

Experiments with mechanically-played violins

C V RAMAN

[Plate I]

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Section I. Introduction

In the first volume (recently published*) of my monograph on the theory of the violin family of instruments, I have discussed on mechanical principles, the relation between the forces exerted by the bow and the steady vibration maintained by it, and the conditions under which the bow is capable of eliciting a sustained musical tone from the instrument. An experimental test of the results indicated by the theory on these points would obviously be of interest. Especially is this the case, as the analysis shows that the yielding of the bridge and the communication of energy from the strings through their supports into the instrument and thence into the air, play a very large part in determining the magnitude of the forces required to be exerted by the bow. An experimental study of the mechanical conditions necessary for obtaining a steady musical tone could thus be expected not merely to throw light on the *modus operandi* of the bow but also to furnish valuable information regarding the instrument itself, its characteristics as a resonator and the emission of energy from it in various circumstances. Further, a study of the kind referred to could be expected also to furnish

*Bull. Indian Assoc. Cultiv. Sci., 1918, No. 15, pages 1-158.

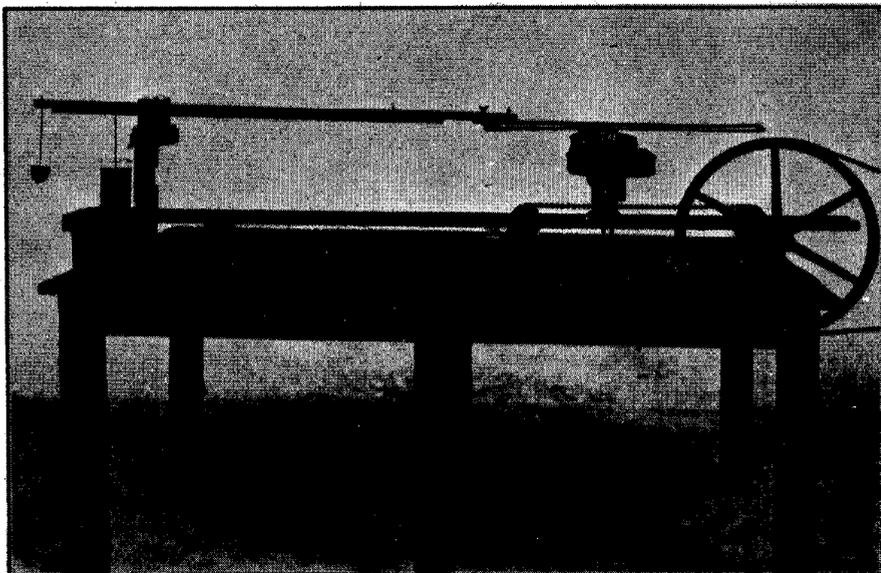
illustrations of the physical laws underlying the technique of the violinist and to put these laws on a precise quantitative basis. The experiments described in the present paper were undertaken with the objects referred to above, and the description of the results now given in these Proceedings is preliminary to a more exhaustive treatment of the subject which it is proposed to give in the second volume of my monograph under preparation for publication as a Bulletin of the Association.

Section II. Description of mechanical player

As the object of the work was to elucidate the theory of production of musical tone from instruments of the violin family, it was decided that the experimental conditions should approximate as closely as practicable to those obtaining in ordinary musical practice. The general principle accordingly held in view in designing the mechanical player was to imitate the technique of the violinist as closely as possible. There was also another reason for adopting this course. It is well known that the bowing of a stringed instrument so as to elicit a good musical tone is an art requiring much practice for its perfect accomplishment. The performance of the same task by purely mechanical appliances under such conditions as would permit of accurate measurements of the pressure and speed of bowing and the discrimination by ear of the effect of varying these factors obviously involves difficulties which it was thought would be best surmounted by imitating the violinist's handling of the bow as closely as the mechanical conditions would permit. A mechanical player designed on this general idea which has fulfilled the requirements of the work is illustrated in plate I.

