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INTRODUCTION TO MODELS OF NEUTRINO MASSES AND MIXINGS

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This review contains an introduction to models of neutrino masses for non-experts. Topics discussed are (*i*) different types of neutrino masses (*ii*) structure of neutrino masses and mixing needed to understand neutrino oscillation results (*iii*) mechanisms to generate neutrino masses in gauge theories and (*iv*) discussion of generic scenarios proposed to realize the required neutrino mass structures.

Key Words: Dirac and Majorana Masses; See-Saw Mechanism; Radiative Mass generation; Grand Unified Theory; Supersymmetry

1 Introduction

Detailed study of neutrino spectrum—their masses and mixing—has now become possible due to significant amount of data from the atmospheric and the solar[1] neutrino experiments. These results are now generally regarded[2] as evidence in favour of non-zero masses for at least two of the three known neutrinos.

The presently available experimental results not only suggest a non-zero neutrino mass but also constrain the patterns of neutrino masses and mixing. These results make it now meaningful to confront various theoretical schemes of neutrino masses with experiments. This review is a short introduction to models of neutrino masses which can explain the presently available results. Since the review is aimed at non-experts, we first discuss the concept of different types of neutrino mass terms. Then we summarize the structure of neutrino masses and mixing demanded by experimental results. Subsequently, we discuss various mechanisms for generation of neutrino masses in gauge theories and finally describe scenarios which are successful in explaining the present day experimental patterns.

2 Neutrino Mass Terms

Neutrino mass corresponds to a Lorentz invariant renormalizable term in the Lagrangian connecting a left and a right-handed field[3,4]. Possible mass terms for neutral fermions can be written in two different ways. These are termed as Dirac and Majorana masses. In order to introduce these mass terms, we need to discuss the charge conjugation property of neutrino fields.

A four component neutrino field ν transforms under charge conjugation as follows:

$$\nu \to \nu^c \equiv \mathbf{C}\nu\mathbf{C}^{-1} = C\overline{\nu}^T \ . \qquad \ldots(1)$$

The Lagrangian for free neutrino field remains invariant if the matrix C is chosen to satisfy

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T \ .$$

C can be chosen to be $i\gamma^2\gamma^0$ in a specific representation for the gamma matrices. In this case one has

$$\nu^c = i\gamma^2\gamma^0\overline{\nu}^T = i\gamma^2\nu^* \ . \qquad \ldots(2)$$

The matrix C satisfies

$$C^\dagger = C^{-1} = C^T = -C \ . \qquad \ldots(3)$$

The following relations are easy to prove using properties of gamma matrices

$$\begin{aligned} (\nu^c)_L &\equiv P_L\nu^c = C(\overline{\nu}_R)^T \equiv (\nu_R)^c \ , \\ (\nu^c)_R &\equiv P_R\nu^c = C(\overline{\nu}_L)^T \equiv (\nu_L)^c \ . \end{aligned} \qquad \ldots(4)$$

It follows that charge conjugate $(\nu_L)^c$ of a left-handed field ν_L is a right handed object and vice versa. Thus if neutrino emitted in a beta decay is left handed then the corresponding anti neutrino would be right handed.

Important point to keep in mind is that although the $(\nu_L)^c$ is right handed, it does not coincide with ν_R which is an independent field with its own dynamical evolution in the massless limit.

A left handed neutrino field ν_L can form a mass term either with its charge conjugate (and hence right handed) field $(\nu_L)^c$ or it can combine with an independent field ν_R. Moreover, ν_R can also combine with its left-handed charge conjugate $(\nu_R)^c$ to give a mass term.

Let us consider a theory containing two fields ν'_L, ν'_R. ν'_R is regarded as an independent field and is different from the charge conjugate of ν'_L. The latter would generally represent any of the neutrino fields corresponding to active (i.e. those having weak interactions) neutrinos $\nu_{e,\mu,\tau}$. ν_R can represent a right handed field unrelated to any of these. Such ν_R would transform as a singlet under $SU(2)\times U(1)$. Alternatively, ν_R can be charge conjugate of any of the active neutrinos, e.g. ν_L may represent ν_{eL} and ν_R may be $(\nu_{\mu L})^c$. We allow both these possibilities.

We can write the following mass terms between ν'_L and ν'_R:

$$\begin{aligned} -\mathscr{L}_{mass} &= \bar{\nu}'_L m_D \nu'_R \\ &+ \frac{1}{2} m_L \overline{\nu'}_L (\nu'_L)^c + \frac{1}{2} m_R (\overline{\nu'}_R)^c \nu'_R + \text{H.c.}\,, \end{aligned} \qquad \ldots(5)$$

where we have used primed fields to distinguish them from the mass eigenstates to be introduced soon. The terms with coefficients $m_{L,R}$ are known as the Majorana mass term and the m_D term is known as the Dirac mass term. The Majorana mass terms are not invariant under global phase changes of the fields since $\nu'_{L,R}$ and $(\overline{\nu'}_{L,R})^c$ transform identically under such changes. In contrast, the Dirac mass term can be made invariant if ν'_L and ν'_R are transformed by the same phase. Thus Majorana mass terms violate Lepton number while the Dirac mass term respects it.

It is seen from eq.5 that neither ν'_L nor ν'_R is a mass eigenstate. Nature of physical neutrino is determined by going to the mass basis. To do this, we rewrite eq.5 as follows:

$$-\mathscr{L}_{\text{mass}} = \frac{1}{2}\begin{pmatrix} \bar{\nu}'_L & ,(\bar{\nu}'_R)^c \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \times \begin{pmatrix} (\nu'_L)^c \\ \nu'_R \end{pmatrix} + H.c. \quad \ldots(6)$$

We have made use of the following relation in writing the above equation.

