

Does a hole squeal: Quantum black holes and information loss*

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Introduction

The 'Vaidya-Raychaudhuri Endowment Award Lecture' for 1994 has the added significance that it is also the 50th year of discovery (invention) of the Vaidya metric. This is the third Vaidya-Raychaudhuri Endowment Award (VREA) lecture. The second VREA lecturer complained of a feeling of inadequacy and noted the difficulty in saying something befitting the stature of these two doyens of relativity: Prof. P. C. Vaidya and Prof. A. K. Raychaudhuri. The person who expressed these feelings was none other than Prof. Jayant Narlikar. If he had such feelings, you can imagine mine. Profs. Vaidya, Raychaudhuri, Mukunda and Narlikar are enough to overawe many of us. As if they are not enough, I find Sir Fred Hoyle in the chair today! My only consolation is that I may be simplifying the task of the next VREA lecturer by reducing the level of expectations. Prof. Dirac once said that the advent of quantum mechanics enabled even mediocre persons to do great work in the 1930s. Taking a cue from that, I thought I would talk about 'Black hole evaporation and unitarity violation' and hope that the excitement of the subject may cover other inadequacies. Recent developments in this area, though largely unsuccessful, have served not only to raise important and exciting issues but seem also to have broken down the 'Berlin Wall' and ended the cold war between particle physicists and general relativists. I have found this problem a fascinating one and also one which seems to have a deep relationship with the foundations of quantum theory.

The black hole solution

A year after Einstein gave the final version of general relativity, Schwarzschild¹ in 1916 gave a solution of Einstein's field equations (now known more generally as the black hole solution) which has still not been

understood fully. The usefulness and importance of the Schwarzschild solution is unquestioned. One is still left marvelling at its various features and the surprises it can provide. One of the first surprises was its incompatibility with Mach's principle, which Einstein thought he had incorporated in his theory. In the second VREA lecture Prof. Narlikar discussed the attempts by him and Prof. Hoyle to incorporate Mach's principle into the theory of gravitation. Even today the Schwarzschild solution continues to be a rich source of study as one tries to reconcile general relativity and quantum theory.

The Schwarzschild solution can be written as the metric (we use units with $G = c = 1$)

$$ds^2 = -(1 - 2M/r) dt^2 + (1 - 2M/r)^{-1} dr^2 + dr^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$

Its use in working out the classical tests like bending of light and precession of the perihelion of Mercury are well known to many. The singularity at $r = 2M$ of the above metric attracted attention right from the beginning. It is well known that it is not a physical singularity, as physical quantities of interest are well behaved at $r = 2M$, and so it must be a coordinate singularity and a change to other suitable coordinates does remove the singularity. However, to a distant or asymptotic observer, the surface $r = 2M$ is still a one-way membrane and nothing can escape to the outside world from inside this surface, usually called the event horizon and more popularly as the black hole.

Almost 45 years later in 1960, Kruskal² gave the coordinate system which provided the natural extension of the Schwarzschild metric and opened the way to many other worlds that the metric contains (see Figure 1). We use

$$U = -4M \exp [(r^* - t)/4M]$$

and

$$V = 4M \exp [(r^* + t)/4M]$$

with

$$\int dr^* = \int dr/(1 - 2M/r),$$

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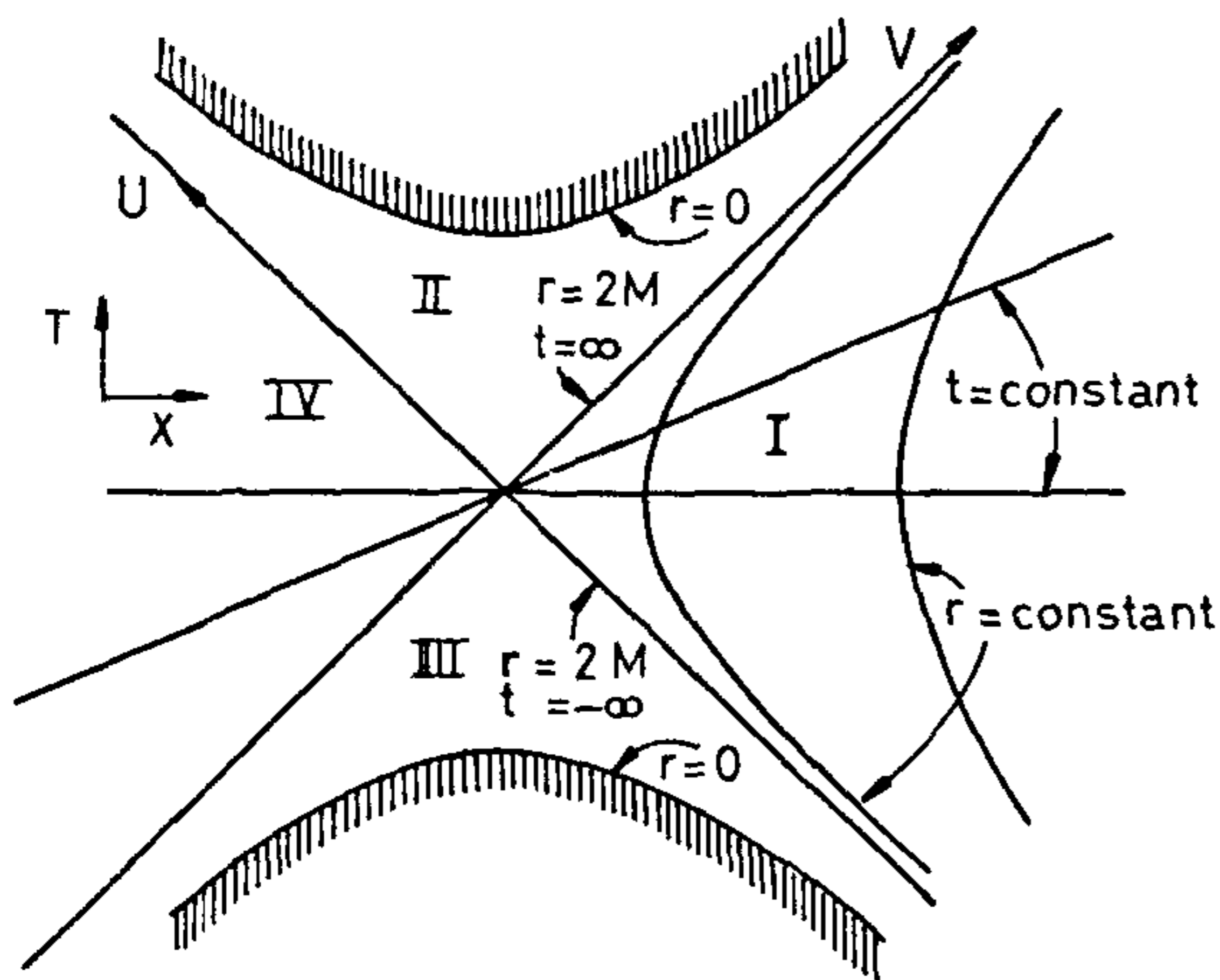


Figure 1. The Kruskal extension of Schwarzschild space-time.

which gives $r^* = r + 2M \ln(r - 2M)$. In the coordinates $T = U + V$ and $R = V - U$, the Kruskal space time is shown in Figure 1. In addition to the familiar region $r > 2M$, where the orbits for the classical tests were studied and which is now called Region I, we have three more regions. While one can go from Region I to Region II, one cannot travel in the opposite direction, thus making the $r = 2M$ surface a one-way membrane. Signals emitted by an observer crossing the horizon are slowed down and a star collapsing to form a black hole seems to take an infinite amount of time to go through the horizon. This created doubts about the very formation of a black hole. One can, however, operationally define, for a given small amount of energy that can be detected, a finite time beyond which no signal is received. There are other unfamiliar or bizarre aspects. There is a time-reversed region (Region IV) and there is a throat or bridge which connects Regions I and IV for a short time³. By using suitable coordinate transformations, Penrose was able to show the infinite regions in a compact way. The Penrose diagram for the Schwarzschild case is shown in Figure 2 (ref. 3).

