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Airworthiness of aircraft.
Part 2. Monte Carlo simulation of fleet performance history

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Abstract. A computer program using the Monte Carlo technique to simulate aircraft performance in fleet operation (modelled in part 1 of this paper) is described. The technique follows the variation in the performance capability of each aircraft in the fleet over its service life. Arbitrary distributions or values can be specified for the input parameters of the stochastic model; in addition, the effect of certification and inspection procedures can also be studied. The output of the simulation includes the performance history of any specific aircraft, fleet performance distribution and statistics (in-service as well as just after overhauls) and the incident rates.

Such a simulation of the single-engine climb gradient of a twin-engined turboprop aircraft leads to the following conclusions for a typical medium-haul airline fleet of 15 aircraft. The fleet mean rather than the standard deviation is generally sensitive to changes in the operating conditions. The time required for the fleet to relax from its new state to near in-service equilibrium conditions is found to increase from 4 months when maintenance is perfect to about 3 years when the maintenance efficiency (a measure of the extent of performance recovery in relation to new aircraft) is 25%. It is found that the gradient considered acceptable at entry into service strongly influences the incident rates, but that the actual testing procedure adopted for clearing aircraft, such as single test, two-best-of-four etc., has hardly any effect. The incident rates are strongly affected by the maintenance efficiency and the flight scatter, moderately by the mean airframe and engine deterioration and the time interval between overhauls, and marginally by propeller deterioration. It is concluded that current airworthiness codes for engine-out take-off climb drawn up in the fifties are today generally conservative because of improved engine reliability, but may still be necessary for engines going through their 'learning' period.

Keywords. Airworthiness; flight safety; stochastic model; simulation.

1. Introduction

In part 1 of this paper (Narasimha & Ananthasayanam 1978), a stochastic model for aircraft performance history was proposed; in particular, the specific example of engine-out climb of a twin-engine aircraft was discussed in some detail. Although it is possible to solve this stochastic model analytically under certain special assumptions (Ananthasayanam & Narasimha 1976a; see also §§4 and 5), it is often necessary in applications to have a more powerful method that can be used, for example, when (i) the input distributions do not follow simple well-known laws, (ii) the number of variables is too large to be handled analytically by simple methods, (iii) special certification or inspection procedures are adopted, (iv) the steady state is not attained

A list of symbols appears at the end of the paper.

(either because the duration of interest is not long enough or because fleet parameters keep changing), (v) the history during the transient is required, and (vi) maintenance is not perfect. Indeed, many other conditions of a similar nature can arise.

In these circumstances, a computer simulation of the fleet using what are known as Monte Carlo techniques (of the type described by Kahn 1956, Hammersley & Handscomb 1964, Shrieder 1966) can provide very useful answers and insights. The basic principle of the technique is to generate, on the computer, direct realisations of a stochastic process from which the required solution for the system in question can be obtained. This paper describes the application of such a technique to study the problem of aircraft performance in a fleet, with particular reference to the take-off climb phase of flight.

2. Outline of procedure

In the present simulation, we follow closely the gradient history of each aircraft in the fleet. Thus we first introduce new aircraft into the fleet at certain time intervals T_I, ensuring that these time intervals follow the appropriate (specified) distribution. Then each aircraft suffers a gradient loss in service due to airframe, engine and propeller deterioration. The values for the quantities governing performance deterioration are again selected from appropriate distributions following either the random slide or the random walk model described in part 1 The aircraft is withdrawn for an airframe check after having been in service for a certain period of time T_A, which is determined by drawing a random value from the distribution followed by T_A. The fractional improvement at a check in the drag standard is also obtained from a suitable distribution, and the aircraft returns to fleet service with a better gradient capability. Similarly the time intervals T_E and T_P between engine and propeller changes respectively, and the gradient capability after the changes, are also determined by using the necessary distributions and relations. The above process is carried out for each aircraft in the fleet till the end of the specified duration of fleet service, thus achieving what we shall call one ‘ fleet history simulation ’. To reduce the statistical fluctuation in the results to acceptably low values, one has to average over an ‘ ensemble ’ containing a sufficiently large number of fleet histories.

Clearly, the important thing to be ensured in such a simulation is that the various gradients, time intervals, deteriorations, the check and change processes etc., all obey the appropriate distribution functions used as inputs to the model.

In the present simulation the fleet gradient distribution is obtained numerically as follows. The expected range of the climb gradient variation in the fleet is divided into n intervals. Thus, if the range is between $G^{(1)}$ and $G^{(n-1)}$ then the n gradient intervals $-\infty < G < G^{(1)}$, $G^{(1)} \leqslant G < G^{(2)}$, ... $G^{(n-1)} \leqslant G < \infty$, are formed, and a set of counters numbered 1 to n respectively are correspondingly assigned to them. Throughout each history, the gradient of all aircraft in the fleet is sampled and the counter within whose range the sampled gradient lies is stepped up at each sampling. During a time interval Δt, two samples of the aircraft gradient, one at the beginning and the other at the end of the time interval are made. Hence, a count means that one aircraft in the fleet has spent a time $\Delta t / 2$ in the corresponding gradient interval. After a lapse of time t, let N_1, N_2, ... N_n denote the number of counts in each of the

Table 1. Distributions assumed for the various quantities characterising the different components during simulation. fleet size=15; total number of simulations=24; duration of fleet history=120 months

No.	Component description and notation	Distributions appropriate for: Performance contribution at production (per cent)	In service wear or contribution (per cent)	Time interval between overhauls (months)	Performance contribution after overhaul (per cent)
1.	Design value G_0	G_0	0	—	G_0
	Gradient variations due to:				
2.	Airframe g_A	$N(0, \sigma(g_A{}^*))$	$N(\mu(a_A)\tau_A \cdot \sigma(a_A)\tau_A)$	$CN(\mu(T_A), \sigma(T_A))$ in $0 \leqslant T_A < \infty$	For each aircraft $g_A{}^*$ as at production
3.	Engine g_E	$N(0, \sigma(g_E{}^*))$	$N(\mu(a_E)\tau_E{}^{1/2}, \sigma(a_E)\tau_E{}^{1/2})$	$CN(\mu(T_E), \sigma(T_E))$ in $0 \leqslant T_E \leqslant \mu(T_E)$	$N(0, \sigma(g_E{}^*))$
4.	Propeller g_P	0	0	$T_P \equiv T_E$ for each aircraft	0
5.	Flight scatter g_S	0	$N(0, k\sigma(g_S))$	—	0