As can be seen from the photograph, a violin and a horse-hair bow of the ordinary type were used in the mechanical player. Instead, however, of moving the bow to and fro, it was found a much simpler matter from the mechanical point of view to keep the bow fixed and to move the violin to and fro with uniform speed. This was arranged by holding the violin lightly fixed in a wooden cradle, the points of support being the neck and the tail piece of the violin as in the ordinary playing of the instrument. The cradle was mounted on a brass slide which moved to and fro noiselessly on a well-oiled cast-iron track. The slide received the necessary movement forward and backward from a pin carried by a moving endless chain and working in a vertical slot carried by the slide. The chain was kept in motion by the rotation of one of the two hubs between which it was stretched, this hub being fixed on the same axis as the driving wheel seen in the plate.*

*The whole of the apparatus was improvised in the laboratory from such materials as were to hand. The slide and cast-iron track were parts of a disused optical bench. The chain and hubs were spare parts purchased from a cycle-dealer. The ball-bearing of the axle of the lever (referred to below) was also part of a cycle. The other fittings were made up in the workshop.



The apparatus was driven by a belt running over a conical pulley which in its turn was driven by a belt passing over the pulley of a shunt-wound electric motor controlled by a rheostat which was allowed to run without any load except the apparatus. Using the rheostat and a Weston Electrical Tachometer, a very constant speed could be maintained during the experiments. Different speeds of motion of the slide carrying the violin were obtained by putting the driving belt of the apparatus on to different parts of the conical pulley, or by adjusting the rheostat.

The mounting of the bow required special attention in order to ensure satisfactory results. As is well known, the violinist in playing his instrument handles the bow in such manner that when it is applied with a light pressure, only a few hairs at the edge touch the string. The bow is held carefully balanced in the fingers of the right hand, the necessary increases or decreases in the pressure of bowing being brought about by increase or decrease of the leverage of the fingers. The suppleness of the wrist of the player and the flaccidity of the muscles of the forearm secures the necessary smoothness of touch. These features are carefully imitated in the mechanical player. The violin bow is held fixed at the end of a wooden lath, an adjustment being provided so that fewer or more hairs of the bow may be made to touch the string of the violin. The lath itself is balanced after the manner of a steelyard, the axis of the lever being mounted on ball-bearings so as to secure the necessary solidity combined with freedom of movement. The weight

of the bow is balanced by a load hung freely near the end of the shorter arm of the lever. The axis of the lever can be raised or lowered to the proper height above the violin such that when the hairs of the bow touch the string, they are perfectly parallel to the cast-iron track along which the violin slides. This is of great importance in order to obtain steady bowing, as otherwise the bow would swing up and down with the movement of the violin along the track, and its inertia would result in a variation of the pressure exerted by it. Any residual oscillations of the bow due to the elasticity of the lever or imperfection in the adjustment referred to above are checked by the damping arrangement shown in the plate. A wire with a number of horizontal disks attached to it at intervals is hung freely from the shorter arm of the lever and dips inside a beaker of water or light oil. This effectually prevents any rapid fluctuations in the pressure of the bow and ensures a smooth movement. The pressure exerted by the bow on the string can be varied by moving a rider along the longer arm of the lever which is graduated. An adjustment is provided by which the block carrying the axis of the lever can be moved by a screw perpendicular to the track, and the distance from the violin-bridge of the point at which the bow touches the string may thus be expeditiously altered.

It will be noticed that with the arrangements described above, the pressure exerted by the bow on the string of the violin would not be absolutely constant throughout, but would vary somewhat as the violin moves along its track from the point nearest to the point farthest from the axis of the lever. This is not however a serious difficulty as the lever is fairly long and the variation of pressure is thus not excessive. Further, the observations of the character of the tone are always made for a particular position and direction of movement of the violin and no ambiguity or error due to the cause referred to above arises.

The speed of bowing may be readily determined from the readings of the Electrical Tachometer or directly by noting on a stop-watch the time taken for a number of strokes to and fro of the violin on its track.

