Why does the Sun's torsional oscillation begin before the sunspot cycle?

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(Dated: July 28, 2009)

Although the Sun's torsional oscillation is believed to be driven by the Lorentz force associated with the sunspot cycle, this oscillation begins 2–3 yr before the sunspot cycle. We provide a theoretical explanation of this with the help of a solar dynamo model having a meridional circulation penetrating slightly below the bottom of the convection zone, because only in such dynamo models the strong toroidal field forms a few years before the sunspot cycle and at a higher latitude.

PACS numbers:

There is a small periodic variation in the Sun's rotation with the sunspot cycle, called torsional oscillations. While this was first discovered on the Sun's surface [1], the nature of torsional oscillations inside the solar convection zone was later determined from helioseismology [2-8]. Several authors [9-12] developed theoretical models of torsional oscillations by assuming that they are driven by the Lorentz force of the Sun's cyclically varying magnetic field associated with the sunspot cycle. If this is true, then one would expect the torsional oscillations to follow the sunspot cycles. The puzzling fact, however, is that the torsional oscillations of a cycle begin a couple of years before the sunspots of that cycle appear and at a latitude higher than where the first sunspots are subsequently seen. At first sight, this looks like a violation of causality—a classic case of the effect preceding the cause! Our aim is to explain this puzzling observation, for which no previous theoretical model offered any explanation. In the models of Covas et al. [10] and Rempel [12], the theoretical butterfly diagrams extend to unrealistically high latitudes of about 60° and the low-latitude branches of torsional oscillations follow the butterfly diagrams closely, not starting at higher latitudes.

Let us summarize some of the other important characteristics of torsional oscillations, which a theoretical model should try to explain. (1) Apart from the equatorward-propagating branch which moves with the sunspot belt after the sunspots start appearing, there is also a poleward-propagating branch at high latitudes. (2) The amplitude of torsional oscillations near the surface is of order 5 m s^{-1}. (3) The torsional oscillations seem to be present throughout the convection zone, though they appear more intermittent and less coherent as we go deeper down into the convection zone (see Figs. 4, 5 and 6 in Howe et al.[7]). (4) In the equatorward-propagating branch at low latitudes, the torsional oscillations at the surface seem to have a phase lag of about 2 yr compared to the oscillations at the bottom of the convection zone (see Fig. 7 in Howe et al.[7]).

The last property of torsional oscillations listed above seems to suggest that the bottom of the convection zone is the source of the oscillations, which then propagate upwards. The property (3) then seems puzzling and contrary to the common sense. One would expect the oscillations to be more coherent near the source, becoming more diffuse as they move upward further away from the source. The observations indicate the opposite of this. We shall discuss a possible explanation for this observation as well. Spruit [13] proposed thermal effects near the surface to be the origin of torsional oscillations—an idea which the property (4) seems to rule out [7].

While there may not yet be a complete concensus, the majority of dynamo theorists believe that the sunspot cycle is produced by a flux transport dynamo, in which the meridional circulation carries the toroidal field produced from differential rotation in the tachocline equatorward and carries the poloidal field produced by the Babcock–Leighton mechanism at the surface poleward [14-21]. Since the differential rotation is stronger at higher latitudes in the tachocline than at lower latitudes, the inclusion of solar-like rotation tends to produce a strong toroidal field at high latitudes rather than the latitudes where sunspots are seen [17-18]. Nandy & Choudhuri [19] proposed a hypothesis to overcome this difficulty. According to them, the meridional circulation penetrates slightly below the bottom of the convection zone and the strong toroidal field produced at the high-latitude tachocline is pushed by this circulation into stable layers below the convection zone where magnetic buoyancy is suppressed and sunspots are not formed. Only when the toroidal field is brought into the convection zone by the meridional circulation rising at lower latitudes, magnetic buoyancy takes over and sunspots finally form. It may be noted that there is a controversy at the present time whether the meridional circulation can penetrate below the convection zone—arguments having been advanced both against [22] and for it [23].

If the Nandy–Choudhuri hypothesis (hereafter NC hypothesis) is correct, then the toroidal field of a particular cycle first forms at a relatively high latitude some time before the sunspots of the cycle would start appearing. Assuming that the Lorentz force of the newly formed toroidal field at the high latitude can initiate the torsional oscillations, the NC hypothesis provides a natural way to explain how the torsional oscillations begin at high latitudes before the appearance of the sunspots of the cycle. Our dynamo model based on the NC hypothesis correctly explains the onset of torsional oscillations at the high latitude before the beginning of the sunspot cycle. We, in fact, would like to argue that the early onset of torsional oscillations provides a compelling evidence in support of the NC hypothesis.

