# Why do millisecond pulsars have weaker magnetic fields compared to ordinary pulsars?

**Arnab Rai Choudhuri[†,\*] and Sushan Konar[#]**

[†]Department of Physics, Indian Institute of Science, Bangalore 560 012, India
[#]CTS and Physics, Indian Institute of Technology, Kharagpur 721 302, India

**Millisecond pulsars, with magnetic fields weaker by three to four orders compared to those of ordinary pulsars, are presumed to be neutron stars spun up by binary accretion. We expect the magnetic field to get screened by the accreted material. Our simulation of this screening mechanism shows that the field decreases by a purely geometric factor $\sin^{-7/2} \phi_{P,i}$ before freezing to an asymptotic value, where $\phi_{P,i}$ is the initial angular width of the polar cap. If $\phi_{P,i}$ lies in the range 5–10°, then the magnetic field diminution factor turns out to be $\sim 10^3$–$10^4$, in conformity with observational data.**

PULSARS, which are rotating neutron stars, are seats of strongest magnetic fields known to mankind. Magnetic fields of most pulsars are around $10^{11}$–$10^{13}$ G, whereas the typical rotation periods are about 1 s. However, there exist a handful of known pulsars with considerably smaller rotation periods, which also have much weaker magnetic fields around $10^8$ G. These so-called millisecond pulsars are often found in binary systems[1].

It is believed that most of the neutron stars (barring possible *magnetars*)[2] are born with characteristics typical of ordinary pulsars. If the neutron star happens to be in a binary system, then it is possible for it to accrete matter with angular momentum from the binary companion. Millisecond pulsars are thought to be neutron stars which have been spun up in such a binary accretion process[3]. Since their magnetic fields are weaker by a factor of $10^3$–$10^4$ compared to the magnetic fields of ordinary pulsars, presumably the magnetic field of the neutron star also decreases during the accretion phase. Several alternative scenarios have been proposed to explain how this decrease of magnetic field takes place, starting from models relating the field evolution to the spin evolution to those based on accretion-induced Ohmic decay in the heated crustal layers[4–10]. One possibility is that the magnetic field gets buried under the accreted matter. Although this idea has been around for a while[11–17], we have for the first time carried out a detailed 2D simulation to test the idea. Details of our calculations are presented elsewhere[18,19]. One of the attractive features of this scenario is that the factor $10^3$–$10^4$ by which the magnetic field decreases can be explained in an elegant and simple way as arising purely out of geometric considerations.

*For correspondence. (e-mail: arnab@physics.iisc.ernet.in)

The strong magnetic field of an accreting neutron star channelizes the accreting material to flow through the polar regions. As the magnetic field of the neutron star decreases because of the screening due to accreting material, it is less able to channelize the accretion flow and thereby the polar cap widens. One can easily find out how the angular width $\phi_P$ of the polar cap depends on the surface magnetic field $B_s$ of the neutron star (see, for example, ref. 20). The field line starting from $\phi_P$ at the surface of the neutron star, with a radius $r_s$, is the last closed field line of the dipolar field and passes through the Alfvén radius $r_A$. It easily follows that

$$\sin \phi_P = \left( \frac{r_s}{r_A} \right)^{1/2}. \tag{1}$$

Assuming that the ram pressure of the freely in-falling accreting material at the Alfvén radius equals the magnetic pressure, a few steps of easy algebra give:

$$r_A = (2GM)^{-1/7} r_s^{12/7} B_s^{4/7} \dot{M}^{-2/7}, \tag{2}$$

where $M$ is the mass of the neutron star and $\dot{M}$ the accretion rate. It follows from eqs (1) and (2) that

$$\sin \phi_P \propto B_s^{-2/7}. \tag{3}$$

This is how the polar cap widens with the weakening magnetic field until $\phi_P$ becomes equal to 90°, when eq. (3) obviously ceases to hold. On taking $M = 10^{33}$ g, $\dot{M} = 10^{-8}$ $M_\odot$ yr$^{-1}$, $r_s = 10$ km, $B_s = 10^{12}$ G, we find from eq. (2) that $r_A \approx 300$ km. Substituting this in eq. (1), we conclude that the initial polar cap angle is of the order of 10°.