$$\bar{\nu}'_L \nu'_R = (\bar{\nu}'_R)^c (\nu'_L)^c \,. \qquad \ldots(7)$$

This relation is a special case of a more general identity which is quite useful in many of the algebra related to charge conjugate fields:

$$\bar{\psi}\Gamma_i \chi = \bar{\chi}^c C \Gamma_i^T C^{-1} \psi^c \,, \qquad \ldots(8)$$

where ψ, χ are any two Dirac spinors, Γ_i represents products of the Dirac gamma matrices. Let us rewrite eq.6 as:

$$-\mathscr{L}_{mass} = \frac{1}{2} \bar{n}'_L \mathscr{M}_\nu (n'_L)^c + H.c.\,, \qquad \ldots(9)$$

where $n'_L \equiv (\nu'_L, (\nu'_R)^c)^T$ denotes a column vector for two neutrino states and $\mathscr{M}_\nu$ is a 2×2 matrix:

$$\mathscr{M}_\nu \equiv \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} . \qquad \ldots(10)$$

It is possible to diagonalize $\mathscr{M}_\nu$ through a unitary matrix U

$$U^T \mathscr{M}_\nu U = Diag.(m_1, m_2)\,, \qquad \ldots(11)$$

where $m_{1,2}$ are eigenvalues of $\mathscr{M}_\nu$ given by

$$m_{1,2} = \frac{1}{2}\left(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right), \qquad \ldots(12)$$

Note that $m_{1,2}$ defined above are not necessarily positive. The neutrino mass basis are defined as

$$\begin{pmatrix} \nu'_L \\ (\nu'_R)^c \end{pmatrix} \equiv U \begin{pmatrix} \nu_{1L} \\ (\nu_{2R})^c \end{pmatrix} . \qquad \ldots(13)$$

The new states ν_{1L} and ν_{2R} represent chiral components of two different neutrino states with masses m_1 and m_2 respectively. If CP conservation is assumed U can be taken as an orthogonal matrix which is specified in terms of a mixing angle θ giving us

$$\begin{aligned} \nu_{1L} &= \cos\theta\ \nu'_L - \sin\theta\ (\nu'_R)^c \\ \nu_{2R} &= \sin\theta (\nu'_L)^c + \cos\theta\ \nu'_R \end{aligned} \qquad \ldots(14)$$

with

$$\tan 2\theta = \frac{2m_D}{m_R - m_L} . \qquad \ldots(15)$$

Since masses $m_{1,2}$ can have either sign, let us define $m_i \equiv |m_i|\eta_i$ ($\eta_i = \pm$) and rewrite eq.9 as:

$$-\mathscr{L}_{mass} = \frac{1}{2}(|m_1|\eta_1 \bar{\nu}_{1L}(\nu_{1L})^c + |m_2|\eta_2(\bar{\nu}_{2R})^c \nu_{2R} + H.c.), \quad \ldots(16)$$

where we have made use of eq.11.

We have been writing all mass terms in terms of chiral projections of the fields. We can always define appropriate four component objects and write masses using these new fields. Define

$$\begin{aligned} \chi_1 &= \nu_{1L} + \eta_1(\nu_{1L})^c , \\ \chi_2 &= \nu_{2R} + \eta_2(\nu_{2R})^c . \end{aligned} \quad \ldots(17)$$

eq.16 assumes the following form:

$$-\mathscr{L}_{mass} = \frac{1}{2}(|m_1|\bar{\chi}_1\chi_1 + |m_2|\bar{\chi}_2\chi_2) . \quad \ldots(18)$$

From the definition (eq.1) of the charge conjugation, it is obvious that the fields $\chi_{1,2}$ satisfy

$$\chi^c_{1,2} = \eta_{1,2}\chi_{1,2}$$

Thus both the fields $\chi_{1,2}$ are self-conjugate. Neutrinos described by these fields are called Majorana neutrinos.

We started with two independent two component objects ν_L, ν_R with the most general mass term given by eq.5. This theory could be rewritten in terms of two (four component) objects $\chi_{1,2}$ satisfying Majorana condition. Eq. 5 therefore generically defines Majorana neutrinos. There are three special cases of this equation which are of considerable phenomenological importance. These correspond to the following neutrino mass matrices m_ν:

$$(A1): \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix}; \quad (A2): \begin{pmatrix} m_L & m \\ m & -m_L \end{pmatrix}; \quad (A3): \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}. \quad \ldots(19)$$

The case (A3) with $m \ll M$ is the simplest example of the seesaw mechanism[3,4]. It leads to two masses, one very large $\sim M$ and other m^2/M, suppressed compared to entries in (A3). In particular, one can get the atmospheric mass scale for $m \sim m_t$ and $M \sim 10^{15}$ GeV. The M is the Majorana mass of the field ν'_R and its largeness implies that ν'_R must be an $SU(2) \times U(1)$ singlet field. On the other hand, if all the entries in neutrino mass matrices are small (typically of the order of neutrino masses) then ν'_R can be identified with a charged conjugate of the active left-handed neutrinos. This is exemplified by the cases (A1) and (A2) which are discussed in[2,5–7].

Both (A1) and (A2) have eigenvalues which are equal and opposite corresponding to $\eta_1 = -\eta_2$ in eq.16. We can define in these cases,

$$\psi = \frac{\chi_1 + \chi_2}{\sqrt{2}} .$$

The mass term in eq.18 can then be rewritten as

$$-\mathscr{L}_{mass} = |m_1|\bar{\psi}\psi . \quad \ldots(20)$$

By definition, $\psi \neq \psi^c$ and the above mass term describes a four component Dirac fermion. It is invariant under a phase transformation on ψ. This phase transformation may thus be identified with the lepton number which is conserved by eq.20. The two original fields ν'_L, ν'_R have merged in this case to form a Dirac state ψ.