Our theoretical model is based on a mean field approach. However, we know that the magnetic field is highly intermittent within the convection zone and we need to take account of this fact when calculating the Lorentz force due to the magnetic field. Since the convection cells deeper down are expected to have larger sizes, Choudhuri [24] suggested that the magnetic field within the convection zone would look as shown in Fig. 1 of that paper. Demanding that the vertical flux tubes give rise to horizontal flux tubes with magnetic field 10^5 G (as suggested by flux rise simulations [25-28]) after stretching in the tachocline, the magnetic field inside the vertical flux tubes at the bottom of the convection zone is estimated to be of order 500 G [24]. This scenario provides a natural explanation for the properties (3) and (4) of torsional oscillations listed above. Presumably the torsional oscillation gets initiated in the lower footpoints of the vertical flux tubes, where the Lorentz force builds up due to the production of the azimuthal magnetic field. This perturbation then propagates upward along the vertical flux tubes at the Alfven speed. If the magnetic field inside the flux tubes is 500 G, then the Alfven speed at the bottom of the convection zone is of order 315 cm s^{-1} and the Alfven travel time from the bottom to the top turns out to be exactly of the same order as the phase lag of torsional oscillations between the bottom of the convection zone and the solar surface. We admit that the magnetic scenario sketched in Fig. 1 of Choudhuri [24] and adopted here is not yet established through rigorous dynamical calculations and a proper study of the propagation of disturbances through such complex magnetic structures is unavailable. However, an assumption of net upward propagation of magnetic disturbances in spite of all these complexities is not an unreasonable ansatz, which is justified by the success of the theoretical model in matching otherwise unexplained aspects of observational data. Since the magnetic field at the bottom is highly intermittent and the velocity perturbations associated with the torsional oscillations are likely to be concentrated around the magnetic flux tubes, we expect the torsional oscillations to be spatially intermittent at the bottom of the convection zone, as seen in the observational data [7]. Since the magnetic field near the surface is less intermittent, the torsional oscillation driven by the Lorentz stress also appears more coherent there. We thus have the puzzling situation that the torsional oscillations seem to become more coherent as they move further away from the source at the footpoints of flux tubes at the bottom of the convection zone.

To develop the theoretical model of torsional oscillations, we extend our already published solar dynamo model [20], in which the NC hypothesis has been incorporated. The basic dynamo code SURYA which is extended for the present calculations is available upon request. Apart from the time evolution equations for the toroidal and poloidal components of the magnetic field which have to be solved in the dynamo problem, we also have to solve an additional simultaneous time evolution equation of the toroidal velocity component v_ϕ. This other equation, which is essentially the ϕ component of the Navier–Stokes equation, is

$$\rho \left\{ \frac{\partial v_\phi}{\partial t} + \frac{v_r}{r}\frac{\partial}{\partial r}(r v_\phi) + \frac{v_\theta}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\phi) \right\} = (\mathbf{F}_L)_\phi + \frac{1}{r^3}\frac{\partial}{\partial r}\left[\nu\rho r^4 \frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left[\nu\rho\sin^3\theta\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)\right], \quad (1)$$

where $(\mathbf{F}_L)_\phi$ is the ϕ component of the Lorentz force. We use the stress-free boundary condition $\partial v_\phi/\partial r = 0$ at the solar surface and take $v_\phi = 0$ at the bottom, although the bottom boundary condition has no effect when the bottom of the integration region is taken well below the tachocline as we do. The kinematic viscosity ν is primarily due to turbulence within the convection zone and is expected to be equal to the magnetic diffusivity. We use the exactly same profile of ν as the profile of the diffusivity of poloidal field, which is shown in Fig. 4 of Chatterjee et al. [20]. In other words, we assume the magnetic Prandtl number to be 1. In order to ensure a period of 11 yr, we choose some parameters in the dynamo equations slightly different from what were used by Chatterjee et al. [20], as listed in Table 1 of Choudhuri et el. [29]. For the density ρ appearing in (1), we use the analytical expression used by Choudhuri & Gilman [25], which gives values of density consistent with detailed numerical models of the convection zone.

