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Width effects of the Z resonance and CP-violating asymmetry in e^+e^- annihilation

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Abstract. CP-violating rate asymmetry can be generated in a process only if its amplitude possesses an absorptive part. It is pointed out that such an absorptive part can be provided in e^+e^- annihilation by the presence of a $Z(Z')$ resonance of non-zero width. The CP asymmetry in the process $e^+e^- \rightarrow l_i\bar{l}_j(\bar{l}_i l_j)$, where l_i are charged leptons, is discussed in several models. In a specific two-Z model, large and observable CP asymmetry in $e^+e^- \rightarrow \tau^+e^-(\tau^-e^+)$ is shown to be possible at LEP/SLC energies.

Keywords. CP violation; Z resonance; Z width; two-Z models; leptonic flavor violation.

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There have been several proposals in recent times to look for CP violation outside the neutral kaon system, especially in Z decays (Hou *et al* 1986; Bernabéu *et al* 1986; Rius and Valle 1990; Choudhury *et al* 1991), in view of the currently operational LEP and SLC e^+e^- machines which have the capability of producing a large number of Z's. The suggestions include investigating the rate asymmetry between two processes related to each other by CP and looking for CP-odd correlations in various processes. Of these, rate asymmetries are easier to search for experimentally. But they can arise theoretically only if the relevant amplitude possesses an absorptive part. This requires either the knowledge of strong interaction phases (final-state interactions) or calculation of loop diagrams. The need for an absorptive part may be avoided in the case of CP-odd correlations; processes in which a non-zero signal can occur are, however, quite complicated (Nowakowski and Pilaftsis 1989).

In this brief communication we examine the possibility that the absorptive part required for CP-violating asymmetry comes from the propagator of an unstable particle[1]. The Breit-Wigner form for the propagator has an imaginary part proportional to the width of the unstable particle, and hence the resulting CP-violating asymmetry would be proportional to this width. Since in this case the need for non-perturbative phase shifts or loop diagrams is avoided, the calculation of the asymmetry involves a simple evaluation of tree diagrams.

Although it is possible to think of quark processes in the standard model (SM) with CP violation coming from complex quark-antiquark couplings to unstable $W^{\pm}$, they would need at least two W's and also involve flavor tagging to distinguish

[1]After the completion of this work we became aware of papers which have recently considered this possibility (Nowakowski and Pilaftsis 1991; Eilam *et al* 1991; Hoogeveen and Stodolsky 1988).

between charge-congugate processes to be of practical relevance. We consider here, instead, extensions of SM, where the new mechanism can be illustrated in a somewhat simpler leptonic process, viz., $e^+e^- \to l_i\bar{l}_j(\bar{l}_il_j)$, $(l_1 = e, l_2 = \mu, l_3 = \tau)$. Since lepton flavor detection is simpler, this would be experimentally easier to observe. In $SU(2) \times U(1)$ models with exotic charged leptons, Z couplings can violate leptonic flavor and make this process possible. Difference in the rates for $e^+e^- \to l_i\bar{l}_j$ and $e^+e^- \to \bar{l}_il_j$ would then be a measure of CP violation.

As we shall see, to get an observable asymmetry consistent with known constraints on lepton flavor violations, we need to extend the model further, and we have considered several extensions. Most remarkably, in a certain $SU(2) \times U(1) \times U(1)'$ model, we find in the region of the Z resonance rather large asymmetries, nearing 100%, for a choice of parameters allowed by all known data. This effect, if present, could be easily observable at current e^+e^- colliders. This is to be contrasted with rather small and unobservable rate asymmetries found in the SM (Hou *et al* 1986; Bernabéu *et al* 1986) and its extensions (Rius and Valle 1990; Choudhury *et al* 1991).

Before we look at any specific model, let us consider the necessary conditions for a CP-violating rate asymmetry to occur. The amplitude for a process can be written as

$$M = \sum_m g_m f_m A_m, \tag{1}$$

where g_m are complex products of coupling constants (including mixing angles and phases), f_m are invariant functions of kinematic variables (including propagators and momentum integrals over propagators) and A_m are tensors (which are assumed real). The sum is over the distinct Feynman diagrams in perturbation theory. The amplitude for the CP -conjugate process is then

$$\bar{M} = \sum_m g_m^* f_m A_m. \tag{2}$$

The difference in the rates for the two processes is proportional to

$$|M|^2 - |\bar{M}|^2 = -4 \sum_{m>n} \mathrm{Im}(g_m g_n^*)\,\mathrm{Im}(f_m f_n^*) A_m A_n. \tag{3}$$

It immediately follows that for a non-zero rate asymmetry, (i) at least two terms must contribute, (ii) of which at least one must have complex couplings and complex f_m. (iii) If two or more terms with complex g_m and f_m contribute, then not all g_m and not all f_m should have identical phases.