Both (A1) and (A2) seem to lead to a Dirac neutrino ψ but there is a subtle difference between these two cases. This difference is revealed when charged current interactions are written out[5] in terms of the Dirac field ψ. Explicitly,

$$\begin{aligned} -\mathscr{L}_{ch} &= \frac{g}{2\sqrt{2}}(\bar{e}_L\gamma_\mu (\cos(\theta - \pi/4)\psi + \cos(\theta + \pi/4)\psi^c_L) \\ &\quad + \bar{\mu}_L\gamma_\mu (\cos(\theta + \pi/4)\psi - \cos(\theta - \pi/4)\psi^c_L))W^\mu \\ &\quad + H.c. \end{aligned} \quad \ldots(21)$$

The mixing angle θ appearing above is given by eq.15. It is $\pi/4$ for (A1) while it is arbitrary ($\tan 2\theta = \frac{m}{m_L}$) in case of (A2). It is seen that the W interactions conserve lepton number for (A1) while this conservation is violated for the matrix in (A2) due to simultaneous presence of ψ and ψ^c in eq.21. As a result, components χ_1 and χ_2 of ψ receive different radiative corrections and the Dirac neutrino gets split. Thus the final theory contains a pair of Majorana neutrinos with almost degenerate masses. Such a pair is referred to as pseudo-Dirac neutrino[8].

3 Experimental Constraints on Neutrino Masses and Mixing

Important information on neutrino masses and mixing comes from three sets of experiments (*i*) atmospheric neutrino searches (*ii*) solar neutrino searches and (*iii*) (mostly negative) searches for neutrino oscillations in laboratory. Details of these experiments have been discussed in this volume elsewhere. We give a brief summary of the restrictions on neutrino masses and mixing derived from the detailed analysis of experimental data.

We assume three neutrinos ν_α $(\alpha = e, \mu, \tau)$ mixed with each other. Mixing is determined by a unitary matrix U which is defined through the charged current weak interactions:

$$-\mathscr{L}_{ch} = \frac{g}{2\sqrt{2}}(\bar{l^-_{\alpha L}}\gamma_\mu U_{\alpha i}\nu_{iL}W^\mu + H.C.) \quad \ldots(22)$$

Here l^-_α denote the charged fermions. The flavour neutrino states ν_α are produced in association with l_α and differ from the mass eigenstates ν_i which enter the above equation. The unitary matrix U can be parameterized in terms of three mixing angles θ, ϕ, ω:

$$\begin{aligned} U &\equiv V_l^\dagger V_\nu \\ &= \begin{pmatrix} c_\phi c_\omega & -s_\phi c_\omega & s_\omega \\ c_\phi s_\theta s_\omega + c_\theta s_\phi & c_\theta c_\phi - s_\phi s_\theta s_\omega & -s_\theta c_\omega \\ -c_\phi c_\theta s_\omega + s_\phi s_\theta & s_\phi s_\omega c_\theta + s_\theta c_\phi & c_\theta c_\omega \end{pmatrix}, \end{aligned} \quad \ldots(23)$$

V_l, V_ν are unitary matrices which describe separate mixing among different l^-_α and ν_α respectively. CP violation would introduce three additional phases in this matrix. These phases do not effect the neutrino oscillation probabilities but play an important role in leptogenesis and in determining the effective mass probed through neutrinoless double beta decay experiments. We do not consider these phases in this note.

Neutrino oscillations are sensitive only to the neutrino (mass)^2 differences $\Delta_{ij} \equiv m^2_{\nu_i} - m^2_{\nu_j}$ and there are two independent Δ_{ij} in case of the three generations, $\Delta_\odot \equiv \Delta_{21}$ which control the oscillations of the solar neutrinos and $\Delta_{\mathbf{atm}} \equiv \Delta_{32}$ controlling the oscillations of the atmospheric neutrinos. Experiments demand the hierarchy $\Delta_\odot \ll \Delta_{\mathbf{atm}}$. Specifically, one has

$$\begin{aligned} 4\cdot 10^{-5}\,\mathrm{eV}^2 \le \Delta_\odot \le 2.8\cdot 10^{-4}\,\mathrm{eV}^2; \\ 1.2\cdot 10^{-3}\,\mathrm{eV}^2 \le \Delta_{\mathbf{atm}} \le 5.0\cdot 10^{-3}\,\mathrm{eV}^2. \end{aligned} \quad \ldots(24)$$

The above range in $\Delta_\odot$ corresponds to the large mixing angle (LMA) MSW solution which appears[9] to be the only allowed solution after the positive results from KamLand[10]. Very different values of $\Delta_\odot$ and $\Delta_{\mathbf{atm}}$ lead to a simplification[11]. Due to this hierarchy, the effect of the atmospheric scale gets averaged out in the solar oscillations and the effect of the solar scale is negligible on the oscillations at the atmospheric scale. As a consequence, the oscillation probability in each case is approximately determined by a single mass scale and single mixing angle and assume a simple form in spite of the presence of three generations:

$$\begin{aligned} (P_{ee})_{solar} &\sim 1 - \sin^2 2\phi \sin^2\left(\frac{\Delta m^2_{12}L}{4E}\right) + \mathscr{O}(s^2_\omega)\,, \\ (P_{\mu\mu})_{atm} &\sim 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{23}L}{4E}\right) + \mathscr{O}(s^2_\omega)\,, \\ (P_{ee})_{chooz} &\sim 1 - \sin^2 2\omega \sin^2\left(\frac{\Delta m^2_{23}L}{4E}\right)\,. \end{aligned} \quad \ldots(25)$$

It is seen that the angles θ, ϕ and ω determine the atmospheric, solar and CHOOZ oscillation probabilities respectively. Analysis of the the solar[9] and atmospheric data imply the following restrictions on the mixing:

$$\begin{aligned} \tan^2\phi &\sim 0.2 - 0.8 \\ \sin^2 2\theta &\sim 0.8 - 1 \\ \sin^2\omega &\le 0.04\,. \end{aligned} \quad \ldots(26)$$

The atmospheric neutrino mixing angle is large and can be maximal. In contrast, the strictly maximal solar mixing is not preferred by the data at 3 σ level[9].

