TOWARDS A MEAN-FIELD FORMULATION OF THE BABCOCK–LEIGHTON TYPE SOLAR DYNAMO. I. α-COEFFICIENT VERSUS DURNEY'S DOUBLE RING APPROACH

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ABSTRACT

We develop a model of the solar dynamo in which, on the one hand, we follow the Babcock–Leighton approach to include surface processes like the production of poloidal field from the decay of active regions, and, on the other hand, we attempt to develop a mean field theory that can be studied in quantitative detail. One of the main challenges in developing such models is to treat the buoyant rise of toroidal field and the production of poloidal field from it near the surface. The previous paper by Choudhuri, Schüssler, & Dikpati (1995) did not incorporate buoyancy. We extend this model by two contrasting methods. In one method, we incorporate the generation of the poloidal field near the solar surface by Durney's procedure of double ring eruption. In the second method, the poloidal field generation is treated by a positive α-effect concentrated near the solar surface, coupled with an algorithm for handling buoyancy. The two methods are found to give qualitatively similar results.

Subject headings: MHD — Sun: interior — Sun: magnetic fields

1. Introduction

Historically there have been two theoretical approaches in understanding the origin of the solar magnetic cycle: the Parker–Steenbeck–Krause–Rädler (PSKR) approach (Parker 1955; Steenbeck, Krause & Rädler 1966) and the Babcock–Leighton (BL) approach (Babcock 1961; Leighton 1969). In both these approaches, the toroidal component of the magnetic field is supposed to be generated from the poloidal component by the stretching of field lines due to differential rotation. In order for a self-sustaining dynamo to exist, the poloidal field has to be generated back from the toroidal field. The two approaches

mentioned above attempt to solve this problem in two different ways. In the PSKR approach, the cyclonic turbulence in the interior of the Sun is supposed to twist the toroidal field lines to produce the poloidal field (the so-called α-effect). On the other hand, the BL approach puts more stress on what is happening at the solar surface and assumes that the poloidal field arises out of the decay of tilted bipolar active regions (which result from the magnetic buoyancy of the toroidal field). Various aspects of the generation of poloidal field at the surface have been discussed by Wang & Sheeley (1991) and Durney, De Young, & Roxburgh (1993).

A formal mathematical formulation of the PSKR approach was developed on the basis of the mean field magnetohydrodynamics (Steenbeck, Krause, & Rädler 1966; Moffatt 1978, Chap. 7; Parker 1979, §18.3; Choudhuri 1998, §16.5). In comparison, the BL approach was based on rather heuristic, and often qualitative, arguments. Until recently, most of the detailed mathematical models of the solar dynamo were worked out on the basis of the PSKR approach. Only in the last few years there have at last been attempts of putting the BL approach on a mathematical footing comparable in sophistication to the mathematical theory of the PSKR approach (Choudhuri et al. 1995; Durney 1995, 1996, 1997; Dikpati & Charbonneau 1999). It now appears that the most successful model of the solar cycle will be something which incorporates the best features of both these approaches (Choudhuri 1999).

Since magnetic buoyancy would be particularly destabilizing in the main body of the convection zone (Parker 1975; Moreno-Insertis 1983), several theorists (Spiegel & Weiss 1980; van Ballegooijen 1982; DeLuca & Gilman 1986; Choudhuri 1990) argued that the solar dynamo may be operating in the overshoot layer at the bottom of the convection zone. With the helioseismic discovery of a shear layer at the bottom of the convection zone, it now appears fairly certain that the generation of the strong toroidal field by the stretching

of field lines must be taking place there. However, it seems unlikely that the whole dynamo process (as envisaged in the PSKR approach) occurs at the bottom of the convection zone. The studies of buoyant rise of the toroidal flux from there suggest that the toroidal field at the bottom of the convection zone should be of the order 10^5 G, substantially stronger than the equipartition value (Choudhuri & Gilman 1987; Choudhuri 1989; D'Silva & Choudhuri 1993; Fan, Fisher, & DeLuca 1993; Caligari, Moreno-Insertis, & Schüssler 1995). Such a strong field would completely quench the α-effect of the PSKR approach. To explain the generation of the poloidal field, the most natural way is to invoke the BL idea of the decay of tilted active regions, though there are still some attempts to work within the PSKR approach by considering an interface dynamo (Parker 1993; Charbonneau & MacGregor 1997; Markiel & Thomas 1999). In this paper, we assume that the poloidal field is produced by the decay of tilted active regions near the solar surface.

Although the sunspots migrate equatorward with the solar cycle, the weak diffuse magnetic field on the solar surface migrates poleward (Bumba & Howard 1965; Howard & LaBonte 1981; Makarov, Fatianov, & Sivaraman 1983; Makarov & Sivaraman 1989). Most of the dynamo models based on the PSKR approach (starting from Steenbeck & Krause 1969) mainly concentrated on the sunspots and ignored the poleward migration of the weak diffuse field. The poleward migration has been explained by assuming that the weak diffuse field (which is essentially the poloidal field) is carried by the meridional circulation (Wang, Nash, & Sheeley 1989a, 1989b; Dikpati & Choudhuri 1994, 1995; Choudhuri & Dikpati 1999). If we now accept the BL idea that the poloidal field is produced by the decay of tilted bipolar active regions, then the meridional circulation should play an important role in the dynamo problem by bringing the poloidal field from the surface to the bottom of the convection zone, where the poloidal field is stretched out to produce the toroidal field. The challenge before us now is to develop a new type of dynamo model, in which the surface processes like the production of the poloidal field from the decay of active regions

are important as in the BL approach, but which has the same mathematical sophistication as the PSKR approach. Such a dynamo model should presumably account for both the equatorward migration of sunspots and the poleward migration of the weak diffuse field. An early step in this direction was taken by Wang, Sheeley, & Nash (1991), who averaged over the radial direction to obtain one-dimensional equations. More realistic two-dimensional models have been developed by Choudhuri, Schüssler, & Dikpati (1995), Durney (1995, 1996, 1997) and Dikpati & Charbonneau (1999).

The mathematical theory of the PSKR approach is based on mean field MHD, which leads to closed equations in the first order smoothing approximation. It is not clear if the implicit assumptions in this mathematical theory are fully satisfied in any realistic astrophysical situation. However, if the assumptions are satisfied, then the mathematical theory provides a completely rigorous description of the dynamo process in the PSKR approach. To make a similar rigorous formulation of the BL approach, we need to develop a consistent mean field description of (i) the buoyant rise of the toroidal flux to produce active regions and (ii) the decay of the tilted active regions to produce the poloidal field. In this paper, we focus our attention on comparing two possible formulations of the production of poloidal field from the decay of tilted active regions. Since it is necessary to include magnetic buoyancy to study this problem, we present some discussion of magnetic buoyancy as well.