If the magnetic field is assumed to have the form

$$\mathbf{B} = B(r,\theta,t)\mathbf{e}_\phi + \nabla\times[A(r,\theta,t)\mathbf{e}_\phi], \quad (2)$$

then the Lorentz force is given by the Jacobian

$$4\pi(\mathbf{F}_L)_\phi = \frac{1}{s^3} J\left(\frac{sB_\phi, sA}{r,\theta}\right), \quad (3)$$

where $s = r\sin\theta$. We, however, have to take some special care in averaging this term, since this is the primary quadratic term in the basic variables (A, B, v_ϕ) and has to be averaged differently from all the other linear terms. The effect of v_ϕ on the magnetic field is also quadratic, and has been added to the similar term giving the effect of differential rotation on magnetic fields in the ϕ-component of the induction equation. The ϕ component of the Lorentz force primarily comes from the radial derivative of the magnetic stress $B_r B_\phi/4\pi$ (the term having $B_\theta B_\phi$ involves θ derivative and is smaller). This stress arises when B_r is stretched by differential rotation to produce B_ϕ and should be non-zero only inside the flux tubes. We assume that B_r, B_ϕ are the mean field values, whereas $(B_r)_{\rm ft}, (B_\phi)_{\rm ft}$ are the values of these quantities inside flux tubes. If f is the filling factor, then we have

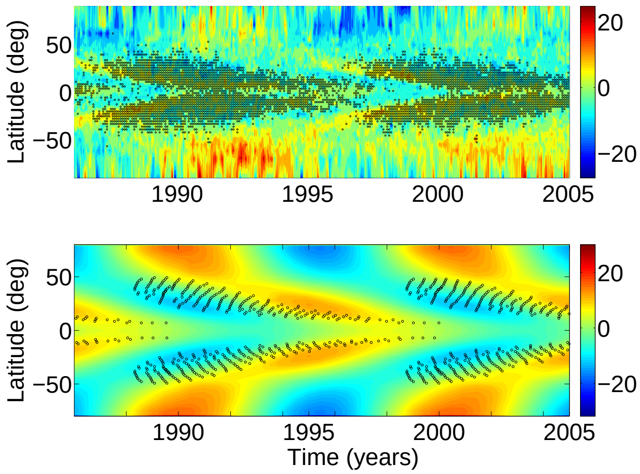

FIG. 1: Comparison between observation and theory. The upper panel superposes the butterfly diagram of sunspots on a time-latitude plot of the observed surface zonal velocity v_ϕ (in m s^{-1}) measured at Mount Wilson Observatory (Courtesy: Roger Ulrich). The comparable theoretical plot is shown in the lower panel, in which the theoretical butterfly diagram from our dynamo model is superposed on the time-latitude plot of theoretically computed v_ϕ (in m s^{-1}) at the surface.

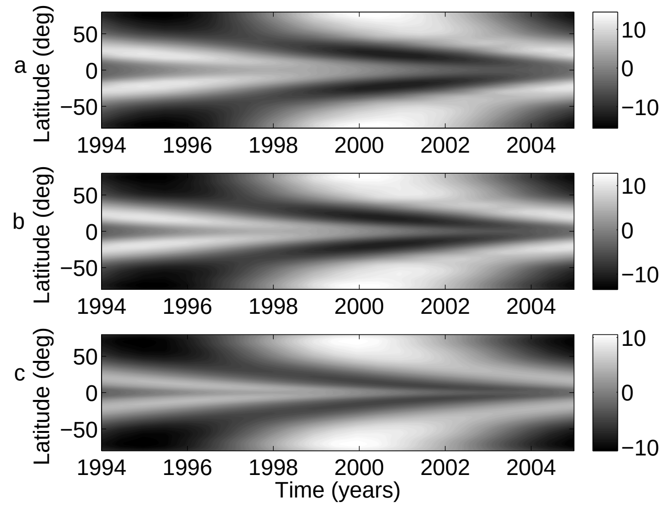

FIG. 2: Theoretical torsional oscillations (v_ϕ in m s^{-1}) in time-latitude plots at different depths of the convection zone: (a) $0.95R_\odot$, (b) $0.9R_\odot$, (c) $0.8R_\odot$.

$B_r = f(B_r)_{\rm ft}$ and $B_\phi = f(B_\phi)_{\rm ft}$, on assuming the same filling factor for both components for the sake of simplicity. It is easy to see that the mean Lorentz stress is

$$f\frac{(B_r)_{\rm ft}(B_\phi)_{\rm ft}}{4\pi} = \frac{B_r B_\phi}{4\pi f}.$$

This suggests that the correct mean field expression for $(\mathbf{F}_L)_\phi$ is given by the expression (3) divided by f.