Section III. Variation of bowing pressure with the position of the bowed region

One of the well-known resources of the violinist is to bring the bow nearer to or to remove it further away from the bridge of the violin, the extreme variation in the position of the bow being from about $\frac{1}{4}$ th to about $\frac{1}{2}$ th of the vibrating length of the string from the bridge. In a recent paper on "The Partial Tones of Bowed Stringed Instruments" published in the *Philos. Mag.* (November 1919), I have discussed in some detail the changes in the amplitudes and phases of the various partials brought about by these changes in the position of the bowed region. In all the cases of musical interest within these limits, the mode of vibration of the string

is practically the same as in the principal Helmholtzian type* in regard to the first three partial components, but differs from it in respect of the higher components to an extent depending on the removal of the bow from the bridge. The ratios of the amplitudes and the relative phases of the first three partials remain practically the same throughout the range, the actual amplitudes for a given speed of the bow varying inversely as the distance of the bowed point from the bridge. The amplitudes of the fourth, fifth and higher partials vary in a similar way with the position of the bowed point provided this is not too far from the bridge, but deviate from this law more and more as the bow is removed further and further from the bridge. The net effect of bringing the bow nearer the bridge (its speed remaining constant) is greatly to increase the intensity of the tone of the instrument, and to make it somewhat more brilliant in character, as is of course well known. Simultaneously with these changes, the pressure with which the bow is applied has to be increased. The mechanical player described above may be used to find experimentally the relation between the bowing pressure and the position of the bow under these conditions.

The graphs in figure 1 (thin lines) represent the results obtained with the player on the D-string of the violin. A few words of explanation are here necessary. As a finite region of the bow is in contact with the string, it is not possible to specify the position of the bow by a single constant. Accordingly, the positions of the inner and outer edges of the region of contact were noted in the observations. The graph therefore shows two curves connecting the positions of the two edges of the bowed region with the magnitude of the bowing pressure. The ordinates of the graphs represent the values of the *minimum* bowing pressure found necessary to elicit a full steady tone with pronounced fundamental. [For bowing pressures smaller than this minimum, the fundamental falls off in intensity, and the prominent partial becomes its octave or twelfth. In certain cases, as for example near the wolf-note pitch, we get 'cyclical' or 'beating tones.']. It will be noticed from the graphs that the bowing pressure necessary increases with great rapidity when the bow is brought near the bridge.

The curve in figure 1 (heavily drawn) lying between the experimental graphs is a representation of the algebraic curve $x^2y = \text{constant}$. (The constant was, of course, suitably chosen). It will be noticed that the graph follows the trend of the experimental values quite closely. In other words, we may say that in the cases studied, the bowing pressure necessary varies practically in inverse proportion to the *square* of the distance of the bow from the bridge. It may be readily shown that this is the result to be expected from theory. In my monograph,* I have shown

*The principal Helmholtzian type is the mode of vibration in which the time displacement graph of every point on the string is a simple two-step zig-zag.

†Bulletin No. 15, pages 73 to 75.

