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The Hemispheric Asymmetry of Solar Activity during the Last Century and the Solar Dynamo

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Abstract We believe the Babcock–Leighton process of poloidal field generation to be the main source of irregularity in the solar cycle. The random nature of this process may make the poloidal field in one hemisphere stronger than that in the other hemisphere at the end of a cycle. We expect this to induce an asymmetry in the next sunspot cycle. We look for evidence of this in the observational data and then model it theoretically with our dynamo code. Since actual polar field measurements exist only from 1970s, we use the polar faculae number data recorded by Sheeley (1991, 2008) as a proxy of the polar field and estimate the hemispheric asymmetry of the polar field in different solar minima during the major part of the twentieth century. This asymmetry is found to have a reasonable correlation with the asymmetry of the next cycle. We then run our dynamo code by feeding information about this asymmetry at the successive minima and compare with observational data. We find that the theoretically computed asymmetries of different cycles compare favourably with the observational data, the correlation coefficient being 0.73. Due to the coupling between the two hemispheres, any hemispheric asymmetry tends to get attenuated with time. The hemispheric asymmetry of a cycle either from observational data or from theoretical calculation statistically tends to be less than the asymmetry in the polar field (as inferred from the faculae data) in the preceding minimum. This reduction factor turns out to be 0.43 and 0.51 respectively in observational data and theoretical simulation.

Key words: Sun: activity — Sun: magnetic fields — sunspots

1 INTRODUCTION

Although solar activity appears reasonably symmetric in the two hemispheres after short-term variations are averaged, some cycles have been known to be stronger in one hemisphere. The aim of the present paper is to analyze the asymmetries of solar cycles during the twentieth century and then to simulate these asymmetries with a solar dynamo model.

The solar magnetic cycle is believed to be produced by a flux transport dynamo operating in the sun's convection zone (Wang, Sheeley & Nash 1991; Choudhuri, Schüssler & Dikpati 1995; Durney 1995; Dikpati & Charbonneau 1999; Nandy & Choudhuri 2001, 2002; Küker, Rüdiger & Schultz 2001; Choudhuri 2003; Guerrero & Muñoz 2004). Fairly sophisticated models of the solar dynamo to explain various regular features of the solar cycle have been constructed. There is, however, not yet a convergence on the values of important parameters. In the model of Chatterjee, Nandy & Choudhuri (2004), the value of turbulent diffusivity for the poloidal field in the interior of the solar convection zone is taken to be 2.4×10^{12} cm^2 s^{-1}. On the other hand, Dikpati & Gilman (2006) take a value about 50 times smaller.

In order to model the hemispheric asymmetry, we need to understand how the irregularities of the solar cycle arise in the flux transport dynamo theory. We believe that the stochastic fluctuations in the dynamo process give rise to the irregularities (Choudhuri 1992). Choudhuri, Chatterjee & Jiang (2007) identify the

Babcock–Leighton process of the production of poloidal field as the main source of randomness in the solar dynamo, whereas other aspects of the dynamo process are assumed to be deterministic. In the Babcock–Leighton process, the poloidal field is produced from the decay of tilted bipolar sunspots. The tilt of bipolar sunspots is caused by the Coriolis force acting on the rising flux tubes (D'Silva & Choudhuri 1993), whereas buffeting of the flux tubes by convective turbulence causes a scatter in the tilt angles around the average given by Joy's law (Longcope & Choudhuri 2002). Because of this scatter in tilt angles, the Babcock–Leighton process appears not to be a deterministic process. Observational data, as plotted in Figure 3 of Jiang, Chatterjee & Choudhuri (2007), also indicate that the polar field produced at the end of a cycle is not correlated with the strength of the cycle. On the other hand, Dikpati & Gilman (2006) use the sunspot area data as the source function for the poloidal field, which amounts to assuming the Babcock–Leighton process to be fully deterministic and which is incorrect in our opinion. Dikpati & Gilman (2006) have predicted that the next cycle 24 will be 30–50% stronger than the last cycle, which is at variance with the prediction of Choudhuri, Chatterjee & Jiang (2007) and Jiang, Chatterjee & Choudhuri (2007) that it will be 30–35% weaker.

Although the polar field produced at the end of a cycle is not correlated with the strength of the cycle, observational data show that the strength of the cycle is correlated quite well with the polar field at the preceding minimum. This is seen in Figure 2 of Jiang, Chatterjee & Choudhuri (2007). In fact, Schatten et al. (1978) proposed long ago that the strength of the polar field at a solar minimum can be used to predict the strength of the next cycle. Svalgaard, Cliver & Kamide (2005) and Schatten (2005) have used the weakness of the present polar field to predict that the next cycle 24 will be weak. Jiang, Chatterjee & Choudhuri (2007) showed that only a reasonably high value of turbulent diffusivity can give rise to the observed correlation between the polar field at the minimum and the strength of the next cycle. How this correlation arises is explained through Figure 1 of Jiang, Chatterjee, and Choudhuri (2007). If the diffusivity is high, then the poloidal field generated at the solar surface by the Babcock–Leighton process diffuses to the tachocline in a few years. Since the next cycle is caused by the toroidal field produced from this poloidal field in the tachocline by differential rotation, it is obvious that the next cycle would appear correlated with the preceding polar field which is formed by the poleward advection of the poloidal field due to meridional circulation. On the other hand, if the diffusivity is low, then the poloidal field produced at the surface cannot diffuse to the tachocline and has to be carried to the tachocline by the meridional circulation. This takes about 20 years so that a particular cycle is not correlated with the polar field in the immediately preceding minimum. Dikpati & Gilman (2007) could predict a strong cycle after a minimum with a weak polar field only because they used a low diffusivity. This would never be possible in a high-diffusivity model. Jiang, Chatterjee & Choudhuri (2007; §5) provided several independent arguments why the diffusivity is likely to have the higher value which they assumed. Yeates, Nandy & Mackay (2007) have recently carried out a thorough study of the effects of diffusivity on a fluctuating dynamo and have confirmed the findings of Jiang, Chatterjee & Choudhuri (2007).

