2.10}$$

We separate the quadratic terms in $S[\Phi]$ as

$$S[\Phi] = \int \mathrm{d}\boldsymbol{x}\, \mathrm{d}t\, \mathrm{d}\bar{\alpha}\, \mathrm{d}\alpha\, [\tfrac{1}{2}\Phi(-\Box + m^2)\Phi + V(\Phi)] \tag{2.11}$$

and to make a contact with Nakazato *et al* (1983), we add a total time derivative $-\frac{1}{2}\Phi(\partial/\partial t)\Phi$ to the exponent in (2.9). We thus obtain

$$Z[\mathscr{J}] = \int \mathscr{D}\Phi \exp\left(\int \mathrm{d}\boldsymbol{x}\, \mathrm{d}t\, \mathrm{d}\bar{\alpha}\, \mathrm{d}\alpha(-\tfrac{1}{2}\Phi K\Phi - V(\Phi) + \mathrm{i}\mathscr{J}\Phi)\right) \tag{2.12}$$

where

$$K = \left((-\Box + m^2) - 2\frac{\partial^2}{\partial\bar{\alpha}\,\partial\alpha} - 2\alpha\frac{\partial}{\partial\alpha}\frac{\partial}{\partial t} + \frac{\partial}{\partial t}\right). \tag{2.13}$$

Restricting $J(x, t)$ to the form

$$J(\boldsymbol{x}, t) = j(\boldsymbol{x})\delta(t) \tag{2.14}$$

$Z[\mathscr{J}]$ in (2.12) generates the steady state equal time correlation functions corresponding to the Langevin equation.

3. Superfield perturbation theory

The propagator for the free superfield Φ satisfies the differential equation

$$\left((-\Box + m^2) - 2\frac{\partial^2}{\partial\bar{\alpha}\,\partial\alpha} - 2\alpha\frac{\partial}{\partial\alpha}\frac{\partial}{\partial t} + \frac{\partial}{\partial t}\right) D(\boldsymbol{x}, t, \alpha, \bar{\alpha}; \boldsymbol{x}', t', \alpha', \bar{\alpha}')$$
$$= \delta(\boldsymbol{x} - \boldsymbol{x}')\delta(t - t')\delta(\alpha - \alpha')\delta(\bar{\alpha} - \bar{\alpha}'). \tag{3.1}$$

In terms of the Fourier transform of D with respect to $\boldsymbol{x}$ and $\boldsymbol{x}'$

$$D(\boldsymbol{x}, t, \alpha, \bar{\alpha}; \boldsymbol{x}', t', \alpha', \bar{\alpha}')$$
$$= (2\pi)^{-4} \int \mathrm{d}\boldsymbol{k}\, \mathrm{d}\boldsymbol{k}' \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x} + \mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{x}')\tilde{D}(\boldsymbol{k}, t, \alpha, \bar{\alpha}; \boldsymbol{k}', t', \alpha', \bar{\alpha}'). \tag{3.2}$$

The solution of (3.1) may be written as

$$\tilde{D}(\boldsymbol{k}, t, \alpha, \bar{\alpha}; \boldsymbol{k}', t', \alpha', \bar{\alpha}')$$
$$= \frac{\delta(\boldsymbol{k}+\boldsymbol{k}')}{(k^2+m^2)} \exp[-(k^2+m^2)|t-t'+\alpha(\bar{\alpha}-\bar{\alpha}')\theta(t'-t)+\alpha'(\bar{\alpha}-\bar{\alpha}')\theta(t-t')|]. \tag{3.3}$$