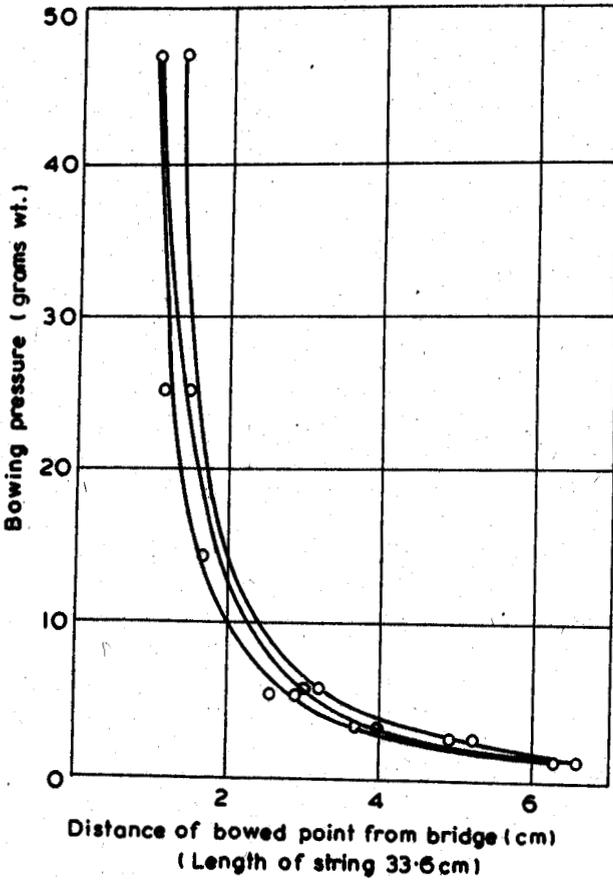


Figure 1

that the minimum bowing pressure P is given by the formula

$$P = \frac{P_{A'} - P_A}{\mu - \mu_A},$$

where $P_{A'}$ is the maximum value at any epoch of the series

$$\sum_{n=1}^{n=\infty} k_n B_n \frac{\sin\left(\frac{2n\pi t}{T} + e_n + e'_n\right)}{\sin \frac{n\pi x_0}{L}},$$

and P_A is the value of the series at the epoch at which the bowed region of the

string slips past the bow. μ is the statical coefficient of friction, and μ_A is the dynamical coefficient of friction during the epoch of slipping. B_1, B_2 , etc. are the amplitudes of the partial vibrations of the string, k_1, k_2 , etc. are numerical constants for the respective partials depending on the instrument, the mass, length and tension of the string, and x_0 is the distance of the bowed point from the end of the string. We have already seen that amplitudes B_n of the first few partials for a given speed of bowing vary in inverse proportion to the distance of the bow from the bridge, and their relative phases remain unaltered. In respect of these partials, the factor

$$1/\sin \frac{n\pi x_0}{L}$$

also varies practically in inverse proportion to x_0 , so long it is a small fraction of l .

To effect a simplification, we may proceed by ignoring the influence of all the partial vibrations except the first few, an assumption which is justifiable in the case under consideration, as by far the greater proportion of the energy of violin-tone is confined to the first few partials. Further, we may for simplicity, treat the difference $(\mu - \mu_A)$ between the statical and dynamical coefficients of friction as practically a constant quantity. This will not introduce serious error, provided the speed of the bow is not very small. For, if the slipping speed be fairly large, any changes in it due to change of the position of the bowed point would not seriously alter the dynamical coefficient of friction. On these simplifying assumptions, it will be seen from the formulæ given above that, within the limits considered, the minimum bowing pressure should vary in inverse proportion to the *square* of the distance of the bow from the bridge, exactly as found in experiment. This relation would, of course, cease to be valid when the bowed point is removed too far from the bridge or when the speed of the bow is very small.

Section IV. Relation between bowing speed and bowing pressure

The changing of the speed of the bow is another of the well known resources of the violinist. The principal effect of this is to alter the intensity of tone. *Pari passu* with the change of speed of the bow, other things remaining the same, the violinist has to alter the pressure of the bow. The relation between these may be readily investigated with the mechanical player. The experimental results for the D-string and for a particular position of the bowed point are shown in figure 2.

The graph shows the following features: (1) for very small bowing speeds, the bowing pressure tends to a finite minimum value; (2) the increase of bowing pressure with speed is at first rather slow; (3) later, it is more rapid, the pressure necessary increasing roughly in proportion to speed, and for large amplitudes of vibration possibly even more than in proportion to the speed of the bow.