Functions and numerical values in the 'reference' case**

λ	3(0)
G_0	1.4
$\sigma(g_A{}^*)$	0.07
$\sigma(g_E{}^*)$	0.14
$\mu(a_A)$	0.03
$\sigma(a_A)$	0.01
$\mu(a_E)$	0.07
$\sigma(a_E)$	0.07
$\mu(T_E)$	12
$\sigma(T_E)$	3(0)
$\mu(T_A)$	6
$\sigma(T_A)$	1(0)
$\sigma(g_S)$	0.125
k	2.0
r_{AL}	1.0
r_{AU}	1.0

Note. $N(a, b) = [1/\sqrt{(2\pi)}b] \exp\{-(x-a)^2\}$, where x is the random variable
Values in bracket refer to changes of the parameter in the 'ideal maintenance' case
The recovery factor r_A obeys a uniform distribution between limits r_{AL} and r_{AU}
The time interval between introduction of aircraft obeys the Poisson distribution $P(\lambda)$ with mean of λ months.

above intervals and N their sum, then the overall-averaged probability density in each interval, defined in equation (18) of part 1, is approximated by

$$\bar{p}\,(G^{(i-1)} \leqslant G < G^{(i)}, t) = \bar{p}_i \simeq N_i/N, \tag{1}$$

and the cumulative distribution by

$$\bar{P}\,(G < G^{(i)}, t) = \sum_{j=1}^{i} \bar{p}_j \simeq \sum_{j=1}^{i} N_j/N. \tag{2}$$

To obtain the distribution $\bar{p}\,(G')$ of the achieved gradient one could draw a random value of the flight scatter g_s from its appropriate distribution (see table 1) and add it to every sampled true gradient G, and then step up the corresponding probability distribution counter. However a more expeditious procedure is to convolute the true gradient distribution $\bar{p}\,(G)$ with the flight scatter distribution $p(g_s)$ and obtain the achieved gradient distribution, which procedure is described in detail by Ramani *et al* (1976).

Based on the above probabilities, such statistics as the mean and standard deviation (s.d.) are easily worked out.

As the results for the post-check and post-change distributions and statistics are useful in validating the simulation model by comparison with flight test data and in estimating the maintenance efficiency of the airframe check (Ananthasayanam & Narasimha 1976b), a provision has also been made in the program to sample separately the gradient values after every airframe and engine overhaul, and to step up special counters provided for the purpose

3. Computer simulation program and flow chart

Figure 1 gives the flow chart of the program.

Initially the following information is read in (see also table 1): the total number of aircraft to be introduced (i.e. fleet size), the duration of the fleet history, the various parameters describing all the distributions governing the components of the gradient, the total number of fleet gradient history simulations to be carried out and as many sets of initial random numbers.

The computer simulates a single fleet history through the following steps.

Step A: At the beginning of each history when the time $t = 0$, as also when a new aircraft (which meets the acceptance standard (see § 6.2)) is introduced, the initial gradient G^* is obtained by drawing values for all its components from their appropriate distributions and adding them. In the computer simulation the gradient change due to deterioration can be taken into account in one of two ways as mentioned in § 6 of part 1. If one is using a random walk model, then the deterioration w_i for the ith component is evaluated by drawing a random value from the distribution followed by $w_i\,(\tau_i)$ where τ_i is the time elapsed after the most recent overhaul. If a random slide model is used, then a value of a_i (equation (13) of part 1) is drawn from its distribution only at overhauls and the deterioration w_i is given by multiplying it

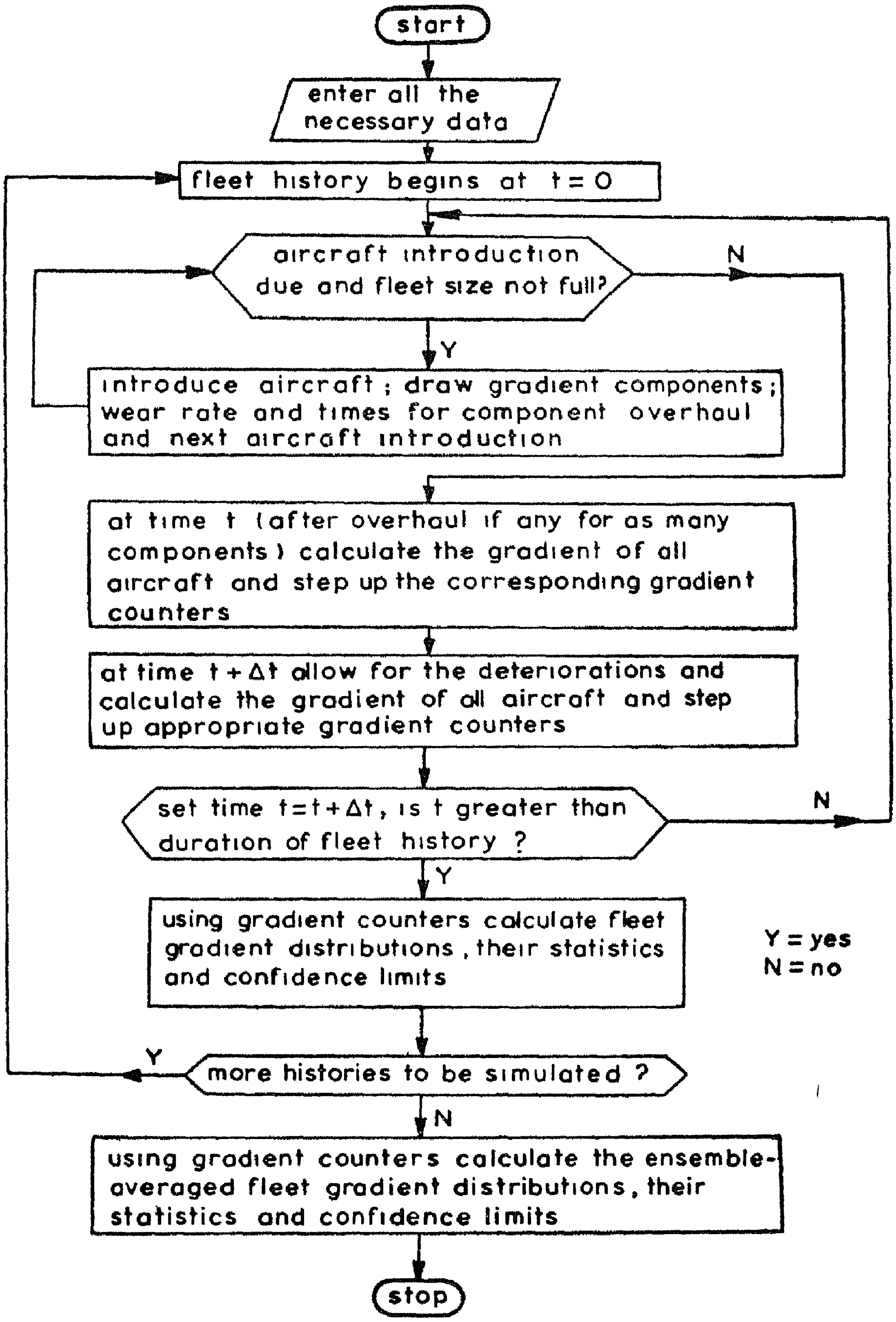

Figure 1. Flow chart of computer program

with $h_i(\tau_i)$, which is a deterministic function of time τ_i. The time intervals T_i before the next overhaul of component i and T_I for the introduction of the next aircraft, are also similarly drawn.