It is not difficult to argue that eq.24 allow the following three mass patterns for neutrinos:

$$\begin{aligned} &\text{(A) Hierarchical Masses:} \\ &\qquad m_1^2 \le m_2^2 \sim \Delta_\odot \ll m_3^2 \sim \Delta_{\mathbf{atm}} \\ &\text{(B) Inverted Hierarchy:} \\ &\qquad m_1^2 \sim m_2^2 \sim \Delta_{\mathbf{atm}} \gg m_3^2; \Delta_\odot \approx m_2^2 - m_1^2 \\ &\text{(C) Almost Degenerate Masses:} \\ &\qquad m_1^2 \sim m_2^2 \sim m_3^2 \gg \Delta_{\mathbf{atm}} \end{aligned} \quad \ldots(27)$$

The overall neutrino mass scale cannot be determined from the oscillation results in case (C). Experiments on single[12] and double beta decay[13] can provide important information on this scale. These experiments can also help in distinguishing between three schemes mentioned above since the mass probed in these experiments can at most be at the atmospheric scale for (A) and (B) while it can be much larger in case of the pattern (C). Possible implications of these experiments on neutrino mass patterns have been thoroughly investigated in the literature[14].

The pattern (B) can be best realized[15] if a neutrino pair forms a pseudo-Dirac neutrino discussed in the earlier section. This can occur accidently[16] or can arise due to imposition of some symmetry, typically some $U(1)$ symmetry. Realization of pattern (C) requires[17] imposition of additional- generally non-Abelian- symmetries. We shall discuss these realizations in a subsequent section.

One particular example of the mixing matrix which describes the experimental results is obtained by taking $s_\omega = 0$ and $s_\phi \sim s_\theta \sim \mathscr{O}(1)$ in eq.23:

$$U \equiv= V_l^\dagger V_\nu = \begin{pmatrix} c_\phi & -s_\phi & 0 \\ c_\theta s_\phi & c_\theta c_\phi & -s_\theta \\ s_\phi s_\theta & s_\theta c_\phi & c_\theta \end{pmatrix}, \ldots (28)$$

This mixing pattern is referred to as bi-large or bi-maximal mixing. The present data seem to favour this mixing matrix.

The above considerations imply that models for neutrino masses should answer three basic questions (*i*) Why neutrino masses are smaller than other fermion masses? (*ii*) How does one realize the basic mass patterns (A-C) demanded by experimental results? (*iii*) What are possible mechanisms to simultaneously obtain two relatively large and one small mixing angle? We discuss possible answers to these questions in section 5 after introducing various mechanisms for neutrino mass generation.

4 Mechanisms for Neutrino Mass Generation

There is extensive literature[2] on how to theoretically realize the mass patterns discussed in the last section. We briefly discuss the ideas involved. All the mechanisms for neutrino mass generations finally lead to an effective mass term for the light neutrinos defined as follows:

$$-\mathscr{L}_{mass} = \frac{1}{2}\bar{\nu}_{iL} m_{\nu ij} \nu^c_{jL} + H.c. \quad \ldots (29)$$

Here i, j are generation indices and we assumed only three light neutrinos. m_ν is a complex symmetric 3×3 matrix.

The operator written above is not $SU(2)$ invariant and could arise only after spontaneous breaking of this symmetry. Moreover, it transforms as an SU(2)-triplet. Thus one needs to generate an effective $SU(2)$-triplet Higgs field. There are various ways of doing this which need extension in the Higgs and/or fermion sector of the standard model. Different mechanisms generate m_ν either at the tree level or through radiative corrections.

Tree Level Neutrino Mass

Tree level mass can be generated by adding either Higgs or fermion or both to the standard model fields. *(i) Additional Fermions*: If neutral fermions are added to the SM fields, then the flavour neutrinos can acquire mass by mixing with them. The additional neutrinos can be $SU(2) \times U(1)$ singlets (*e.g.* right handed neutrinos) or doublets (*e.g.* Higgsino in SUSY) or $SU(2) \times U(1)$ triplets (*e.g.* a Wino). Generation of neutrino masses in all these cases has been discussed in the literature.

Addition of three right handed neutrinos N'_{iR} lead to the seesaw mechanism with the following mass terms:

$$-\mathscr{L}_{mass} = \bar{\nu}'_{iL}(m_D)_{ij} N'_{jR} + \frac{1}{2}(\bar{N'}_{iR'})^c (M_R)_{ij} N'_{jR} + \text{H.c.}, \quad \ldots (30)$$

The above equation gives the mass matrix

$$\mathscr{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} . \quad \ldots (31)$$

Both m_D and M_R are matrices in generation space. When M_R is nonsingular and is given by a scale M much larger than that in m_D, we get

$$m_\nu \sim -m_D M_R^{-1} m_D^T . \quad \ldots (32)$$

All the neutrino masses are automatically suppressed due to the large scale M in M_R. A large M is natural in grand unified $SO(10)$ theories which therefore provide a nice framework to understand small neutrino masses. One gets the following mass hierarchy for a diagonal M_R

$$m_1 : m_2 : m_3 :: m_{D1}^2 : m_{D2}^2 : m_{D3}^2$$

Here m_{Di} are eigenvalues of m_D. As long as these eigenvalues are hierarchical, the neutrino masses also display the hierarchy.

(ii) Additional Higgs: Additional Higgs is required to be an $SU(2)$ triplet in order to generate eq.29 through

renormalizable couplings. Such triplet Higgs occur naturally in the left right symmetric theories and their grand unified extensions. This mechanism becomes attractive if triplet vacuum expectation value (vev) can be made naturally small $\sim \mathcal{O}(m_\nu)$. This happens[18] in the above mentioned theories. The left right symmetry automatically demands the presence of a left-handed triplet field Δ_L once a right handed triplet is used to break the $SU(2)_R$ symmetry. Moreover, the vev for these two triplets are also found to have a seesaw relation when scalar potential for such a model is minimized[4, 18]:

$$\langle \Delta_L \rangle \sim \gamma \frac{M_W^2}{\langle \Delta_R \rangle} ,$$

where $\gamma \sim \mathcal{O}(1)$. Because of the above relation, the left-handed triplet automatically acquires a very small vev if the right handed one is at a large scale.[a]

Typical left right symmetric theory will have contribution from Δ_L as well as the seesaw contribution mentioned above. The effective neutrino mass matrix in this case is given by

$$m_\nu \sim \gamma f \frac{M_W^2}{\langle \Delta_R \rangle} - m_D \frac{f^{-1}}{\langle \Delta_R \rangle} m_D^T . \qquad \ldots(33)$$

f here denotes the Yukawa coupling matrix for the left (right) handed neutrinos to Δ_L (Δ_R). This version of the seesaw mechanism[18] is known as the type II seesaw model.