Twenty years ago, in 1974 came the biggest surprise. Hawking⁴ showed that black hole is not really black. It emits thermal radiation, now called Hawking radiation. This was the culmination of several related developments, now known collectively as black hole thermodynamics. These developments pointed to a close connection between the area of the event horizon and its entropy, which in turn gives a certain temperature to the black hole.

After a few years of intensive study, general relativists seem to have accepted the correctness of Hawking's arguments. There is, however, no hope of an observa-

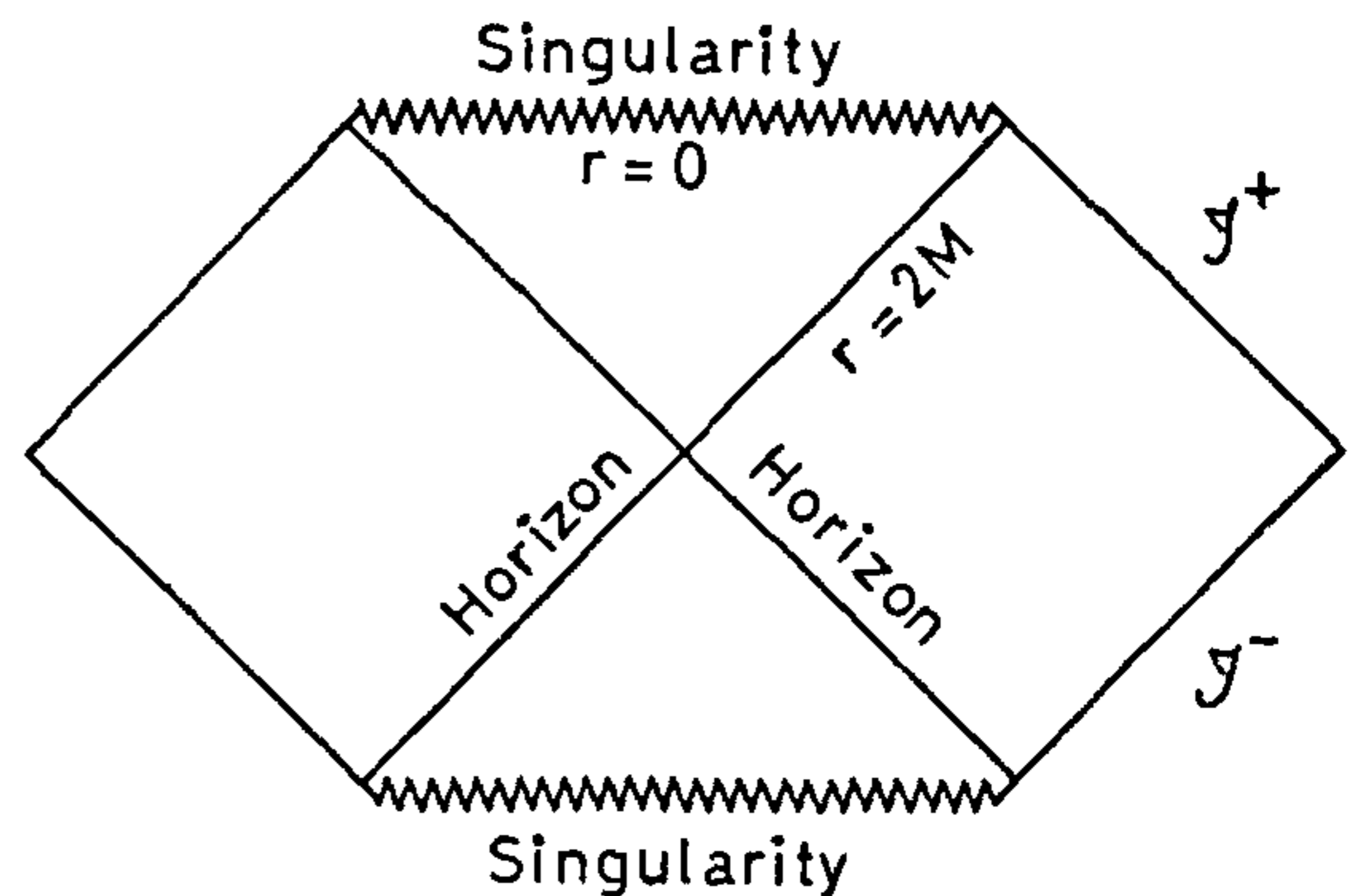


Figure 2. Penrose diagram.

tional verification in the near future, which makes the subject closer to 'mathematics' than to 'physics'. Hawking's derivation is based on semiclassical quantum field theory, which treats gravity as a classical field while treating the other fields using quantum mechanics.

Vaidya (metric) and Raychaudhuri (equation) to the rescue

It may be appropriate to digress a little bit and describe the way Vaidya metric⁵ and the Raychaudhuri equation⁶ were used to settle one of the controversial points about Hawking radiation in 1980-81. In 1980, Tipler⁷ questioned the 'static' approximation under which Hawking had derived the presence of radiation. Tipler argued that due to back reaction the collapse of the radiating black hole takes place either in a very short time (of the order of 1 s for a black hole of one solar mass) or does not take place at all! He concluded that Hawking radiation was not a realistic phenomenon. For his arguments, he used for the time dependence of the horizon an equation obtained from the Raychaudhuri equation which can be reduced, when shear and vorticity are absent, to the form

$$d^2r/dt^2 = 2L dr/dt - 4\pi \langle T_{ab} \rangle l^a l^b r,$$

where $L = 1/8M$ and l^a is the tangent vector to a null geodesic generator of the horizon. Using the fact that initially the horizon is static, he took $d^2r/dt^2 = 0$ (incorrectly as it turned out), implying $dr/dt = \text{const}$, so that

$$2L dr/dt = 4\pi \langle T_{ab} \rangle l^a l^b r.$$

For a Hawking black hole, T_{ab} at the horizon is known to be negative. At the horizon there is an ingoing flux of negative energy which balances the outgoing Hawking radiation at large distances. Tipler then argued that

very soon d^2r/dt^2 becomes positive as r decreases, as $L = 1/r$ and $\langle T_{ab} \rangle \approx 1/r^4$, and the horizon would start expanding again unless the singularity is reached before that. This gave a lifetime of the order of a second and made the static approximation questionable. Tipler concluded that a black hole does not evaporate.