The simplest extensions of SM satisfying conditions (i) and (ii) above are $SU(2)_L \times U(1)$ models which include at least one generation of charged leptons, transforming under $SU(2)_L$ as (A) left-handed (LH) and right-handed (RH) singlets (vector singlets), or (B) LH ad RH doublets (vector doublets), or (C) LH singlets and RH doublets (mirrors). In all these models, the mixing of ordinary with exotic leptons violate the leptonic Glashow-Iliopoulos-Maiani (GIM) mechanism, and therefore induce flavor-changing couplings which are complex in general. The two diagrams needed for a CP asymmetry in $e^+e^- \to l^+e^-, l^-e^+ (l = \mu, \tau)$ are those involving s- and t-channel exchanges of the Z, with the width of the Z providing an imaginary part to the propagator. However, the condition (iii) is satisfied only in model C. In models

A (*B*) the flavor-violating couplings are only RH(LH), and the *s*- and *t*- channel contributions have identical phases for the couplings. In model C both LH and RH couplings occur with different phases, and their interference can give rise to a rate asymmetry.

It turns out, however, that this asymmetry is constrained to be extremely small for two reasons. Since in model C both LH and RH couplings enter, there have to be two helicity flips, suppressing the effect by a factor of at least $m_e m_l/M_Z^2$. Moreover, the mixing angles in model C are severely constrained by the experimental limit on the electric dipole moment of the electron (hipura 1991a). As a result, the CP asymmetry turns out to be unobservably small. We do not discuss this case any further.

Another class of popular models in which the conditions (i) and (ii) can be satisfied is based on E_6 as a grand unification group (Hewett and Rizzo 1989). In these models, when E_6 is broken to a low energy group $SU(2) \times [U(1)]^2$ or $SU(2) \times [U(1)]^3$, there is at least one extra neutral gauge boson (Z'), and vector doublets of exotic leptons, which mix with the ordinary leptons. The presence of Z' provides the additional amplitude needed to satisfy condition (i). This has the advantage that one no longer needs to invoke the interference between the LH and the RH fermionic coupling as in the previous case. The consequent mass suppression can be avoided as a result and the asymmetries could be potentially large. Moreover, one could now obtain asymmetries in the processes $e^+e^- \to f_i \bar{f}_j$ (f = any quark or lepton) as *t*-channel exchanges are not required. The presence of exotic leptons also generates the flavor-changing, in general complex, couplings to Z and Z'. Relative phases between the Z and Z' couplings depend upon the way the fermions transform under the gauge group. As we will see from the explicit expression given later, these phases are zero unless the Z (Z') distinguishes between various generations of the exotic (ordinary or exotic) fermions[2]. This does not happen in E_6-based models and consequently condition (iii) needed to get non-zero asymmetry is not satisfied in these models.

The above considerations lead us to study $SU(2) \times U(1) \times U(1)'$ models with the $U(1)'$ acting horizontally on the ordinary fermions. The requirement of anomaly cancellation restricts the choice of $U(1)'$ considerably. In particular, if $U(1)'$ acts only on leptons, then only three choices are allowed in the absence of exotic fermions (He *et al* 1991). While other choices of $U(1)'$ would be possible once one introduces exotic fermions or allows quarks to transform non-trivially under $U(1)'$, we shall concentrate for illustrative purpose on a specific $SU(2) \times U(1) \times U(1)'$ model with the $U(1)'$ hypercharge Y' identified with $L_e - L_\tau$. L_e, L_τ represent here the lepton numbers of e and τ respectively. The phenomenology of this and two other related models is quite interesting and has been extensively discussed (He *et al* 1991). We shall consider here a specific extension which allows large flavor violation in the e-τ sector. It is possible to satisfy all the three constraints in this model and obtain large CP-violating rate asymmetry in the process $e^+e^- \to \tau^+e^-(\tau^-e^+)$.

