Quasi-cycles in monsoon rainfall by wavelet analysis

Sudarsh V. Kailas* and R. Narasimha[†,§]

*Centre for Civil Aircraft Design and Development, National Aerospace Laboratories, Bangalore 560 017, India
[†]Fluid Dynamics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Jakkur Campus, Bangalore 560 064 and National Institute of Advanced Studies, Bangalore 560 012, India

Indian rainfall data over the period 1870 to 1990 have been analysed using continuous wavelet transforms. Results obtained are found to be particularly revealing with regard to the temporal structure of the time series if local scaling is used for the transform coefficients, normalizing their values at any wavelet scale with the maximum at that particular scale. Tracking the maxima in such normalized values leads to a picture of seven prominent modes, which are modulated in wavelet scale (and hence also frequency) and in amplitude. Because the modes wander over the parameters of the wavelet map, they cannot always be easily detected by classical spectral analysis. The quasi-cyclic modes so identified exhibit appreciable jitter, and have *average* periods varying from a little less than 2 years to nearly 80 years.

THE nature and causes of the spatial and temporal variability of Indian monsoon rainfall and its interaction with global circulation continue to be among the most interesting questions in climate research. Since the first studies of the fluctuations of Indian rainfall[1], data stretches (of length varying from 25 to 120 years) have continued to be examined for oscillations, short- and long-term trends, distribution of extreme events like droughts and floods, and other characteristics like epochs in rainfall[2–8].

In particular, possible periodicities in the rainfall time series have been sought to be unravelled from different observational data sets as well as numerical simulations. Such periodicities are generally inferred either by classical harmonic analysis using Fourier transforms and associated filtering techniques, or by determination of correlations between the rainfall and other periodic local or global parameters. Among the various cycles so determined are those corresponding to 11 and 22-year periods[4], suggesting a close relation to the sunspot cycle, the quasi-biennial (recently termed 'tropical' biennial) oscillation (QBO)[6] with a period between 2.3 and 2.8 years, and a prominent 60-year cycle found in decadal averages[8].

It is well known, however, that many correlations have varied over time, sometimes even changing sign[6]. This has led to attempts to identify different regimes or 'epochs' from time-series data of monsoon meteorological variables.

However, despite a variety of such studies, the issue of possible periodicities remains controversial[9], and there is as yet no clear picture of the nature of the complex nonlinear interactions that must be responsible for the observed variability. In a different application of wavelet techniques, Torrence and Webster[10,11] have recently used wavelet power spectra and variance time series to study correlations between the El-Niño–Southern Oscillation (ENSO) indices and monsoon rainfall, and have drawn interesting conclusions about the inter-decadal changes in the ENSO-monsoon system. However, they have concentrated on the absolute value of the wavelet coefficients, while we have attempted to identify the various modes in the monsoon system using local scaling on the real part of the wavelet transform co-efficients.

In this first report on a series of wavelet analyses of Indian rainfall, we use the data set of Parthasarathy *et al.*[8], who have formulated the concept of the 'homogeneous Indian monsoon' (HIM) region and compiled the appropriate data on a monthly basis over the period 1871 to 1990. The range of scales over which this data set can be analysed is determined at the lower end by the resolution (not more than 12 data points a year), and at the upper end by its limited length (120 years).

The HIM rainfall data set provides area-weighted rainfall over the northwestern and central parts of India (covering about 55% of the area of the country), known to exhibit a significant degree of spatial coherence. While an appreciable part of the country is thus left out of consideration in this data set, it is also well known that rainfall in different parts of the Indian subcontinent are sometimes anti-correlated; the use of HIM data may therefore be expected to contain the strongest signal present in the Indian monsoons.

Since its introduction by Grossmann and Morlet[12], an extensive literature on the wavelet transform is now available, with many excellent books on this subject[13,14]. Mathematically, the continuous wavelet transform of the rainfall signal $R(t)$ results from its convolution product with a dilated and translated mother wavelet $\psi(t)$,

$$\tilde{R}(t,a) = \frac{1}{\sqrt{a}} \int \psi((t-t')/a) R(t')\, dt'.$$

Here a is a scale parameter and t' is the translation parameter. The mother wavelet $\psi(t)$ is any oscillating mathematical function that fulfills certain 'admissibility' conditions, which are that it must have compact support and zero mean. Many such functions have been proposed and used in the literature. The usual normalization of the wavelet shown in the above formula preserves the $L^2(R)$ norm (or 'energy') and is what is used in the analysis below.