Hajicek and Israel⁸ and Bardeen⁹ obtained the equation for r as a function of time directly from Vaidya's radiating metric⁵

$$- [1 - 2m(r)/r] dt^2 + 2 dv dr + r^2 dr,$$

and showed that d^2r/dt^2 never becomes zero but stays negative all the time. This showed that taking $d^2r/dt^2 = 0$ in Raychaudhuri's equation was not justified. Thus, the wrong use of Raychaudhuri's equation was discovered by the correct use of Vaidya's equation! It is not only at the IAGRG meetings that these two doyens cooperate to help in the amicable settlement of disputes! There is also an unusual derivation of Hawking radiation using the quasinormal modes of the Vaidya metric by York¹⁰. But I shall not discuss it here.

Information loss problem

Hawking was among the first to realize that black-hole evaporation poses a serious problem in preserving unitarity in time evolution of a quantum-mechanical state. It had been known earlier that information that went into a black hole was lost, but the presence of an event horizon made this unobjectionable. Black-hole evaporation (without any remnant singularity) would remove the event horizon, but does it also give back the information that went into the black hole? A purely thermal Hawking radiation cannot carry and give back any information and so in the process of black-hole evaporation we expect to lose information. In quantum-mechanical language, a pure state goes into a mixed state and there is violation of unitarity (Figure 3).

One might think that in a macroscopic phenomenon like black-hole evaporation, it is difficult to keep track of all the degrees of freedom involved. But we know that in the absence of the effects of gravitation this can, in principle, be done, like when a large block of ice melts or a bomb explodes. According to the standard rules of quantum field theory, in a fixed Minkowski space-time the time evolution of any system from a given initial state is described unambiguously by a unitary transformation acting on the state. This implies that there is no loss of fundamental, fine-grained information. Hawking argued that this is no longer true in the presence of a black hole. The main problem is to know what happens to the black hole when all its mass is radiated away, and what happens to all the information

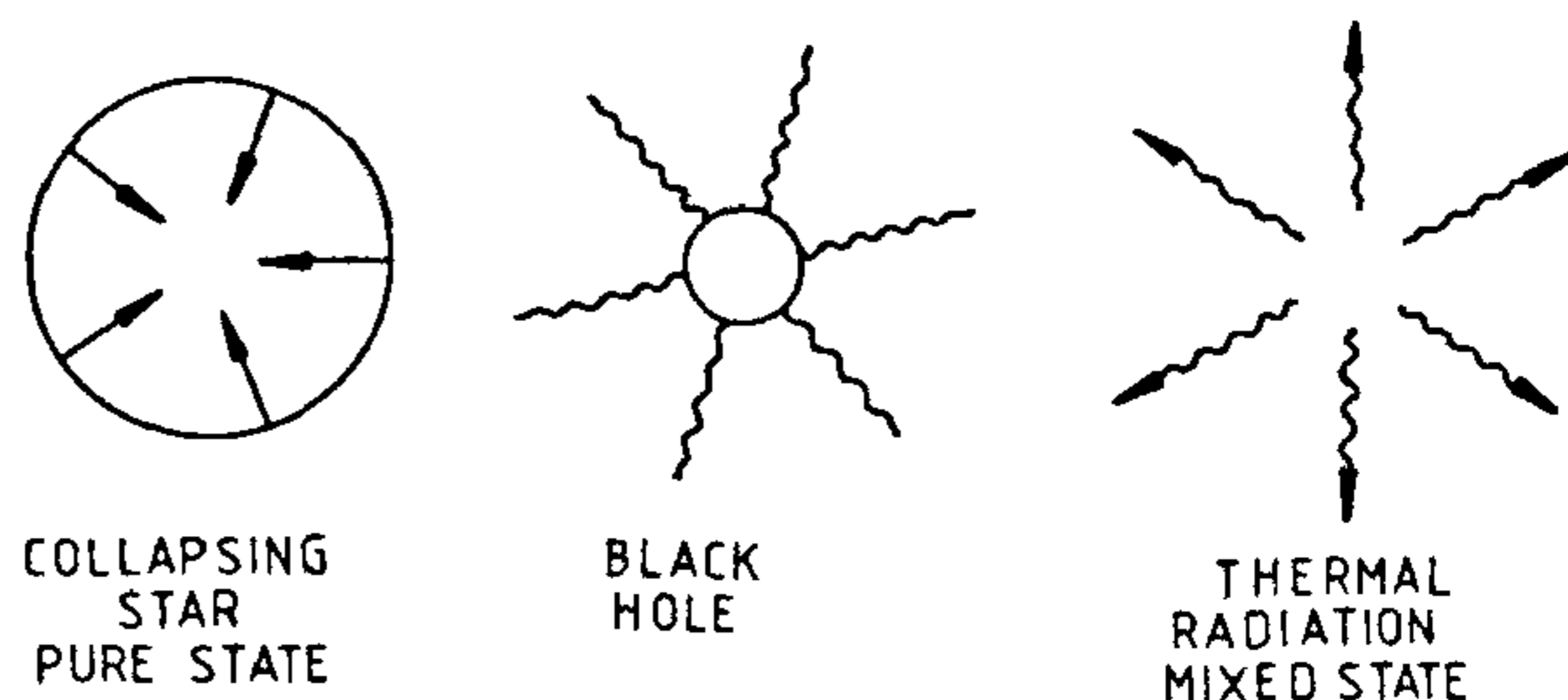


Figure 3. Pure state going into a mixed state

that has gone into the black hole through the one-way membrane but has not been able to come out.

Normally, one equates unitarity with conservation of probability. Hawking¹¹ proposed a change in the basic structure of time evolution in quantum mechanics, conserving probability but allowing for unitarity violation. He proposed replacing the usual time evolution operator $U = \exp(-iHt)$ by another operator $\$$ acting linearly on matrices, and taking ρ the density matrix to $\$ \rho$. Here $\$$ is called the superscattering operator. It can conserve probability but generically violates unitarity.

However, Banks *et al.*¹² showed that violation of unitarity necessarily implies violation of conservation of energy. Information transfer seems to require some energy transfer along with it. So Hawking's way of dealing with the problem by introducing a superscattering operator does not seem to be correct.

In general, one has the following possibilities, when one wonders about the information that went into the black hole: information is (1) lost, (2) reemitted and/or (3) retained in some remnant of the black hole. In the first case, loss could also mean going to another universe through a wormhole. In the case of reemission, it could take place either before or after the matter crosses the horizon. The former, called 'bleaching', is generally considered not possible in view of the fact that nothing out of the ordinary is expected to happen to matter freely falling into black hole at the horizon (as all physical quantities are finite there). To get information after the matter has crossed the horizon would imply noncausal behaviour as the time inside the horizon is infinite in the distant observer's frame and information will have to travel back in time. If a remnant of the hole carries all the information and radiates it out, it must be a long-living one as a lot of information has to be sent out by an object of small mass (of the order of Planck mass). So, at first sight, the second and third alternatives do not seem very feasible. A discussion of the whole problem requires a knowledge of the back reaction on the metric due to emission of Hawking radiation. One has not been able to work this out yet.