Radiative Models

It can happen that some symmetry (e.g. lepton number) forbids neutrino mass term at the tree level even after extending the standard model fields. Soft breaking of this symmetry may however radiatively induce a finite neutrino mass at one or two loop level. This provides an attractive mechanism for understanding the smallness of neutrino masses. The radiatively induced contribution to neutrino masses may be present in addition to the tree level mass. The presence of radiative contribution in this case may explain the hierarchy among neutrino masses. This happens in case of the supersymmetric theories containing explicit lepton number violating interactions.

Radiative mechanism for neutrino mass generation can be implemented in a number of ways. Nice classification of all these possibilities can be found in ref.[20].Many of these models are variations of the basic mechanism proposed by Zee[21]. This mechanism needs an extended Higgs sector containing a charged singlet field h^+ and a double field ϕ_2 in addition to the standard Higgs doublet ϕ_1. This allows the following additional terms in the SM Lagrangian:

$$\begin{aligned} -\mathcal{L}_{Zee} &= f_{\alpha\beta} \overline{l^c_\alpha} l_\beta h^+ + H.c. , \\ V &= \mu \phi_1 \phi_2 h^+ + H.c. \end{aligned} \qquad \ldots(34)$$

The first equation by itself cannot generate neutrino mass at tree level since it does not contain a bilinear term in neutrino fields. It cannot give rise to radiative mass also since this term conserves lepton number with appropriately defined lepton number for the field h^+. It is not possible to define a conserved lepton number when both the terms displayed above and the charged lepton Yukawa couplings are simultaneously present. In the presence of all these terms, neutrinos obtain radiative masses.

The general structure of neutrino mass matrix obtained in this model is given by

$$\mathbf{m}_{zee} = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix} , \qquad \ldots(35)$$

where

$$m_{\alpha\beta} = C f_{\alpha\beta} (m_\beta^2 - m_\alpha^2)$$

and C is a constant which depends upon the Higgs masses and mixing.

The consequences of this mass matrix have been extensively studied[22, 25, 26]. If one of the parameters $m_{\alpha\beta}$ is suppressed compared to the other two then the above mass matrix has approximate symmetry of the type $L_\alpha - L_\beta - L_\gamma$. The above mass matrix then leads to a pseudo Dirac and an almost massless neutrino desired for phenomenology. But not all such cases can give the correct mixing pattern. For example if $f_{\alpha\beta}$ are of similar magnitudes then $m_{e\mu} \ll m_{\mu\tau} \sim m_{e\tau}$. This case leads to the explanation[25] of the atmospheric neutrino deficit but cannot solve the solar neutrino problem. By allowing hierarchy in $f_{\alpha\beta}$, it is possible to

[a] It is trivial to generalize the above considerations to non-left right symmetric theories also and obtain naturally small vev for the left-handed triplet if a high mass scale is introduced in the theory, see ref.[19].

obtain bi-large pattern and simultaneous solutions to both anomalies[11, 22] but solar mixing angle stays very close to maximal[11, 26] in contrast to the non-maximal mixing required in the LMA solution.

A variation of the above model is obtained by replacing h^+ with a doubly charged field[21, 27] κ^{++}. Such a model does not need addition field ϕ_2 and leads to neutrino masses at two loop level. Phenomenological consequences of this type of model are studied in ref.[28].

5 Neutrino Mass Models

We now give a brief summary of how the above mechanisms are used in arriving at successful model for neutrino masses. Rather then discussing any specific model, we discuss basic scenarios that have been realized through explicit models in the literature. Our choice below is subjective and there exists various alternative scenarios[21, 27, 28] which too can explain the present day neutrino anomalies.

Seesaw Mechanism in Grand Unified Theories

The attractive feature of seesaw mechanism is a strong link existing between neutrino and other fermion masses in grand unified theories. Generic expectations based on this picture are hierarchical neutrino masses and mixing matrix similar to the quark sector. Since the neutrino mixing matrix is known to contain large mixing angle(s) one has to look for ways which break this link without sacrificing inherent quark lepton symmetry. This can be done in a natural fashion in the following ways:

- The strong link between neutrino and other fermion masses existing in type I seesaw, eq.31 does not hold in case of the type II seesaw model if the first term in eq.33 dominates. This dominance occurs quite naturally[4, 18]. The light neutrino masses get related in this case to the structure of M_R rather than to m_D. This structure can be non-trivial allowing large neutrino mixing to coexist with small quark mixing in $SO(10)$. It has recently been argued that large atmospheric mixing arise naturally in this framework and is strongly linked to the $b-\tau$ unification[29]. The neutrino mass hierarchy can also be very different from the type I seesaw models. One can naturally get[17] almost degenerate neutrino masses in this scenario.

- Difference in the quark and lepton mixing matrices can arise naturally in GUT if the charge lepton mass matrix is chosen to be highly asymmetrical[30]. The $SU(5)$ symmetry relates the left-handed charged leptonic mixing to the right-handed down quark mixing. The latter can contain large mixing angles without conflicting with phenomenology. This can happen with asymmetrical M_l. Such choices of the charged lepton and down quark mixing matrix can simultaneously explain the small mixing in the quark sector and the large mixing among leptons. A number of models based on such asymmetrical matrices have been constructed[30].

- In the context of the conventional (type I) seesaw, strong departure between the neutrino and quark masses and mixings get generated in the presence of some structure and/or of different mass scales[31] in M_R. The simplest example is dominance of a single RH neutrino which can lead to large mixing. Both the mixing and mass hierarchy can get altered drastically with a simple texture in M_R. An illustrative and phenomenologically useful example is provided by the following 2×2 texture:

$$M_R = \begin{pmatrix} 0 & M \\ M & M' \end{pmatrix} \qquad m_D = \begin{pmatrix} m_c & 0 \\ 0 & m_t \end{pmatrix}. \qquad \ldots(36)$$

The Dirac matrix m_D has the conventional form typical of any GUT. But the texture of M_R not only leads to large mixing but also changes neutrino mass hierarchy completely. Due to hierarchy in the up quark masses, the light neutrino mass matrix automatically displays[16] an approximate $U(1)$ symmetry and as a consequence, neutrinos are now pseudo-Dirac with almost equal and opposite masses.