It was pointed out by Stix (1974) that the mathematical formulation of the BL approach is in some ways analogous to the α-effect of the PSKR approach. Choudhuri, Schüssler, & Dikpati (1995) modeled the decay of titled active regions to produce the poloidal field by invoking an α-coefficient which is concentrated near the solar surface. Durney (1995, 1996, 1997) followed Leighton (1969) more closely and treated the same by introducing a double ring of flux at the surface where the eruption takes place. Introducing

an α-coefficient concentrated near the surface is certainly a very approximate way of incorporating the main idea of the BL approach into the mathematical theory of the PSKR approach. Justifying this procedure rigorously is even more difficult than justifying the α-coefficient in the PSKR approach. However, this procedure produces the desired effect of generating the poloidal field where we want to generate it. If magnetic buoyancy is included in some way to bring the strong toroidal field from the bottom to the top and then the concentrated α-effect acts on it, the net result is similar to what happens in Durney's double ring method. Since this procedure is easier to implement than Durney's double ring method, one important question is whether this procedure is at least as good as Durney's double ring method. In this paper, we take a simple dynamo model and present calculations done with both the methods. We show that the results are qualitatively similar. It may be noted that it is not our aim to build realistic models of the solar cycle in this paper. For example, we have presented a contrasting study of these methods by assuming a differential rotation which does not vary with latitude as in Choudhuri, Schüssler, & Dikpati (1995). This simplification allows the specific features of the two methods to be seen clearly. A realistic differential rotation makes the results immensely more complicated, which we shall discuss in our next paper in which an attempt will be made to model the solar cycle properly.

Durney (1995, 1995, 1997) allowed flux eruption to take place only at one latitude at a time. In Durney's model, it is difficult to allow simultaneous eruptions in a band of latitudes, which happens in the real Sun. The model of Choudhuri, Schüssler, & Dikpati (1995) did not incorporate magnetic buoyancy and allowed the toroidal field to be brought to the surface from the bottom by meridional circulation. To make comparisons with Durney's double ring method, we now include magnetic buoyancy in that model by allowing the magnetic field to erupt whenever it has a value larger than a critical value. It may be noted that incorporating magnetic buoyancy in the PSKR approach was relatively easier,

since magnetic buoyancy there merely removed the flux from the dynamo region and played the role of a dissipative process. Some authors treated magnetic buoyancy by putting a simple loss term in the dynamo equation (DeLuca & Gilman 1986; Schmitt & Schüssler 1989), whereas others included a general upward flow due to magnetic buoyancy (Moss, Tuominen, & Brandenburg 1990a, 1990b). We have to go beyond such simple prescriptions in a BL approach, where magnetic buoyancy is a more integral part of the dynamo process and is not just a flux removal mechanism. In our BL model, magnetic buoyancy removes the flux from the bottom layer where the toroidal field is generated and then brings the flux to the top of the convection zone where the poloidal field is produced from it. Earlier, Choudhuri & Dikpati (1999) and Dikpati & Charbonneau (1999) incorporated the effect of magnetic buoyancy by including a dynamo source term near the surface which is a product of the α-coefficient and the toroidal magnetic field *at the bottom* of the convection zone.

From the observation that the following spots in active regions appear at higher latitudes on the solar surface, it is easy to figure out that α has to be positive in the northern hemisphere. This is also clear from the expression of α-coefficient obtained by Stix [1974; Eq. (8)] by recasting the equations of Leighton (1969). The positive sign of α gives a new twist to the problem. It is well known that the product of α and the vertical gradient of differential rotation has to be negative in the northern hemisphere for the equatorward propagation of the dynamo wave (Parker 1955; see Choudhuri 1998, §16.6). Even if α and the velocity gradient are concentrated in two different layers, this condition still remains valid (Moffatt 1978, §9.7). Since the vertical gradient of differential rotation in the lower latitudes, as found by helioseismology, is positive, its product with α is positive and one would expect a poleward propagation of the dynamo wave. It was demonstrated by Choudhuri, Schüssler, & Dikpati (1995) that an equatorward propagation is still possible in this situation, if the time scale of meridional circulation is shorter than the time scale of diffusion between the layers of α and velocity shear. This opens up the possibility of

building models of the solar dynamo in which we have a positive α near the surface and a positive gradient of differential rotation at the bottom of the convection zone. The meridional circulation has to play a very crucial role in such models in ensuring the desired behaviour. While using Durney's double ring method, the signs of the magnetic field in the two rings have to be chosen such that there is a correspondence with the positive α situation. With the double ring method also, we found that the dynamo wave at the bottom of the convection zone propagates equatorward only when there is a strong meridional flow and propagates poleward when this flow is switched off.

In §2 we discuss the details of our model. Then we go on to present our main results in §3. Our conclusions are summarised in §4.

2. The model

We assume axisymmetry in all our calculations. The magnetic and velocity fields can be written as

$$\mathbf{B} = B\mathbf{e}_\phi + \nabla \times (A\mathbf{e}_\phi), \tag{1}$$

$$\mathbf{v} = \mathbf{v}_p + r\sin\theta\Omega\mathbf{e}_\phi. \tag{2}$$

where B and A respectively represent the toroidal and poloidal components of the magnetic field; Ω is the angular velocity, and $\mathbf{v}_p = v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta$ is the meridional circulation. We substitute equations (1) and (2) in the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta\nabla^2\mathbf{B}, \tag{3}$$

where η is the coefficient of turbulent diffusion. This gives

$$\frac{\partial A}{\partial t} + \frac{1}{s}(\mathbf{v}_p.\nabla)(sA) = \eta\left(\nabla^2 - \frac{1}{s^2}\right)A + Q, \tag{4}$$

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rv_rB) + \frac{\partial}{\partial\theta}(v_\theta B)\right] = \eta\left(\nabla^2 - \frac{1}{s^2}\right)B + s(\mathbf{B}_p.\nabla)\Omega, \tag{5}$$

where $s = r\sin\theta$ and $\mathbf{B}_p = \nabla \times (A\mathbf{e}_\phi)$. We have added one extra term Q on the right-hand side of equation (4), which does not follow from the induction equation (3). It is a term which describes the generation of the poloidal field. The usual $\alpha\Omega$ dynamo is given by the equations (4) and (5), where Q is simply

$$Q = \alpha B. \tag{6}$$

To incorporate the effect of magnetic buoyancy and the decay of tilted active regions, we have to allow for changes in B due to the rise of magnetic flux from the bottom of the convection zone to the top and specify Q appropriately. Before describing how we incorporate Durney's double ring method as well as our method of concentrated α-effect near the surface, let us discuss a few general points which hold for both cases.