If the Babcock–Leighton process of poloidal field generation is the source of randomness in the solar dynamo, then a theoretical model based on mean field equations has to be corrected by feeding the actual value of the observed polar field at the solar minimum (Choudhuri, Chatterjee & Jiang 2007). Since reliable polar field measurements are available only from mid-1970s, Choudhuri, Chatterjee & Jiang (2007) and Jiang, Chatterjee & Choudhuri (2007) attempted to model only the last three solar cycles. As these last three cycles were only weakly asymmetrical between the hemispheres, they are not particularly convenient in studying the physics of hemispheric asymmetry, although Jiang, Chatterjee & Choudhuri (2007) presented some calculations of hemispheric asymmetry. Jiang, Chatterjee & Choudhuri (2007) pointed out two other works which provide proxies for the polar field at earlier minima: (i) the polar faculae numbers analyzed by Sheeley (1991); and (ii) large-scale magnetic moments obtained by Makarov et al. (2001) from the positions of dark filaments. While Jiang, Chatterjee & Choudhuri (2007) carried out some correlation analyses based on these data, they were not used in dynamo modelling. Since Sheeley (1991) has provided both the north and south polar faculae numbers during 1906–1990, we can use this to estimate the asymmetries in the polar field during the various solar minima of the twentieth century. Jiang, Chatterjee & Choudhuri (2007) stressed the fact that polar fields inferred from the faculae data may not always be reliable. Since it is still the best that we can do to model the asymmetries of earlier cycles, it is instructive to see what we get from this approach. While revising this paper in response to the referee's report, we came across a recent paper

Cycle Number	Polar faculae number at beginning of cycle			Total sunspot area during the cycle		
	F_N	F_S	F_{AS}	A_N	A_S	A_{AS}
15	28.3	31.6	-0.057	4.33×10^4	3.57×10^4	0.097
16	53.9	49.4	0.043	4.65×10^4	3.91×10^4	0.087
17	25.2	30.6	-0.097	6.00×10^4	5.96×10^4	0.003
18	51.5	33.0	0.219	7.43×10^4	7.03×10^4	0.027
19	64.8	44.1	0.190	10.55×10^4	7.39×10^4	0.176
20	66.2	36.9	0.284	6.94×10^4	4.91×10^4	0.171
21	24.5	29.2	-0.086	7.51×10^4	7.76×10^4	-0.017
22	23.6	26.3	-0.053	6.38×10^4	7.24×10^4	-0.063
23	16.0	16.3	-0.010	5.61×10^4	6.44×10^4	-0.069
24	6.7	10.8	-0.231	-	-	-

Table 1 Polar faculae numbers and total sunspot areas in two hemispheres during the various cycles.

of Sheeley (2008) in which he extends his polar faculae analysis till 2006 and we have been able to include these additional data in this revised version of the paper.

The randomness of the Babcock–Leighton process may give rise to a stronger poloidal field in one hemisphere compared to the other. Just as the polar field at the minimum gives an indication of the strength of the next cycle, we may expect that a hemispheric asymmetry in the polar field may be indicative of a hemispheric asymmetry in the solar activity during the next cycle. We find a reasonably good correlation in the observational data. The theoretical dynamo model with our assumed value of diffusivity reproduces this correlation qualitatively. In spite of a large scatter in the data, we can clearly see some interesting patterns.

We present a discussion of hemispheric asymmetry seen in the observational data in §2. Then §3 presents calculations from our dynamo model in which magnetic field values in the two poles during the minima are fed. The theoretical results of hemispheric asymmetry are discussed in §4. Our conclusions are summarized in §5.

2 OBSERVATIONAL DATA

We use Figure 1 of Sheeley (1991) to estimate the north polar faculae number (F_N) and the south polar faculae number (F_S) at successive solar minima when the polar faculae numbers had their maxima, except that we use Figure 2 of Sheeley (2008) for the last two solar minima. The values of F_N and F_S at the beginnings of various cycles are listed in Table 1 along with the asymmetry factor