It is clear from the above that the seesaw mechanism can easily lead to correct phenomenology without sacrificing the inherent quark lepton universality.

SUSY and Neutrino Masses

Just like grand unified theories, supersymmetry was proposed for reasons unrelated to neutrino masses. But like grand unified $SO(10)$, the minimal supersymmetric standard model (MSSM) automatically leads to verifiable and economical predictions for neutrino masses. Neutrino masses in the most popular version of MSSM are related to the the quark and lepton masses as in case of GUTs but unlike them, the mixing angles among neutrinos are delinked from the quark mixing angles in MSSM. Thus one can have simultaneous explanation of the large neutrino mixing and hierarchical masses[32].

Lepton number violation occur automatically when the standard electroweak model is generalized to the MSSM. The simplest lepton number violating terms are bilinear terms connecting leptonic doublets L_i to a Higgs superfield H_2 with opposite hypercharge:

$$W_\varepsilon = \varepsilon_i L_i H_2\,, \qquad \ldots(37)$$

The three dimensionful parameters ε_i along with parameters of MSSM control three neutrino masses and three mixing angles. It can be shown[33] that the above terms become unphysical and do not lead to any neutrino masses if (1) slepton and Higgs masses and their B parameters are universal at a high scale and (2) if the down type quarks and the charged leptons are massless. The universality is assumed at some scale in major studies of MSSM and it follows automatically in case of the gauge mediated supersymmetric breaking. These versions of MSSM automatically lead to predictive framework for neutrino masses. Universality violations and hence neutrino masses are generated in these theories by the charged lepton and the down type quark masses which determine neutrino masses. Neutrino masses are thus determined by these masses. These have been extensively studied[32,34–37].Neutrinos obtain their masses in two different ways. Universality violation lead to vev for sneutrinos. This generates mixing of neutrinos to neutralinos and leads to seesaw type[38] contribution for one of the neutrinos. The others get their masses radiatively at the one loop. Relative importance of these determine neutrino mass hierarchy. It is found that models with universal boundary conditions can lead to correct hierarchy and large atmospheric mixing naturally but cannot give a large solar mixing angle. Violation of universality is an essential ingredient[36] needed to obtain a large solar mixing angle. It is possible to construct viable models assuming some universality violation[39].

An alternative source of neutrino masses in MSSM is provided by the trilinear lepton number violating couplings in the superpotential. The number of such terms is large but still it is possible to make meaningful statements on the neutrino mass pattern. Purely trilinear couplings also lead to two different types of mass terms, sneutrino vev induced seesaw type contribution[40] and 1-loop contribution. Both these terms play important role in determining neutrino mass hierarchy. The neutrino spectrum can be predictive without making any specific assumption other than assuming that all trilinear couplings are similar in magnitudes. One can get the hierarchical masses and required mixing pattern under this assumption in case of the trilinear couplings[35] $\lambda'_{ijk} L_i Q_j D^c_k$ but the $\lambda_{ijk} L_i L_j E^c_k$ couplings cannot lead to the correct pattern if all are similar[35, 36].

Neutrino Masses and Electroweak Gauge Interactions

The standard electroweak interactions do not violate lepton number. But if some source of lepton number violation and hence of neutrino masses is present then the electroweak interactions can substantially modify the structure of neutrino mass matrix. We already discussed a simple example of this in section 2. As shown there, the neutrinos described by the mass matrix (A2) in eq.19 are degenerate but the splitting between them arises due to the standard gauge interactions. The electroweak interactions in this way can account for the solar neutrino oscillations.

The modification of the neutrino mass spectrum arising due to the electroweak interaction has been extensively discussed[41], see ref.[42] for a review. It is supposed that some source of lepton number violation, e.g. the seesaw mechanism generates light neutrino mass matrix at a high scale. The modification of this matrix arising due to the electroweak interactions in SM or MSSM can be studied using the relevant renormalization group equations[43, 44]. This mechanism has been used to understand some of the puzzling features of neutrino mass spectrum.

- *Large Mixing Angle:* Electroweak interactions can offer a possible answer to why the quarks and lepton mixing angles differ. It was shown[43, 45] that these interactions can amplify a small mixing angle between two neutrino mass states if they have (almost) equal masses and the same CP parity.

- *Small solar mass scale:* Radiative corrections can also explain the smallness of the ratio $\frac{\Delta_\odot}{\Delta_{\mathbf{atm}}}$. One can start with three neutrino mass matrix in which two of the neutrinos are degenerate. In this situation, the atmospheric scale originates in high energy theory but the solar scale gets generated radiatively. This can explain the smallness of the solar scale. This mechanism is found to be quite predictive. If high scale values of $\Delta_\odot$ and U_{e3} are zero (as it happens in several models for neutrino masses[15]) then the radiatively generated solar scale and U_{e3} are insensitive to the detailed structure of leptonic mass matrices and are predicted[6,7] in terms of the low energy variables, e.g.,

$$\Delta_\odot \approx 4\delta_\tau \sin^2\theta_A \cos 2\theta_\odot |m_{ee}|^2 \quad \ldots (38)$$

The above equation correlates the LMA solution with the Majorana mass m_{ee} probed in neutrinoless double beta decay and requires m_{ee} to be close to its present experimental limits.

Summary

We have tried to present an overview of neutrino masses in a simple way. We discussed the neutrino mass patterns, mechanisms to generate them and possible theoretical frameworks which explain unique features of the neutrino mass spectrum. We have come close to determining neutrino masses and leptonic mixing matrix but we are still far from determining the framework which leads to this spectrum. This has to be searched through other possible signals of the basic framework. Grand unified theories and supersymmetry are two possible mechanisms which can explain the neutrino mass spectrum in a natural way. Observing R violation of the right magnitudes would indirectly support supersymmetry as a possible origin. On the other hand it is likely that neutrino masses may originate due to physics at the electroweak scale itself[21,28]. This would leave signals in terms of exotic particles like the singly or doubly charged Higgs singlets.