The equations (4) and (5) have to be solved in the northern quadrant of the convection zone as usual (i.e. within $R_b = 0.7R_\odot \le r \le R_\odot$, $0 \le \theta \le \pi/2$). The boundary conditions are discussed in previous papers (Dikpati & Choudhuri 1994; Choudhuri, Schüssler, & Dikpati 1995). They are

$$\text{At } \theta = 0: \quad A = 0,\ B = 0,$$

$$\text{At } \theta = \frac{\pi}{2}: \quad \frac{\partial A}{\partial \theta} = 0,\ B = 0,$$

$$\text{At } r = R_b: \quad A = 0,\ \frac{\partial}{\partial r}(rB) = 0,$$

$$\text{At } r = R_\odot: \quad B = 0,$$

the boundary condition for A at the top $r = R_\odot$ being that it has to match a smooth potential field outside. See §3 of Dikpati & Choudhuri (1994) for a detailed discussion of how this is implemented.

To solve equations (4) and (5) with these boundary conditions, we need to specify η, Ω, $\mathbf{v}_p$ and Q. As in Choudhuri, Schüssler, & Dikpati (1995), we assume the turbulent diffusion

to have the constant value $\eta = 1.1 \times 10^7$ m^2 s^{-1}. For the angular velocity Ω also, we use the same expression as used in that paper:

$$\Omega = \Omega_0\{0.9294 + 0.0353 \left[1 + \mathrm{erf}\left(\frac{r - r_3}{d_3}\right)\right]\} \tag{7}$$

with $r_3 = 0.7R_\odot$, $d_3 = 0.1R_\odot$, $\Omega_0 = 2.7 \times 10^{-6}$ s^{-1}. This latitude-independent angular velocity roughly corresponds to the helioseismologically determined rotation profile near the solar equator, with $\partial\Omega/\partial r$ positive. For the meridional circulation $\mathbf{v}_p$, we again use the expression used previously (Dikpati & Choudhuri 1995; Choudhuri, Schüssler, & Dikpati 1995). In other words, we take

$$\rho\mathbf{v}_p = \nabla \times (\psi\mathbf{e}_\phi) \tag{8}$$

with ψ given by

$$\psi r \sin\theta = \psi_0 \sin\left[\frac{\pi(r - R_b)}{(R_\odot - R_b)}\right] \{1 - e^{-\beta_1 r\theta^\epsilon}\} \times \{1 - e^{\beta_2 r(\theta - \pi/2)}\} e^{-((r-r_0)/\Gamma)^2} \tag{9}$$

and ρ given by

$$\rho(r) = C\left(\frac{R_\odot}{r} - \gamma\right)^m \tag{10}$$

The values of the parameters used are $\beta_1 = 1.4 \times 10^{-8}$ m^{-1}, $\beta_2 = 2.7 \times 10^{-8}$ m^{-1}, $\epsilon = 2.0000001$, $r_0 = (R_\odot - R_b)/5$, $\Gamma = 3.3 \times 10^8$ m, $\gamma = 0.9$, $m = 3/2$. The pattern of meridional circulation for these values of parameters is shown in Figure 3a of Dikpati & Choudhuri (1995). The amplitude of the meridional circulation is fixed by taking $\psi_0/C = -7.9 \times 10^8$ m^2 s^{-1}, which corresponds to a maximum surface velocity (v_0) of about 7.0 m s^{-1} in the mid-latitudes.

In the paper of Choudhuri, Schüssler, & Dikpati (1995), the source term Q was given by equation (6) with α taken in the form

$$\alpha = \frac{\alpha_0}{1 + B^2} \cos\theta \frac{1}{4}\left[1 + \mathrm{erf}\left(\frac{r - r_1}{d_1}\right)\right] \times \left[1 - \mathrm{erf}\left(\frac{r - r_2}{d_2}\right)\right]. \tag{11}$$

The parameters are $r_1 = 0.95R_\odot$, $r_2 = R_\odot$, $d_1 = d_2 = 0.025R_\odot$, making sure that the α effect is concentrated in the top layer $0.95R_\odot \leq r \leq R_\odot$. The α-quenching factor $1 + B^2$

included in the denominator helps the system to relax to periodic solutions with amplitude $B \sim 1$. This essentially means that we are choosing the unit of B in such a way that the nonlinear feedback becomes important when B is of order unity or larger. Choudhuri, Schüssler, & Dikpati (1995) took $\alpha_0 = 3$ m s^{-1} and found that it gave rise to marginally critical oscillations. When magnetic buoyancy is included, we find that this value of α_0 often gives decaying solutions. To ensure that the solutions do not decay, we take $\alpha_0 = 10$ m s^{-1} in most of the calculations in the present paper.

2.1. Incorporating the double ring

After time intervals τ, we find the co-latitude $\theta_{\rm er}$ where the toroidal field is maximum and allow the flux to erupt above in the form of the double ring, if this maximum value exceeds a specified critical field B_c. Following Figure 1 of Durney (1997), we show the two emergent flux rings in Figure 1. One ring of positive magnetic field $K/\sin\theta$ is put between the co-latitudes θ_1, θ_2, whereas the other ring of negative magnetic field $-K/\sin\theta$ is between θ_3, θ_4. The factor $\sin\theta$ ensures that the flux through one ring balances the flux through the other ring. In Durney's notation, θ_1, θ_2, θ_3 and θ_4 will be

$$\theta_1 = \theta_{\rm er} - \frac{\chi + \Lambda}{2},$$

$$\theta_2 = \theta_{\rm er} - \frac{\chi - \Lambda}{2},$$

$$\theta_3 = \theta_{\rm er} + \frac{\chi - \Lambda}{2},$$

$$\theta_4 = \theta_{\rm er} + \frac{\chi + \Lambda}{2}.$$

As in Durney (1997), we make the somewhat unphysical assumption that these rings extend only from $R_\odot$ to $R_\odot - \Delta r$, where the field lines end abruptly. At the time of eruption, then, in the region $R_\odot - \Delta r \le r \le R_\odot$, we put the magnetic field

$$\begin{aligned} \Delta B_r &= \frac{K}{\sin\theta} \text{ if } \theta_1 \le \theta \le \theta_2 \\ &= \frac{-K}{\sin\theta} \text{ if } \theta_3 \le \theta \le \theta_4 \\ &= 0 \text{ elsewhere} \end{aligned} \tag{12}$$

Putting this magnetic field is equivalent to adding the vector potential ΔA given by

$$\Delta B_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\Delta A),$$

from which

$$\Delta A = \frac{r}{\sin\theta}\int_0^\theta \sin\theta'\Delta B_r d\theta' \tag{13}$$

if we do not consider the variation of ΔA in r. Substituting for ΔB_r from (12), we conclude that ΔA can be non-zero only in the range $R_\odot - \Delta r \le r \le R_\odot$, where we have

$$\begin{aligned} \Delta A \sin\theta &= 0 && \text{for } 0 \le \theta \le \theta_1 \\ &= R_\odot K(\theta - \theta_1) && \text{for } \theta_1 \le \theta \le \theta_2 \\ &= R_\odot K(\theta_2 - \theta_1) && \text{for } \theta_2 \le \theta \le \theta_3 \\ &= R_\odot K[(\theta_2 - \theta_1) - (\theta - \theta_3)] && \text{for } \theta_3 \le \theta \le \theta_4 \\ &= 0 && \text{for } \theta_4 \le \theta \le \frac{\pi}{2} \end{aligned} \tag{14}$$

Adding this ΔA to A leads to a discontinuity in A at $R_\odot - \Delta r$. Durney (1997) writes, "Such an expression for the vector potential generates latitudinal magnetic fields (associated with the closure of magnetic lines of force)", but also claims that these discontinuities "are numerically inconsequential".