As pointed out by Chatterjee et al. [20], the only nonlinearity in the dynamo equations comes from the critical magnetic field B_c above which the toroidal field within the convection zone is supposed to be unstable due to magnetic buoyancy. Jiang et al. [30] found that we have to take $B_c = 108$ G (which is the critical value of the mean toroidal field and not the toroidal field inside flux tubes) to ensure that the poloidal field at the surface has correct values. Once the amplitude of the magnetic field gets fixed this way, we find that only for a particular value of the filling factor f the amplitude of the torsional oscillations matches observational values. Our calculations give a filling factor $f \approx 0.067$, which is higher compared to the earlier estimate of $f \approx 0.02$ by Choudhuri [23]. Theoretical values of velocity in all our figures are computed by using $f = 0.067$. Apart from the usual meridional circulation used in our model [20], we include a constant upward velocity $v_r = 300$ cm s^{-1} in (2) to account for the upward transport by Alfven waves when solving our basic equation (1) for v_ϕ. Note that this additional v_r does not represent any actual mass motion and does not have to satisfy the continuity equation which the meridional circulation satisfies. Because of our lack of knowledge about this upward transport, we assume the upward velocity to be independent of depth and allow it to transport the magnetic stresses from the bottom to the surface where they freely move out due to the stress-free boundary condition, mimicking what we believe must be happening in the real Sun.

Figure 1 presents a comparison of theory with observations by putting the butterfly diagram of sunspots on the time-latitude plot of torsional oscillations at the surface. The theoretical plot correctly reproduces the initiation of the low-latitude branch of torsional oscillations about 2 yr before the sunspot cycle, starting at a latitude higher than typical sunspot latitudes. Apart from the NC hypothesis, the assumption of the upward advection of the perturbations at Alfven speed is crucial. On switching off the Alfven wave, even though the torsional oscillations begin at a high latitude at the bottom of the convection zone before the starting of the sunspot cycle, the disturbance has to reach the surface through diffusion and we do not see the correct initiation of torsional oscillations at the surface. We also note that the phase of the torsional oscillations (i.e. regions of positive and negative v_ϕ in the time-latitude plot) with respect to the sunspot cycle is reproduced quite well. On decreasing (increasing) the Alfven speed, the phase of the torsional oscillations with respect to the butterfly diagram gets shifted towards the right (left). While our main aim was to explain the properties of the low-latitude branch of torsional oscillations, our theoretical model has reproduced the high-latitude branch as well, without our having to do anything special for it. The physics behind this branch will become clear when we discuss Figure 3 later. Figure 2 showing torsional oscillations at different depths has to be compared with the observational Figs. 4–6 of Howe et al. [7]. A careful look shows a small phase delay in the upper lay-

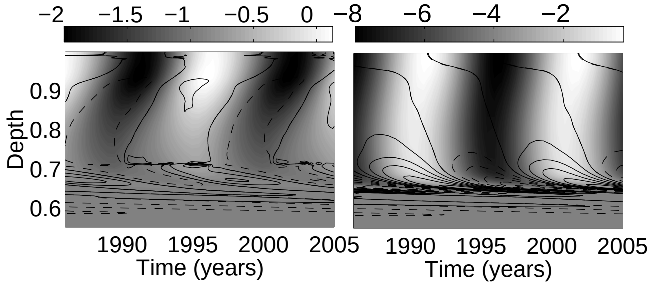

FIG. 3: Theoretical torsional oscillations (v_ϕ in m s^{-1}) as functions of depth and time at latitudes 20° (left) and 70° (right). The contours indicate the Lorentz force $(\mathbf{F}_L)_\phi$, the solid and dashed lines denoting positive and negative values.

ers compared to the lower layers. The observational plots become more intermittent at the greater depths due to the more intermittent nature of the magnetic field there. This is not reproduced in the theoretical model based on mean field equations.