The accreting material falling through the two polar caps flows horizontally towards the equator in both the hemispheres. At the equator, the opposing materials flowing in from the two poles meet, sink underneath the surface (inducing a counter-flow underneath the equator-ward flow at the surface) and eventually settle radially on the neutron star core. With a suitably specified flow having these characteristics, we have studied kinematically how the magnetic field evolves with time, taking into account the fact that the polar cap width changes with the evolution of the magnetic field, thereby altering the velocity field also. Figure 1 a and b shows the velocity field at an early stage and at a late stage respectively. We find that the equator-ward flow near the surface seen in Figure 1 a is quite efficient in burying the magnetic field underneath the surface. However, when the polar cap opens to 90°, the accretion becomes spherical and radial, as seen in Figure 1 b. It is found that such accretion is not efficient in burying the magnetic field any further. Figure 2 shows magnetic field lines at different stages of evolution, whereas Figure 3 plots the surface magnetic field at 45° as a function of time. Clearly, the magnetic field at the surface of the neutron star keeps decreasing until the polar
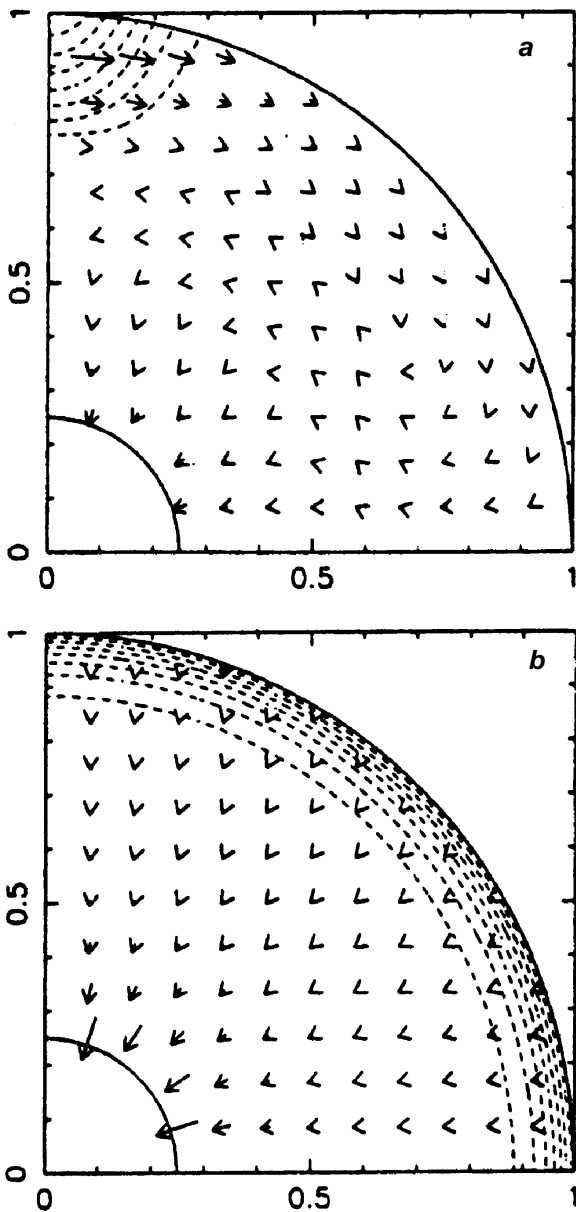
**Figure 1.** Velocity fields induced inside the neutron star due to accreting material settling on the surface. Arrows indicate the flow velocities, whereas dashed lines are contours of constant $\nabla.(r\mathbf{v})$. The velocity field is specified in the code in such a way that it keeps changing with evolution of the magnetic field. Details of how we do this are provided elsewhere[19]. Here we show the velocity fields at two different instants: (*a*) when the magnetic field is strong and the polar cap angle is small (at a relatively early stage); and (*b*) when the magnetic field has become much weaker and the polar cap has opened up (at a relatively late stage). Dashed lines (contours of constant $\nabla.(r\mathbf{v})$) indicate regions that are sources of new material due to accretion. We see in (*a*) that the new material is dumped in a narrow polar cap, inducing an equator-ward flow just below the surface. After reaching the equator where this flow meets the oppositely-directed flow from the other pole, the flow sinks underneath the surface, induces a counter-flow and eventually settles on the core of the neutron star. On the other hand, we see in (*b*) that the new material falls isotropically all over the surface and the induced flow is radially inward. The horizontal flow shown in (*a*) is expected in reality to be confined only within 1% of the neutron star radius immediately below its surface[18]. Here we have inflated that layer to 10% of radius for easy visualization.
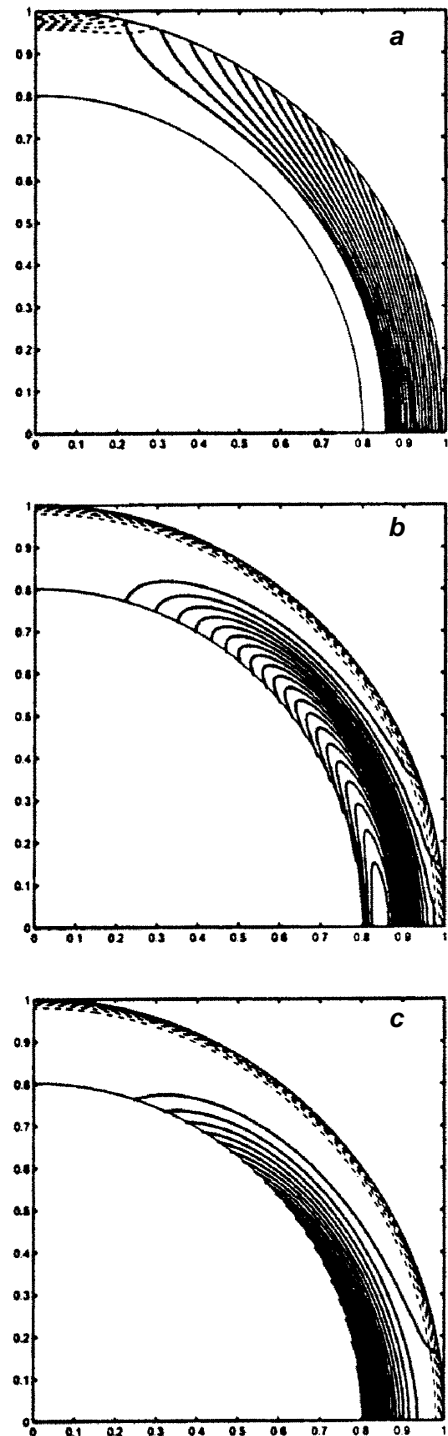