Durney (1995, 1996, 1997) took the separation between the rings, χ, to be proportional to $\cos\theta_{\rm er}$. To keep the numerical computations simpler, we instead take K appearing in (14) to be proportional to $\cos\theta_{\rm er}$, while keeping the separation between the rings fixed.

This has the same physical effect, except at the low latitudes where the two rings may overlap. We, however, find that flux eruption remains restricted to higher latitudes where this overlap is unimportant. If we take K to be proportional to the toroidal magnetic field B at the bottom (from which the flux rings originate), then our problem becomes linear in magnetic field and one has to make many runs to find the marginally growing solution. We circumvent this problem by including something like α-quenching in the following fashion:

$$K = K' \frac{B_{\rm max} \cos\theta_{\rm er}}{1 + |B_{\rm max}|^2}, \tag{15}$$

where $B_{\rm max}$ is the toroidal magnetic field at the bottom of the convection zone at the latitude where it is maximum. The justification behind this is the fact that a stronger toroidal field is less affected by the Coriolis force (D'Silva and Choudhuri 1993; Howard 1993) and hence is less efficient in generating the poloidal field. It can easily be seen from (14) and (15) that K' is a dimensionless quantity.

It is seen on the solar surface that the magnetic field of the higher-latitude sunspot is positive when the toroidal field underneath the surface is positive. It should be clear from (12) and (15) that this is achieved by taking K' positive, which is the case in all our calculations. It now follows from (14) and (15) that a positive B at the bottom of the convection zone would imply a positive increment in A at the surface where the magnetic flux emerges. This is something like a positive α-effect, which obviously corresponds to a positive value of K'.

We solve (4) and (5) with differential rotation and meridional circulation as given by (7)–(10). The source term Q in (4) is given by (6) and (11). However, in addition to this usual source term, we allow for possible changes in the value of A abruptly, in the double ring regions of the surface, at intervals of τ, to take account of magnetic buoyancy. We run our code to find the maximum value of B after intervals τ. If this exceeds the critical field B_c and occurs at the co-latitude $\theta_{\rm er}$, then we consider two rings situated on two sides of this

co-latitude and add ΔA as given by (3) to A. The control parameter in our problem is K' appearing in (4). When K' is zero, there is no double ring formation and we get the model of Choudhuri, Schüssler, & Dikpati (1995). On the other hand, when K' is sufficiently large, the net effect of double ring formation at intervals of τ becomes much more important than the source term Q in (4) and we have the model of Durney (1997). Thus, in the two opposite limits of the control parameter K', our model is respectively reduced to the models of Choudhuri, Schüssler, & Dikpati (1995) or Durney (1997).

2.2. Concentrated α-effect with magnetic buoyancy

We wish to argue that the double ring method is similar to allowing magnetic flux to rise due to magnetic buoyancy and then letting the α-effect concentrated near the surface to act on it. In this method also, we solve (4) and (5) in conjunction with (6)–(11). However, instead of having double ring formations at intervals of τ (leading to abrupt changes of A as seen from [14]), we now allow B to change abruptly at intervals of τ to take account of flux rise due to magnetic buoyancy. This is done in the following way.

We assume that the toroidal field B becomes buoyant when its value crosses a critical value B_c. After intervals of time τ, we check if B has become larger than B_c at certain points. Then, at those points, B is reduced by a factor $1 - f$, i.e.

$$B \rightarrow B(1 - f).$$

The flux removed from these points is taken vertically above and deposited near the surface by increasing B there in such a fashion that the total flux remains conserved in the transfer process. Since the equations are numerically solved on a $N \times M$ grid, the simplest procedure is to deposit all the flux at the grid point just below the surface. For example, if B crosses B_c only at one grid point on the radial line at a fixed latitude, then we have

to decrease B by fB there and the toroidal field at the grid point just below the surface has to be increased by an amount $f'B$. Since the grid size at the surface corresponds to a greater distance in the latitudinal direction than that at the bottom, we need to take $f' = f(R_i/R_f)$ to ensure the conservation of magnetic flux (here, R_i is the radius near the bottom where the flux is depleted and R_f is the radius near the surface where the flux is deposited). We have also made some runs in which the flux taken up from one grid point is distributed within a few grid points near the top instead of all the flux being deposited in one grid point, and the results turn out to be qualitatively similar. The strength of magnetic buoyancy is increased by increasing the control parameter f. In the limit $f = 0$, we get back the model of Choudhuri, Schüssler, & Dikpati (1995), in which there was no magnetic buoyancy and the toroidal field was brought to the surface by the meridional circulation. When f is made sufficiently large (even though it has to remain less than 1), magnetic buoyancy is found to dominate and the system has a limiting behavior.

Compared to the double ring method, this method has some attractive features. Firstly, here the eruption at any instant takes place over a range of latitude rather than at one point as in the double ring method. This corresponds to the real Sun more closely. It is not easy to extend the double ring method to handle simultaneous flux eruptions at more than one point. If we simultaneously put several double rings in a range of latitudes, then the positive ring of an intermediate double ring will cancel with the negative ring of the next double ring and we shall be left with a positive ring and a negative ring at a wide separation. It follows from (14) that this will mean adding to A over a wide range of latitude. This would make the model more similar to the mean field model and the special character of the original double ring model would be completely lost. Also, we now allow for the toroidal flux to be depleted at the bottom of the convection zone due to magnetic buoyancy. As we shall argue later, we believe this to be quite important. In fact, we shall present some results with the double ring method with the toroidal flux at the bottom

depleted parametrically.

3. Results

We now present and compare results obtained by the two methods described above. As we saw, K' and f happen to be the respective control parameters in these two methods. On setting these control parameters equal to 0, both these methods are reduced to the model of Choudhuri, Schüssler, & Dikpati (1995, hereafter CSD model). All our calculations are done on a 64×64 grid. We allow the eruptions to take place after times $\tau = 8.8 \times 10^5$ s and use a value $B_c = 1$ for the critical field in all our calculations. When we start our calculations with any arbitrary magnetic field configuration, the code relaxes to a periodic solution for a proper set of parameters. What we discuss below are properties of such relaxed periodic solutions.

