Pramāṇa, Vol. 12, No. 3, March 1979, pp. 227–234, © printed in India

# On the colour contribution to effective weak vertex in broken colour gauge theories

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MS received 15 September 1978

**Abstract.** Treating the breaking of colour symmetry via the mixing between the colour gluons and weak bosons (a la Rajasekaran and Roy) it is observed that the colour contribution to the effective weak vertex of a quark at zero momentum transfer is zero upto $O(\alpha)$.

**Keywords.** Colour gluons; $O(\alpha)$ corrections; broken colour symmetry.

## 1. Introduction

Within the framework of unified gauge theories of strong, weak and electromagnetic interactions with integrally charged quarks (Pati and Salam 1973; Rajasekaran and Roy 1975) colour gluons are not neutral to weak and electromagnetic processes as in theories with fractionally charged quarks. To achieve the weak and electromagnetic interactions of colour objects the colour symmetry must be broken spontaneously. This results in making colour gluons massive, weak and electromagnetic currents acquiring octet pieces and the colour gluons mixing with the weak bosons and the photon. The final breaking to $U_Q(1)$ makes weak bosons massive (with the Weinberg-Salam mixing) and leaves the photon massless. The broken colour can manifest itself above colour threshold and also in higher order processes of colour singlet objects where more than one colour changing gluon vertex can produce a colour singlet effect. To study the effect of the latter we consider below the $O(\alpha)$ radiative corrections arising from gluon exchanges to the colour singlet weak vertex of a quark (which is what is effective in weak decays below colour threshold) at zero momentum transfer. We show that such contributions are zero upto $O(\alpha)$.

## 2. The model

In the $\mathrm{SU}(3)_{\mathrm{col.}} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)$ theory, the following choice of physical gluon fields

$$\begin{aligned} G^i_\mu &= V^i_\mu - g/f\, W^i_\mu \qquad & i = 1, \ldots, 3, \\ G^i_\mu &= V^i_\mu & i = 4, \ldots, 7, \\ G^8_\mu &= V^8_\mu - \frac{1}{(\sqrt{3})}\, g'/f\, U_\mu, \end{aligned} \tag{1}$$

can be made which renders vector Boson-Mass matrix diagonal and diagonalises the Weinberg-Salam part by the usual choice of orthogonal fields $Z_\mu$ and $A_\mu$ (Rajasekaran and Roy 1975), where $V_\mu{}^l$ ($l=1, \ldots, 8$), $W^i_\mu$ ($i=1, \ldots, 3$) and $U_\mu$ are the gauge bosons corresponding to the group SU $(3)_{\text{col.}}$, SU $(2)_L$ and U(1) respectively and $f$, $g$ and $g'$ are the respective coupling constants. The non-orthogonality of $G^i_\mu$ ($i=1$, 2, 3, 8) to $W_\mu$, $Z_\mu$ and $A_\mu$ generates momentum-dependent quadratic couplings providing the mixing of $G^i_\mu$ with $W_\mu$, $Z_\mu$ and $A_\mu$. This mixing leads to vector-dominance kind of vertices as shown in figure 1. The mixing vertex is given by $(-1/f) \times$ (appropriate semi-weak coupling constant) $\times (q^2 g_{\mu\nu} - q_\mu q_\nu)$. The $q_\mu q_\nu$ terms will in general contribute to off shell quark lines. Using the propagator for the gluon the effective vertex (for $W$ for e.g.) is $-g/f\,(q^2 g_{\rho\nu} - q_\rho q_\nu)/(q^2 - m_g{}^2)$, where $m_g$ is the mass of the gluon.

Also, the weak currents acquire octet pieces and in particular the charge changing weak current (which is what we consider below) is

$$j^{w^\pm}_\mu = j^\pm_\mu + j'^\pm_\mu \equiv (j^1_\mu \pm i j^2_\mu) + (j'^1_\mu \pm i j'^2_\mu), \tag{2}$$

where

$$j^\pm_\mu = \sum_{i=1}^{3} \left\{ (\bar{p}_i\ \bar{n}_i)\, \frac{\tau^\pm}{2}\, \gamma_{\mu L} \begin{pmatrix} p_i \\ n_i \end{pmatrix} + (\bar{c}_i\ \lambda_i)\, \frac{\tau^\pm}{2}\, \gamma_{\mu L} \begin{pmatrix} c_i \\ \lambda_i \end{pmatrix} \right\},$$

$$j'^\pm_\mu = \sum_{q=p,n,c,\lambda} \bar{q}\, \frac{\lambda^\pm}{2}\, \gamma_\mu\, q,$$

$$\gamma_{\mu_L} = \tfrac{1}{2}\, \gamma_\mu (1-\gamma_5);\ \ \tau^\pm = \tau^1 \pm i\,\tau^2;\ \ \lambda^\pm = \lambda^1 \pm i\,\lambda^2.$$