The perturbation theory diagrams for $Z[\mathscr{J}]$ in terms of the superfield Φ are in a one-to-one correspondence with those of the Euclidean field theory. This superspace formulation bypasses the need for the rearrangement done by Nakazato *et al* (1983) of the stochastic perturbations theory based on the Langevin equation (1.2). Comparing our propagator (3.3) with the corresponding expression of Nakazato *et al*, we find that the propagator (3.3) has an extra term $[\alpha'\bar{\alpha}\theta(t-t')-\alpha\bar{\alpha}'\theta(t'-t)]$ appearing inside the modulus sign in the exponent of (3.3). However, as long as one restricts oneself to the Green function of the φ field alone—a restriction enforced by the choice of $\mathscr{J}$ as in (2.10)—the above mentioned extra terms do not contribute. This can be seen as follows. Apart from energy and momentum conserving δ functions for each internal vertex and integrations over loop momenta, the contribution of an arbitrary diagram Γ to the Green function $\langle\varphi(\boldsymbol{x}_1, t_1)\varphi(\boldsymbol{x}_2, t_2)\ldots\varphi(\boldsymbol{x}_n, t_n)\rangle_0$ is given by

$$\int \prod_{\text{ext vertices}} \mathrm{d}\bar{\alpha}_k\, \mathrm{d}\alpha_k\, \delta(\alpha_k)\delta(\bar{\alpha}_k) \prod_{\text{int vertices}} \mathrm{d}t\, \mathrm{d}\bar{\alpha}_l\, \mathrm{d}\alpha_l \prod_{\text{lines of }\Gamma} \tilde{D}_{ij}. \tag{3.4}$$

The propagator $\tilde{D}_{ij}$ for a line joining two vertices i and j and carrying momentum $\boldsymbol{k}$ can be split as

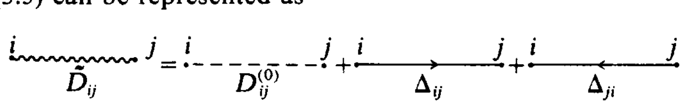
$$\tilde{D}_{ij}(t_i, \alpha_i, \bar{\alpha}_i; t_j, \alpha_j, \bar{\alpha}_j; k^2)$$
$$= a^{-1}\exp(-a|t_i-t_j|)[1+a\alpha_i\bar{\alpha}_i\theta(t_j-t_i)+\alpha_j\bar{\alpha}_j\theta(t_i-t_j)$$
$$-a\alpha_i\bar{\alpha}_j\theta(t_j-t_i)-a\alpha_j\bar{\alpha}_i\theta(t_i-t_j)]$$
$$\equiv D_{ij}^{(0)}+\Delta_{ij}+\Delta_{ji} \tag{3.5}$$

where

$$D_{ij}^{(0)} = a^{-1}\exp(-a|t_i-t_j|)[1+a\alpha_i\bar{\alpha}_i\theta(t_j-t_i)+a\alpha_j\bar{\alpha}_j\theta(t_i-t_j)], \tag{3.6}$$

$$\Delta_{ij} = \exp(-a|t_i-t_j|)\alpha_i\bar{\alpha}_j\theta(t_j-t_i), \qquad a=(k^2+m^2). \tag{3.7}$$

Graphically (3.5) can be represented as

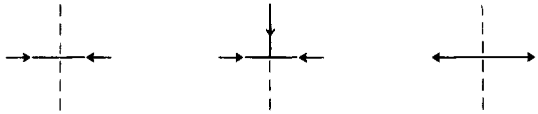


Substituting the right-hand side of (3.5) for the propagators in (3.4) and expanding the product, we get a sum of terms each of which is the contribution of a diagram obtained by replacing the original D_{ij} line by the D_{ij}^{0}, Δ_{ij} and Δ_{ji} lines in all possible ways. We now note the following.

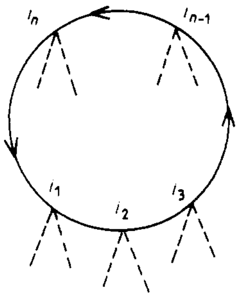
(i) Graphs in which a Δ line ends at an external vertex contribute zero because of the presence of terms $\delta(\alpha_k)\delta(\bar{\alpha}_k)$ for the external vertices.

(ii) Graphs in which a single Δ line ends at an internal vertex also do not contribute because of relations (2.7).