3.1. Results with the double ring method

Durney (1997) did not allow the toroidal flux to be depleted at the bottom of the convection zone due to magnetic buoyancy. To study the effect of flux depletion, we present some calculations in which we allow flux depletion in the following simple manner. At the times of eruption after interval τ, we find out at which point the toroidal field has the maximum value $B_{\rm max}$ ($> B_c$). While putting the two flux rings at the top, we also decrease $B_{\rm max}$ by an amount $f_d B_{\rm max}$ at the maximum point. Then f_d becomes a second parameter in the problem in addition to K' in our problem. After finding the co-latitude $\theta_{\rm er}$ where the toroidal field is maximum, the next two poleward grid points are taken as θ_1, θ_2, and the next two equatorward grid points are taken as θ_3, θ_4. The flux rings are assumed to go 3 grid points deep (i.e. Δr is taken 3 grid points below the surface).

Figure 2 shows how the dynamo period T_d changes with the parameter K' when f_d is held constant. The different curves correspond to different values of f_d. When we go to the limit of CSD model by putting $K' = 0$, we find the period to be 66 yrs. When $f_d = 0$ (i.e. there is no flux depletion at the bottom), we find that the change in the period with K' does not follow any particular trend. T_d at first increases slightly with increasing K' and then comes down to a value close to that of the CSD model. This behaviour for $f_d = 0$ may result from the fact that in this case we are actually creating flux (in the form of erupted double rings) without any depletion. More meaningful behaviour follows for the other values of f_d (such as 0.25, 0.5, 0.75). The period decreases with increasing K' and tends to saturate at some asymptotic value for large K'. To understand what is happening, let us look at Figure 3 which shows the evolution of magnetic field during a half-period for the case $K' = 1000$, $f_d = 0.5$. In the plots of poloidal field, we have indicated the latitudes of last flux eruption with small arrows. However, the individual double rings are not usually discernable. That is not surprising. Flux eruption in the form of double rings keeps occuring at intervals of τ. Hence the latest double ring is merely superposed on the field created by the previous double rings and does not stand out against the background of previously created field. On looking at the plots of the toroidal field, it is clear that the toroidal field keeps weakening as we go to lower latitudes. This weakening of toroidal field at lower latitudes becomes more prominent as we make f_d larger. This implies that flux eruption never takes place at very low latitudes and the dynamo process is basically confined to higher latitudes. Since it takes less time to transport magnetic flux through a limited range of latitudes, the dynamo period is shorter for non-zero f_d. In combination with this effect, an increasing K' will make the erupted double rings stronger, thus recycling toroidal flux to poloidal flux more efficiently. This reduces the time period of the dynamo as compared to the period in the limit of the CSD model, in which the toroidal field is brought to the surface by the meridional flow only near the equator and the whole range

of latitudes is involved. It may be noted that Durney (1997) did not present any plots of magnetic field configurations in his paper. However, we do get a deeper insight into the problem by looking at such field configuration plots. For example, note that the direction of the poloidal field (clockwise or anti-clockwise) starts reversing at the time when we have an extended belt of strong toroidal field.

Durney (1997) has presented several plots showing how the eruption latitude changes with time (Figures 7–10 in his paper). We present a similar plot in Figure 4 for the case $K' = 1000$, $f_d = 0$, corresponding to no flux depletion at the bottom as in the calculations of Durney (1997). Here, we see that eruptions continue near the pole for some time at the beginning of a cycle and then progressively move to lower latitudes. This plot looks very much like the plots presented by Durney (1997) — especially his Figure 7. This is certainly very reassuring, since the numerical techniques employed by us and by Durney (1997) are completely different. Apart from the production of the double rings, our code allows for the toroidal flux to be brought to the surface by meridional circulation and then to be acted upon by α-coefficient (an effect not present in Durney's calculations). However, when K' is made as large as 1000, this effect is insignificant. In fact, we made some runs with $\alpha = 0$ and found that the results for zero or non-zero α are virtually indistinguishable when $K' = 1000$. For example, the plots of eruption latitude against time and the butterfly diagrams look identical in both the cases.

We have already mentioned that a positive K' is like a positive α-effect concentrated near the surface. Choudhuri, Schüssler, & Dikpati (1995) showed that a positive α concentrated near the surface leads to a poleward propagation of the dynamo wave when the meridional flow is switched off. We find exactly the same result in the double ring approach with positive K' if we switch off the meridional flow. Figure 5 shows a time-latitude plot of the toroidal field at the bottom of the convection zone with meridional flow for the case

$K' = 1000$, $f_d = 0.5$, whereas Figure 6 is a similar plot without meridional flow keeping all the other parameters the same. We see clear indication of poleward migration in Figure 6.

3.2. Results for concentrated α with buoyancy

For contrast, we now present results obtained by the method described in § 2.2. As we have seen, the control parameter in this problem is $f(< 1)$, which measures the strength of magnetic buoyancy. Figure 7 shows how the dynamo period changes on increasing f. As in Figure 2, we begin with a period of 66 yrs in the limit $f = 0$ corresponding to the CSD model. On making the effect of buoyancy stronger (by increasing f), the flux transport (from the bottom of the convection zone to the top) takes place more efficiently and also the toroidal flux gets depleted quickly. This results in the dynamo period reducing with increasing f, until it reaches an asymptotic value of about 25 yrs. We may point out here that we did some runs for this method without depleting the field at the bottom, which would correspond to the case $f_d = 0$ for the double ring method. We found that even in this case, there is no decrease in the time period with increasing f (which in this case corresponds to only field addition at the top) and T_d more or less hovers around the CSD limit of 66 years.

Since the two methods discussed by us are sufficiently different, it is not obvious which value of K' in the first method would correspond to a certain value of f in the second method. In both methods, however, the dynamo periods saturate to asymptotic values when these control parameters are sufficiently large. So the most sensible thing is to compare results of the two methods when the control parameters are large enough to ensure that the dynamo period has the asymptotic value. Figure 8 shows the time evolution of the magnetic field during a half period for the parameters $f = 0.05$ (i.e. the magnetic buoyancy is strong enough to saturate the period to its asymptotic value). On comparing

with Figure 3, we find that the broad features of the magnetic field distribution are very similar. The main difference is that one sees some toroidal field distributed near the top of the convection zone in Figure 8, whereas such fields are not present in Figure 3. The reason behind this is obvious. In the double ring method, we directly put double rings above regions of strong toroidal field and this contributes directly to the poloidal field. When we introduce the intermediate step of the toroidal field first rising due to buoyancy and then being acted upon by α-effect, then we get toroidal field at the top of the convection zone also, as in Figure 8. The other difference between Figures 3 and 8 is that often the field lines in Figure 3 in some places (especially near the surface) are not as smooth as the field lines are everywhere in Figure 8. This is certainly due to double ring formations in Figure 3, which are concentrated local effects. As in Figure 3, here also we find that the direction of poloidal field reverses at around the time the strong toroidal field belt is maximally extended.