If the fleet has not reached its full size and another aircraft introduction is due, step A is repeated.

Step B: Here the following operations are carried out for all the components of all the aircraft in fleet service.

If a component overhaul is not due, its contribution to the aircraft gradient remains unchanged.

If an overhaul is due (for example on the airframe), a value for the recovery factor r_A is drawn from its own distribution, and the gradient contribution after overhaul is calculated. A new value for a_A (or w_A as the case may be) and the time T_A for the next check are both selected. Similarly if an engine change is due, the value of the relevant gradient contribution from the new engine is drawn from the prescribed distribution, as also the new value for the engine deterioration parameter a_E (or w_E) and the time T_E for the next engine change. A similar process is also carried out when a propeller change is due.

If however both airframe and engine overhauls are simultaneously due for an aircraft, the order in which these are carried out is determined randomly with a probability of $\frac{1}{2}$ for each of the two possibilities. (This is a special but not an essential assumption in the present work.)

After the above operation the aircraft gradient G based on the various contributions is calculated and the appropriate gradient counters are stepped up.

Step C: The time is advanced by $\triangle t$. The changed gradient values of the various components are calculated using the assumed random walk or random slide model. The gradient G of each aircraft in the fleet at this time is then calculated by adding up the changed gradient values of the various components. Then using the values of G of various aircraft, the appropriate counters are once again stepped up.

Step D: Steps A, B and C are repeated for the whole duration of the fleet history. Then the overall-averaged density and cumulative distributions at the end of the history are obtained by using (1) and (2) The fleet statistics such as mean and s.d., and the confidence limits for these parameters as well as for the distributions at various gradient values, are all worked out (Ramani *et al* 1976)

Step E: To reduce the statistical fluctuation in the fleet gradient distributions and their statistics to acceptable limits, one has to average over a large number (ensemble) of fleet histories: 24 were found to suffice in the present case (see § 5).

4. Typical histories of aircraft gradient

Figure 2 shows typical aircraft gradient histories in fleet service using random slide models for the airframe and engine deteriorations. The improvement in the true gradient capability after an airframe check is evident, since some amount of deterioration is always made up. At an engine change, generally there is an improvement,

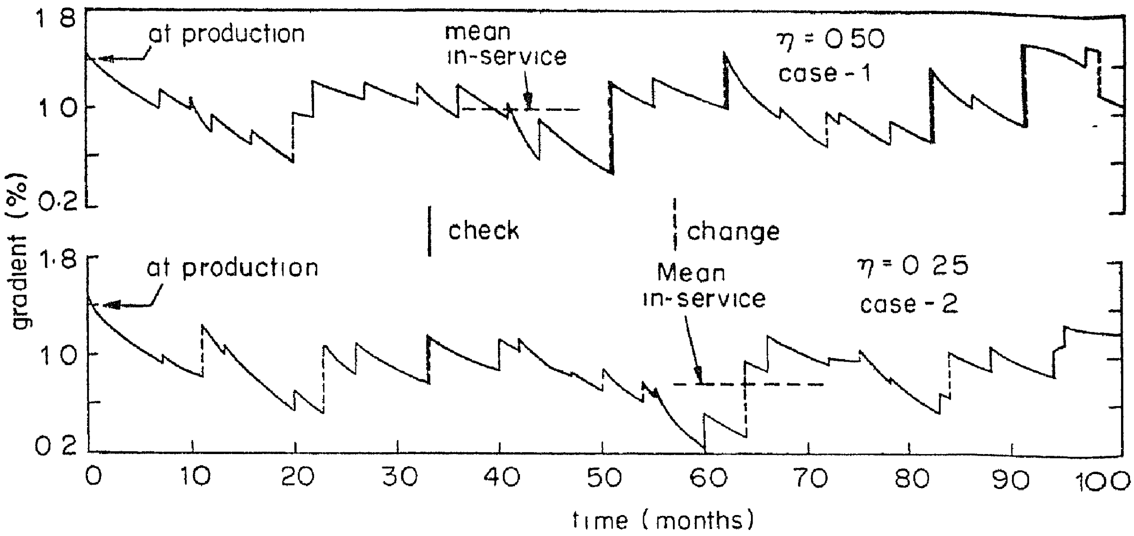

Figure 2. Typical histories of aircraft gradient

but occasionally as is shown in the first trace around $t = 98$ months, the post-change performance is inferior to the pre-change performance. This is because even though the engine deterioration is generally overcome by replacing the deteriorated engine, the new engine could occasionally have a poor initial performance Also an occasional improvement during service (of the kind occurring around $t = 94$ months in Case 2) may also be observed: note that the distributions describing aircraft or engine gradient change do allow, although with a small probability, an improvement in gradient with time. Indeed such changes cannot be ruled out and are not impossible, but they have little effect on the final results (Narasimha and Ramani 1975, unpublished).

Corresponding to the two cases whose mean efficiencies (see (5)) at a check are 0·50 and 0·25 respectively, the mean performance of the fleet both at production and in-service taken from Ananthasayanam & Narasimha (1976c) are also marked in figure 2. One may note that qualitatively the gradient history of each aircraft in both cases is around the in-service values. However, the gradient wanders quite a bit, the reason being that the fleet s.d. is of the order of 0·3 %.

5. Validation of the simulation

Validation has to be of two kinds: one in which the computational accuracy of the results is established, and the second in which the physical basis of the proposed model for the process is tested against observed or measured data.