$$F_{AS} = \frac{F_N - F_S}{F_N + F_S} \quad (1)$$

It should be noted that the polar faculae number plotted in Figure 1 of Sheeley (1991) is often noisy near the solar minima when this number has maximum values. So, when using F_{AS} as a proxy for the asymmetry in the polar field, the possibility of significant errors should be kept in mind. Since actual measurements of polar field from WSO were available since 1976, Sheeley (1991, 2008) presented comparisons of actual polar field values and the faculae numbers during the period when both types of data were available. While the correlation between the two appears reasonably good, it is certainly not extremely tight. Jiang, Chatterjee & Choudhuri (2007) pointed out that the polar field inferred from the faculae number data of Sheeley (1991) did not always agree with the polar field inferred from the parameter $A(t)$ computed by Makarov et al. (2001) from the positions of dark filaments. It may be noted that Li et al. (2002) also estimated the asymmetry of polar faculae data of Sheeley (1991) by a slightly different method. Instead of using the maximum value of polar faculae number as we have done, they used the total number of polar faculae in the two hemispheres. The hemispheric asymmetries computed by them are sometimes slightly different from our F_{AS}. Since we are interested in the asymmetry of the polar field at the beginning of the cycle, we believe that F_{AS} is a better proxy for that than the asymmetry computed by Li et al. (2002).

To compute asymmetries of sunspot cycles, we use the sunspot area data from the archive of Royal Greenwich Observatory available at the website:

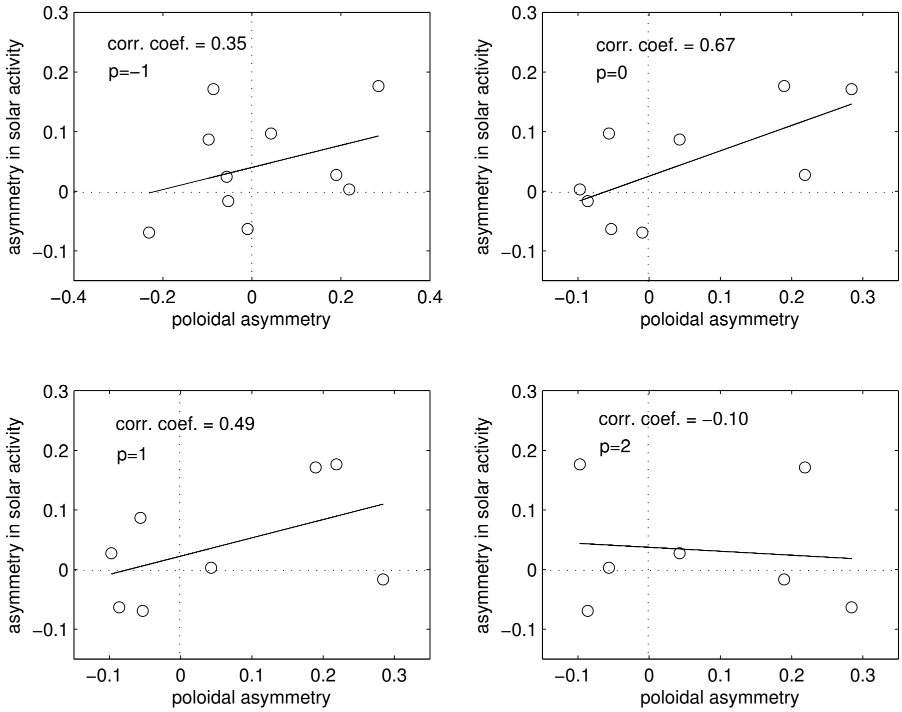

Fig. 1 The observed asymmetry in sunspot area A_{AS} of cycle $n+p$ is plotted against the polar faculae asymmetry F_{AS} at the beginning of the cycle n.

http://solarscience.msfc.nasa.gov/greenwch.shtml.
Monthly averages of daily sunspot areas for the northern and southern hemispheres are available at this website. We add up the monthly sunspot areas over one sunspot cycle to get a 'total' sunspot area during the cycle in one hemisphere. Let us denote these 'total' sunspot areas in the two hemispheres summed over sunspot cycles by A_N and A_S. Table 1 also lists the values of A_N and A_S for various sunspot cycles along with the asymmetry factor

$$A_{AS} = \frac{A_N - A_S}{A_N + A_S} \tag{2}$$

We will look into the behaviour of the sunspot area asymmetry A_{AS} of cycle $n+p$ against the polar faculae asymmetry F_{AS} at the beginning of the cycle n.

Figure 1 shows plots of A_{AS} of cycle $n+p$ against F_{AS} at the beginning of the cycle n for four values of p: -1, 0, 1 and 2. When we are dealing with a small number of data points, it should be kept in mind that the value of the correlation coefficient r may not always have a high statistical significance. Even in the case of 'null hypothesis' that there is actually no correlation, the probability that N data points would give a correlation coefficient higher than $|r|$ is given by

$$\mathrm{erfc}\left(\frac{|r|\sqrt{N}}{\sqrt{2}}\right)$$

(see, for example, Press et al. 1988, §13.7). For the four panels in Figure 1 corresponding to $p = -1, 0, 1, 2$, the probability of the null hypothesis turns out to be 27%, 4.4%, 17% and 79% respectively. We have the best correlation when $p = 0$. There may be a weak correlations in the cases $p = 1$ or $p = -1$, though we cannot make any definitive statement on the basis of the limited data. It is clear that the correlation disappears for $p = 2$. In normal data analysis, the null hypothesis is usually ruled out if its probability is less than 5%. Only in the case $p = 0$ we can rule out the null hypothesis and conclude with a reasonable degree of confidence that there is a real correlation.