Toy models (two-dimensional)

Recently, there was a lot of excitement when hopes were raised that a toy model based on string theory in two dimensions can be solved exactly, even including quantum effects. This claim has not been sustained, but in the resulting activity one has still learnt a lot. It all began in 1991 with the discovery of a black hole in two-dimensional string model by Mandal *et al.*¹³ and independently by Witten¹⁴. The string black hole is quite similar to the Schwarzschild one. The metric is

$$ds^2 = dr^2 - \tanh^2 r dt.$$

It appears to have a singularity at $r = 0$, but the scalar curvature R has no singularity there as $R = 4/\cosh^2 r$.

So, one makes a Kruskal-like transformation to $2u = \exp(r' - t)$ and $2v = \exp(r' + t)$, where r' is the tortoise-like coordinate $r' = r + \ln[1 - \exp(-2r)]$ and $dr' = \coth r dr$ ($r' = -\infty$ when $r = 0$), to get

$$ds^2 = -du dv / (1 - uv).$$

While the horizon is at $uv = 0$ ($r = 0$, $r' = -\infty$), $uv = 1$ is a real singularity as $\cosh^2 r = 0$ there. If we take

$$\begin{aligned} \phi &= -\frac{1}{2} \ln(1 - uv) = \ln(\cosh^2 r) \\ &= \ln[1 + \exp(2r')/4], \end{aligned}$$

we have $ds^2 = -\exp(2\phi) du dv$. The Kruskal diagram for this case is shown in Figure 4.

Callan *et al.*¹⁵ have proposed a model similar to this and claimed that it was solvable, which was what led to some excitement. In their model,

$$ds^2 = -d\sigma^+ d\sigma^- / (1 + M \exp(\sigma^- - \sigma^+))$$

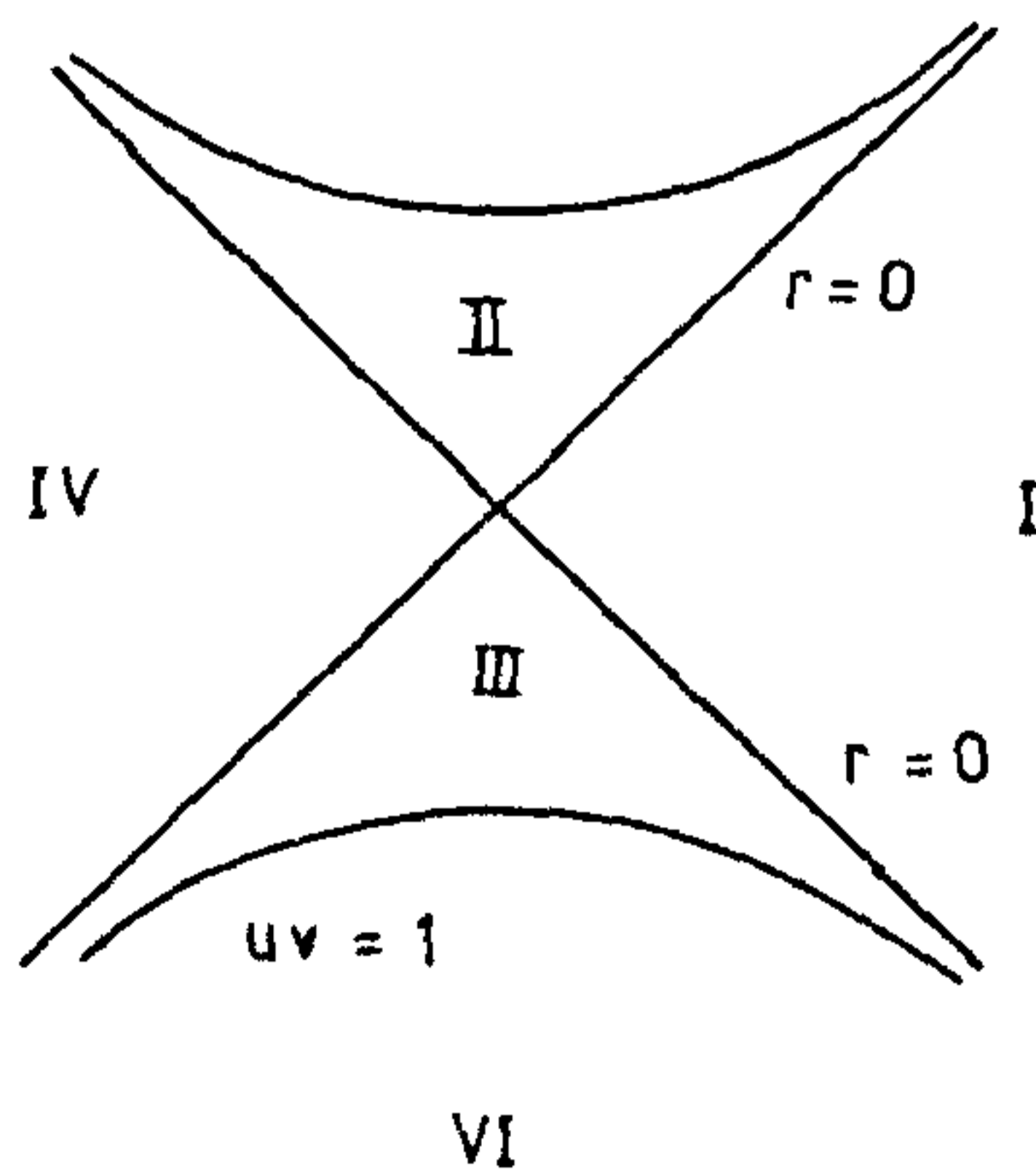


Figure 4. Penrose diagram for a two-dimensional black hole.

$$= (d\sigma^2 - dr^2) / [1 + M \exp(-2\sigma)],$$

with $\sigma^\pm = \tau \pm \sigma$.

Writing

$$\phi = -\frac{1}{2} \ln[M + \exp(2\sigma)],$$

we get

$$ds^2 = \exp(2\phi + 2\sigma) (d\sigma^2 - d\tau^2).$$

The presence of field ϕ makes this a metric of dilaton gravity. The action is given by

$$\begin{aligned} S &= \frac{1}{2\pi} \int d^2\sigma \sqrt{-g} [\exp(-2\phi) \{R + 4(\nabla\phi)^2 + 4\lambda^2\} \\ &\quad - \frac{1}{2} \sum_i (\nabla f_i)^2]. \end{aligned}$$

The last term is due to the N matter fields that are present. The summation is over the N fields f_1, \dots, f_N . For $M = 0$, $\phi = -\sigma$ and we have a flat metric and the space is called linear dilaton. For $M > 0$, the solution is a black hole given by the Penrose diagram (Figure 5). The horizon is at $\sigma^+ - \sigma^- = 2\sigma \rightarrow -\infty$ (σ corresponds to r' here) and we have a black hole with singularities of the type that a string black hole had. At the horizon, $\exp(2\phi)_{\text{hor}} = 1/M$. For $M < 0$, it is a naked singularity.