The continuous wavelet transform provides generously abundant information on the signal, often revealing many facets otherwise impossible to determine (as we propose to show). Use of a complex wavelet provides for additional information on the phase of the signal, useful in determining the presence of cycles[15].

[§]For correspondence. (e-mail: roddam@caos.iisc.ernet.in)

The wavelet transforms are determined using NALLETS, a software package developed in VC++ at the National Aerospace Laboratories primarily for analysis of complex flow-turbulence data. For complex wavelets, the package provides a display of either the real or the imaginary part of the wavelet coefficients, or their phase or modulus, or any combinations thereof. The package has been extensively used by the authors in both 1D and 2D applications[16–18].

In the two plots that we shall show below, the wavelet scale *a* is quoted in terms of the support of the wavelet in years (which is a *measure* of the stretch of data windowed by the wavelet at that scale). Furthermore, as the data stretch considered is of finite length (120 years), it is continued on either side of its end-points by the usual technique of periodic mirroring, in order to obtain wavelet coefficients for the whole duration of the data stretch[19]. This procedure permits us to obtain an idea of the 'larger-scale' transform coefficients. Strictly speaking, the Morlet wavelet picks up a cycle of period unity at a wavelet scale about 6.5 times the period (as can be verified from the wavelet transform coefficients reproducing the annual period). Hence, for this wavelet, a wavelet scale of up to six times the length of the data set could be utilized to obtain meaningful information, but we have restricted the analysis here to a maximum wavelet scale of 480 years to ensure that there are no residual effects of so extending the data set.

In displaying contour maps of the wavelet transform coefficients in the (t, a) plane – we shall call this the wavelet map – we have found it convenient to adopt what we shall call 'local' scaling. Here the coefficient is normalized at each value of a with respect to its maximum value at that particular value of a (in contrast to the more common 'global' scaling, where the normalization is with respect to the maximum value of the transform over all a and t). Thus the structure present at each value of a becomes visible. However, interpretation of the wavelet map with local scaling must be made with caution, as a prominent 'high' on local scaling may be quite weak in an overall sense. The identity of the wavelets that make dominant contributions to the original signal cannot therefore be directly inferred from such locally scaled maps.

Figure 1 shows a continuous wavelet map obtained using the Morlet wavelet on the annual HIM data. The top panel shows the actual rainfall. The wavelet map is colour-coded to show contours of the absolute value of the real part of the (complex) wavelet transform coefficient, using local scaling. (The imaginary part behaves very similarly, but with a slight phase shift. We use the absolute value to highlight the quasi-cyclic behaviour of the signal.) The wavelet scale axis is logarithmic, in order to display inter-scale variations better.

One striking feature of the map is the existence of strong 'vertical' organization at various times, best seen usually between the wavelet scales of 10 to 100 years (and occasionally beyond). ('Vertical' here refers to features oriented perpendicular to the time axis in the wavelet map, while 'horizontal' will refer to orientations roughly