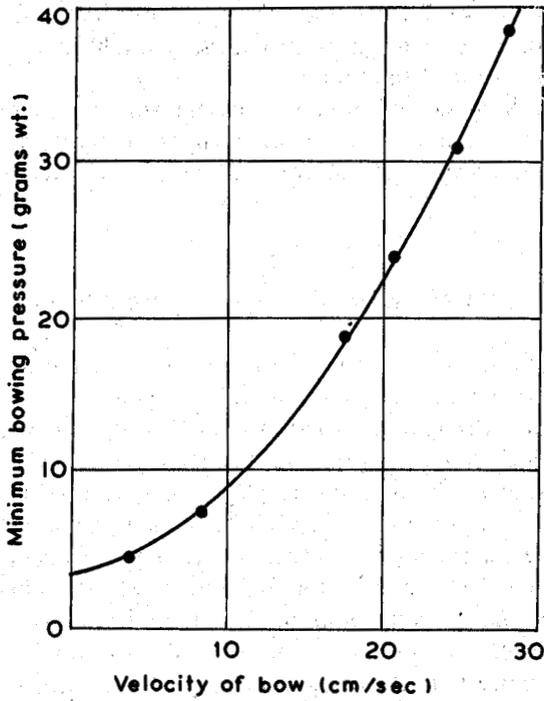


Figure 2

The foregoing results are, broadly speaking, in agreement with what might be expected on theoretical grounds.* This can be seen from the formula for the bowing pressure referred to in section III. With increasing speed of the bow, the amplitudes B_n of the partial vibrations increase in proportion, so that if the difference $\mu - \mu_A$ between the static and dynamical coefficients of friction be regarded as a constant, the bowing pressure necessary should vary directly as the speed of the bow. For very small speeds of the bow, however, it is not correct to take $\mu - \mu_A$ as constant, and it would be nearer the mark for such speeds to take $\mu - \mu_A$ as proportional to the velocity of slip, that is, as proportional to the speed of the bow. Thus, for very small speeds of the bow, the pressure necessary should be nearly independent of the speed, that is, should converge to a finite minimum speed. For larger speeds of the bow, it would be correct to take $\mu - \mu_A$ as constant and the bowing pressure should then vary proportionately with the speed. For very large speeds, the theory of small oscillations would no longer be applicable,

*Bulletin No. 15, pages 151-153.

and the quantities k_1, k_2, k_3 , etc. might increase with the speed of the bow. For such large speeds, the bowing pressure necessary might increase more than in proportion to the speed of the bow.

A more precise discussion of the experimental results would be possible on the basis of quantitative data as to the manner in which the coefficient of friction between rosined horse-hair and catgut varies with the velocity of slip at different pressures.

Section V. Variation of bowing pressure with pitch

The pitch of violin tone depends on (1) the linear density of the bowed string, (2) its length, and (3) its tension, and may be varied by varying any one or other of these factors. In practice, the violinist varies the pitch by (1) altering the vibrating length by "stopping" down the string on the fingerboard, or (2) by passing from one string to another. The mechanical player may be used to investigate the dependence of bowing pressure upon pitch when the latter is varied in any of the ways that may be suggested. Obviously, the sequence of phenomena observed would not be exactly the same for the four strings of the violin as these are of different densities and tension, communicate their vibrations to the body of the instrument at different points of the bridge and also vibrate in considerably different planes relatively to the bridge and belly when excited by the bow in the usual way. In the experimental work now to be described, a particular string of the violin, e.g. the 4th or G-string, was used, and the pitch was varied as in the ordinary playing of the instrument by 'stopping' the string at different points. This was arranged by clamping the string down to the fingerboard, with a light but strong brass clamp shaped like an arch which could be put across the fingerboard, passed down upon it and then lightly fixed to it by two set-screws at the two ends. The inner face of the clamp was lined with leather to imitate the ball of the fingers of the violinist and to prevent damage to the strings.

A few remarks are here necessary. In actual practice, when the violinist stops down the string so as to elicit a note of higher pitch, he generally takes the bow up rather nearer the bridge so as to preserve the relationship between the vibrating length of the string and the distance of the bow from the bridge. Strictly speaking, this should also have been done in the present investigation. But as it would have been somewhat troublesome and involved the risk of errors in the adjustment of the position of the bow, it was decided to keep the bow in a fixed position somewhat close to the bridge and to find the relationship between the bowing pressure and the pitch of the string under these conditions. We have already seen that when the bow is fairly close to the bridge, the bowing pressure necessary varies practically in inverse proportion to the square of the distance of the bow from the bridge. Accordingly, the effect of keeping the bow in a fixed position when the pitch is altered, instead of it bringing it nearer the bridge at each stage, is

to decrease the bowing pressure necessary in a progressive and calculable ratio. This effect does not accordingly interfere with our observation of the characteristic changes of bowing pressure with pitch, which are connected with the changes in the forced vibration of the bridge and belly of the violin brought about by the change in the frequency of excitation.