If we have infalling matter $f_i = F(\sigma^+)$ falling into the linear dilaton vacuum, it forms a black hole as shown in Figure 5.

The solutions before and after the infall are given by

$$ds^2 = -d\sigma^+ d\sigma^-, \quad \phi = -\sigma \quad (\text{linear dilaton}),$$

$$ds^2 = -d\sigma^+ d\sigma^- / [1 + M \exp(\sigma^- - \sigma^+) - \Delta \exp(\sigma^-)],$$

$$\exp(-2\phi) = M + \exp(\sigma^+) [-\exp(\sigma^-) - \Delta].$$

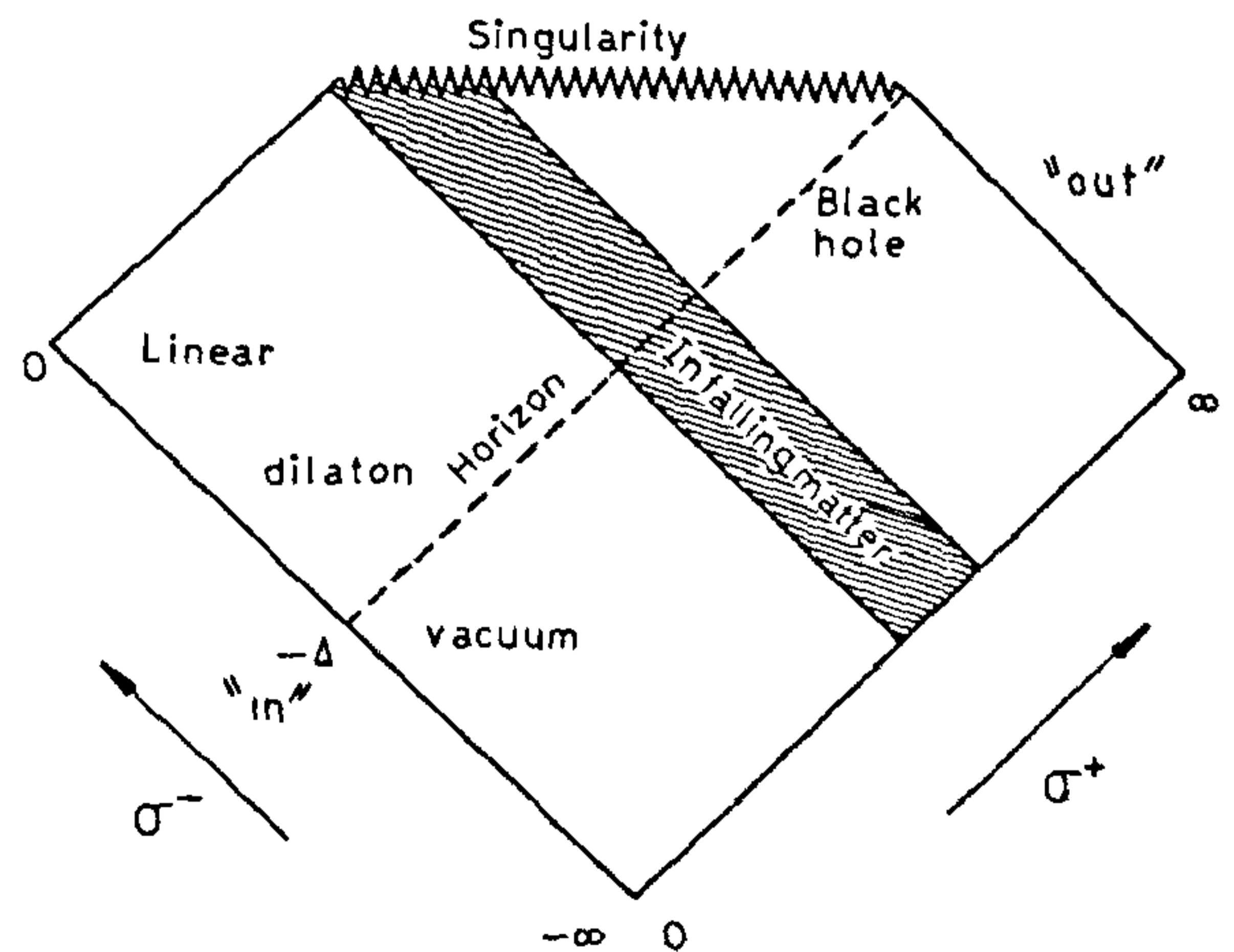


Figure 5. The Penrose diagram for a collapsing black hole formed from a left-moving matter distribution

$$T_{++} = \frac{1}{2} (\partial_+ F)^2$$

$$M = \int d\sigma^+ T_{++}, \quad \Delta = \int d\sigma^+ \exp(-\sigma^+) T_{++}.$$

Using $\xi^- = -\ln[\exp(-\sigma^-) - \Delta]$, $\xi^+ = \sigma^+$; $ds^2 = -d\xi^+ d\xi^- / [1 + M \exp(\xi^- - \xi^+)]$.

Hawking radiation is described by $\exp(-i\omega\xi^-)$ as positive frequency. This is a combination of $\exp(-i\omega\sigma^-)$ and $\exp(i\omega\sigma^-)$. That is a mixture of positive and negative frequencies of an asymptotic observer, which leads to emission with Hawking temperature in the usual way.

Back reaction

The back reaction problem seemed solvable under the assumption that dilaton and metric fluctuations are negligible compared to the fluctuations of the matter fields f_i . Quantization is considered via the functional integral

$$\int Dg D\phi \exp(iS_{\text{grav}}[g, \phi]) \int Df \exp(-i/4\pi)$$

$$\times \int d^2\sigma \sqrt{-g} \sum (\nabla f_i)^2.$$

By using methods of string theory like Polyakov-Liouville action, trace anomaly, we can show

$$\langle T_{--}^{\text{Haw}} \rangle = \frac{N}{48} \left[1 - \frac{1}{[1 + \Delta \exp(\xi^-)]^2} \right] = \langle T_{--}^{\text{Matter}} \rangle,$$

thus providing consistent energy-momentum balance between infalling matter and the emitted Hawking radiation.

Disaster

Calculation of collapse along these lines¹⁶, unfortunately, develops a singularity (kinetic operator degenerates) at ϕ_c , where $\exp(2\phi_c) = 12/N$. This singularity is hidden behind an apparent horizon, defined by $[\nabla \exp(-\phi)]^2 = 0$. Here $\exp(-\phi)$ is like a radius. So, the two-dimensional model seems as unsolvable as the four-dimensional one. This singularity is present even for the linear dilaton. So, this model is also not solvable and we are not much wiser about the problem of unitarity violation (Figure 6).

Giddings¹⁷ still argues that no information comes out in order-by-order calculation in $1/N$ approximation. He uses the fact that Hawking radiation emerges at weak coupling before ϕ becomes critical. In this theory $\exp(2\phi)$ corresponds to the gravitational coupling.