Although we may not be able to assert this with a very high degree of statistical significance, it appears that the asymmetry of the poloidal field produced at the end of a sunspot cycle is the major factor determining the asymmetry of the next cycle. This would be possible only if the information about the poloidal field asymmetry at the solar surface can be communicated within a few years (≈ 5 years) to the tachocline which is the breeding ground for the sunspots in the next cycle. As argued by Jiang, Chatterjee & Choudhuri (2007) and confirmed by Yeates, Nandy & Mackay (2007), this requires a diffusivity of the order 2.4×10^{12} cm^2 s^{-1} as used by Chatterjee, Nandy & Choudhuri (2004) and Choudhuri, Chatterjee & Jiang (2007). If the diffusivity is assumed to be 50 times smaller as in Dikpati & Gilman (2006), then diffusion cannot carry an information from the solar surface to the tachocline in a reasonable time. This has to be done by the meridional circulation, which has an advection time of about 20 years. On using such a low value of diffusivity in their numerical simulations, Charbonneau & Dikpati (2000) found that the polar field at the beginning of a cycle n had the maximum correlation with the strength of the cycle $n + 2$, there being virtually no correlation with the cycle n (see their Figure 9).

Choudhuri, Chatterjee & Jiang (2007) found that the effect of a sudden disturbance in their dynamo model persists for about 15–20 years (see their Figure 2). It is intriguing to note that a 'memory' of 15–20 years is consistent with what we see in Figure 1. If we tentatively allow that there is a weak correlation in the $p = 1$ case (which is suggested in the plot, but not established at a high level of statistical significance), then we conclude that the correlation becomes weaker from the $p = 0$ to the $p = 1$ case and disappears in the $p = 2$ case. This would suggest a 'memory' of order 15–20 years. While it may be unlikely that all the parameters used by Chatterjee, Nandy & Choudhuri (2004), Choudhuri, Chatterjee & Jiang (2007) and Jiang, Chatterjee & Choudhuri (2007) have the exactly correct values, the values of quantities like diffusivity probably have been chosen correct within a factor of 2 or 3, since 'memory' from this model is in good agreement with the limited observational data that we have. If the 'memory' is longer than a cycle, then the randomness introduced by the Babcock–Leighton process at the end of a cycle does not erase all the effects of the previous cycle completely. Bushby & Tobias (2007) have argued against very long memories in a complex nonlinear system like the solar dynamo. On the other hand, Charbonneau, Beaubien & St-Jean (2007) suggested that the 'even-odd' effect in the solar cycle is caused by period doubling, which would imply a memory which is at least as long as what we are suggesting.

The last important point to note in the observational data is that the correlation line for the $p = 0$ case in Figure 1 has a slope of 0.43. Even if the polar field asymmetry at a minimum is the primary cause of the asymmetry in the next cycle, it seems that the asymmetry in the cycle is statistically expected to be only 0.43 times the polar asymmetry. In other words, the asymmetry tends to get reduced as the cycle progresses. Chatterjee & Choudhuri (2006) studied the coupling between the two hemispheres and showed that, for a dynamo with high diffusivity, the two hemispheres remain coupled even after the introduction of asymmetries. So we expect that the hemispheric asymmetries continously get washed away until the randomness in the Babcock–Leighton process towards the end of a cycle creates fresh asymmetries.

One may wonder whether we would get plots similar to what we see in Figure 1 when we try to correlate the total polar faculae number $F_N + F_S$ at the beginning of cycle n with the 'total' sunspot area $A_N + A_S$ during cycle $n + p$. Some plots of this kind are shown in Figs. 2 and 3 of Jiang, Chatterjee & Choudhuri (2007). When we estimate the polar field at the beginning of cycle n from the value of $A(t)$ computed by Makarov et el. (2001) and correlate it with the 'total' sunspot area of cycle $n + p$, we get plots very similar to the plots in Figure 1. However, when we carry out such an exercise by taking $F_N + F_S$ as a proxy of the polar field, we do not get very clear plots. Even in the case $p = 0$, we do not find a good correlation. It is intriguing that we get the interesting plots of Figure 1 by correlating the asymmetries F_{AS} and A_{AS}, but we do not get such plots when we try to correlate $F_N + F_S$ and $A_N + A_S$ for different cycles. While we do not have a proper explanation for this, we offer a tentative hypothesis. The counting of faculae numbers involves some degree of human judgment. When Sheeley counted faculae numbers in the same plates once in 1990 and again in 2007, the results differed by a factor of 1.38 (Sheeley 2008). So, even when the same human being does all the countings, the faculae numbers of different cycles may be counted at different times and the results may not be very standardized. On the other hand, the asymmetry of a cycle is obtained from faculae counts of north and south poles within a few months. Presumably these are counted by a human being within a short interval of time and more or less in the same way. To calculate the asymmetry F_{AS} as given by (1), we have to take a ratio of the faculae numbers such that any lack of standardization gets eliminated. So it is possible that the asymmetry F_{AS} is a better proxy for the polar field asymmetry than $F_N + F_S$ is a proxy for the average polar field. In support of this hypothesis, we draw the reader's attention to Table 1 of Li et al. (2002), where the polar faculae data of Sheeley (1991) from Mount Wilson Observatory (MWO) are listed along with the polar faculae data from National Astronomical Observatory of Japan (NAOJ) for four cycles. It seems that the total number of faculae decreased from cycle 20 to cycle 21 according to the MWO data, but increased according to the NAOJ data. However, the asymmetries for cycles 20 and 21 computed from these two different data sets turned out to be very similar!