(iii) The contribution of graphs in which two or more Δ lines point towards or away from a vertex also vanishes because $\alpha^2=\bar{\alpha}^2=0$. Thus, for example, all graphs having an internal vertex of one of the following types do not contribute:



(iv) The only other graphs with Δ lines which may give non-zero contribution are those having a closed loop of Δ lines. These diagrams must have the form



where $D^{(0)}$ lines may be joined to other parts of the diagram. Let $i_1 \ldots i_n$ be the vertices on a loop of Δ lines. The contribution of this diagram is proportional to

$$\theta(t_{i_2}-t_{i_1})\theta(t_{i_3}-t_{i_2})\ldots\theta(t_{i_{n-1}}-t_{i_n})\theta(t_{i_n}-t_{i_1})$$

and hence vanishes.

Thus we have proved that as far as the n-point functions of φ alone are concerned we may replace $\tilde{D}$ by $D^{(0)}$, thereby proving that the superfield perturbation theory gives for n-point functions, expressions which are identical with expressions obtained by Nakazato *et al* (1983) by rearrangement of stochastic diagrams generated by the Langevin equation (1.2).

4. Stochastic quantisation of gauge theories

For non-abelian gauge theories the Euclidean action is

$$S_{\mathrm{Cl}} = \tfrac{1}{4}\int \mathrm{d}\boldsymbol{x}\, F_{\mu\nu}^{\ a} F_{\mu\nu}^{\ a} \tag{4.1}$$

with

$$F_{\mu\nu}^{\ a} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc} A_\mu^b A_\nu^c. \tag{4.2}$$

It has been demonstrated by Parisi and Wu that in the stochastic quantisation method for gauge theories there is no need for any gauge fixing procedure and that the gauge invariant quantities can be obtained by computing the steady state limit of the corresponding equal time correlation functions from the Langevin equation

$$\partial A_\mu^a(\boldsymbol{x}, t)/\partial t = -\delta S_{\mathrm{Cl}}/\delta A_\mu^a(\boldsymbol{x}, t) + \eta_\mu^a(\boldsymbol{x}, t). \tag{4.3}$$

Here $\eta_\mu^a(\boldsymbol{x}, t)$ are Gaussian noise sources with

$$\langle \eta_\mu^a(\boldsymbol{x}, t)\rangle = 0, \qquad \langle \eta_\mu^a(\boldsymbol{x}, t)\eta_\nu^b(y, t')\rangle = 2\delta_{\mu\nu}\delta^{ab}\delta(\boldsymbol{x}-y)\delta(t-t'). \tag{4.4}$$

Note that, owing to the gauge symmetry of S_{cl}, the 'free part' of the Langevin equation for the longitudinal component

$$\partial A_\mu^{a\mathrm{L}}(\boldsymbol{x}, t)/\partial t = \eta_\mu^{a\mathrm{L}}(\boldsymbol{x}, t) \tag{4.5}$$

has no damping term and as a consequence quantities such as $\langle A_\mu^{a\mathrm{L}}(\boldsymbol{x}, t) A_\nu^{b\mathrm{L}}(\boldsymbol{y}, t)\rangle$ diverge as $t \to \infty$. Thus, while the steady limit of gauge invariant quantities is well defined, it does not exist for gauge non-invariant quantities. One of the ways of avoiding this undesirable feature is to damp out the longitudinal modes by suitably modifying the Langevin equation (4.3). A suggestion due to Nakano (1983) consists in adding a term $-\lambda A_\mu^a$ to the RHS of (4.3). This modification alters the steady state distribution and thereby makes the gauge invariant quantities dependent on λ. The correct results for the gauge invariant quantities are then recovered in the $\lambda \to 0$ limit. Another method due to Nakagoshi *et al* (1983), based on the work of Baulieu and Zwanziger (1981), is to start with the following Langevin equation,

$$\frac{\partial}{\partial t} A_\mu^a(\boldsymbol{x}, t) = -\frac{\delta S_{\mathrm{cl}}}{\delta A_\mu^a(\boldsymbol{x}, t)} + \lambda^{-1} D_\mu^{ab}(\partial_\nu A_\nu^b) + \eta_\mu^a(\boldsymbol{x}, t), \tag{4.6}$$

where D_μ^{ab} is the covariant derivative

$$D_\mu^{ab} = \delta^{ab}\partial_\mu - g f^{abc} A_\mu^c . \tag{4.7}$$

The structure of the added damping term ensures that the steady state distribution of the gauge invariant sector remains independent of λ. Further, this modification has the advantage of being interpretable in terms of the gauge fixing procedure and the associated Faddeev–Popov ghosts in the conventional field theory.