Finally Figure 9 presents a time-latitude plot of the toroidal field at the bottom for the same case which is presented in Figure 8. Again, this figure looks qualitatively similar to Figure 5, the main difference being the fact that the toroidal field has become much weaker near the equator in Figure 8 due to more efficient flux depletion at the bottom, which takes place naturally in this method.

4. Conclusion

Following Choudhuri, Schüssler, and Dikpati (1995) and Durney (1997), we build a hybrid model of the solar dynamo, in which the best features of both the PSKR and the BL approaches are combined. The aim is to include the surface processes emphasized in BL models into a model as suitable for detailed quantitative study as the PSKR models. We study two possible methods of achieving this. One is to introduce double rings above the

region where the toroidal field is maximum, as done by Durney (1995, 1996, 1997). The second method is to make the toroidal field rise when it is above a critical value and then allow it to be acted upon by an α-coefficient concentrated near the surface. It is reassuring that the results obtained by the two methods are qualitatively similar.

We believe that the depletion of toroidal flux by magnetic buoyancy is an important process. Flux tube calculations (Choudhuri & Gilman 1987; Choudhuri 1989; D'Silva & Choudhuri 1993; Fan, Fisher, & DeLuca 1993; Caligari, Moreno-Insertis, & Schüssler 1995) suggest that the toroidal field at the bottom of the convection zone has a value of 10^5 G—much stronger than the equipartition value. After the belt of strong toroidal field reaches the equator, it disappears and the next half-cycle of the dynamo begins. If the field is so strong, then turbulent diffusion will be completely suppressed and will not be effective in destroying the strong toroidal field. The only way to annihilate this belt of strong toroidal field is to expect magnetic buoyancy to deplete its strength sufficiently by the time this belt propagates to the equator. In our second method, this flux depletion automatically takes place. In the double ring method, we have included the possibility of toroidal flux depletion as an extra effect, which was not taken into account by Durney (1997). When the toroidal flux is depleted appropriately, both the methods make the period of the dynamo decrease on increasing the control parameters (K' or f) and saturate at some asymptotic value. This decrease of period is due to the efficient and rapid transport of toroidal flux by magnetic buoyancy. It is true that the decrease of period is more pronounced in the second method, where the flux depletion is more prominent. However, a difference by a factor 2 or 3 in the asymptotic period is probably not such a significant uncertainty compared to many other factors. The magnetic field configurations obtained in the two methods, as seen in Figures 3 and 8, are also quite similar, with the poloidal field reversing its direction for the same configuration of the toroidal field. Then, very importantly, the dynamo wave is found to propagate poleward when the meridional circulation is switched

off in the double ring method. In other words, the double ring method with positive K' has characteristics quite similar to a model with positive α-effect concentrated near the surface. The results obtained by the two methods are not exactly identical. However, given the many uncertainties plaguing the solar dynamo theory at the present time, representing the generation of poloidal field near the surface by a concentrated α-effect acting on erupted toroidal field seems like a good enough approximation.

We should point out that there are several logistic problems in numerically handling the double ring, which are not there if we use an α-effect instead. Firstly, to properly create rings of latitudinal size similar to sunspot size with appropriate separation, one has to either use at least 500 grid points in the θ direction or use a special code which employs a finer mesh in the region where eruption takes place. Durney (1997) used 101×101 grid, which corresponds to a grid size of about 11,000 km in the latitudinal direction at the Sun's surface. The width of the double ring has to be at least 4 times this, i.e. about 44,000 km—definitely inadequate to resolve the north-south polarity separation of a typical active region. To ensure whether our results have converged with respect to grid size, we repeated some calculations on 32×32 grid and compared the results with those obtained on our usual 64×64 grid. We found that results obtained by our second method of concentrated α-effect were so close in the two cases that various plots looked indistinguishable. However, results obtained by the double ring method, in which important source terms are taken at the limit of grid resolution, while remaining qualitatively similar on halving the grid size, showed some changes. Our grid size is comparable to what other researchers (Durney 1997, Dikpati & Charbonneau 1999) have used on similar problems. We believe that the grid size has to be reduced considerably to properly resolve double rings and to give results completely invariant with grid size. Since the ring separation was at the limit of grid resolution, we kept the ring separation fixed and made our constant K proportional to $\cos\theta_{\rm er}$ (see [15] and the discussion preceding that). Durney (1997) claims to have made the ring separation

proportional to $\cos\theta_{\rm er}$, but never explains in his paper how this could be done with only 101 grid points in the θ direction. Another important consideration is that the double ring method is easy to implement when we allow flux eruption only at one point at one time, but it is not easy to generalize if multiple flux eruptions are allowed. In reality, we find that, at a certain time, several active regions emerge in a belt of latitudes—with the different active regions usually separated in longitude. If one could use an appropriately resolved 3D code in which active regions of realistic size were made to emerge in different latitudes and longitudes, then certainly that would have been a much more satisfactory calculation than what we are doing now. We hope that future computers will be used by researchers more numerically capable than us to tackle this problem. When one uses rings to replace active regions through an averaging over longitude and uses a grid not fine enough to resolve individual sunspots, one already introduces some drastic averaging. Introducing an α-coefficient concentrated near the surface instead of using double rings may not be such a big step after that.

Let us end with a comment on what we mean by a Babcock-Leighton model, since this term often creates some confusion. Babcock (1961) and Leighton (1969) emphasized the surface process of poloidal field generation from tilted active regions—in contrast to the usual mean field MHD where the poloidal field is supposed to be produced in the interior region of turbulence (see, for example, Choudhuri 1998). Hence any dynamo model in which the poloidal field is generated in a thin layer near the solar surface should be called a Babcock-Leighton model. Dikpati & Charbonneau (1999) also use the term in this sense. Durney (1995, 1996, 1997) followed Leighton (1969) more closely in incorporating the Babcock-Leighton idea through the double ring method. Introducing a phenomenological α-coefficient concentrated near the surface is another way of representing the Babcock-Leighton idea. One should, however, be careful not to interpret this α-coefficient in the way it is interpreted in the mean field MHD. For example, the α-coefficient here is not obviously

related to the average helicity of turbulence as in mean field MHD (see, for example, Choudhuri 1998, §16.5). This coefficient merely provides a phenomenological description of the production of poloidal field from the decay of tilted active regions, which is obvious in the formulation of the BL model by Stix (1974). Wang & Sheeley (1991) also referred to this process as an "α-effect" in exactly the same sense as us, even though they never used the symbol α in their actual equations!

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Fig. 1.— A sketch of the erupted double ring at the solar surface.