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[2]This statement applies to E_6-based models containing the SM times various $U(1)$ groups. If Z' arises from a non-Abelian structure, for example, $SU(2) \times U(1) \times SU(2)$, then mixing between Z and Z' can involve *complex* coefficients. In that case, the relative phase in the flavor-changing couplings of the mass eigenstate gauge bosons could be nonzero. We shall not consider this possibility here.

The couplings of the ordinary fermions to Z bosons $Z_m (m = 1, 2)$ can be parametrized in general as

$$\mathscr{L}_Z = -((G^L_m)_{ij} \bar{f}_{iL} \gamma_\mu f_{jL} Z^\mu_m + L \leftrightarrow R), \tag{4}$$

where Z_m are mass eigenstates related to Z and Z' by a 2×2 orthogonal matrix with the mixing angle ϕ. $f_{ia} (a = L, R)$ denote the chiral projections of the mass eigenstates of the ordinary fermions. They are related to the weak eigenstates of the ordinary (f'_{ia}) and exotic $(F'_{\alpha a})$ fermions through unitary matrices U^a. If the couplings of $f'_{ia}(F'_{\alpha a})$ to Z and Z' are denoted by $z_a(z_{a\alpha})$ and $z'_a(z'_{a\alpha})$, respectively, then

$$\begin{aligned}(G^a_1)_{ij} &= \cos(\phi)(\delta_{ij} z_a + (z_{a\alpha} - z_a) U^a_{\alpha j} U^{a*}_{\alpha i}) \\ &\quad - \sin(\phi)(z'_{ak} U^a_{kj} U^{a*}_{ki} + z'_{a\alpha} U^a_{\alpha j} U^{a*}_{\alpha i}). \quad (a = L, R)\end{aligned} \tag{5}$$

$(G^a_2)_{ij}$ are obtained from the above by the interchange $\cos(\phi) \to \sin(\phi)$, $-\sin(\phi) \to \cos(\phi)$. Here the index $k(\alpha)$ refers to the ordinary (exotic) fermions.

We now specialize to charged leptons, through many of the following arguments would go through for quarks as well. The cross section for $e^+ e^- \to l_i \bar{l}_j (i, j = 1, 3)$ including s- and t- channel Z_1 and Z_2 exchanges is given by

$$\begin{aligned}\sigma(e^+ e^- \to l_i \bar{l}_j) = \int \frac{dt}{16\pi s^2} &[(G^L_n)^*_{ij} (G^L_m)_{ij} (G^L_m)_{11} (G^L_n)_{11} (s + t)^2 (f_m(s) + f_m(t)) \\ &\times (f_n(s) + f_n(t))^* + (G^L_n)^*_{ij} (G^L_m)_{ij} (G^R_m)_{11} (G^R_n)_{11} (s^2 f_m(t) f^*_n(t) \\ &+ t^2 f_m(s) f^*_n(s)) + L \leftrightarrow R].\end{aligned} \tag{6}$$

Lepton masses have been neglected and s, t are the usual Mandelstam variables. f_m are the Breit-Wigner functions occurring in the $Z_{1,2}$ propagators[3],

$$f_m(s) = \frac{1}{s - M^2_m + iM_m \Gamma_m}, \tag{7}$$

and likewise for $f_m(t)$. $M_m(\Gamma_m)$ denotes the mass (total width) of Z_m.

The difference in $l_i \bar{l}_j$ and $\bar{l}_i l_j$ production cross sections following from (6) can be seen to be proportional to $\mathrm{Im}\,((G^a_1)_{ij} (G^a_2)^*_{ij})$. This difference, which is a measure of CP violation, is zero if $(G^a_1)_{ij}$ is relatively real with respect to $(G^a_2)_{ij}$ for both $a = L, R$. From (5), one obtains, for $i \neq j$,

$$\begin{aligned}\mathrm{Im}((G^a_1)_{ij} (G^a_2)^*_{ij}) &= z'_{ak}(z_{a\alpha} - z_a)\,\mathrm{Im}(U^{a*}_{kj} U^a_{ki} U^{a*}_{\alpha j} U^a_{\alpha i}) \\ &\quad + z'_{a\beta}(z_{a\alpha} - z_a)\,\mathrm{Im}(U^{a*}_{\beta j} U^a_{\beta i} U^{a*}_{\alpha j} U^a_{\alpha i}).\end{aligned} \tag{8}$$