5. Superspace formulation of the stochastic quantisation for gauge theories

We now present a superspace formulation of (4.6) closely following the scalar case.

The steady generating functional corresponding to the Langevin equation (4.6) may be written as

$$Z[J] = \int \mathscr{D}\eta \exp\left(\int \mathrm{d}\boldsymbol{x}\, \mathrm{d}t[-\tfrac{1}{4}\eta_\mu^a(\boldsymbol{x}, t)\eta_\mu^a(\boldsymbol{x}, t) + \mathrm{i}J_\mu^a(\boldsymbol{x}, t) A_{\mu\eta}^a(\boldsymbol{x}, t)]\right) \tag{5.1}$$

where $A_{\mu\eta}^a$ is a solution of the Langevin equation with initial value specified at $t = -\infty$. The above expression for $Z[J]$ may be written as

$$Z[J] = \int \mathscr{D}\eta\, \mathscr{D}A\, \mathrm{Det}\, M\, \delta\left(\frac{\partial A_\mu^a}{\partial t} + \frac{\delta S_{\mathrm{cl}}}{\delta A_\mu^a} - \lambda^{-1} D_\mu^{ab}(\partial_\nu A_\nu^b) - \eta_\mu^a\right)$$
$$\times \exp\left(\int \mathrm{d}\boldsymbol{x}\, \mathrm{d}t(-\tfrac{1}{4}\eta_\mu^a\eta_\mu^a + \mathrm{i}J_\mu^a A_\mu^a)\right) \tag{5.2}$$

where

$$M = \delta_{\mu\rho}\,\delta^{ar}\frac{\partial}{\partial t} + \frac{\delta^2 S_{\mathrm{cl}}}{\delta A_\rho^r\, \delta A_\mu^a} - \lambda^{-1}\frac{\delta}{\delta A_\rho^r} D_\mu^{ab}(\partial_\nu A_\nu^b). \tag{5.3}$$

We now introduce functional integral representations for the δ function and for Det M

with the help of fields ω_μ^a, ψ_μ^a, $\bar{\psi}_\mu^a$ and integrate over η_μ^a to get

$$Z[J]=\int \mathscr{D}A\,\mathscr{D}\omega\,\mathscr{D}\psi\,\mathscr{D}\bar{\psi}\,\exp\bigg\{\int \mathrm{d}\boldsymbol{x}\,\mathrm{d}t\bigg[-\omega_\mu\cdot\omega_\mu-\mathrm{i}\omega_\mu\cdot\bigg(\frac{\partial A_\mu}{\partial t}+\frac{\delta S_{\mathrm{cl}}}{\delta A_\mu}\bigg)$$
$$+\frac{\mathrm{i}}{\lambda}\,\omega_\mu\cdot D_\mu(\partial_\nu A_\nu)-\bar{\psi}_\mu^a\bigg(\frac{\partial}{\partial t}\,\delta^{ab}+\frac{\delta^2 S_{\mathrm{cl}}}{\delta A_\mu^a\delta A_\nu^b}\bigg)\psi_\nu^b$$
$$+\lambda^{-1}(\bar{\psi}_\mu\cdot\partial_\mu\,\partial_\nu\psi+g(\partial_\nu\bar{\psi}_\nu)\cdot\psi_\mu\times A_\mu-g\bar{\psi}_\mu\cdot\psi_\mu\times\partial_\nu A_\nu+\mathrm{i}J_\mu\cdot A_\mu)\bigg]\bigg\}. \tag{5.4}$$