Black hole complementarity

Susskind *et al.*¹⁸ have come up with an attractive idea

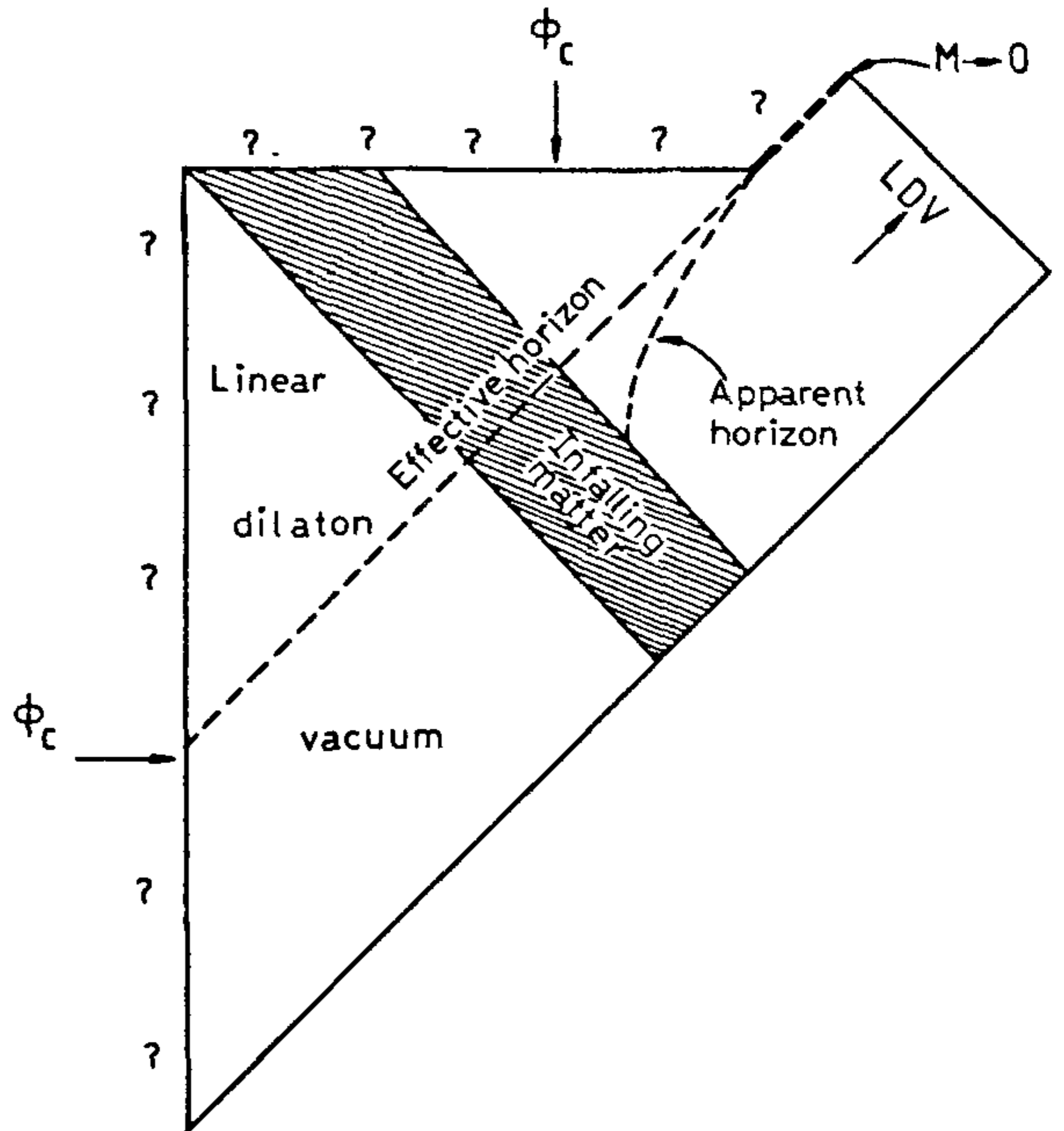


Figure 6. Singularity line at $\phi = \phi_c$.

to analyse the black hole formation and evaporation based on the two-dimensional model. This approach is based on the idea that one should not attempt to describe in the same framework the situation as seen by a freely falling observer and an asymptotic stationary observer. These views are complementary. In this approach, unitary evolution is demanded by assumption and formulated as a postulate. When this idea is pushed to its logical limit, we find that it leads to prediction of some form of 'bleaching' or information reemission. This approach seems easier to formulate in terms of the 'membrane' idea pioneered by Thorne *et al.*¹⁹ in the astrophysical context. The event horizon is very important for the distant or asymptotic observer. Nothing can come outside of it. It is normally claimed that there cannot be any drastic change at the horizon as all the known physical quantities of interest, like curvature, are finite there. Though this may be true for the infalling observer (locally), for the asymptotic observer the event horizon, or more exactly, the membrane which is very close to the event horizon, may play an important physical role. The idea is expressed in the form of three postulates¹⁸.

Postulate 1. The formation and evaporation of black holes as viewed by the distant observer can be described within the context of standard quantum theory. There exists a unitary S-matrix which describes the evolution from infalling matter to outgoing Hawking-like radiation.

Postulate 2. Outside the stretched horizon of a black

hole, physics can be described to a good approximation by a set of semiclassical field equations.

Postulate 3. To a distant observer the black hole appears to be a quantum system with discrete energy levels. The dimension of the subspace of states describing a black hole of mass M is the exponential of the entropy $S(M)$.

Specifically, it is assumed that the origin of thermodynamic behaviour of black hole is the coarse graining of a large, complex, ergodic but conventionally quantum system. It is also accepted that a freely falling observer experiences nothing out of the ordinary when crossing the horizon as required by equivalence principle. It might seem contradictory to postulate one in the following way.

If space-time is foliated with a family of Cauchy surfaces Σ as shown in Figure 7, which shows the Penrose diagram for the evaporating black hole, the S -matrix relates the surface below and above the point P (where global event horizon intersects the curvature singularity). The Hilbert space of states can be written as a tensor product of black hole and outside Hilbert spaces. If there is a unitary operator which relates the outside state before the formation of black hole to the outside state after the evaporation of black hole, this would imply that there is no net information transfer to the black hole. So, any information received must have been sent back from the horizon or some membrane outside it.