The graph in figure 3 represents the relationship between bowing pressure and pitch within a part of the range of tone of the violin which includes the first three of the natural frequencies of vibration of the body of the instrument. The particular violin used was of German make, marked copy of Antonius

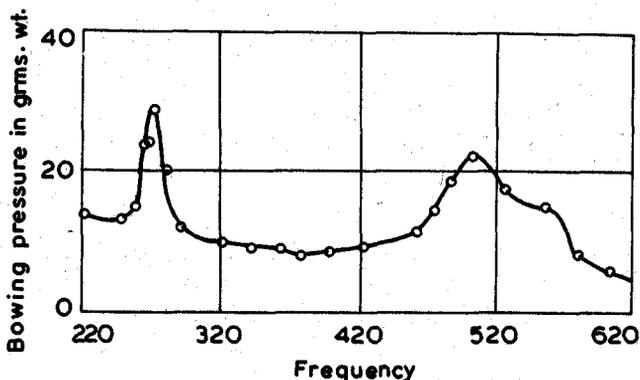


Figure 3. Relation between bowing pressure and pitch (without mute).

Straduaris, the bridge being of the usual Straduaris model. The experiments were made on the 4th or G-string, stopped down to various pitches. It will be noticed that the graph for the bowing pressure shows pronounced maxima and minima. There is a strong maximum at 270, another maximum between 470 and 520, and a distinct hump between 520 and 570. These maxima pretty nearly coincide in pitch with the first three maxima of intensity of the fundamental in the tone of the violin as estimated by ear, in other words with the frequencies of maximum resonance of the instrument to the gravest component of the force exerted on it by the vibrating string. The maximum lying between 470 and 520 is specially interesting as this region exhibits the well-known phenomenon of the 'wolf-note'. In the ascending part of this portion of the graph, and especially at and near the peak of the curve, it is found that when the pressure of the bow is less than the minimum required to elicit a steady tone with a well-sustained fundamental component, we get 'cyclical' or beating tones of the kind described and illustrated by me in previous papers.* The rapidity of the beats depends on

*Bulletin No. 15, also *Philos. Mag.* October 1916.

the pitch of the tone which it is attempted to elicit and also on the pressure and speed of the bow. A similar tendency to production of a 'wolf-note' though not so striking, is also manifested in the part of the graph between 520 and 570. The maximum in the region of 250 to 285 does not show a similar tendency, at any rate to any appreciable extent. It would appear that the gravest resonance of the violin chiefly involves a vigorous oscillation of the air within the belly of the instrument, but not so vigorous an oscillation of the bridge and belly as in the second and third natural modes of vibration which show the wolf-note phenomenon. Further evidence on this point is furnished by experiments on the effect of putting a load or mute on the bridge of the violin as will be referred to in the following section.

The formula for the bowing pressure quoted on page 413 enables the variation of bowing pressure shown in figure 3 to be explained. In the series

$$\sum_{n=1}^{n=\infty} K_n B_n \frac{\sin\left(\frac{2n\pi t}{T} + e_n + e'_n\right)}{\sin\frac{n\pi x_0}{L}},$$

of which the graph practically determines the bowing pressure required, the B_n 's stand for the amplitudes of the different partial vibrations of the string, and the K_n 's are quantities which are practically proportional to the corresponding partial components of the forced vibration of the bridge transverse to the string at its extremity. In the case of a string bowed near the end, $B_n/\sin(n\pi x_0/L)$ varies nearly as $1/n^3$ and thus decreases rapidly as n increases. Further, in the part of the range of violin-tone covered by the graph in figure 3, the fundamental component of the vibration of the bridge should obviously be well marked, and K_1 would therefore be of the same order of quantities as K_2, K_3 , etc. or even larger. Hence the value of the series given above would be principally determined by its leading term proportional to K_1 , and the variation of bowing pressure with pitch would practically follow the fluctuations of K_1 , in other words would follow the variations in the amplitude of the fundamental component in the forced vibrations of the bridge and pass through a series of maximum values at the successive frequencies of resonance of the instrument. This is practically what is shown by the experimental results for the bowing pressure appearing in figure 3, and the graph gives us an idea of the *sharpness* of the resonance of the instrument at each of the frequencies referred to. It must be remembered, however, that the bowing pressure required is also influenced by the terms proportional to K_2, K_3 , etc. that is by the resonance of the instrument to the second, third and higher partial components of the vibration, and some evidence of this also appears in the graph in figure 3. For instance, though the first resonance of the instrument was actually found to be at a pitch of 284, the peak of the curve for the bowing pressure is at about 270 as can be seen from figure 3. This appears to be a consequence of the fact that the course of the curve is to some extent modified by the resonance of the *octave*. It is obvious that corresponding to the resonance of the instrument to