The accuracy of computation has been established by comparison with the exact solution available for the ' ideal process ' (Ananthasayanam 1976) in which all aircraft enter fleet service at time $t = 0$, airframe checks occur at time intervals T_0 and engine changes at $2T_0$ (every other check therefore coincides with a change), and the performance recovery factor at every airframe check is assumed to be unity. Further details regarding this process and the simulation are given in table 1. The following conclusions are drawn for a 15-aircraft fleet with a service life of 120 months. A sampling time interval Δt = 1 month (which would correspond to nearly 250 hr c flight, typical of a medium-haul airliner) is optimum: a smaller value for Δt woul increase the required computing time in inverse proportion, and a larger valu would become comparable with the time interval between overhauls. Further, random slide model for the deteriorations is preferable to the random walk model a it saves computer time. A gradient span truncated to lie in the range of $-1{\cdot}2\%$ to $2{\cdot}7\%$ (in steps of $0{\cdot}1\%$) is adequate for the study of the enroute performance Also 24 fleet history simulations are required to achieve 5% accuracy in $P(G'_{cr})$ which is the probability of performance failure (see equation (1) of part 1), whence the confidence limits for the fleet mean and s.d. turned out to be $0{\cdot}025\%$ and $0{\cdot}01\%$ respectively. Figure 3 compares the exact results for $P(G')$ and the simulation and the agreement can be seen to be excellent.

As for the validation of the physical model, for a given aircraft performance model and operational pattern, the only parameter available to the modeller for adjustment is the recovery factor at an airframe check, which may be chosen to simulate either an observed history or the post-overhaul performance. Indeed a fairly quick method of estimating r_A from airline performance records is available and such a validation was found to be satisfactory (Narasimha 1976, Ananthasayanam & Narasimha 1976b).

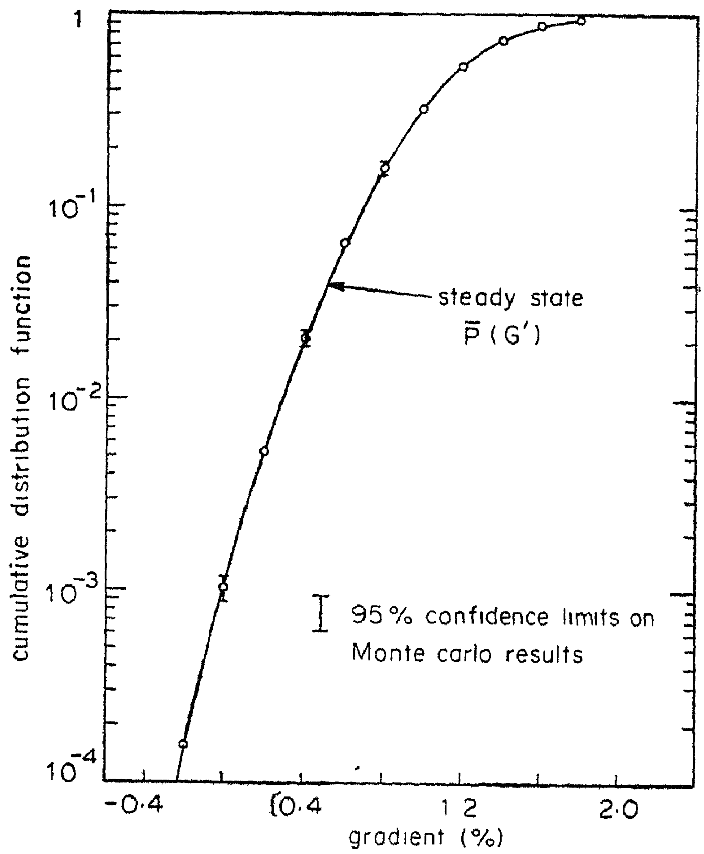

Figure 3. Comparison of Monte Carlo result with exact steady state solution

The analysis of a typical fleet performance using Monte Carlo simulation is presented in the next section.

6. Analysis of a typical airline operation

As illustrative examples of the results obtainable from the simulation program, we consider the operational practices of a typical medium-haul airline. Table 1 lists the distributions based on recorded and inferred data from Narasimha & Ramani (1975, unpublished) which we have assumed here for the parameters and all the other input data for simulation for what we call a ' reference airline' process characteristic of such an airline.

We now discuss in turn the effects of various parameters on the enroute climb performance.

6.1. *Effect of rate of introduction of aircraft into fleet service*

When the parameter T_I was varied to follow a Poisson distribution with mean 0, 3 and 12 months respectively, with all other distributions being held fixed, the resulting achieved fleet gradient distributions showed hardly any difference, and thus we conclude that T_I in general has a negligible effect on the fleet performance (Ananthasayanam 1976).

6.2. *Acceptance procedures for entry into fleet service*

Production aircraft are usually flight-tested before they are accepted for fleet service. If G^* and $(G^*)'$ denote the true and achieved gradient capabilities and G_A the acceptance standard, then some of the possible acceptance procedures are:

(i) any value of G^* (which requires no test!),
(ii) $(G^*)' \geqslant G_A$ in a single test,
(iii) $(G^*)' \geqslant G_A$ in two out of four tests,
(iv) $G^* \geqslant G_A$.

The first procedure needs little discussion. In the next two criteria, the flight scatter during tests would on many occasions help the aircraft to achieve a better performance than its true capability. But in the last criterion, only those aircraft whose true gradient capability is greater than or equal to the critical value is accepted for service, and hence is the most stringent of all However, determination of the true gradient capability of the aircraft calls for a large number of flight tests (see § 5 of part 1). Fortunately, as we shall see below, this criterion when compared with the others does not lead to any significant changes in the fleet performance.

If for the sake of illustration we neglect the gradient changes at production due to differences among propellers (these changes are quite small, in any case; see also table 1), we obtain

$$G^* = G_0 + g_A{}^* + g_E{}^* , \tag{3}$$

and if $\quad G_A = G_0,$

then $$g_A{}^* + g_E{}^* \geqslant 0 \tag{4}$$

in the last criterion.

This means that better airframe and engine combinations are initially selected But if, in subsequent engine changes the performance of the engines installed obeys the same distribution as when new, the ultimate gain in performance can arise only from the choice of better airframes at production. However as the drag variations are generally low, one has hardly any beneficial effects from such discrimination of airframes through various acceptance procedures (Ananthasayanam & Narasimha 1976c).

We must not conclude from the above result that acceptance flight tests can never serve any useful purpose. They can, for example, determine the mean performance, which (as we shall see below) does strongly influence incident rates (see equation (2) of part 1); but if the mean performance were reasonably well known already, the selective advantages of the different procedures are insignificant.