We introduce a superfield $\mathscr{A}_\mu^a$ thus:

$$\mathscr{A}_\mu^a=A_\mu^a+\psi_\mu\bar{\alpha}+\alpha\bar{\psi}_\mu+\mathrm{i}\alpha\bar{\alpha}\omega_\mu. \tag{5.5}$$

The generating functional (5.4) may be written as

$$Z[\mathscr{J}]=\int \mathscr{D}\mathscr{A}_\mu\,\exp\bigg\{-\int \mathrm{d}\boldsymbol{x}\,\mathrm{d}t\,\mathrm{d}\bar{\alpha}\,\mathrm{d}\alpha\bigg[\mathscr{A}_\mu\cdot\bigg(\frac{-\partial^2}{\partial\alpha\,\partial\bar{\alpha}}-\alpha\frac{\partial}{\partial\alpha}\frac{\partial}{\partial t}\bigg)\mathscr{A}_\mu$$
$$+\frac{1}{2\lambda}\,\mathscr{A}_\mu\cdot\bigg(\bar{\alpha}\frac{\partial}{\partial\bar{\alpha}}-\frac{\partial}{\partial\bar{\alpha}}\,\bar{\alpha}\bigg)\mathscr{D}_\mu\,\partial_\nu\mathscr{A}_\nu$$
$$-\frac{1}{\lambda}\,\mathscr{A}_\mu\cdot\mathscr{D}_\mu\,\partial_\nu\bar{\alpha}\frac{\partial}{\partial\bar{\alpha}}\,\mathscr{A}_\nu-\mathrm{i}\,\mathscr{J}_\mu\cdot\mathscr{A}_\mu\bigg]-S_{\mathrm{cl}}[\mathscr{A}]\bigg\} \tag{5.6}$$

where $\mathscr{D}_\mu$ now stands for the super covariant derivative

$$\mathscr{D}_\mu=\partial_\mu-gA_\mu\times\ldots \tag{5.7}$$

and

$$\mathscr{J}_\mu=J_\mu(\boldsymbol{x},t)\delta(\alpha)\delta(\bar{\alpha}). \tag{5.8}$$

Adding a total time derivative to the exponent in (5.6), the generating functional becomes

$$Z[\mathscr{J}]=\int \mathscr{D}\mathscr{A}_\mu\,\exp\bigg(-\int \mathrm{d}\boldsymbol{x}\,\mathrm{d}t\,\mathrm{d}\bar{\alpha}\,\mathrm{d}\alpha\,\mathscr{L}\bigg) \tag{5.9}$$

where

$$\mathscr{L}=\tfrac{1}{4}(\partial_\mu\mathscr{A}_\nu-\partial_\nu\mathscr{A}_\mu-g\mathscr{A}_\mu\times\mathscr{A}_\nu)^2+\frac{1}{2\lambda}\,\mathscr{A}_\mu\cdot\bigg(\bar{\alpha}\frac{\partial}{\partial\bar{\alpha}}-\frac{\partial}{\partial\bar{\alpha}}\,\bar{\alpha}\bigg)\mathscr{D}_\mu\,\partial_\nu\mathscr{A}_\nu$$
$$-\frac{1}{\lambda}\,\mathscr{A}_\mu\cdot\mathscr{D}_\mu\partial_\nu\bar{\alpha}\frac{\partial}{\partial\bar{\alpha}}\,\mathscr{A}_\nu-\mathscr{A}_\mu\cdot\bigg(\frac{\partial^2}{\partial\alpha\,\partial\bar{\alpha}}+\alpha\frac{\partial}{\partial\alpha}\frac{\partial}{\partial t}-\frac{1}{2}\frac{\partial}{\partial t}\bigg)\mathscr{A}_\mu-\mathrm{i}\mathscr{J}_\mu\cdot\mathscr{A}_\mu. \tag{5.10}$$