3 THE NUMERICAL DYNAMO MODEL

We now carry out an analysis of the asymmetry in solar activity on the basis of the standard dynamo model presented by Nandy & Choudhuri (2002) and Chatterjee, Nandy & Choudhuri (2004). The basic equations for the standard axisymmetric $\alpha\Omega$ solar dynamo model can be found in Chatterjee, Nandy & Choudhuri (2004). In order to solve these governing equations, we make use of the solar dynamo code SURYA developed by the research group at the Indian Institute of Science. This code and a detailed guide (Choudhuri 2005) can be availed upon request by sending an e-mail to Arnab Rai Choudhuri (email address: arnab@physics.iisc.ernet.in). The code SURYA has been the basis for dynamo calculations presented in several papers (Chatterjee, Nandy & Choudhuri 2004; Choudhuri, Chatterjee & Nandy 2004; Chatterjee & Choudhuri 2006; Choudhuri, Chatterjee & Jiang 2007; Jiang, Choudhuri & Wang 2007; Jiang, Chatterjee & Choudhuri 2007; Yeates, Nandy & McKay 2007).

As discussed earlier, the Babcock-Leighton process of poloidal field generation from the decay of tilted bipolar sunspot pairs involves randomness. Hence, in order to analyze the irregularities of the solar cycles, we have to force-feed the observational data for the poloidal field at the solar minima. To accomplish this, Choudhuri, Chatterjee & Jiang (2007) adopted the following method. Cycle 22 was chosen as the average cycle and the observed value of the polar field at a solar minimum was divided by the value of the polar field at the beginning of cycle 22 to arrive at a numerical factor γ. This factor γ is essentially a measure of the observed poloidal field at a solar minimum. Now let $\overline{A_{\min}}$ be the amplitude of the scalar function $A(r, \theta)$ which gives the poloidal field at the minima of a relaxed solution of the dynamo code. The code was stopped at successive minima, when $A(r, \theta)$ above $r > 0.8R\odot$ would be multiplied by a constant factor such that its amplitude becomes equal to $\gamma\overline{A_{\min}}$, where γ is the numerical factor appropriate for that minimum. Values of $A(r, \theta)$ below $r < 0.8R\odot$ were left unchanged to ensure that only the poloidal field created in the previous cycle would be updated, but any poloidal field created in still earlier cycles which may be present at the bottom of the convection zone was not changed. Choudhuri, Chatterjee & Jiang (2007) used a single γ for the whole Sun at every minimum. On the other hand, Jiang, Chatterjee & Choudhuri (2007) used a function $\gamma(\theta)$ of the latitude obtained from WSO data of poloidal field at different latitudes. We now follow the procedure of assigning two different γ_N and γ_S for the two hemispheres obtained from the north and south polar faculae numbers during the minima. If we again take the cycle 22 as an average cycle, we see in Table 1 that the average value of polar faculae number (i.e. the average of north and south poles) at the beginning of that cycle was 24.95. Dividing the numbers in the second and third columns of Table 1 by this, we get the values of γ_N and γ_S.

On the basis of this methodology, we carry out simulations for cycles 15–23 by updating the poloidal field at the minima with the help of the polar faculae number data of Sheeley (1991, 2008). Before presenting the results of asymmetry, we show a theoretical sunspot number plot in Figure 2 along with the observational data. As already pointed out by Choudhuri, Chatterjee & Jiang (2007) and Jiang, Chatterjee & Choudhuri (2007), the absolute value of the theoretical sunspot number does not have a particular physical significance. So we have scaled it appropriately to produce a good fit with the observational data. We found that the theoretically calculated cycles vary in duration slightly if we feed the poloidal field data at the minima by our procedure. It is believed that the duration of a cycle is set by the time scale of the meridional circulation (Charbonneau & Dikpati 2000; Hathaway et al. 2003), and helioseismology gives us information about the variation of meridional circulation only from 1996 onwards. Most probably, it is the variation of meridional circulation with time which is the primary cause of variation in the observed durations of cycles. Since

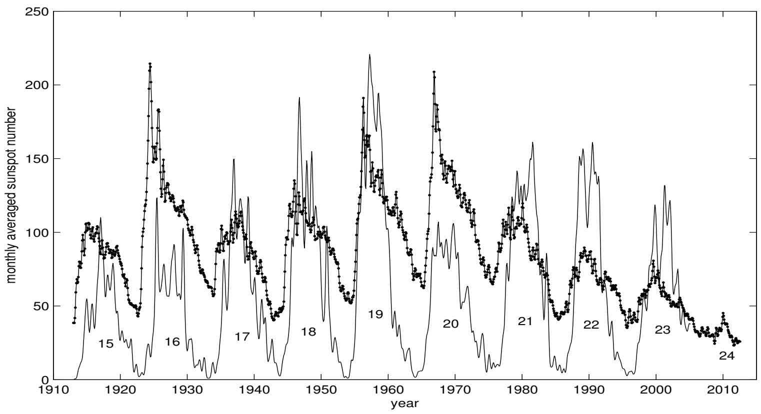

Fig. 2 The solid line represents the monthly averaged sunspot numbers from observation, while the dash-dotted line represents the theoretical monthly averaged sunspot number calculated by feeding the polar faculae data of Sheeley (1991, 2008) in the dynamo code.

we do not have any information of meridional circulation variation at earlier times, we take the meridional circulation to be constant in our model and do not try to match the observed variation of cycle durations. The total duration of cycles 15–22 in our theoretical model turned out to slightly longer than the observed duration. We had to shrink the time axis in the theoretical model by a factor 0.86 to produce Figure 2.