Thus, all distinctions between initial states of infalling

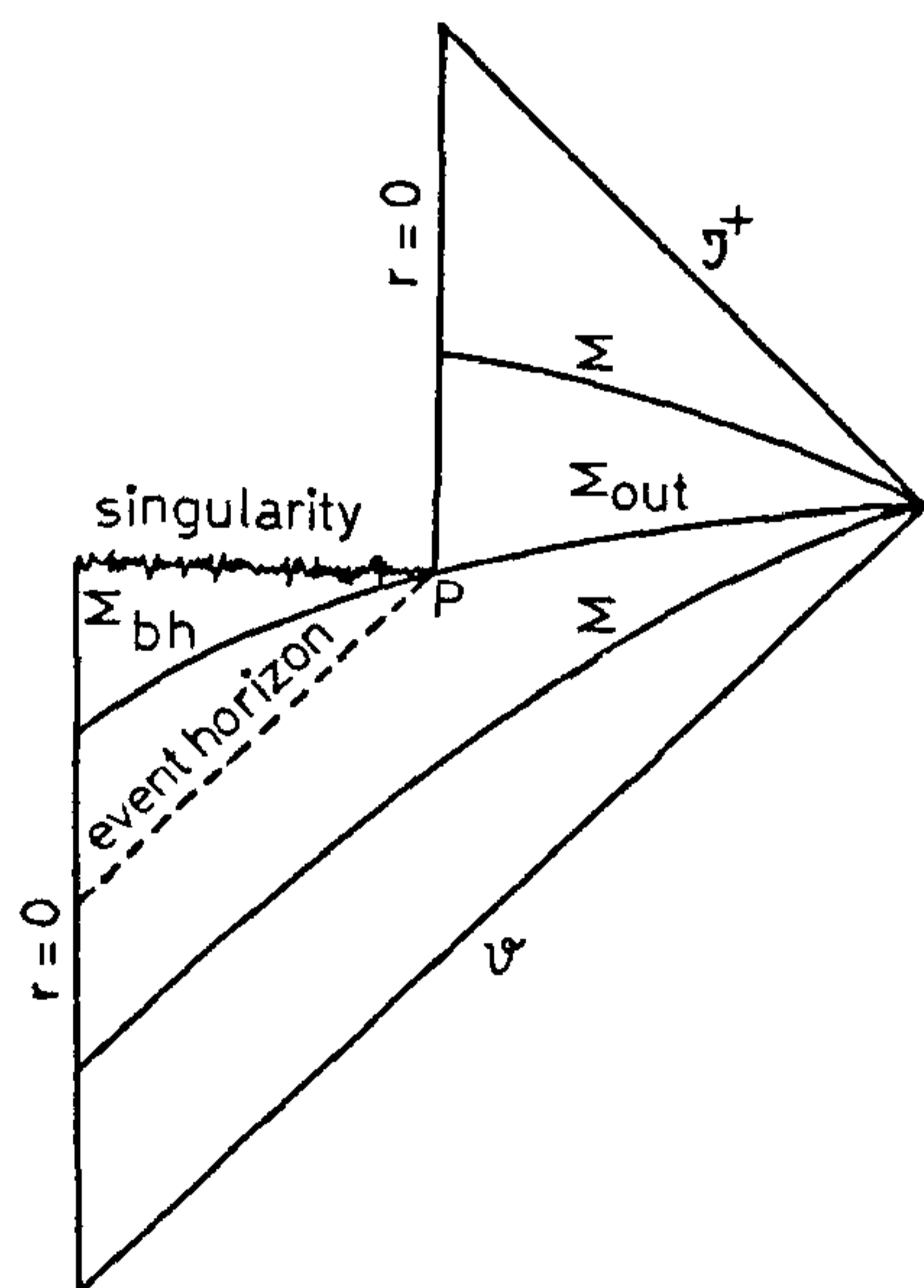


Figure 7. Penrose diagram for black-hole evolution

matter must be obliterated before the state crosses the global horizon. But this is an unreasonable violation of the equivalence principle that nothing out of the ordinary happens at the event horizon. According to these authors¹⁸, this conclusion is not correct, as a state describing interior and exterior together is unphysical because this implies correlations which have no operational meaning as no information can come out from inside. Only a superobserver outside our universe (GOD!) can make use of the product Hilbert space. So, it is claimed that the assumptions that (i) distant observer sees all infalling information returned in Hawking-like radiation and (ii) infalling observer sees nothing unusual at event horizon are not contradictory. If one demands a standard quantum theory valid for both observers, it is inconsistent with the postulates. One can call this a sophisticated 'bleaching' scenario, which many of us may find attractive.

In discussing this idea in the context of two-dimensional models, Susskind *et al.*¹⁸ avoid the problem of singularity at ϕ_{cr} by imposing suitable boundary conditions, which is somewhat unsatisfactory. They feel that discussion in terms of the membrane or stretched horizon is more physical and satisfactory though the treatment is still qualitative.

Stretched horizon and two kinds of entropy

Classically, quasistationary black holes can be described by outside observers in terms of a 'stretched horizon'¹⁹ which behaves like a physical membrane with certain mechanical, electrical and thermal properties. The description is coarse-grained in character. It has time-irreversibility and dissipation properties of a system described by ordinary thermodynamics. The membrane is very real to the outside observer. If he or she is suspended just above the stretched horizon, an intense flux of energetic radiation will be observed, apparently emanating from the membrane. He or she will also see other electrical, mechanical and thermal properties. If, however, the observer lets go the suspension and falls freely, the membrane will disappear and they cannot even report this fact to the outside world. In this sense, there is a complementarity between observations made by infalling observers and distant observers.

To implement the postulates, it is assumed that the coarse-grained thermodynamic description of the membrane has an underlying microphysical basis. The microphysical degrees of freedom appear in the quantum Hamiltonian used to describe the observable world. They must be of sufficient complexity to behave ergodically and lead to a coarse-grained description.

Lagrangian mechanics and thermodynamics are quite different descriptions of a system. In Lagrangian mechanics, the motion of any system is reversible and the

concept of heat and entropy has no place. Thermodynamics is the theory of irreversible dissipation of organized energy into heat. The thermodynamic description arises from the coarse graining of the mechanical description. In thermodynamics, configurations that are macroscopically similar are considered identical.

To discuss black hole formation and evaporation, it is useful to distinguish the two kinds of entropy that normally arise: entropy of entanglement and entropy of ignorance (or thermal entropy). The former is of quantum origin. Consider a quantum system composed of two parts or subsystems A and B . In our discussions these two subsystems will refer to the stretched horizon and the radiation field outside the stretched horizon.

Let the total Hilbert space be a product of the two sub Hilbert spaces. $H = H_A \times H_B$. If $\{|a\rangle\}$, $\{|b\rangle\}$ are orthonormal bases for H_A and H_B , respectively, a general ket $|\psi\rangle$ in H may be written as $|\psi\rangle = \sum \psi(a, b) |a\rangle \times |b\rangle$.