If the Z and Z' couplings of the weak eigenstates are independent of the flavor (index k or α above) then unitarity of U^a can be used to show that the RHS of (8) is zero. Hence unless Z or Z' distinguishes between the fermionic generations, one cannot obtain CP-violating rate asymmetry for massless leptons. E_6-based models

[3]Strictly speaking, the Breit-Wigner form gives the correct absorptive part of the amplitude only for s-channel exchange (and even then, Γ should be a function of s). However, our numerical results do not change significantly by dropping the imaginary part of $f_m(t)$.

do not contain any horizontal subgroups and hence cannot lead to the desired asymmetry.

We now turn to the specific $SU(2)\times U(1)\times U(1)'$ model with $Y'=L_e-L_\tau$. The presence of the gauged $U(1)'$ is by itself not sufficient to generate flavor-changing Z couplings and we need to add exotic leptons. We add one $SU(2)$ vector singlet lepton E of charge -1 and $Y'=1$. We also need an $SU(2)$ singlet neutral scalar with $Y'=2$ whose vacuum expectation value breaks $U(1)'$ and mixes E with e and τ. It can be shown that the most general charged-lepton mass matrix, including a direct $SU(2)\times U(1)\times U(1)'$ singlet mass term for E, gives arbitrary masses to all leptons and generates flavor-changing τe couplings to Z and Z'. μ does not mix with e,τ or E since L_μ remains an exact symmetry in the model. With the above Higgs content, the Z and Z' do not mix and coincide respectively with Z_1 and Z_2. Consequently, the LEP results on the Z mass do not restrict the mass of Z'. This together with the absence of quark couplings to Z' allows the latter to be even lighter than the Z (He *et al* 1991).

The fermionic couplings (5) to $Z\equiv Z_1$ and $Z'\equiv Z_2$ are now given by

$$(G_1^L)_{ij}=\frac{g}{\cos\theta_W}[(-\tfrac{1}{2}+\sin^2\theta_W)\delta_{ij}+\tfrac{1}{2}U_{4i}^{L*}U_{4j}^L], \tag{9}$$

$$(G_1^R)_{ij}=\frac{g}{\cos\theta_W}[\sin^2\theta_W\delta_{ij}], \tag{10}$$

$$(G_2^a)_{ij}=g'[\delta_{ij}-2U_{3i}^{a*}U_{3j}^a].\ (i,j\neq 2) \tag{11}$$

$i,j=1,3$ label the e and τ respectively, while the index 4 corresponds to E. $(G_2^a)_{ij}$ is zero when i or j is 2. g,g' are respectively $SU(2)$, $U(1)'$ coupling constants and θ_W is the SM weak mixing angle. The observed leptonic universality of the Z decay requires $|U_{41}|$ and $|U_{42}|$ to be small compared to 1. In this case, it is convenient to adopt the following parametrization for the elements of U^a:

$$U_{11}^a=U_{33}^a=\cos\theta_a;\ U_{13}^a=-U_{31}^a=\sin\theta_a;\ U_{43}^a=|U_{43}^a|e^{i\delta_a}; \tag{12}$$

with U_{41}^a chosen real. The difference and the sum of the cross sections for $e^+e^-\to e^-\tau^+$, $e^+\tau^-$ follow from eqs (6) and (9–12). The former is as follows:

$$\sigma(e^+e^-\to e^-\tau^+)-\sigma(e^+e^-\to e^+\tau^-)=\int\frac{\mathrm{d}t-g^2g'^2}{8\pi s^2\cos^2\theta_W}|U_{41}^L U_{43}^L|\sin 2\theta_L\sin\delta_L$$
$$\times\{(-\tfrac{1}{2}+\sin^2\theta_W)\cos 2\theta_L(s+t)^2\,\mathrm{Im}[(f_1(s)+f_1(t))(f_2(s)+f_2(t))^*]$$
$$+\sin^2\theta_W\cos 2\theta_R\,\mathrm{Im}[t^2f_1(s)f_2(s)^*+s^2f_1(t)f_2(t)^*]\}. \tag{13}$$

The above equation can be used to evaluate the asymmetry

$$A=\frac{\sigma(e^+e^-\to e^-\tau^+)-\sigma(e^+e^-\to e^+\tau^-)}{\sigma(e^+e^-\to e^-\tau^+)+\sigma(e^+e^-\to e^+\tau^-)}. \tag{14}$$

Before presenting the results, a discussion of constraints on various parameters in the model is in order. The constraints on the model from various experiments involving flavor-diagonal neutral currents have been discussed in great detail by He *et al*

(1991), and those continue to apply. However, since the model considered here has an additional sector with flavor violation, there are further constraints coming from non-observation of leptonic flavor-violating processes, which put limits on the mixing of ordinary with heavy leptons.