6.3. *Acceptable gradient for certification at production*

When the acceptable gradient at production is itself increased, with the maintenance being still assumed to be perfect, incident rates obviously ought to decrease. Such

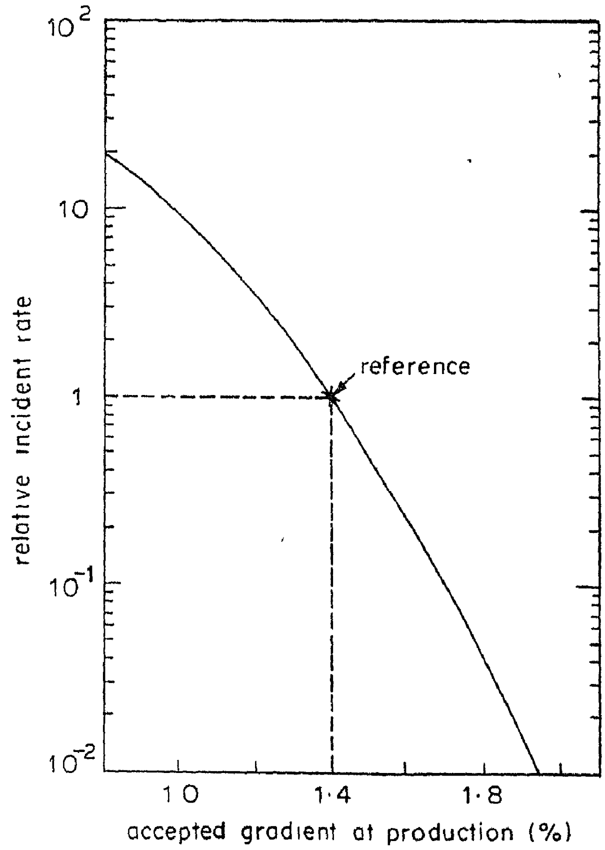

Figure 4. Effect of acceptable gradient at production on relative incident rates

effects of varying G_A are shown in figure 4. When the acceptable gradient is increased by 0·1 %, 0·2 %, 0·3 % and 0·4 % there is a decrease in the performance failure probability by about 2·2, 4·5, 10 and 25 times respectively. Also, a lowering of G_A by the same extent as indicated above leads to a corresponding increase in the performance failure probability of about 2, 3·5, 6 and 9 times. This indicates that the incident rates achieved by a fleet are strongly influenced by the performance considered acceptable for entry into fleet service.

6.4. *Airframe, engine and propeller deteriorations*

6.4a. *Fleet statistics* To study the effect of airframe and engine deterioration each of the parameters governing the distributions characterising these deteriorations was varied from zero to four times the reference value given in table 1, keeping all other parameters fixed at their respective values. The analysis is carried out in terms of the parameters $\mu(a_i)$, $\sigma(a_i)$ describing the normal distributions which the deteriorations are assumed to follow; similarly the $\mu(T_i)$, $\sigma(T_i)$ describing the truncated normal distributions which the time intervals between overhauls are assumed to obey.

We first discuss the variation of the mean gradients. It is seen from figure 5 that changing $\mu(a_A)$ (keeping all other parameters fixed) results in a proportional variation of the mean in-service and post-change gradients. As the mean gradient loss due to airframe deterioration is independent of $\sigma(a_A)$, so are the mean gradients. Increasing the parameter $\mu(T_A)$, which increases the mean time interval between checks, leads to a nearly linear decrease of the mean gradients. As T_A is distributed symmetrically (except for the cut-off at zero) about $\mu(T_A)$, the parameter $\sigma(T_A)$ hardly affects the mean gradients.

Figure 6, referring to parameters governing engine deterioration, has qualitative features very similar to those shown for airframe deterioration. Further, we note from figures 5 and 6 that within the range of all the airframe and engine variables considered, the change in fleet s.d. (square root of the variance) is of the order of one-tenth percent; this is quite small when compared to the general percentage changes in fleet mean.

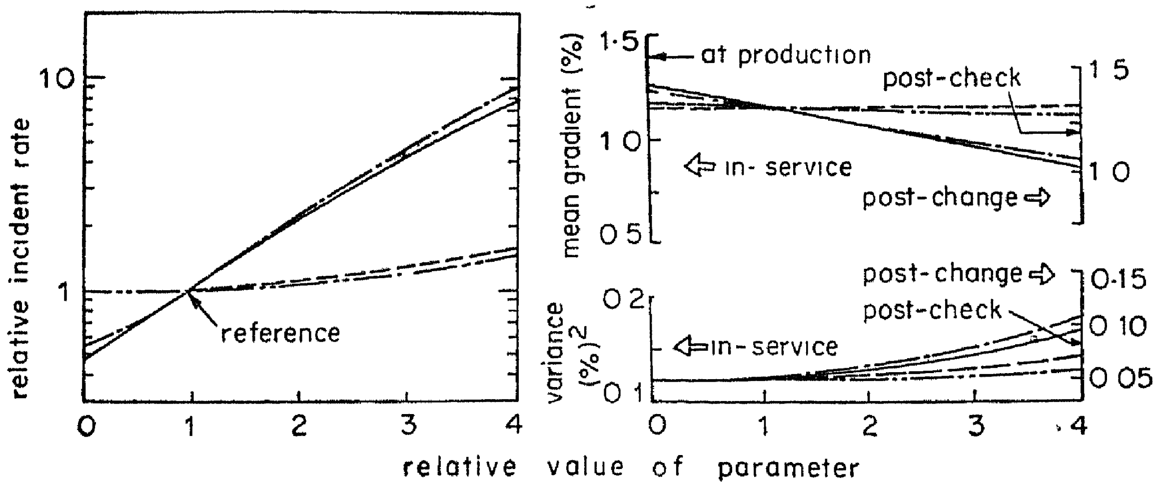

Figure 5. Effect of variables controlling airframe deterioration;—$\mu(a_A)$; - - - $\sigma(a_A)$;—·— $\mu(T_A)$; —··— $\sigma(T_A)$.

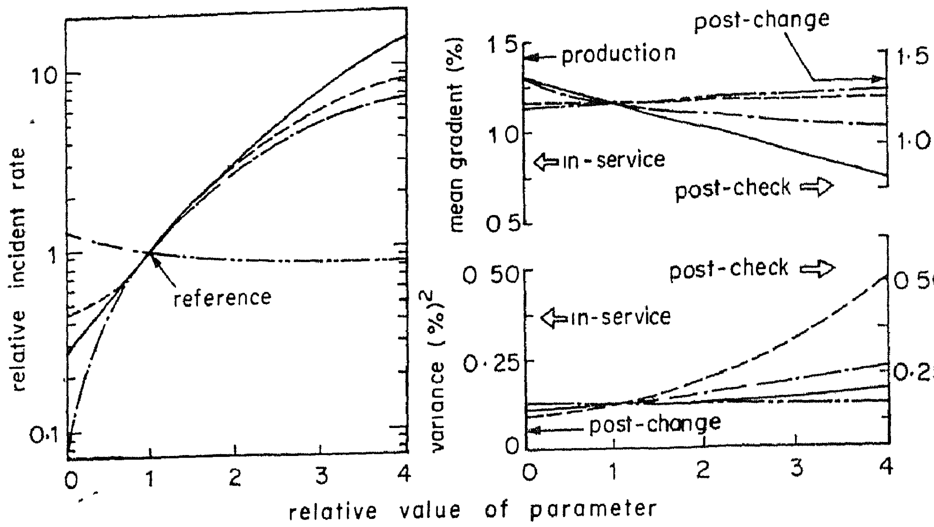

Figure 6. Effect of variables controlling engine deterioration; ——$\mu(a_E)$; - - - $\sigma(a_E)$; —·— $\mu(T_E)$; —··—$\sigma(T_E)$