It was mentioned by Chatterjee, Nandy & Choudhuri (2004) that one of the limitations of their model (which we use here) is that the theoretical sunspot number at the minima remained significantly non-zero. We see in Figure 2 that there is no good match between theory and observations during the solar minima. This was the case in the results of Choudhuri, Chatterjee & Jiang (2007) and Jiang, Chatterjee & Choudhuri (2007) as well. The fits between theory and observations during the maxima of most of the cycles seem reasonable, except the two weak cycles 16 and 20, as well as the last cycle 22. The two weak cycles 16 and 20 correspond to the two data points in Figure 2(b) of Jiang, Chatterjee & Choudhuri (2007) which are quite a bit away from the correlation line. As pointed out by Jiang, Chatterjee & Choudhuri (2007), these two weak cycles were preceded by fairly high values of polar faculae number suggesting a strong polar field in the previous minimum, whereas the polar field inferred from the value of $A(t)$ as computed by Makarov et al. (2001) is on the lower side.

For the sake of comparison, we also carried out a calculation of cycles 16–23 by feeding the polar field data at the minima inferred from the values of $A(t)$ given by Makarov et al. (2001). The result is shown in Figure 3. Note that, for this calculation, a single value of γ was used at each minimum, which was taken to be proportional to $A(t)$ at that minimum. We see that the fit between theory and observation is better in this case. This was expected because the correlation plot given in Figure 2(a) of Jiang, Chatterjee & Choudhuri (2007) based on the data of Makarov et al. (2001) shows a tighter correlation than the correlation plot given in Figure 2(b) based on the polar faculae data of Sheeley (1991, 2008).

4 THE ASYMMETRY CALCULATION

We now present the asymmetry calculations. Although polar field data are available from mid-1970s and one can calculate the polar field asymmetry from these data for the last few solar minima, we use the faculae data throughout, for the sake of uniformity and consistency. Asymmetry calculations for the last 3 cycles based on the polar field data have been presented by Jiang, Chatterjee and Choudhuri (2007). However, as we already pointed out, these cycles were only mildly asymmetric and are not so useful in the study of the physics of hemispheric asymmetry.

The upper panel of Figure 4 shows the theoretical sunspot numbers in the two hemispheres from our dynamo simulation as functions of time for the cycles 15–23. The theoretical curve shown in Figure 2 is

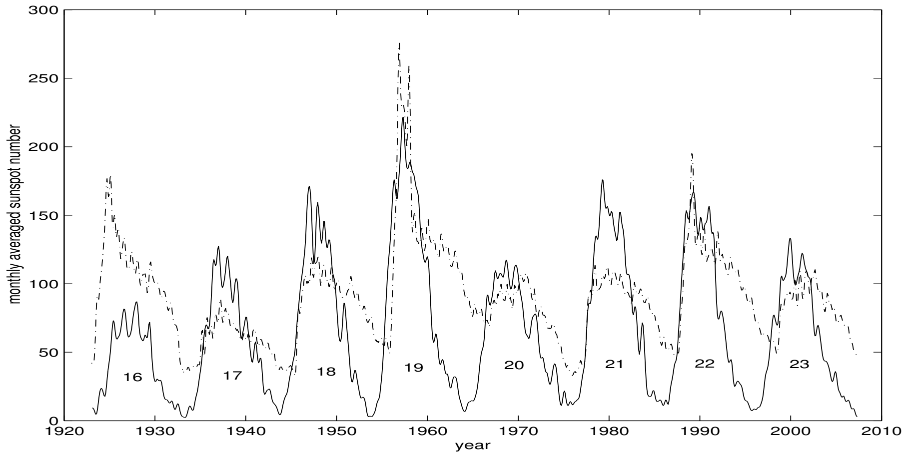

Fig. 3 The solid line represents the monthly averaged sunspot numbers from observation, while the dash-dotted line represents the theoretical monthly averaged sunspot number calculated by feeding the polar field value inferred from the data of Makarov et al. (2001) in the dynamo code.