**Figure 2.** Magnetic field lines during different phases of evolution. Dashed lines indicate the contours of constant $\nabla.(r\mathbf{v})$ at the same instant. *a*, Initial magnetic field which is evolved by solving the induction equation numerically, with the specified velocity field which keeps changing as the magnetic field weakens. (The velocity fields at an early stage and at a late stage are shown in Figure 1). Details of the numerical code are given in the appendix of our earlier paper[18], which also discusses the various boundary conditions used. The code has been adopted from one which was used extensively for studying the evolution of the solar magnetic fields[21,22].
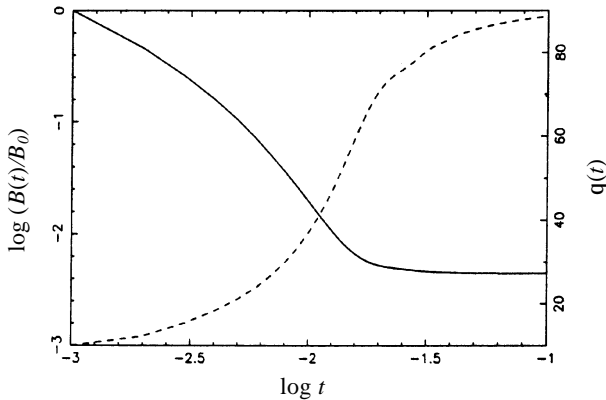
**Figure 3.** Magnetic field at the surface at 45° latitude changing with time (solid line). Evolution of the polar cap angle $\phi_P$ with time is also indicated (dashed line). It is clear the magnetic field decreases rapidly till the polar cap opens to about 90°, after which the decay of the magnetic field is significantly halted.

cap opens to 90°, after which the magnetic field is essentially frozen, since the radial accretion cannot screen it any further. If $\phi_{P,i}$ is the initial polar cap width, then it follows from eq. (3) that the magnetic field would decrease by a factor $(\sin 90°/\sin \phi_{P,i})^{7/2}$ from its initial value, before it is frozen to an asymptotic value. On taking $\phi_{P,i}$ in the range 5–10°, this factor turns out to be about $10^3$–$10^4$, exactly the factor by which the magnetic fields of millisecond pulsars are weaker compared to the magnetic fields of ordinary pulsars.