the fundamental tone in the range of pitch from 470 to 570, the octave should be strongly reinforced when the pitch lies within the range 235 to 285; hence the peak of the curve for the bowing pressure instead of being at 284 is actually shifted towards a lower frequency (270) as is seen from figure 3.

Obviously, the experiments described in this section may be extended in various directions. The curves for the three other strings of the violin, and especially over the whole of the possible range of pitch of the tone of the instrument, and the differences between the curves obtained with the different strings would deserve investigation. The differences between different violins could obviously be studied by this method, and the constants K_1 , K_2 , etc. for any particular violin and string and for various pitches may be found experimentally by study of the free and forced oscillations of the bridge, and used for a theoretical calculation and comparison with experiment of the bowing pressure required for exciting the tone of the instrument.

Section VI. Effect of muting on bowing pressure

Perhaps the best illustration of the close relation existing between the forces required to be exerted by the bow and the communication of vibrations from the string to the bridge and belly of the instrument and thence to the air, is furnished by the effect of putting a mute on the bridge on the bowing pressure required for eliciting a musical tone. Very striking results on this point may be obtained with the aid of the mechanical player. Figure 4 illustrates the relation between bowing

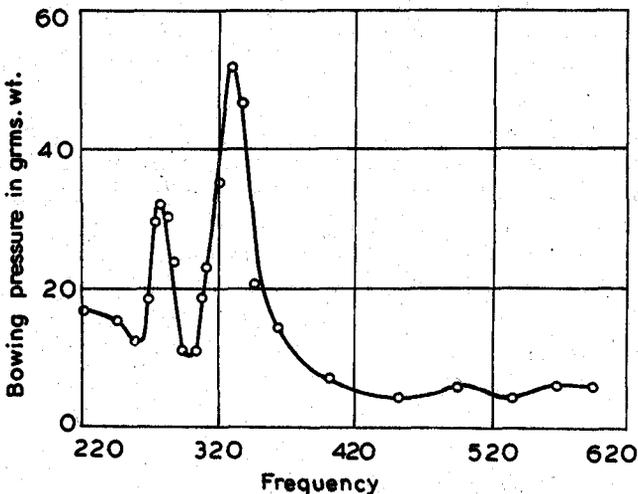


Figure 4. Relation between bowing pressure and pitch (with mute).

pitch and bowing pressure observed experimentally, the G-string of the violin being used as in figure 3, and the results shown in figure 3 and figure 4 being obtained under the same conditions except that in the latter case, a brass mute weighing 12.4 grammes was clamped to the bridge, while in the former case, the bridge was unmuted. The great difference between the two cases is obvious, and the change in the form of the graph for the bowing pressure shows a very close analogy with the change in the character of the forced vibration of the instrument and the intensity of the tone of the violin over the whole range of pitch produced by application of the mute.

In view of the discussion of the graph for the bowing pressure contained in the preceding section, and the theoretical treatment of the effect of muting already given by me in previous papers* it is perhaps not necessary to enter here into a detailed examination of the subject, and it may suffice briefly to draw attention to some of the features appearing in the graph in figure 4. It will be noticed that there is a peak in the curve at a frequency of about 280. This is the pitch of the first resonance of the instrument, the fundamental component of the vibration being reinforced, and in this case, the position of the peak in the curve for the bowing pressure is not appreciably influenced by the resonance of the octave as in figure 3. It is clear that the pitch of the first resonance of the instrument is hardly influenced at all by the application of a load of 12.4 grammes to the bridge, and the bowing pressure required at the first peak of the curve is nearly the same as in figure 3. The first natural mode of vibration of the violin does not therefore appear to involve any very large vibration of the bridge. Following the peak at 280, we have in figure 4 a very high peak near 330 which is the pitch of the 'wolf-note' as lowered by the mute of 12.4 grammes. The great lowering of pitch (from 490 to 330) shows that the second mode of vibration of the body of the violin involves a very large vibration of the bridge, and the enormous increase of bowing pressure at the peak of the curve is also noteworthy. This can no doubt be explained on dynamical principles as due to the very greatly increased amplitude of the forced vibration of the bridge due to the loading. At higher pitches, the bowing pressure necessary falls off very rapidly, though one or two minor maxima (due to the resonance of the instrument in its higher modes as altered by the loading) are also obtained. The tone of the instrument in the higher ranges of pitch when muted is extremely feeble.