References

1 S Fukuda *et al* (Super-Kamiokande collaboration) *Phys Rev Lett* **86** (2001) 5651; *Phys Rev Lett* **86** (2001) 5656

2 For reviews of various models see A Yu Smirnov hep-ph/9901208; R N Mohapatra hep-ph/9910365 and hep-ph/0211252; A S Joshipura and Pramana **54** (2000) 119; G Altarelli and F Feruglio hep-ph/0206077; M C Gonzalez-Garcia and Y Nir hep-ph/0206077; A Yu Smirnov hep-ph/0209131

3 E D Commins and P H Bucksbum *Weak Interactions of leptons and quarks* **Cambridge Univ Press** (1983); B Kayser, G Debu and F Perrier *The physics of massive neutrinos* **World Scientific** (1989); C W Kim and A Pevsner *Neutrinos in physics and astrophysics* **Harwood academic publishers** (1993)

4 R N Mohapatra and P B Pal *Massive neutrinos in physics and astrophysics* **World Scientific** (1991)

5 Anjan S Joshipura and S D Rindani *Phys Lett* B **494** (2000) 114

6 Anjan S Joshipura, *Phys Lett* B **543** 2002 276 [hep-ph/0205038]

7 Anjan S Joshipura and S D Rindani, hep-ph/0202064; hep-ph/0211404; Anjan S Joshipura, S D Rindani and N Nimai Singh hep-ph/0211378

8 L Wolfenstein *Nucl Phys* B **186** (1981) 147; S T Petcov *Phys Lett* B **110** (1982) 245; C N Leung and S T Petcov *Phys Lett* B **125** (1983) 461

9 A Bandyopadhyay *et al* hep-ph/00204286 and hep-ph/0212146; V Barger *et al* hep-ph/0204253 and hep-ph/0212126; P C de Holanda and A Yu Smirnov hep-ph/0205241 and hep-ph/0212270; J N Bahcall *et al* hep-ph/0204314 and hep-ph/0212147; G L Fogli, E Lisi and D Montanino *Phys Rev* D **64** 093007 (2001) [arXiv:hep-ph/0106247]; G L Fogli *et al* hep-ph/0212127; C V K Baba, D Indumathi and M V Murthy hep-ph/0201031

10 K Eguchi *et al* KamLand Collaboration hep-ex/0212021

11 A S Joshipura and P Krastev D *Phys Rev* D **50** (1994) 31

12 J Bonn *et al Nucl Phys Proc Suppl* **91** (2001) 273; Extensive discussion can be found in Y Farzan, O Peres and A Yu Smirnov hep-ph/0105105; F Feruglio, A Strumia and F Vissani hep-ph/0201291

13 H V Klapdor-Kleingrothaus *et al Eur Phys J* A **12** (2001) 147; H V Klapdor-Kleingrothaus *et al Mod Phys Lett* A **16** (2001) 2409; C E Aalseth *et al* hep-ex/0202018; H L Harney hep-ph/0205293 and H V Klapdor-Kleingrothaus hep-ph/0205228

14 An incomplete list of the references is given by H V Klapdor-Kleingrothaus, H Pas and A Yu Smirnov *Phys Rev* D **63** (() 2001)73005; H Minakata and O Yasuda hep-ph/9609276; W Rodejohann *Nucl Phys* B **597** (2001) 110 and hep-ph/0203214; F Vissani hep-ph/9904349; S M Bilenky *et al Phys Lett* B **465** (1999) 193; S M Bilenky, S Pascoli and S T Petcov *Phys Rev* D **64** (2001) 053010; R Adhikari and G Rajasekaran *Phys Rev* D **61** (2000) 031301; H Minakata and H Sugiyama hep-ph/0111269, hep-ph/0202003 and hep-ph/0212240; S Pascoli and S T Petcov hep-ph/0205022; S Pascoli, S T Petcov and W Rodejohann hep-ph/0212113 and hep-ph/0209059

15 Anjan S Joshipura *Phys Rev* D **60** (1999) 053002; R N Mohapatra, A Perez-Lorenzana and C A de Pires *Phys Lett* B **474** (2000) 355; R Barbieri *et al Phys Lett* B **445** (1999) 239; Anjan S Joshipura and S D Rindani *Phys Lett* B **464** (1999) 239 and *Eur Phys J* C **14** (2000) 85; H S Goh, R N Mohapatra and S P Ng hep-ph/0205131; L Lavoura *Phys Rev* D **62** (2000) 093011; W Grimus and L Lavoura *Phys Rev* D **62** (2000) 093012 and *J of High Energy Phys* **07** (2001) 045; R N Mohapatra *Phys Rev* D **64** (2001) 091301 hep-ph/0107274; K S Babu and R N Mohapatra *Phys Lett* B **532** (2002) 77

16 G Dutta and A S Joshipura *Phys Rev* D **51** (1995) 5271

17 D Caldwell and R N Mohapatra *Phys Rev* D **48** (1993) 3259; A S Joshipura *Zeit fur Physik* C **64** (1994) 31; A Ioannisian and J W F Valle *Phys Lett* B **332** (1994) 93; E Ma *Phys Rev Lett* **83** (1999) 2514; E J Chun and S K Kang *Phys Rev* D **64** (2001) 09790 and hep-ph/9912524; E Ma and G Rajasekaran *Phys Rev* D **64** (2001) 113012; E Ma *Phys Rev Lett* **83** (1999) 2514 and hep-ph/0201225; R Adhikari, E Ma and G Rajsekaran *Phys Lett* B **486** (2000) 134; M Fukugita, M Tanimoto and T Yanagida *Phys Rev* D **57** (1998) 4429; M Tanimoto *Phys Lett* B **483** (2000) 417 and *Phys Rev* D **59** (1999) 017304; H Minakata and O Yasuda hep-ph/9609276; F Vissani hep-ph/9708483; E J Chun and S Pokorski *Phys Rev* D **62** (2000) 053001