Flavor-violating Z decays into leptons have been looked for at LEP (Akrawy *et al* 1991), and their non-observation limits the couplings $(G^a)_{ij}(i \neq j)$ in our model. However, more stringent bounds (Choudhury *et al* 1991; Joshipura 1991b) on the mixing come from the flavor-diagonal Z decays into leptons measured at LEP, together with the Schwarz inequality for $(U^a)_{\alpha i}$. These can be shown (Joshipura 1991b) to imply the upper limit $|U^L_{41} U^L_{43}| \leqslant 0{\cdot}0106$. We have assumed the maximum value $|U^L_{41} U^L_{43}| = 0{\cdot}0106$ for our numerical estimates. This corresponds (Joshipura 1991b) to a branching ratio for $Z \to \tau e$ a factor of about 10 less than the bound put by the direct search (Akrawy *et al* 1991).

A further constraint comes from limits on the branching ratio for $\tau^- \to e^- e^- e^+$, which can occur in the model via Z and Z' exchange. Using the experimental limit BR $(\tau \to eee) \leqslant 3{\cdot}8 \times 10^{-5}$ we have calculated the lower limit on $M_{Z'}$ as a function of g'/g, which is shown in figure 1[4]. Also shown in figure 1 is the asymmetry A for this minimum $M_{Z'}$ at $\sqrt{s} = M_Z$ for different values of g'/g. The continuous and dashed lines correspond respectively to $\theta_L = 80°$ and $\theta_L = 45°$. We have taken $\theta_R = 0$, $\delta_L = 90°$, $\sin^2\theta_W = 0{\cdot}23$ and the fine structure constant $\alpha(M_Z^2) = 1/128$. The Z resonance parameters chosen are $M_Z = 91{\cdot}16$ GeV and $\Gamma_Z = 2{\cdot}55$ GeV. The Z' total width in the model is given by

$$\Gamma_{Z'} = \frac{g'^2}{4\pi} M_{Z'}. \tag{15}$$

It is seen from figure 1 that the asymmetry is generally quite large (about 20–25%) near $\sqrt{s} = M_Z$, and can even approach 100% for $\theta_L = 80°$ and small g'/g.

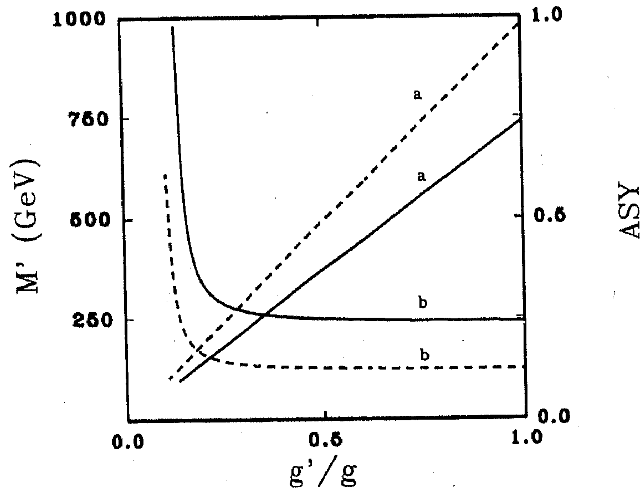

Figure 1. (a) The lower limit M' on $M_{Z'}$ coming from the experimental limit on $\tau \to eee$, and (b) the value of the asymmetry A (labelled ASY) corresponding to this minimum value of $M_{Z'}$ plotted against g'/g. The solid and dashed curves correspond respectively to $\theta_L = 80°$ and $\theta_L = 45°$.

[4]We consider only $M_{Z'} > M_Z$, since for $M_{Z'} \approx M_Z$ the large Z' contribution to the cross section for $e^+e^- \to e^+e^-$ for $\sqrt{s} \approx M_Z$ is in conflict with experiment.