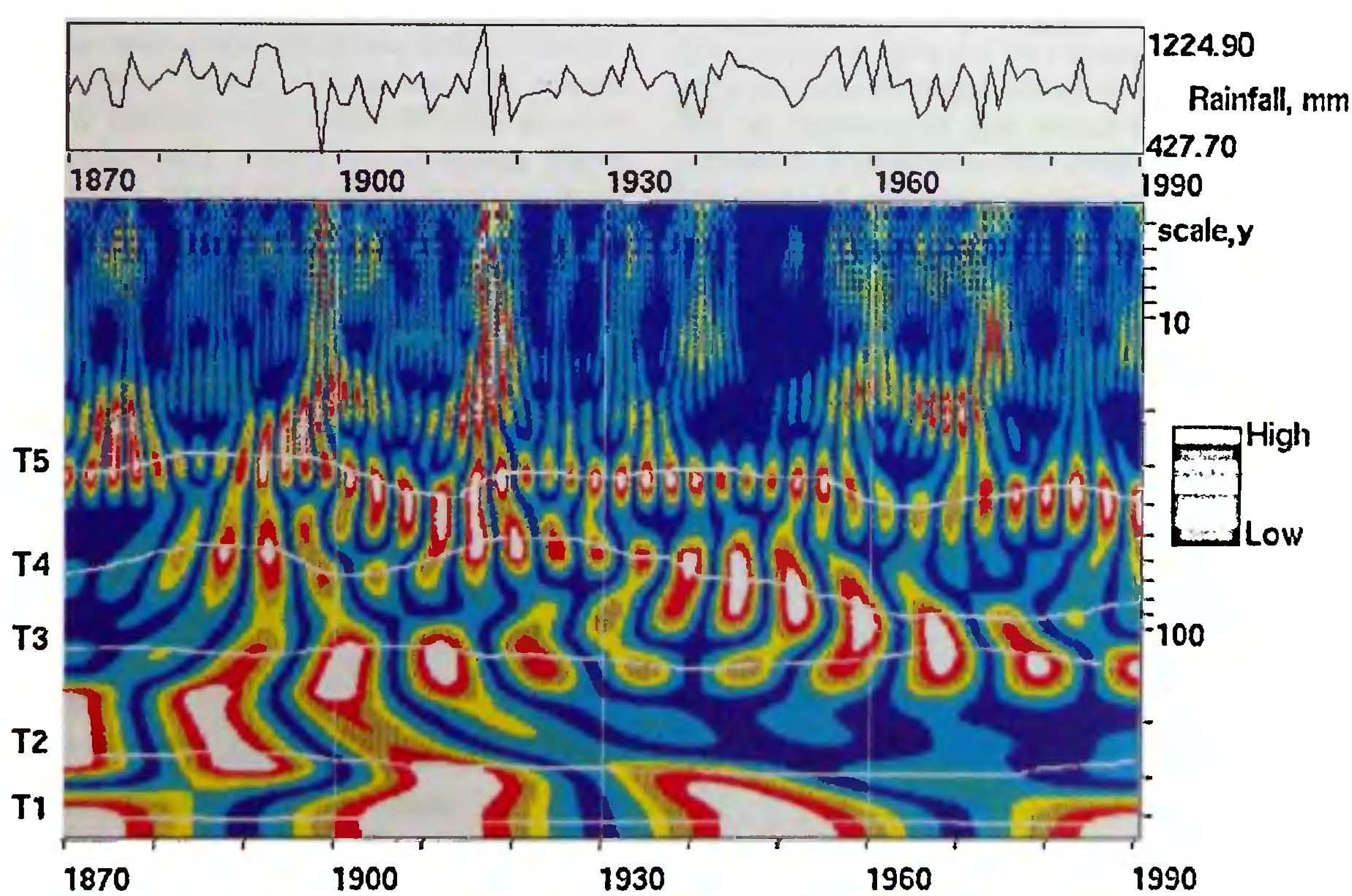

Figure 1. Wavelet map of annual homogeneous Indian monsoon rainfall showing trajectories of the first 5 modes.

parallel to the time axis.) Thus, the drought of 1898, the flood–drought dipole of 1916–17, the flood of 1960 and the drought of 1971 all have strong vertical organization, suggesting the cooperative interaction of what we shall call the modes revealed at various wavelet scales. Other events with weaker organization may also be noticed in Figure 1.

Even more interestingly, the wavelet map shows a striking degree of quasi-horizontal organization. One easily notices regions of high 'activity' which fall into distinct groups roughly aligned along weakly meandering rows; within each group the active regions occur at approximately but not strictly equal intervals. We shall call these groups quasi-cycles. Such quasi-cycles can be missed in classical spectral analysis, as both interval between successive crests in the quasi-cycles as well as their location on the time scale axis vary, so that they do not correspond to a well-defined spectral line.

To identify the major quasi-cycle groups (or modes) revealed in Figure 1, we string together adjacent points of peak activity in each half-cycle (the coefficient shown in Figure 1 being an absolute value, it will possess two crests per cycle). This process yields what we shall call modal trajectories, shown as an overlay in Figure 1. These trajectories are obtained by joining each maximum in the locally scaled wavelet coefficient to its neighbour in the same nearly horizontal row first through a series of short straight lines, and later by a smooth curve generated through a curve-fitting routine.

As can be seen from Figure 1, in many cases the generation of such a curve is surprisingly unambiguous, suggesting strongly the presence of the quasi-cycles already mentioned. There are, however, certain isolated maxima where the situation is not clear (e.g. those around the 300 year wavelet scale); in these cases examination of numerical values often reveals a maximum that may be concealed in the *coarser* contours. A point that must be kept in mind is that a maximum in the locally scaled transform may not necessarily correspond to a maximum with global scaling; in this case the trajectory determined by the present procedure is picking out a preferred point along a *ridge* in the globally-scaled map.