Put another way, the magnetic field freezes when the Alfvén radius becomes equal to the neutron-star radius. The asymptotic value of the surface magnetic field can be found directly from eq. (2) by setting $r_A$ equal to $r_s$, which gives

$$B_{asymp} = (2GM)^{1/4} \dot{M}^{1/2} r_s^{-5/4}. \tag{4}$$

Using the various standard values mentioned before, we find $B_{asymp} \approx 10^8$ G. When the magnetic field falls to this value, it can no longer channelize the accretion flow, resulting in the flow becoming isotropic. Such a flow is unable to screen the magnetic field any further. After the accretion phase is over, the neutron star appears as a millisecond pulsar with this magnetic field. We propose this as the reason why millisecond pulsars are found with magnetic fields of order $10^8$ G.

1. Lorimer, D. R., Binary and Millisecond Pulsars at the New Millennium. *Living Rev. Relativity*, 2001, **4**, 5.

2. Kouveliotou, C. *et al.*, An X-ray pulsar with a superstrong magnetic field in the soft gamma-ray repeater SGR 1806-20. *Nature*, 1998, **393**, 235.
3. Bhattacharya, D., Models for the Evolution of Neutron Star Magnetic Fields. *Pulsar Timing, General Relativity and the Internal Structure of Neutron Stars* (ed. Arzoumanian, Z.), Royal Netherlands Academy of Arts and Sciences, 1999, pp. 235–243.
4. Jahan Miri, M. and Bhattacharya, D., Magnetic Evolution of Neutron Stars in Wide Low-Mass Binary Systems. *Mon. Not. R. Astron. Soc.*, 1994, **269**, 455.
5. Jahan Miri, M., Magnetic and spin evolution of pulsars. *Mon. Not. R. Astron. Soc.*, 1996, **283**, 1214.
6. Bhattacharya, D. and Datta, B., Ohmic decay of magnetic flux expelled from neutron star interiors. *Mon. Not. R. Astron. Soc.*, 1996, **282**, 1059.
7. Geppert, U. and Urpin, V., Accretion-driven magnetic field decay in neutron stars. *Mon. Not. R. Astron. Soc.*, 1994, **271**, 490.
8. Urpin, V. and Geppert, U., Accretion and evolution of the neutron star magnetic field. *Mon. Not. R. Astron. Soc.*, 1995, **275**, 1117.
9. Konar, S. and Bhattacharya, D., Magnetic field evolution of accreting neutron stars. *Mon. Not. R. Astron. Soc.*, 1997, **284**, 311.
10. Konar, S. and Bhattacharya, D., Magnetic field evolution of accreting neutron stars-II. *Mon. Not. R. Astron. Soc.*, 1999, **303**, 588.
11. Bisnovatyi-Kogan, G. S. and Komberg, B. V., Pulsars and close binary systems. *Sov. Astron.*, 1974, **18**, 217.
12. Blandford, R. D., De Campli, W. M. and Königl, A., Unsteady Accretion onto Magnetized Neutron Stars. *Bull. Am. Astron. Soc.*, 1979, **11**, 703.
13. Taam, R. E. and van den Heuvel, E. P. J., Magnetic field decay and the origin of neutron star binaries. *Astrophys. J.*, **305**, 1986, 235.
14. Romani, R. W., A unified model of neutron-star magnetic fields. *Nature*, **347**, 1990, 741.
15. Romani, R. W., Neutron star magnetic fields. *Isolated Pulsars* (eds van Riper, K., Epstein, R. and Ho, C.), Cambridge University Press, 1995, pp. 75–82.
16. Cumming, A., Zweibel, E. and Bildsten, L., Magnetic Screening in Accreting Neutron Stars. *Astrophys. J.*, 2001, **557**, 958.
17. Melatos, A. and Phinney, E. S., Hydromagnetic Structure of a Neutron Star Accreting at Its Polar Caps. *Publ. Astron. Soc. Aust.*, 2001, **18**, 421.
18. Choudhuri, A. R., and Konar, S., Diamagnetic screening of the magnetic field in accreting neutron stars. *Mon. Not. R. Astron. Soc.*, 2002, **332**, 933.
19. Konar, S. and Choudhuri, A. R., Diamagnetic screening of the magnetic field in accreting neutron stars II – The effect of polar cap widening. *Mon. Not. R. Astron. Soc.*, astro-ph/0304490 (in press).
20. Shapiro, S. L. and Teukolsky, S. A., *Black Holes, White Dwarfs and Neutron Stars*, John Wiley, 1983.
21. Dikpati, M. and Choudhuri, A. R., The evolution of the Sun's poloidal field. *Astron. Astrophys.*, 1994, **291**, 975.
22. Nandy, D. and Choudhuri, A. R., Explaining the Latitudinal Distribution of Sunspots with Deep Meridional Flow. *Science*, **296**, 2002, 1671.