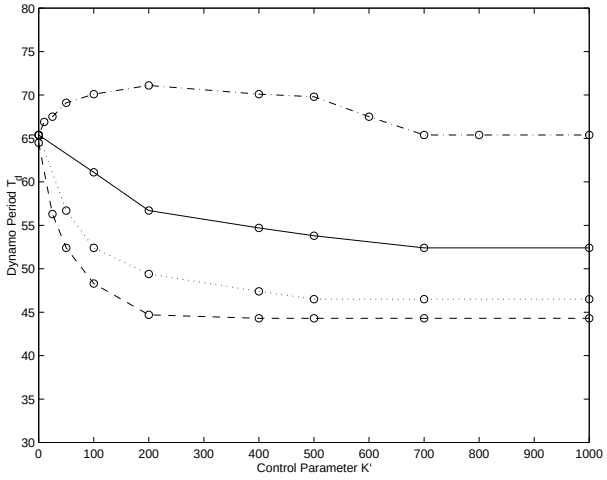

Fig. 2.— The variation in the dynamo period (in units of years) with the control parameter K' for four different f_d values. The dash-dotted line corresponds to $f_d = 0$, the solid line to $f_d = 0.25$, the dotted line to $f_d = 0.5$ and the dashed line to $f_d = 0.75$.

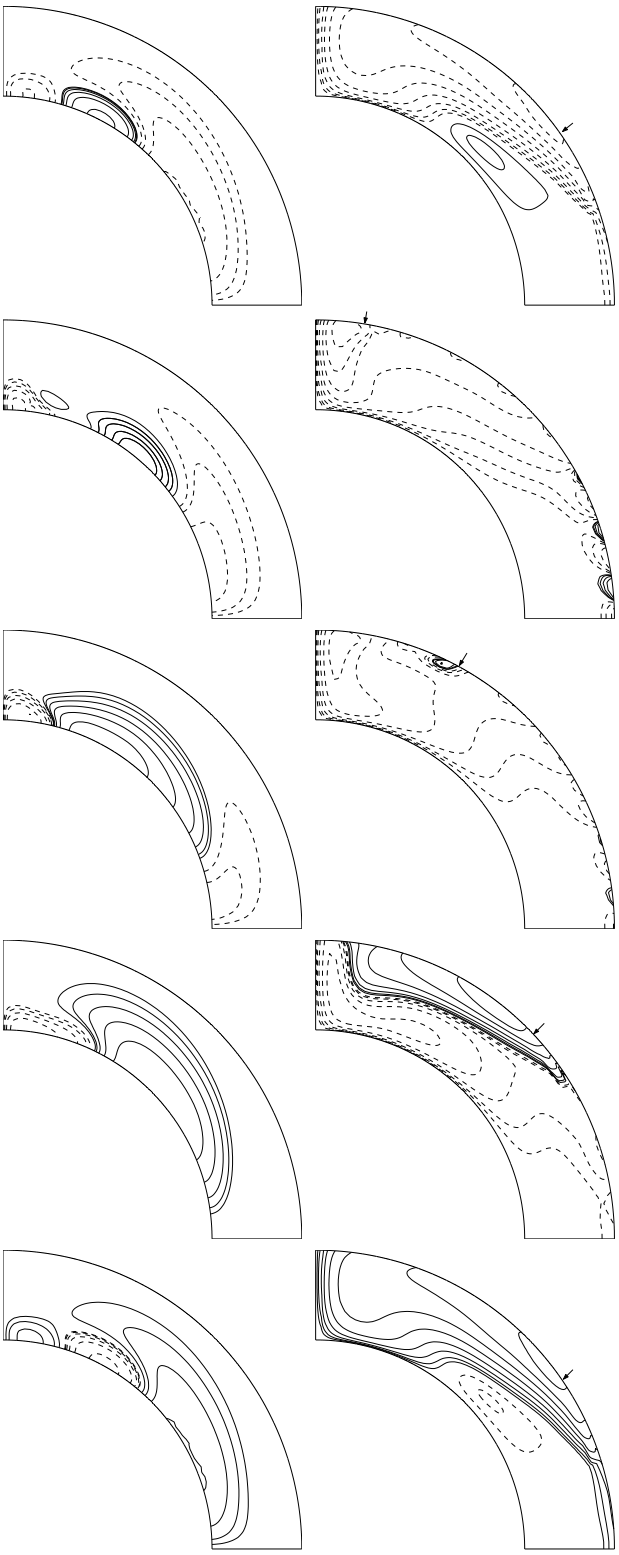

Fig. 3.— Time evolution of the toroidal field (left hand column) and poloidal field (right hand column) configuration in a meridional cut of the northern quadrant of the solar convection zone ($0.7R_\odot, \leq r \leq R_\odot$, $0 \leq \theta \leq \pi/2$) for the case with $K' = 1000$, $f_d = 0.5$. The whole set covers a dynamo half period. That is from top to bottom $t = 0$, $T_d/8$, $T_d/4$, $3T_d/8$, $T_d/2$.

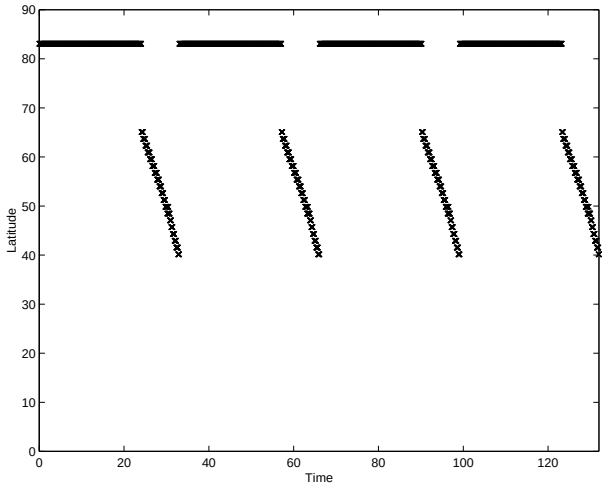

Fig. 4.— The variation in the eruption latitude with time (in years) for $K' = 1000$, $f_d = 0$.

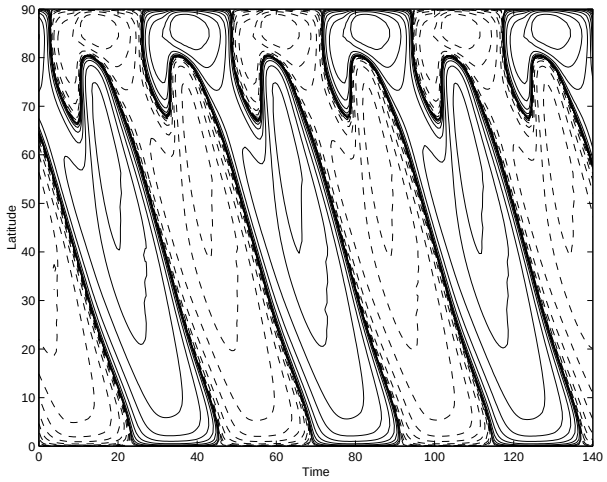

Fig. 5.— Time-latitude plot of the contours of constant toroidal field B at the bottom of the convection zone, for the case with $K' = 1000$, $f_d = 0.5$ and $v_0 = 7.0ms^{-1}$. The solid lines denote positive B and the dashed lines negative B. Time is in years.