The density matrix of a subsystem A in the basis $\{|a\rangle\}$ is

$$\rho_A(a, a') = \sum_b \psi(a, b) \psi^*(a', b)$$

and that of B is

$$\rho_B(b, b') = \sum_a \psi(a, b) \psi^*(a, b')$$

Note that the composite system $A \cup B$ is in a pure state. The entropies of entanglement of subsystems A and B are defined by

$$S_E(A) = -\text{Tr}[\rho_A \ln \rho_A] \quad \text{and} \quad S_E(B) = -\text{Tr}[\rho_B \ln \rho_B].$$

$S_E(A) = S_E(B)$ if composite system is in a pure state as A and B together are in our case. $S_E = 0$ only if the ket $|\psi\rangle$ is an uncorrelated product state. The entropy of entanglement S_E is not subject to the second law of thermodynamics. It can increase or decrease with time. If H_B is of dimension D_B and H_A of dimension D_A then

$$S_E(B)_{\text{max}} = -\ln(D_B) = S_E(A)_{\text{max}}, \quad \text{where } D_B \leq D_A.$$

Entropy of ignorance or thermal entropy arises as we have to assign a density matrix to a system not because it is quantum-entangled with a second system but because we are ignorant about its state. We assign a probability to each state. If we know nothing, we take ρ proportional to 1. If we know only its energy, we take $\rho \neq 0$ only in allowed energy space. Thermal entropy arises because of the practical inability to follow the fine-grained details of a system. For a system in thermal equilibrium with a reservoir

$$\rho_{\text{Max-Boltzmann}} = \rho_{\text{MB}} = Z^{-1} \exp(-\beta H)$$

and

$$S_T = -\text{Tr}[\rho_{\text{MB}} \ln(\rho_{\text{MB}})].$$

Formation and evaporation of black hole

Now let us consider the formation and evaporation of a two-dimensional black hole. The evolution of entropy with time is shown in Figure 8. Initially, the stretched horizon is in the ground state with minimum area and radiation is in a pure state, so $S_E = 0$. When the area of horizon increases because of infalling matter, Hawking radiation in the form of f-quanta are emitted. The states of f-quanta are correlated to the state of the horizon and so S_E increases. But $S_E(H_s) < S_T(H_s) = A(t)$. So, S_E is bounded and must return to zero as the horizon goes to the vacuum value. Page²⁰ has shown that S_E follows S_T in the beginning. He also showed that the dependence on the parameter m_{pl}/M is nonanalytic so that Giddings conclusion, mentioned earlier, that in weak-coupling approximation no information comes out, may not be valid.

The final outgoing radiation is different from thermal. To see this, notice that halfway through the evaporation process $S_E = S_T$ and fine-grain total entropy is zero. But radiation is correlated to the degrees of freedom of the horizon (H_s). As more time passes, the horizon emits more quanta and the earlier correlation between horizon and radiation is replaced by a correlation between the earlier and the newly emitted later quanta. Because of the transfer of these correlations to the radiation itself, the S_E goes to zero and the horizon is no longer correlated to the radiation. Local properties will be thermal but there are correlations spread over entire time occupied by the outgoing flux energy. In this way information is sent back to the outside system and no loss of unitarity is there.

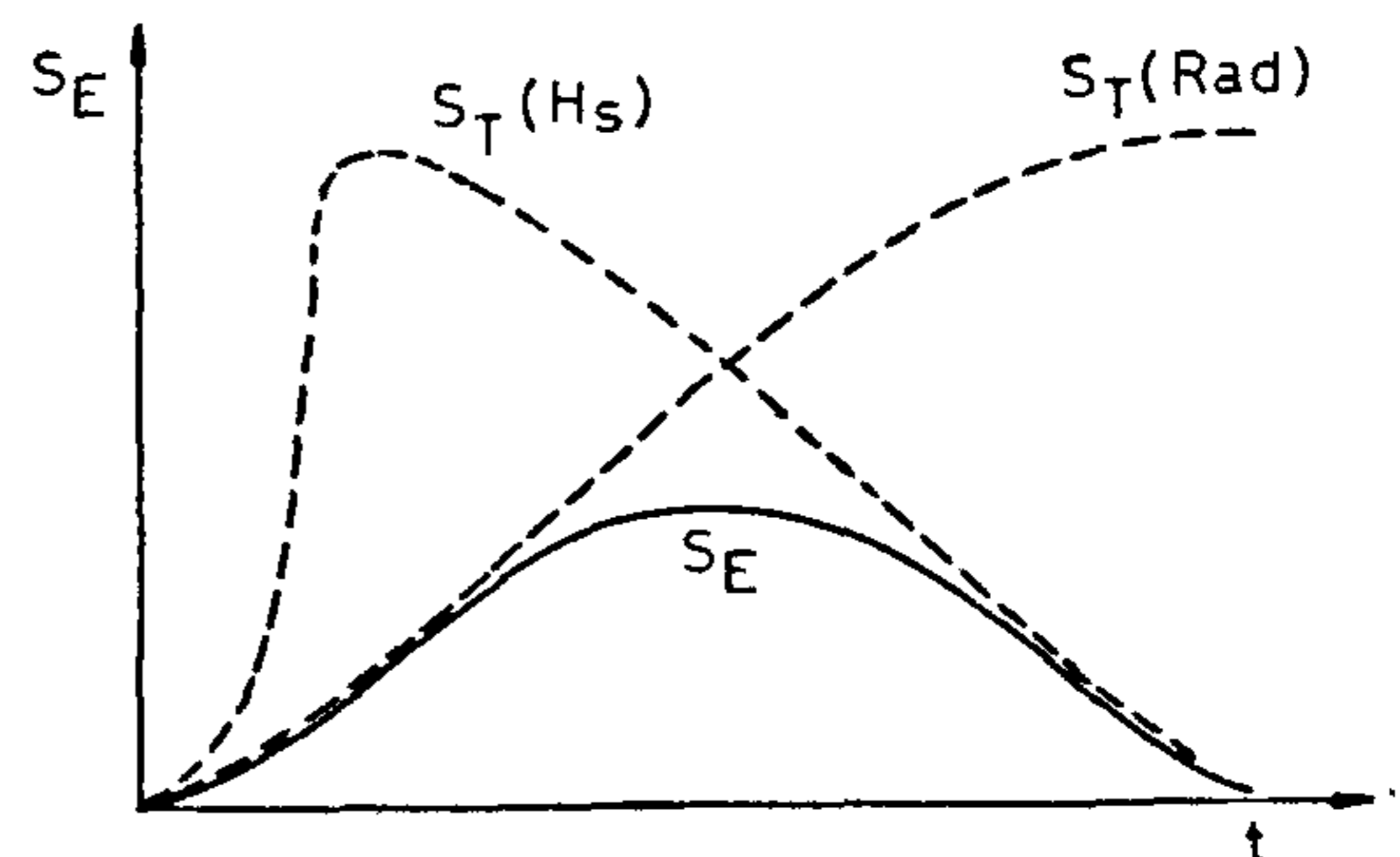


Figure 8. Entanglement entropy of radiation and stretched horizon. The dashed curves indicate the thermal entropies.

General remarks

A strange ambiguity seems to prevail when one discusses the event horizon of a black hole. On the one hand, it is a monster gobbling up things which will never return. On the other, it is a harmless region, as curvature and other physical variables are finite there for the freely falling observer. Is the event horizon eventful or a neutral spectator? The contradiction arises due to the need for reconciling two opposite points of view: that of the freely falling observer and the asymptotic observer. The suggestion that they are complementary and we cannot listen to both, thus, seems an attractive idea. When all the dust has settled down, what has been chalked up on the board? We are no wiser as even the two-dimensional models have turned out to be not solvable in closed form. Some new ideas and a lot of new techniques have come up in the process. The black-hole complementarity seems a very attractive idea. However, the theorists working in this area (both particle physicists and general relativists) have to get a lot more confidence in their mathematical techniques before a consensus emerges. Can an observable prediction emerge? One can always hope. Maybe in the area of cosmology, where too we have an event horizon, a prediction may emerge!

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