6.4b. *Relative incident rates* For the enroute case, since $G'_{cr} = 0{\cdot}4\%$ ARB 1966 ICAO 1974, the pr (perf. flr) is evaluated at the above value of the gradient. Fo purposes of discussion, it is useful to define the relative incident rate as

$$\frac{\text{pr (perf. flr. for a given value of the parameter)}}{\text{pr (perf. flr. in the reference case)}}.$$

Figure 5 shows that $\mu(a_A)$ and $\mu(T_A)$ which control the mean airframe deterioration have a reasonable influence on the relative incident rate. $\sigma(a_A)$ has a weak influence presumably because of its small value. Change in $\sigma(T_A)$ appear to balance the effect of checks carried out before and after $\mu(T_A)$ and hence $\sigma(T_A)$ also induces little change in the incident rate.

Figure 6 shows that $\mu(a_E)$ and $\sigma(T_E)$, governing the engine deterioration, have on the other hand, a substantial effect on the incident rate, like the similar airframe deterioration parameters $\mu(a_A)$ and $\mu(T_A)$. But in addition the parameter $\sigma(a_E)$ also has a reasonable effect unlike $\sigma(a_A)$, because $\sigma(a_E)$ is comparable to $\mu(a_E)$ in the reference case. Increase of $\sigma(T_E)$ would lead to more engine changes before $\mu(T_E)$ and hence the incident rate is slightly decreased, and vice versa.

As the propeller is always open to inspection, and its deterioration is chiefly from aerodynamic causes (such as pitting, damage to deicing boots or leading edge strips, etc.), it is reasonable to assume that high deterioration will be immediately noticed. Hence, a uniform distribution was adopted for the gradient loss rate due to this cause and the propeller was also assumed to be changed at every engine change, so that $T_P \equiv T_E$. (This again is a special assumption made in the present work, but is by no means essential.) The performance failure probability with propeller deterioration was found to be about 60% higher than without it, and hence the effect of the propeller on fleet incident rates may be considered at best only marginal (Ananthasayanam & Narasimha 1976c).

7. Initial variations and flight scatter

In general, since g_A^*, g_E^* and g_S obey similar time-independent distributions, their effects are also similar Figure 7 shows that when $\sigma(g_A^*)$, representing the drag differences among new airplanes, is increased nearly four times, the incident rate is roughly tripled Since $\sigma(g_E^*)$ is greater than $\sigma(g_A^*)$ in the reference case, when changed by the same factor it induces a correspondingly larger variation of the incident rate.

When the flight scatter is zero, i.e. if all the airplanes exactly achieve their true gradient capability in flight, then the incident rate is nearly one-tenth of the reference case. Doubling and quadrupling the flight scatter s.d. from its reference value increases the relative incident rate by nearly 5 and 15 times respectively. This means that as the s.d. of flight scatter varies from 0 to 0·5% the incident rate rises by nearly 120 times. For in-service conditions, changing the surprise factor k is equivalent to changing $\sigma(g_S)$ proportionately.

Thus the flight scatter is the variable that has the largest effect on the fleet incident rate. Unfortunately there appears to be very little chance of controlling flight scatter. Further, very little information is available regarding the surprise factor k, which is taken to be a constant. It appears worthwhile to carry out simulator as well as flight tests to know more about the nature of this factor.

8. Effect of airframe recovery factor on fleet performance

Even when the maintenance is perfect there would be some average airframe deterioration present in the fleet. Further, with the present day aeroplanes following a system of equalised maintenance checks in which at best only a fraction of the total deterioration existing in the airframe (since production) is made up, there would be some additional airframe deterioration present even during steady state operational

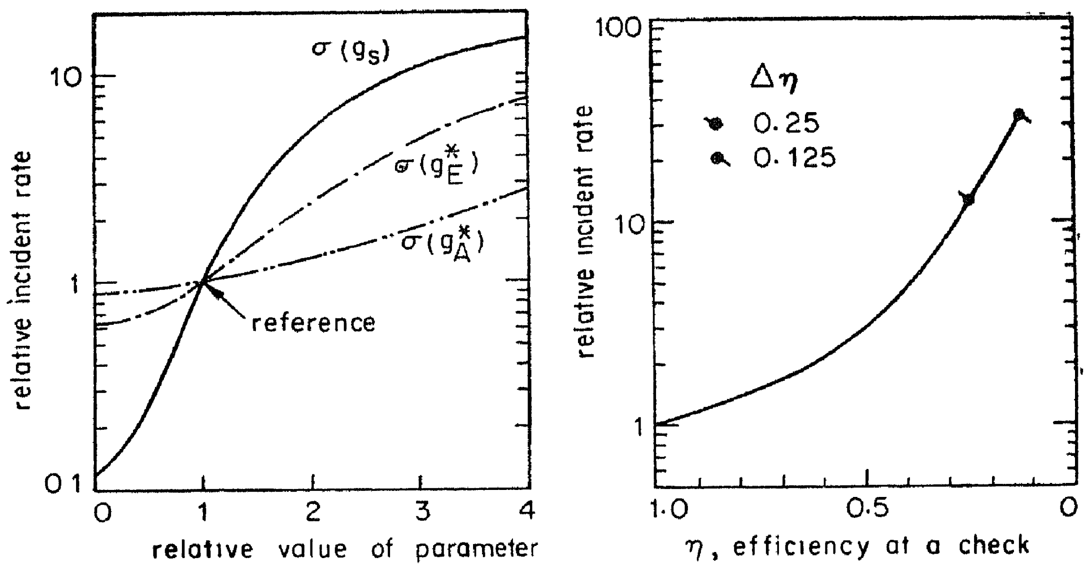

Figure 7. Effect of initial variations, flight scatter and maintenance efficiency

conditions. This additional airframe deterioration due to imperfect maintenai may be called 'drag set'. To study the effect of such maintenance checks, compu simulations were made with the recovery factor r_A (i) having fixed values, in t range $0{\cdot}125 \leqslant r_A \leqslant 1$ and (ii) obeying a uniform distribution between the limits r and r_{AU}. It is useful to consider these results in terms of the variables

$$\eta = m(r_A),\ \Delta\eta = \tfrac{1}{2}(r_{AU} - r_{AL}), \qquad ($$

called the mean maintenance efficiency and its spread respectively.