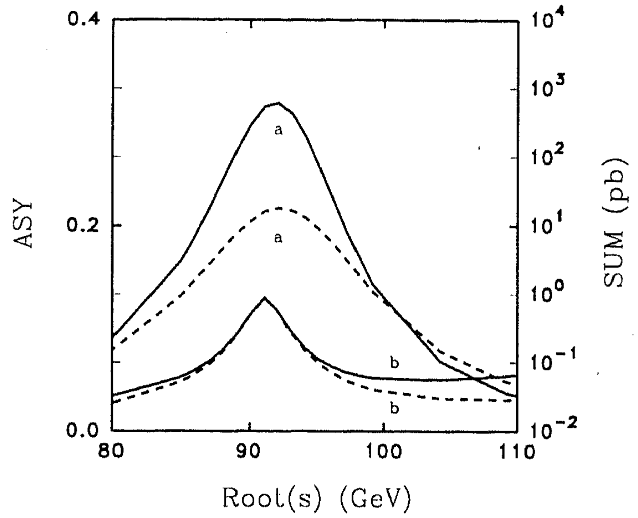

Figure 2. (a) The asymmetry and (b) the sum of $\tau^- e^+$ and $\tau^+ e^-$ cross sections in e^+e^- annihilation plotted against $\sqrt{s}$ for $M_{Z'} = 150\,\text{GeV}$ and $g'/g = 0{\cdot}2$ (solid curve) and for $M_{Z'} = 400\,\text{GeV}$ and $g'/g = 0{\cdot}5$ (dashed curve). Both correspond to $\theta_L = 80°$ and $\theta_R = 0$.

In figure 2, the asymmetry is plotted as a function of $\sqrt{s}$ for somewhat larger values of $M_{Z'}$ compared to the minimum allowed, and for $\theta_L = 80°$. These are more conservative cases. The asymmetry peaks at $\sqrt{s} = M_Z$, where it is fairly large. To get an idea of the number of events expected, the sum of $\tau^- e^+$ and $\tau^+ e^-$ production is also plotted in figure 2 as a function of $\sqrt{s}$. Though the cross section is considerably higher at the Z' peak than at the Z peak, and can provide large flavor violation, the asymmetry is large only at $\sqrt{s} = M_Z$.

The cross section for τe production at the Z peak is in the picobarn range, which should be compared to the total peak cross section, which is about 50 nb. Assuming an overall τe detection efficiency of 0·1, observation of a τe cross section at this level would be possible with $\geqslant 5 \times 10^5$ Z events. The CP asymmetry A would then be observable with a minimum number N_Z of Z events given by $N_Z = [A^2/5 \times 10^5]^{-1}$. For $A \approx 0{\cdot}2$, for example, $N_Z \approx 1{\cdot}25 \times 10^7$. We know however, from figures 1 and 2, that much higher asymmetries are possible. These would easily be observable in future runs of LEP.

We have included the effect of the width of the Z' assuming that it is constant. This is not valid in general[3]. However, the contribution of the $\Gamma_{Z'}$ to asymmetry is negligible at $\sqrt{s}$ near M_Z where the main contribution comes from Γ_Z and Re f_2. Hence, the inclusion of the s dependence of $\Gamma_{Z'}$ does not alter figures 1 and 2. Interestingly, as displayed in figure 2, the asymmetry can be large even if $M_{Z'}$ is significantly higher than M_Z[5]. Hence, even if Z' is not directly observable at present energies, its effect in the CP asymmetry could be observed.

In conclusion, we have explored a novel type of contribution to the CP-violating rate asymmetry in $e^+e^- \to l_i \bar{l}_j$ due to the nonzero width of the Z boson. In models

[3] Strictly speaking, the Breit-Wigner form gives the correct absorptive part of the amplitude only for s-channel exchange (and even then, Γ should be a function of s). However, our numerical results do not change significantly by dropping the imaginary part of $f_m(t)$.

[5] Large asymmetries can result for large $M_{Z'}$ by choosing a correspondingly large g'. We do not need a g' larger than 1 even for $M_{Z'}$ of order TeV in order to get asymmetries at a few per cent level.

with large leptonic flavor violation, observable asymmetries can rise. This effect, if present, should be easily observable at LEP. It is worthwhile emphasizing that similar effects would be present in other processes and models, and should be investigated.

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