We have shown in Figure 1 five such modes, labelled T1 to T5. To examine shorter-period quasi-cycles, we analyse the monthly data; at wavelet scales greater than 30 years the monthly and annual data yield virtually the same transforms, so Figure 2 is confined to values from smaller wavelet scales. This is seen to lead to a clearer identification of the sixth and seventh modes, indicated as T6 and T7 in Figure 2; these modes are present in Figure 1 as well, but cannot be unambiguously identified there. In general these short-period modes are extremely jittery. In drawing their modal trajectories, we have adopted the following rules. (i) Identify the maxima in the locally-scaled wavelet coefficient nearest to the next lower modal trajectory (e.g. T5 when drawing T6). (ii) Where the maxima are not clear, explore the neighbourhood in detail, and select the maxima whose separation is closest to the inter-crest intervals found on parts of the trajectory that can be unambiguously identified. (iii) Join the maxima so found, and draw a smooth curve through them.

The *average* period of the modes so determined can be obtained by counting crests in the locally normalized wavelet coefficient. The periods so identified are respectively 79, 48, 22, 12.0, 5.8, 3.0 and 1.8 years for trajectories T1 through T7. The seventh mode is extremely jittery but is close to a biennial oscillation. It is intriguing that the seventh and sixth modes have *average* periods towards the limits usually quoted for the QBO; similary the sixth and fifth modes cover between them the periods usually quoted for the ENSO. This makes it difficult to associate any of the 5th–7th modes exclusively with the