8.1. *Relaxation of fleet statistics*

The variations of the instantaneous and the time-averaged fleet statistics are shown figure 8 for different efficiencies at a check. Once again, as found in § 6.4a, tl change in the mean is larger than the change in the s.d. for various efficiencies. Wi lower maintenance recovery factors, the fleet achieves lower values for the steac state mean gradient due to the increasing drag set.

A comparison of the exact steady state values (Ananthasayanam & Narasimh 1976a) and the time-averaged fleet statistics are also shown in figure 8. There

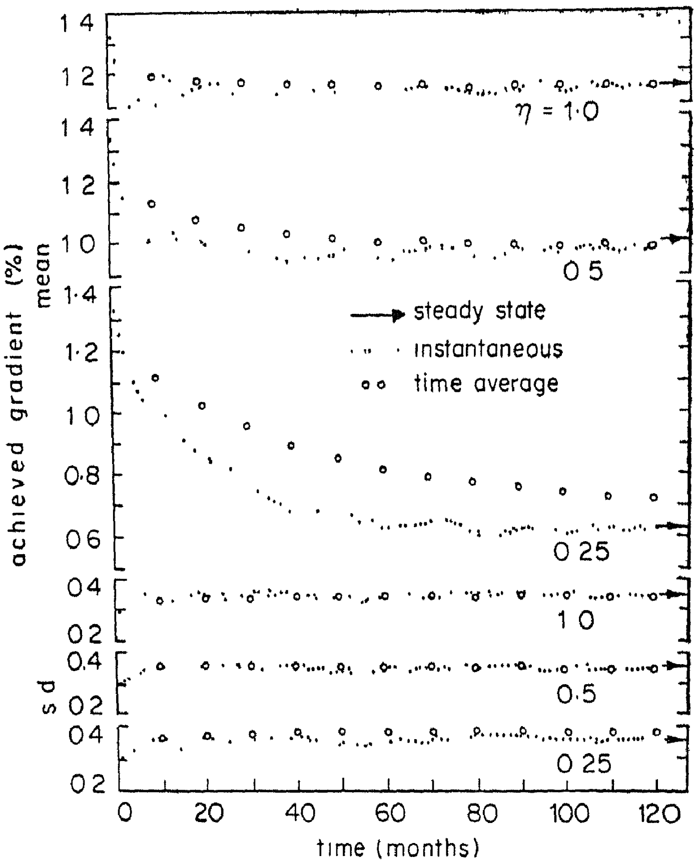

Figure 8. History of fleet statistics for different maintenance efficiencies

agreement between the two results at least upto $\eta = 0{\cdot}5$. For lower values of η, the simulation result has a higher value for the time-averaged mean since the fleet has spent a considerable time during its service life in relaxing to its steady state condition.

To obtain a measure of the time taken to reach the steady state it is interesting and useful to define a relaxation time for the fleet. One possible and convenient definition is the time taken for the fleet performance to reach a specified neighbourhood of the steady state, e.g. within 10% of the difference between the mean gradient at production and in the steady state. It may be seen from figure 8 that the relaxation time varies from about 4 months when $\eta = 1{\cdot}0$ to 3 years when $\eta = 0{\cdot}25$. Also one may note that in all cases the fleet s.d. appears to settle down more quickly than the mean does to the steady state value.

8.2. *Relative incident rates for different recovery factors*

The relative incident rate for different values of η and $\triangle\eta$ are also shown in figure 7. The rapidly increasing performance failure probability with decreasing η (and consequently a higher drag set) is evident. The figure also shows that for a fixed η the spread in the recovery factor $\triangle\eta$ at a check has very little influence on the relative incident rate.

Thus the efficiency at a maintenance check is seen to be a very important parameter controlling the achieved fleet incident rates. It may be noted that one has some control over this parameter unlike the flight scatter

9. Implications for airworthiness codes

The incident rates in the ‘ reference airline process ’ are shown in figure 9 for different engine failure rates (failures include all shutdowns, as mentioned in § 3 of part 1). At the rates of about 3×10^{-4} per hour, which according to ICAO (1953) were characteristic of the power units on the DC-3, the most widely used aircraft at the time of the formulation of the codes, it may be seen that the incident rate of 5×10^{-6} corresponding to $G'_{cr} = 0{\cdot}4\%$ is in the range considered acceptable by ICAO, namely 2×10^{-6} to 7×10^{-6}. However, as may be seen from figure 10, engine failure rates have decreased over the years to values as low as $0{\cdot}12 \times 10^{-4}$ (Anon 1976). Improvements in engine reliability of this kind, which have occurred consistently over the last 20 years, could in principle be used to reduce the performance margin, and hence also the ‘ gross ’ performance requirements. However it should not be forgotten that during the fifties when turbine engines were introduced to power aeroplanes, their failure rates were of the same order as for propeller engines (ICAO 1953). Further, many turbofan engines have had relatively high shutdown rates in the initial stages of operation (Anon 1974).

We may therefore conclude that while the margins required by current codes imply incident rates somewhat below the acceptable limit on engines of proven reliability, and so tend to be conservative, they may nevertheless be necessary to allow for engines whose reliability has not been established, and in particular when new engine types are being introduced.