Collecting the bilinear terms in $\mathscr{L}$, the differential equation for the superfield propagator is seen to be

$$\bigg\{\bigg(-\Box-2\frac{\partial^2}{\partial\bar{\alpha}\,\partial\alpha}-2\alpha\frac{\partial}{\partial\alpha}\frac{\partial}{\partial t}+\frac{\partial}{\partial t}\bigg)\delta_{\mu\rho}$$
$$+\bigg[1-\frac{1}{\lambda}\bigg(\bar{\alpha}\frac{\partial}{\partial\bar{\alpha}}+\frac{\partial}{\partial\bar{\alpha}}\,\bar{\alpha}\bigg)\bigg]\partial_\mu\,\partial_\rho\bigg\}D_{\rho\nu}^{ab}(\boldsymbol{x},t,\alpha,\bar{\alpha};\boldsymbol{x}',t',\alpha',\bar{\alpha}')$$
$$=\delta_{\mu\nu}\,\delta^{ab}\,\delta(\boldsymbol{x}-\boldsymbol{x}')\,\delta(t-t')\,\delta(\alpha-\alpha')\,\delta(\bar{\alpha}-\bar{\alpha}'). \tag{5.11}$$

Defining the Fourier transform of $D^{ab}_{\mu\nu}$ with respect to x and x' as follows

$$D^{ab}_{\mu\nu}(x, t, \alpha, \bar{\alpha}; x', t', \alpha', \bar{\alpha}') = (2\pi)^{-4} \int \mathrm{d}k\, \mathrm{d}k' \exp(\mathrm{i}k \cdot x + \mathrm{i}k' \cdot x') \tilde{D}^{ab}_{\mu\nu}(k, t, \alpha, \bar{\alpha}; k', t', \alpha', \bar{\alpha}'), \tag{5.12}$$

it is easily checked that the solution of (5.11) is given by

$$\begin{aligned}\tilde{D}^{ab}_{\mu\nu}&(k, t, \alpha, \bar{\alpha}; k', t', \alpha', \bar{\alpha}') \\ &= \delta^{ab}\, \delta(k + k') \bigg[\frac{1}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \\ &\quad \times \exp[-k^2 |t - t' + \alpha(\bar{\alpha} - \bar{\alpha}')\theta(t' - t) + \alpha'(\bar{\alpha} - \bar{\alpha}')\theta(t - t')|] \\ &\quad + \frac{\lambda k_\mu k_\nu}{k^2} \exp\left(-\frac{k^2}{\lambda} |t - t' + \alpha(\bar{\alpha} - \bar{\alpha}')\theta(t - t') + \alpha'(\bar{\alpha} - \bar{\alpha}')\theta(t - t')| \right) \bigg].\end{aligned} \tag{5.13}$$

This completes the superspace formulation of the stochastic quantisation method for gauge theories. We would like to note here that the form of the action written in terms of the superfields is not unique.

The perturbation theory in terms of the superfields is now straightforward and circumvents the need for rearranging the stochastic diagrams generated by the Langevin equation (4.6) as has been done by Nakagoshi *et al* (1983). The propagator (5.13) differs from the corresponding expression given by Nakagoshi *et al* in that it has an extra term $[\alpha'\bar{\alpha}\theta(t - t') - \alpha\bar{\alpha}'\theta(t' - t)]$ appearing inside the modulus sign in the exponent of (5.13). These differences, however, do not contribute to the Green functions of the gauge fields.

6. Conclusions

The superspace formulation of the stochastic quantisation method presented here permits us to construct stochastic regularisation schemes and to discuss Ward identities of the regularised theory along the lines suggested by Niemi and Wijewardhana (1982).

The formulation developed here is of interest not only for quantum field theories but also for investigating steady state properties of physical phenomena described by Langevin equations. This includes a large number of systems that have been investigated in the area of dynamical critical phenomena. In this context this formalism in our opinion provides a very elegant way of computing steady state equal and unequal time correlation functions.

It is also of great interest to investigate the invariances of the action in terms of superfields and to study the consequences thereof by deriving the corresponding Ward identities. This is being currently investigated and will be discussed elsewhere.

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