nothing but the sum of the two curves shown in Figure 4. For the sake of comparison, the observational data of monthly sunspot areas in the two hemispheres as functions of time are shown in the bottom panel of Figure 4. Both in the theoretical and observational plots, the northern hemisphere is found considerably more active than the southern hemisphere during cycles 19 and 20. These were the cycles with the strongest asymmetry during the twentieth century. The areas below the curves in the top panel of Figure 4 for a particular cycle give the theoretical total sunspot numbers N_N and N_S in the two hemispheres for that cycle. We can then calculate the theoretical asymmetry of a cycle in the usual way:

$$N_{AS} = \frac{N_N - N_S}{N_N + N_S} \tag{3}$$

The theoretically calculated values of asymmetry N_{AS} for various cycles are listed in Table 2, along with the values of observed asymmetry A_{AS} which were already listed in the last column of Table 1. Then the fourth column of Table 2 gives the ratio of the theoretical asymmetry to the observed asymmetry, whereas the last column lists the difference between them. For the cycles which had sufficient observed asymmetry (i.e. more than 10%), we find this ratio to be of order 1. However, when the asymmetry is small (i.e. less than 10%), it does not have much statistical significance and sometimes the theoretical and observational asymmetries even have opposite signs. Only for the cycle 17 which had the weakest observed asymmetry of only 0.3%, the ratio given in the third column of Table 2 is off from 1 by more than an order of magnitude. However, we find in the last column that the difference between theoretical and observed asymmetries in this case is quite small. We conclude that our theoretical dynamo model produces the approximately correct value of asymmetry when it is sufficiently large.

In Figure 5 we plot the theoretically calculated asymmetry N_{AS} for cycle n against the asymmetry F_{AS} in the polar faculae number at the beginning of the cycle n, which is essentially the asymmetry between γ_N and γ_S values that have been fed into the code. The null hypothesis in this case has a probability of only 0.6%. We have to compare the theoretical Figure 5 with the corresponding observational figure which is the plot for $p = 0$ in Figure 1. Compared to the slope 0.43 in that figure, the slope in Figure 5 has a somewhat higher value of 0.51. We consider this to be a remarkable agreement between theory and observations. As we pointed out in §2, the coupling between the hemispheres tends to reduce any asymmetry between the hemispheres. Hence we find that the observed asymmetry A_{AS} of a cycle is less than the asymmetry F_{AS} of polar faculae number at the beginning of that cycle, which is an indication of the source of asymmetry in the cycle. We now find that the theoretically calculated asymmetry N_{AS} of the cycle is also reduced compared to F_{AS} at the beginning of the cycle and the reduction is by a factor which is comparable to the factor

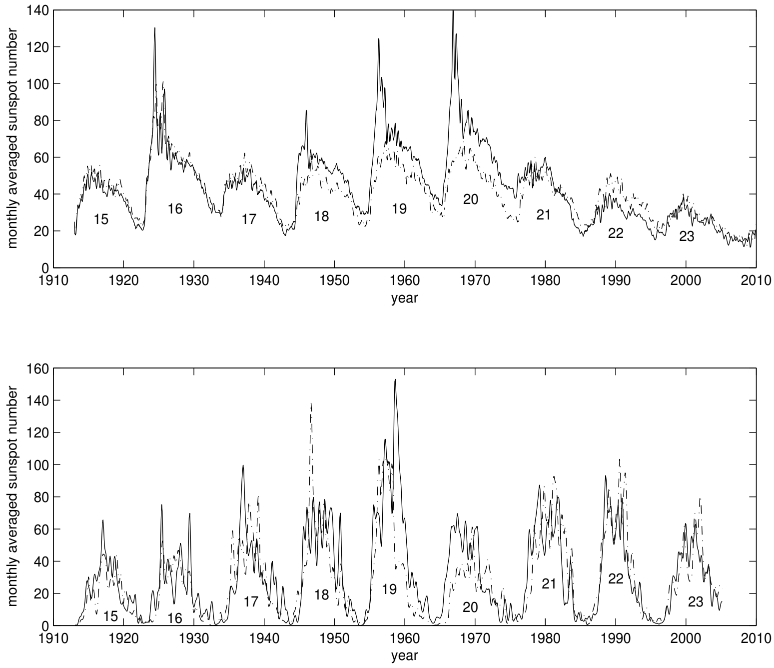

Fig. 4 The top panel plots the theoretical monthly averaged sunspot numbers in the northern (solid line) and the southern (dash-dotted line) hemispheres. The bottom panel shows the observational plot for the same.

Cycle number	N_{AS}	A_{AS}	$\frac{N_{AS}}{A_{AS}}$	$N_{AS} - A_{AS}$
15	-0.044	0.097	-0.45	-0.141
16	-0.016	0.087	-0.18	-0.103
17	-0.047	0.003	-15.6	-0.050
18	0.103	0.027	3.81	0.076
19	0.132	0.176	0.75	-0.044
20	0.182	0.171	1.06	0.011
21	0.017	-0.017	-1.00	0.034
22	-0.115	-0.063	1.83	-0.052
23	-0.034	-0.069	0.49	0.035
24	-0.056	-	-	-

Table 2 Theoretical (N_{AS}) and observed (A_{AS}) asymmetries in solar activity.

we see in the observational data. We believe that this is again an indication that parameters like diffusivity which are responsible for the coupling between the hemispheres probably have values in the correct ball park in our dynamo model. Figure 6 plots theoretical asymmetry N_{AS} against the observational asymmetry A_{AS} for different cycles. The correlation coefficient of 0.73 is quite remarkable, judging by the fact that considerable uncertainties are involved in using the polar faculae number as the proxy of the polar field. The probability of the null hypothesis is only 2.9%, leading us to conclude that the theoretical results agree well with observational data.