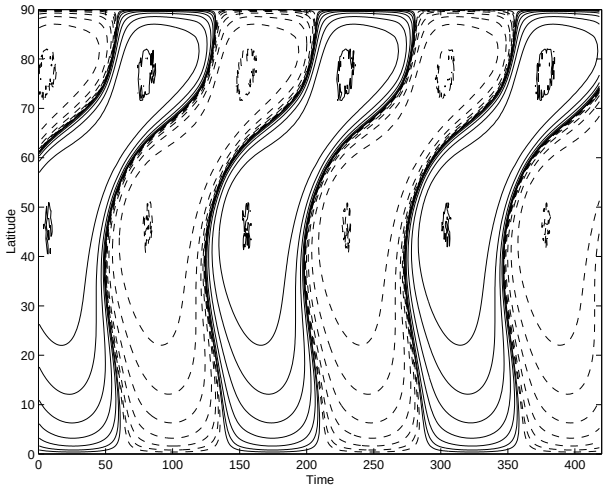

Fig. 6.— Time-latitude plot of the contours of constant toroidal field B at the bottom of the convection zone, for the case with $v_0 = 0.0$. The other parameter are the same as in Figure 5.

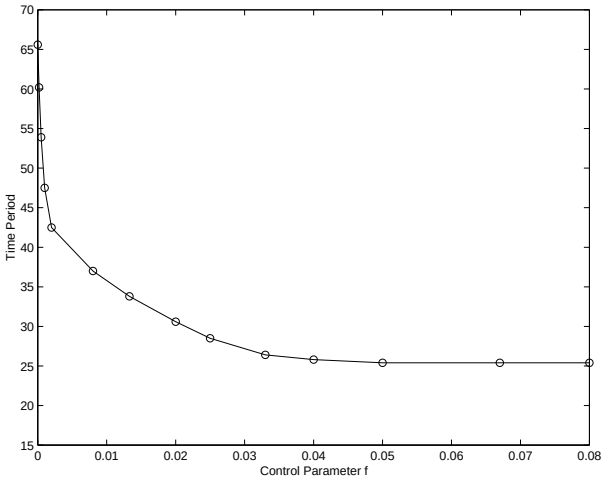

Fig. 7.— The variation in the dynamo period (in units of years) with the control parameter f for our second method - concentrated α effect with buoyancy.

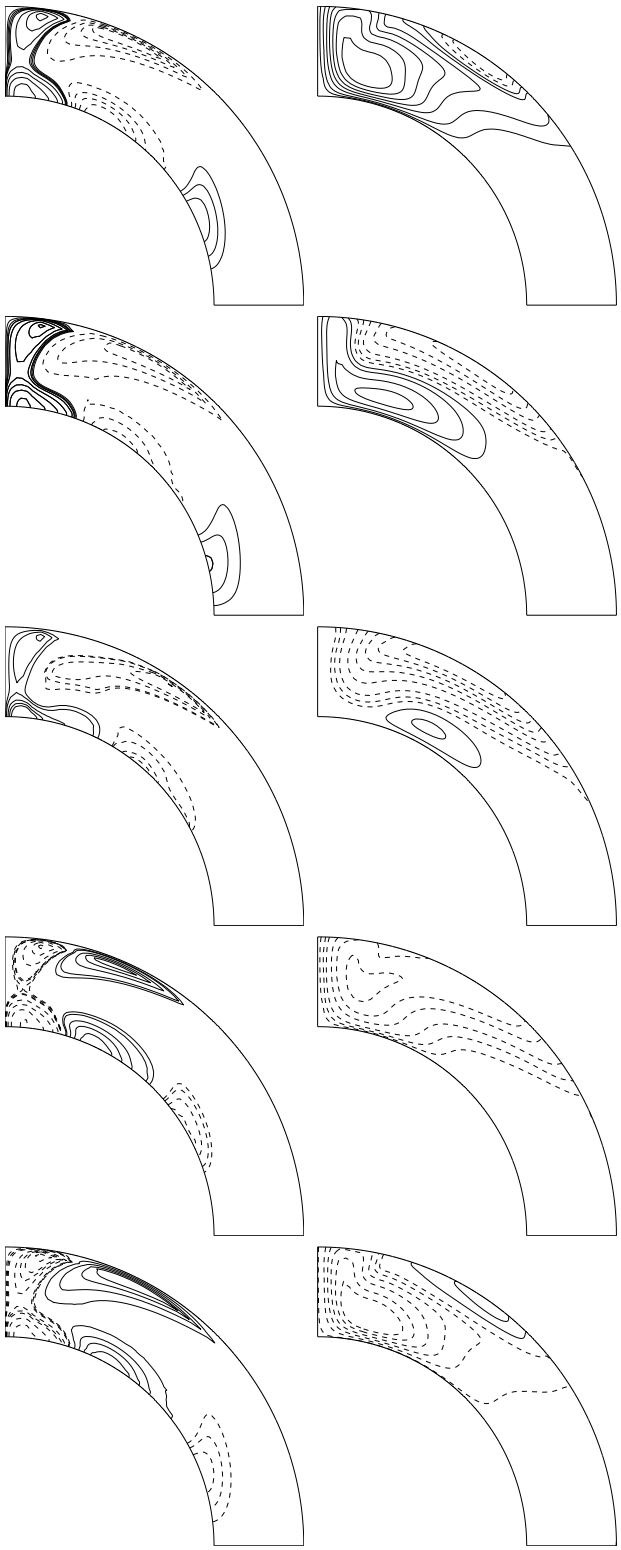

Fig. 8.— Time evolution of the toroidal field (left hand column) and poloidal field (right hand column) configuration for the concentrated α effect with buoyancy method with $f = 0.05$. The convention followed is the same as in Figure 3.

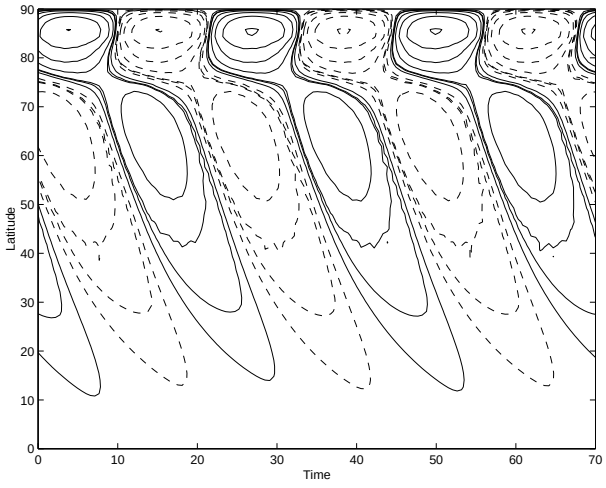

Fig. 9.— Time-latitude plot of the contours of constant toroidal field B at the bottom of the convection zone for the concentrated α effect with buoyancy method, with $f = 0.05$. Time is in years.