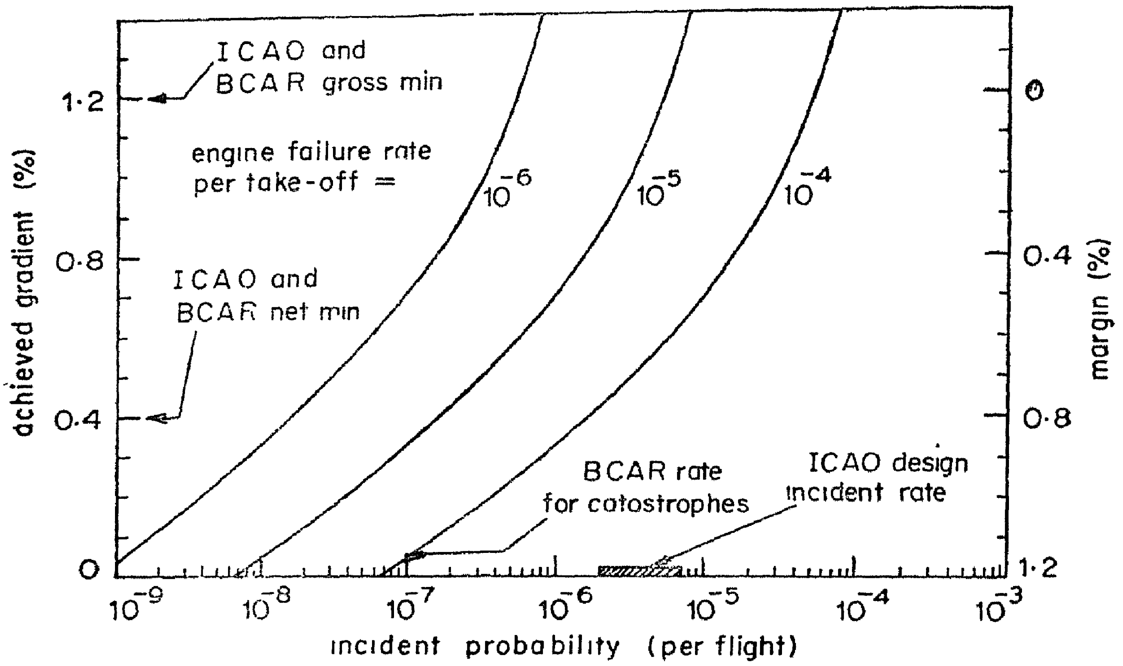

Figure 9. Incident rates in enroute climb

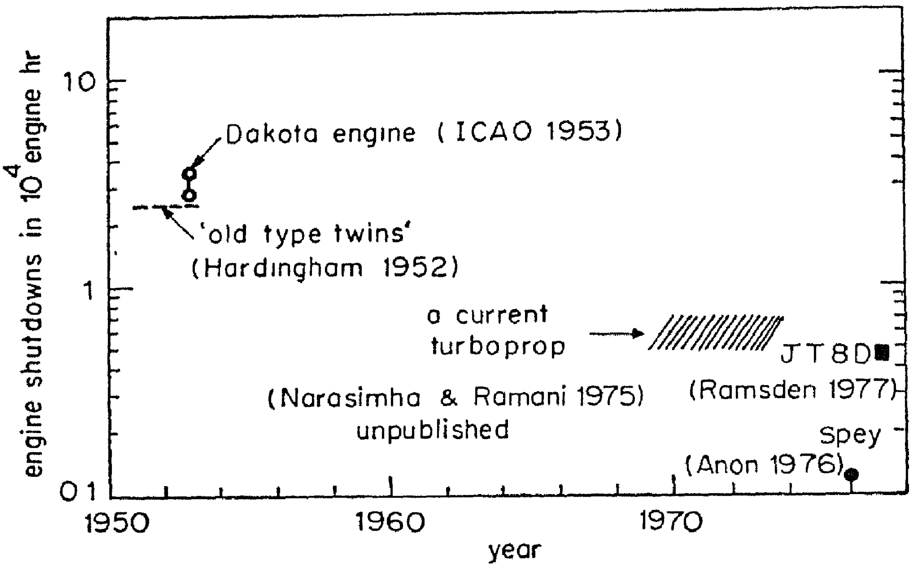

Figure 10. Improvement in engine reliability

10. Conclusions

Though analytical solutions are possible in some simple cases for the stochastic model of take-off climb gradient performance of an aircraft fleet in service, to handle the more general problem of an actual airline operation, a computer simulation of the performance using the Monte Carlo technique is proposed. For an airline fleet of 15 aircraft whose service life is 10 years, with the input parameters obeying appropriate distributions, it is possible to obtain results for the in-service, post-check and post-change gradient distributions and their statistics with acceptable accuracy after 24 fleet history simulations on a computer. It takes about 6 min on the IBM 360/44 computer to generate such simulation results for a single set of input parameters governing such an airline fleet.

When a detailed analysis is carried out of a typical medium-haul airline's operational practices the following picture emerges. The fleet incident rate is strongly governed by the gradient at production considered acceptable for entry into service, but is hardly affected by acceptance flight test procedures such as the single test, two-best-of-four etc. (Such test procedures may still be necessary, however, to ensure that the performance does not actually fall below the acceptable level!). The fleet mean climb gradient is generally more sensitive than the standard deviation to changes in operational parameters like the time interval between overhauls, the mean rate of engine, airframe and propeller deterioration, the maintenance efficiency (as measured by the performance maintained relative to that when new), etc. The mean airframe and engine deteriorations and the mean time interval between checks and changes substantially affect the incident rates, whereas the propeller deterioration has at best only a marginal effect. The efficiency at an airframe maintenance check, and the flight scatter (due to piloting techniques and the flying conditions) are the parameters that have the largest effect on the fleet incident rates. An operator has some control over the former, but unluckily little or none over the latter. Further simulator and flight studies could throw more light on the nature of the surprise factor.

The airworthiness codes formulated in the fifties appear relatively conservative due to the improvement in reliability of the engines over the years, but are still necessary for airplanes flying with engines experiencing ' teething ' troubles and going through their learning period.

The work reported here was inspired by an aircraft evaluation project conducted by Professor S Dhawan; we are grateful to him and to many colleagues who assisted on this project for their criticism. The above work forms part of a Ph.D. thesis by MRA who is especially thankful to the authorities of the Indian Institute of Science and to the staff of the IISc computer centre for their encouragement and help.

List of symbols

a	random variable of the slide model in equation (13) of part 1
C	normalisation constant for the truncated distributions in table 1
G	true climb gradient
G_0	the basic value of the gradient for new aircraft
G'_{cr}	minimum required ' net ' gradient
G'	achieved climb gradient

$G^{(k)}$	gradient values to fix the range of gradient counters $k=0, 1, 2, \ldots n-1$
G^*	climb gradient at production
g	in-service gradient contribution to G
g_S	flight scatter
g^*	gradient contribution to G^* when new
k	surprise factor
$m(x)$	mean value of the variable x
$N(\mu, \sigma)$	normal distribution with mean μ and standard deviation σ
$\mathrm{P}(x)$	cumulative distribution of x
$\mathrm{p}(x)$	probability density function of x
$\mathrm{pr}(X)$	probability of X
r_A	recovery factor at an airframe check
r_{in}	recovery factor for the ith component at the nth check
s.d.	standard deviation
$s(x)$	standard deviation of x
T	time interval between overhauls
T_I	time interval between introduction of aircraft into fleet service
t	time
w_i	performance loss at age τ_i

Greek

δ	Dirac delta function
$\triangle$	small change of quantities
ε	incident probability
τ	age after overhaul

Superscripts

$'$	achieved gradient
$*$	value for new aircraft

Crown

$\bar{\ }$	steady state values

Subscripts

A	airframe
E	engine
P	propeller
i	summation index over A, E or P

Abbreviations

ARB	Air Registration Board
AWTM	Airworthiness Technical Manual
BCAR	British civil airworthiness requirements
ICAO	International Civil Aviation Organisation

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