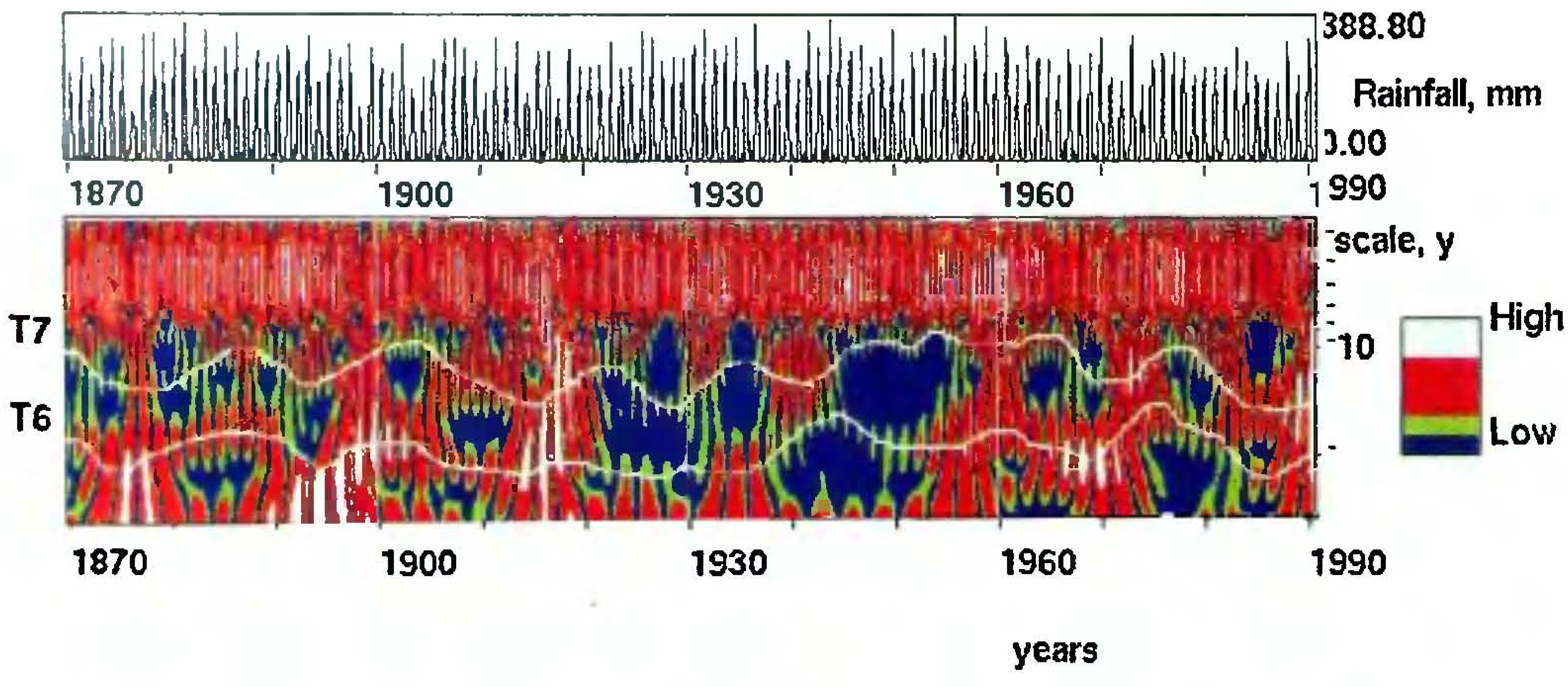

Figure 2. Wavelet map of homogeneous Indian monsoon monthly rainfall up to 30-year wavelet scales, with trajectories of the 6th and 7th modes.

QBO or the ENSO. The second mode is close to the 60-year cycle noted in earlier work. The present analysis suggests the existence of an 80-year quasi-cycle as well; those with average periods of 12 and 22 years invite an obvious association with sunspot cycles.

It is significant that the 120-year rainfall time series data analysed here shows clear evidence of 7 prominent modes in the monsoon rainfall. None of these is strictly periodic, but the allowance for scale modulation in wavelet analysis permits us to identify quasi-cycles (with jittery periods) more clearly than with any spectral method.

A more detailed analysis, showing the evolution of each quasi-cycle over time, and the identification of different regimes in the rainfall time series in different regions of India, will be published elsewhere[20].

1. Blanford, H. F., *India Met. Mem.*, no. 3, 1886.
2. Walker, G. T., *Selected Papers*, Indian Meteorological Society, New Delhi, 1986.
3. Mooley, D. A. and Parthasarathy, B., *Mon. Weather Rev.*, 1983, **111**, 967–978.
4. Ananthakrishnan, R. and Parthasarathy, B., *J. Climatol.*, 1984, **4**, 149–169.
5. Joseph, P. V., Proceedings of the Symposium on Tropical Monsoons, IITM, Pune, 1976, pp. 378–387.
6. Krishna Kumar, K., Ph D thesis, IITM, Pune, 1997.
7. Shukla, J., in *Monsoons* (eds Fein, J. S. and Stephens, P. L.), Wiley, New York, 1987, pp. 399–464.
8. Parthasarathy, B., Rupa Kumar, K. and Munot, A. A., *Proc. Indian Acad. Sci.* (*Earth Planet. Sci.*), 1993, **102**, 121–155.
9. Burroughs, W. J., *Weather Cycles*, Cambridge University Press, Cambridge, 1992.
10. Torrence, C. and Webster, P. J., *QJRMS*, 1998, **124**, 1985–2004.
11. Torrence, C. and Webster, P. J., *J. Climate*, 1999, **12**, 2679–2690.
12. Grossman, A. and Morlet, J., *SIAM J. Math. Anal.*, 1984, **15**, 723–736.
13. Chui, C. K., *An Introduction to Wavelets*, Academic Press, 1992.
14. Meyer, Y., *Wavelets: Algorithms and Applications*, SIAM, Philadelphia, 1993.
15. Kronland-Martinet, R., Morlet, J. and Grossmann, A., *Int. J. Pattern Recog. Artif. Intell.*, 1987, **2**, 97–126.
16. Anandkumar, K. and Kailas, S. V., *J. Aerospace Engg., Proc. Intn. Mech. Engg., Part G*, 1999, **213**, 143–162.
17. Kailas, S. V., Bhat, G. S. and Kalavathi, K., Proceedings of the 6th Asian Congress on Fluid Mechanics, Singapore, 1995, pp. 993–997.
18. Kailas, S. V. and Narasimha, R., *Expts. Fluids*, 1999, **27**, 167–174.
19. Barlaud, M. (ed.), *Wavelets in Image Communication*, Elsevier, Amsterdam, 1994, pp. 99, 196.
20. Kailas, S. V. and Narasimha, R., (to be published).

ACKNOWLEDGEMENTS. We are grateful to Profs. S. Gadgil, J. Srinivasan and B. N. Goswami for their helpful comments on this work.

Received 12 October 1999; revised accepted 19 January 2000