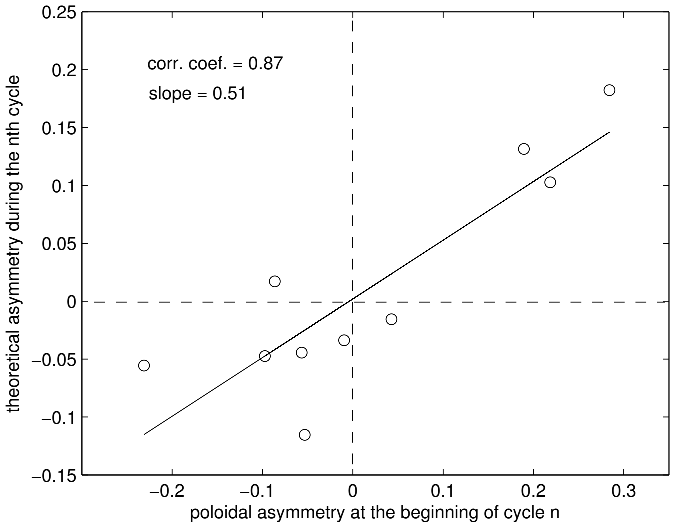

Fig. 5 Theoretically calculated asymmetry in sunspot activity during the cycle n is plotted against the observed asymmetry in the polar faculae number at the beginning of the cycle.

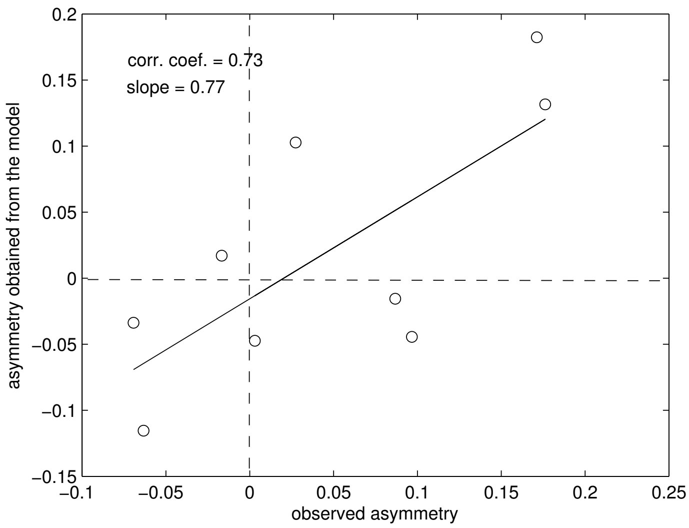

Fig. 6 The theoretical asymmetry N_{AS} of various cycles is plotted against the observational asymmetry A_{AS}.

5 CONCLUSION

During the twentieth century, some cycles had hemispheric asymmetry larger than 17% as seen in Table 1. It is possible that the hemispheric asymmetry of the solar activity plays an important role in determining the character of the solar cycle. For example, there is some observational evidence that the there was a large hemispheric asymmetry at the time of the onset of the Maunder minimum (Sokoloff & Nesme-Ribes 1994) and this asymmetry may even have played some role in inducing the Maunder minimum (Charbonneau 2005). However, to the best of our knowledge, not much systematic effort has been made previously to study the asymmetry of solar activity with the help of dynamo models. While we have taken the point of view that the Babcock–Leighton process is the source of irregularities in the solar cycle, it should be remembered that it is not yet clear whether this alone can explain the Maunder minimum or if something else is needed.

The randomness of the Babcock–Leighton process can make the poloidal field in one hemisphere stronger than the other and we suggest that this induces an asymmetry in the solar cycle. We have direct poloidal field data only from mid-1970s. Cycles from that time onwards have been only mildly asymmetric and hence are not particularly suitable for studying hemispheric asymmetry. Also, we need a larger data set to draw any statistically significant conclusions. So we use the polar faculae number reported by Sheeley (1991, 2008) as the proxy of the polar field. In spite of uncertainties involved in this procedure, we find that the asymmetry in the polar faculae number during a solar minimum is correlated with the hemispheric asymmetry of the next cycle. The correlation becomes weaker with succeeding cycles, suggesting a memory of about 15–20 years. We point out that this type of correlation is possible only if we assume a relatively high value of diffusivity like 2.4×10^{12} cm^2 s^{-1} (Chatterjee, Nandy & and Choudhuri 2004). A diffusivity of this order gives the right kind of memory when the dynamo is subjected to a disturbance in the poloidal field generation (Choudhuri, Chatterjee & Jiang 2007).

When we run our dynamo code by feeding the appropriate asymmetry at successive minima and model the sunspot cycles during the twentieth century, we get a qualitative agreement between theory and observations. We know that the cross-hemispheric coupling tries to reduce any asymmetry between the two hemispheres (Chatterjee & Choudhuri 2006). Both in observational data and theoretical simulations, we find that the asymmetry of a cycle statistically tends to be less than the asymmetry in the faculae number during the preceding minimum. The reduction factors also turn out to be comparable in the observational data and theoretical simulation. This is quite a remarkable agreement, given the many uncertainties involved in our analysis. Solar physicists may have to wait for about half a century to be able to carry out an analysis like the present analysis based on the actual measured polar field asymmetries rather than using proxies like the polar faculae number. Such an analysis will be more satisfactory than the present analysis, provided there will be some strongly asymmetric cycles in the next half century. We, however, hope that our methodology will provide the framework for any such future analysis.

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