18 R N Mohapatra and G Senjanovic *Phys Rev* D **23** (1981) 165

19 E Ma and U Sarkar *Phys Rev Lett* **80** (1998) 5716

20 K S Babu and E Ma *Mod Phys Lett* A **4** (1989) 1975

21 A Zee *Phys Lett* **93** (1980) 389

22 Y Koide *Nucl Phys Proc Suppl* **111** (2002) 294 [arXiv:hep-ph/0201250]

23 P H Frampton, M C Oh and T Yoshikawa *Phys Rev* D **65** (2002) 073014 [arXiv:hep-ph/0110300]; E Mitsuda and K Sasaki *Phys Lett* B **516** (2001) 47; A Ghosal, Y Koide and H Fusaoka *Phys Rev* D **64** (2001) 053012; K R Balaji, W Grimus and T Schwetz *Phys Lett* B **508** (2001) 301 [arXiv:hep-ph/0104035]; M Matsuda, C Jarlskog, S Skadhauge and M Tanimoto arXiv:hep-ph/0005147; A S Joshipura and S D Rindani *Phys Lett* B **464** (1999) 239 [arXiv:hep-ph/9907390]

24 A Y Smirnov and Z J Tao *Nucl Phys* B **426** (1994) 415 [arXiv:hep-ph/9403311]

25 A Y Smirnov and M Tanimoto *Phys Rev* D **55** (1997) 1665 [arXiv:hep-ph/9604370]

26 B Brahmachari and A S Choubey *Phys Lett* B **531** (2002) 99

27 K S Babu *Phys Lett* B **203** (1988) 132; A Zee *Nucl Phys* B **264** (1986) 99

28 K S Babu and C Macesanu hep-ph/0212058

29 B Bajc, G Senjanovic and F Vissani hep-ph/0210207

30 K S Babu and S M Barr *Phys Lett* B **381** (1996) 202; J Sato and T Yanagida *Phys Lett* B **430** (1998) 127; C H Albright, K S Babu and S M Barr *Phys Rev Lett* **81** (1998) 1167

31 A Yu Smirnov *Phys Rev* D **48** (1993) 3264

32 A S Joshipura and S K Vempati *Phys Rev* D **60** (1999) 095009 [arXiv:hep-ph/9808232]

33 K S Babu and A S Joshipura Talks at the Trieste Conf on ‘Quarks and Leptons: Masses and Mixing’, Oct 1996, Oct 1997

34 R Hempfling *Nucl Phys* B **478** (1996) 3; M A Diaz, J C Romao and J W Valle *Nucl Phys* B **524** (1998) 23 [arXiv:hep-ph/9706315]; S Roy and B Mukhopadhyaya *Phys Rev* D **55** (1997) 7020 [arXiv:hep-ph/9612447]; M Bisset, O C Kong, C Macesanu and L H Orr *Phys Rev* D **62** (2000) 035001 [arXiv:hep-ph/9811498]; K Choi, E J Chun and K Hwang *Phys Lett* B **488** (2000) 145 [arXiv:hep-ph/0005262]; A Abada, S Davidson and M Losada hep-ph/0111332; J C Romao, M A Diaz, M Hirsch, W Porod and J W Valle *Phys Rev* D **61** (2000) 071703 [arXiv:hep-ph/9907499] ; M Hirsch, M A Diaz, W Porod, J C Romao and J W Valle *Phys Rev* D **62** (2000) 113008; E J Chun and S K Kang *Phys Rev* D **61** (2000) 075012 [arXiv:hep-ph/9909429]

35 A S Joshipura and S K Vempati *Phys Rev* D **60** (1999) 111303

36 A S Joshipura, R D Vaidya and S K Vempati *Phys Rev* D **65** (2002) 053018 [arXiv:hep-ph/0107204]

37 A S Joshipura, R D Vaidya and S K Vempati *Nucl Phys* B **639** (2002) 290 [arXiv:hep-ph/0203182]

38 A S Joshipura and M Nowakowski *Phys Rev* D **51** (1995) 2421 [arXiv:hep-ph/9408224]

39 E J Chun, D W Jung and J D Park hep-ph/0211310

40 B de Carlos and P L White *Phys Rev* D **54** (1996) 3427 [arXiv:hep-ph/9602381]; A S Joshipura, V Ravindran and S K Vempati *Phys Lett* B **451** (1999) 98; E Nardi *Phys Rev* D **55** (1997) 5772 [arXiv:hep-ph/9610540]

41 J Ellis *et al Eur Phys J* C **9** (1999) 310; J Ellis and S Lola *Phys Lett* **110** **458** (1999) 389; A Casas *et al Nucl Phys* B **556** (1999) 3; *Nucl Phys* B **569** (1999) 82; *Nucl Phys* B **573** (2000) 652; *JHEP* **9909** (1999) 015; A Dighe and A S Joshipura hep-ph/0010079; N Haba *et al Phys Lett* B **489** (2000) 184; N Haba and N Okamura *Eur Phys J* C **14** (2000) 347; E J Chun *Phys Lett* B **505** (2001) 155; G Dutta hep-ph/0202097 and hep-ph/0203222; N Haba and N Okumura hep-ph/0202097 and hep-ph/0203222; N Haba, N Okumura and M Sugiura *Prog Theor Phys* **103** (2000) 367; S F King and N Nimai Singh *Nucl Phys* B **591** (2000) 3 and *Nucl Phys* B **596** (2001) 81; M Frigerio and A Yu Smirnov hep-ph/02122263

42 P H Chankowski and S Pokorski *Int J of Mod Phys* A **17** (2002) 575

43 K S Babu, C N Leung and J Pantaleone *Phys Lett* B **139** (1993) 191

44 P H Chankowaski and Z Pluciennik *Phys Lett* B **316** (1993) 312; S Antusch *et al Phys Lett* B **519** (2001) 238 and *Phys Lett* B **525** (2002) 130

45 K R S Balaji *et al Phys Rev Lett* **84** (2000) 5034 and *Phys Lett* B **481** (2000) 33