Further investigations which are worthy of being carried out would be the study of the effect of gradually increasing the mass of the mute on the graph for the bowing pressure, and also of putting the mute at different places on the bridge. In view of what has been stated above, it is clear that we may expect the changes in

**Philos. Mag.* October 1916, *Nature (London)* October 1917, *Philos. Mag.* June 1918, and *Bulletin No. 15*, pages 143-151.

the form of the graph to follow closely the changes in the pitch of resonance of the instrument in its various modes produced by the loading.

Section VII. Other applications of the mechanical player

The investigations described in the preceding sections may be extended in various directions. Some indications have already been given on these points, and it will suffice here to suggest some of the other possible applications of the mechanical player. As the instrument affords a means of bowing the violin at precisely measurable speeds and pressures, it furnishes a means by which the intensity of violin-tone and its variations with pitch may be quantitatively determined and compared with the indications of mathematical theory. Various questions, such as for instance the effect of heavier or lighter stringing, the effect of varying the pressure, speed, and the width of the region of contact of the bow, and the position of the bowed region on the tone-quality of the violin may be quantitatively studied with a degree of accuracy that cannot be approached in manual playing with its undetermined conditions. Further, the study of the tone-intensity, of the bowing pressure curves, and of the vibration-curves of bridge and belly of the instrument under quantitative conditions made possible by mechanical playing may be expected speedily to clear up various structural problems relating to the construction of the violin, e.g. the effect of the peculiar form of the Stradivarius bridge, the influence of its position, the function of the sound-post and base-bar, the shape of the air-holes, the thickness, curvature and shape of the elastic plates composing the violin, and the influence of various kinds of varnish. The dynamical specification of the constants determining the tone-quality of any violin over the whole range of pitch may be regarded as one of the aims towards which these investigations are directed.

Section VIII. Synopsis

The paper describes the construction of a mechanical violin-player intended for study of the acoustics of the instrument, and some of the investigations in which it has been applied. The principal feature in the player which is worthy of notice is that the conditions obtaining in ordinary musical practice are imitated with all the fidelity possible in mechanical playing, and the results obtained with it may therefore be confidently regarded as applicable under the ordinary conditions of manual playing. The following is a summary of the results obtained in the four investigations described in the present paper: (1) *Effect of the position of the bowed region on the bowing pressure*: it is shown that provided the speed of the bow is not too small, the bowing pressure necessary within the ordinary musical range of bowing varies inversely as the *square* of the distance of the bow from the bridge.

(2) *Relation between bowing speed and bowing pressure*: it is shown that for very small bowing speeds, the bowing pressure necessary tends to a finite minimum value, and the increase of bowing pressure with speed is at first rather slow, but later becomes more rapid. (3) *Variation of bowing pressure with pitch*: the graph for the bowing pressure for different frequencies shows a series of maxima which approximately coincide in position with the frequencies of resonance of the instrument. (4) *Effect of muting on bowing pressure*: it is found that the mute produces profound alterations in the form of the graph. The bowing pressure necessary is increased in the lower parts of the scale and decreased in the higher parts of the scale. The peaks in the graph shift towards the lower frequencies in consequence of the alteration in the natural frequencies of resonance of the violin produced by the loading, and the change in the form of the graph is closely analogous to the change of the intensity of the fundamental tone of the instrument produced by the muting.

Some further possible applications of the mechanical player are also indicated in the paper.