Figure 3 shows how torsional oscillations evolve in depth and time at 2 different latitudes. The plot for latitude 20° compares very well with Fig. 4(D) of Vorontsov et al. [5] or Fig. 7 of Howe et al. [7]. It is clear in the plot of 20° that the Lorentz force is concentrated in the tachocline at $0.7R_\odot$, where the low-latitude torsional oscillations are launched to propagate upward. The physics of the high-latitude branch is, however, very different, with the Lorentz force contours for latitude 70° indicating a downward propagation and not a particularly strong concentration at the tachocline. As the poloidal field sinks with the downward meridional circulation at the high latitudes, the latitudinal shear $d\Omega/d\theta$ in the convection zone acts on it to create the toroidal component [21] and thereby the Lorentz stress. With the downward advection of the poloidal field, the region of Lorentz stress moves downward. In the case of the low-latitude branch, the plot for latitude 20° shows that the amplitude of the torsional oscillations becomes larger near the surface due to the perturbations propagating into regions of lower density, which is consistent with observational data [5]. If the upward Alfven propagation is switched off, then the disturbances from the bottom of the convection zone reach the top by diffusion (the diffusion time being about 5 yr in our model), but the amplitude of torsional oscillations in the upper layers of the convection zone generally falls to very low values.

Compared to the earlier theoretical models of torsional oscillations, the two novel aspects of our model are (1) the NC hypothesis, which allows the formation of strong toroidal field in the high-latitude tachocline before the beginning of the sunspot cycle; and (2) the assumption that the perturbations propagate upward along flux tubes at the Alfven speed. With these two assumptions incorporated, our theoretical model readily explains most aspects of torsional oscillations without requiring any changes in the parameters of the original dynamo model [29]. Both these assumptions seem essential if we want to match theory with observations in detail.

SC thanks C.S.I.R. (India) for financial support through an SRF. Partial support from a DST project No.SR/S2/HEP-15/2007 is acknowledged.

[1] R. Howard, & B. J. LaBonte, Astrophys. J. **239**, L33 (1980).
[2] A.G. Kosovichev, & J. Schou, Astrophys. J. **482**, L207 (1997).
[3] R. Howe, *et al.* Astrophys. J. **533**, L163 (2000).
[4] H. M. Antia, & S. Basu, Astrophys. J. **541**, 442 (2000).
[5] S.V. Vorontsov, *et al.* Science **296**, 101 (2002)
[6] S. Basu, & H. M. Antia, Astrophys. J. **585**, 553 (2003)
[7] R. Howe, *et al.*, 1995-2004. Astrophys. J. **634**, 1405 (2005).
[8] R. Howe, *et al.* Sol. Phys. **235**, 1 (2006).
[9] B. R. Durney, Sol. Phys. **196**, 1 (2000).
[10] E. Covas, R. Tavakol, D. Moss, & A. Tworkowski, Astron. Astrophys. **360**, L21 (2000).
[11] P.J. Bushby, Mon. Not. R. Astron. Soc. **371**, 772 (2006).
[12] M. Rempel, Astrophys. J. **647**, 662 (2006).
[13] H. C. Spruit, Sol. Phys.**34**, 277 (1974).
[14] Y.-M. Wang, N. R., Jr. Sheeley, & A. G. Nash, Astrophys. J. **383**, 431 (1991).
[15] A. R. Choudhuri, , M. Schussler, & M. Dikpati, Astron. Astrophys. **303**, L29 (1995).
[16] B. R. Durney, Sol. Phys. **160**, 213 (1995).
[17] M. Dikpati, & P. Charbonneau, Astrophys. J. **518**, 508 (1999).
[18] M. Küker, G. Rüdiger, & M. Schultz, Astron. Astrophys. **374**, 301 (2001).
[19] D. Nandy, & A. R. Choudhuri, Science **296**, 1671 (2002).
[20] P. Chatterjee, D. Nandy, & A. R. Choudhuri, Astron. Astrophys. **427**, 1019 (2004).
[21] G. Guerrero, & E. M. de Gouveia Dal Pino, Astron. Astrophys. **464**, 341 (2007).
[22] P. A. Gilman, & M. S. Miesch, Astrophys. J. **611**, 568 (2004).
[23] P. Garaud, & N. H. Brummel, Astrophys. J. **674**, 498 (2008).
[24] A. R. Choudhuri, Sol. Phys. **215**, 31 (2003).
[25] A. R. Choudhuri, & P. A. Gilman, Astrophys. J. **316**, 788 (1987).
[26] A. R. Choudhuri, Sol. Phys. **123**, 217 (1989).
[27] S. D'Silva, & A. R. Choudhuri, Astron. Astrophys. **272**, 621 (1993).
[28] Y. Fan, G. H. Fisher, & E. E.Deluca, Astrophys. J. **406**, 390 (1993).
[29] A. R. Choudhuri, P. Chatterjee, & J. Jiang, Phy. Rev. Lett. **98**, 131103 (2007).
[30] J. Jiang, , P. Chatterjee, & A. R. Choudhuri, Mon. Not. R. Astron. Soc. **381**, 1527 (2007).