The fermion vector boson interaction contributing to the charge changing vertex becomes

$$\mathscr{L}^{\text{FV}}_{\text{c.c}} = \frac{g}{\sqrt{2}}\, (j^{W-}_\mu W^\mu + j^{W+}_\mu W^{\mu\dagger}) + f\,(j'^-_\mu G^\mu + j'^+_\mu + G^{\mu\dagger}). \tag{3}$$

The first term contributes to the charge changing vertex via the direct coupling of $W^\pm$ to the fermion current (which has an octet piece) and the second term gives a pure octet contribution due to direct coupling of gluons to the octet currents and then going over to $W^\pm$ through the above mentioned mixing.

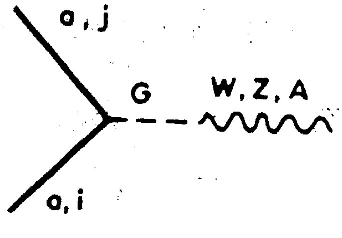


**Figure 1.** Quark-weak boson coupling via gluon-weak boson mixing.

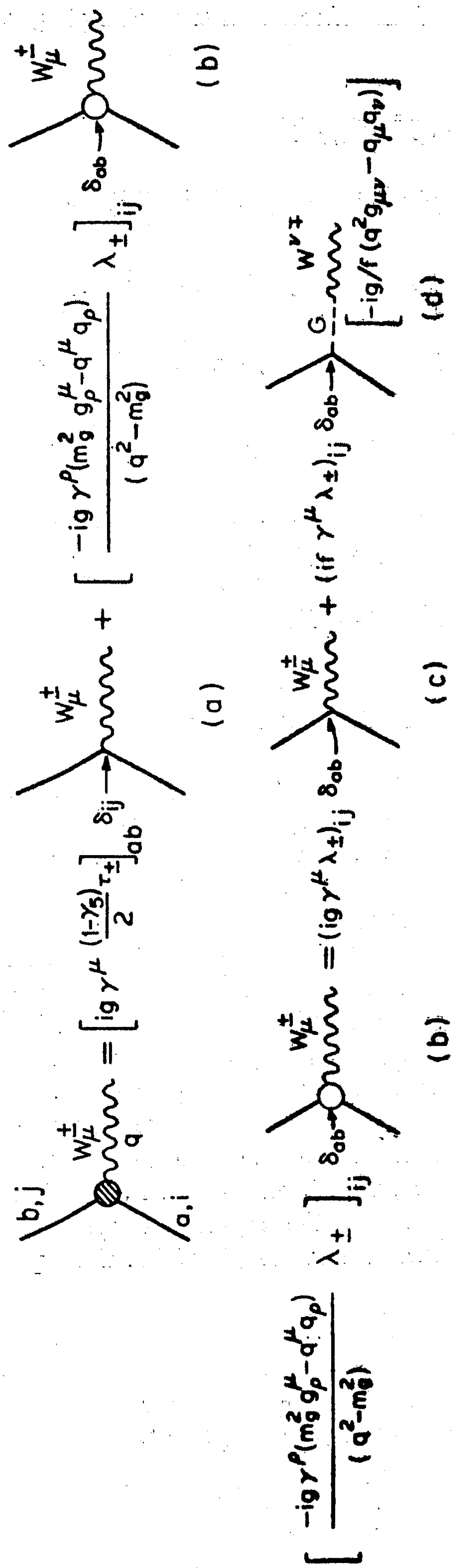


**Figure 2.** The complete effective quark-weak boson vertex.

The matrix element, of the effective charge changing weak current $J_\mu^{W\pm}$ will, therefore, be given by:

$$\langle Q_{a,i}(p') \,|\, g J_\mu^{W\pm} \,|\, Q_{b,j}(p)\rangle$$

$$= g\,\bar{q}_{ai}(p')\,\{(\tau^\pm)_{ab}\,\delta_{ij}\,\gamma_{\mu L} + \delta_{ab}\,(\lambda^\pm)_{ij}\,\gamma_\rho$$

$$\times\,[g^\rho{}_\mu - (q^2\,g^\rho_\mu - q^\rho\,q_\mu)/(q^2 - m_g^2)]\}\,q_{bj}(p),$$

$$= g\,\bar{q}_{ai}(p')\,\{(\tau_\pm)_{ab}\,\delta_{ij}\,\gamma_{\mu L} + \delta_{ab}\,(\lambda^\pm)_{ij}$$

$$\times\,\gamma_\rho\,(q^\rho\,q_\mu - m_g^2\,g^\rho_\mu)/(q^2 - m_g^2)\,\}\,q_{bj}(p);\; q = p - p'. \qquad (4)$$

The two terms in (4) correspond to (a) and (b) of figure 2 where, (a) is the coupling of colour singlet current to $W_\mu$ (the vertex as in a gauge theory without broken colour) and (b) is the coupling of colour octet current. (b) is the sum of the octet contribution of $W_\mu^\pm$ and $G_\mu^\pm$ ((c)+(d)).

## 3. Colour gluon contributions to $O(\alpha)$ corrections

We wish to find the octet contribution to the $O(\alpha)$ radiative corrections to the quark-$W$-boson vertex, which is diagonal in colour space—for e.g., the coupling constant $g_{n_i p_i}\,W$. This will contribute to the $\beta$-decay constant of colour singlet objects, for instance.

The various octet contributions to $O(\alpha)$ corrections are the vertex corrections and self energies of the quark lines to 1-loop level. These fall into two classes: (I) in which the main vertex is colour singlet and the octet currents contributing to the singlet part are in the loop. (II) in which the main vertex is a colour octet and one of the vertices in the loop is also an octet, together giving a singlet contribution.

Diagrams of class I (figure 3) are expected to give zero contribution because of the conservation of the weak current not being broken by the octet currents i.e. the current at the main vertex commutes with the octet currents of the loop. A Ward identity will hold for the currents involved and in the limit $q \to 0$, therefore, the diagrams cancel. This is same as the argument for non-renormalisation of weak interaction by strong interaction in the limit of massless quarks.

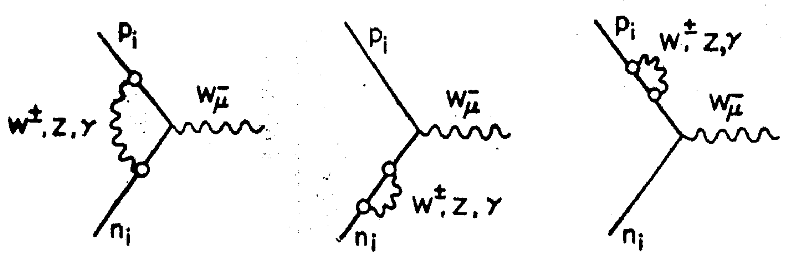


**Figure 3.** Diagrams with colour singlet main vertex.

From diagrams of class II, those which have $W$-loops can give non-zero contribution since the relevant currents do not commute only for these. However, it turns out that even these give zero contribution at the one-loop level as indicated below.

Since the self-energies in this class of diagrams are not diagonal in colour space, there are two kinds of self-energy insertions on each of the external legs corresponding to different order in which the octet vertex and the singlet vertex occur in the self energies (figures 4c & 4d and 4e & 4f). And corresponding to this order there are two kinds of vertex diagrams 4a and 4b. 4a cancels with 4c & 4d and 4b cancels with 4e & 4f in a Ward-Identity-like fashion. Since the self-energies now are $\gamma_5$-dependent the wave function renormalisations are to be done correctly. For this we follow the procedure of Bollini *et al* (1973) and Hiida (1963).

## 4. The amplitudes and their cancellation

We will write down the $O(\alpha)$ corrections for this class of diagrams, for quark of colour $i$, in the limit $m \to 0$; $q \to 0$.

$$M_{\mathrm{II}}^{(a)} = \frac{(ig)^2}{2} \cos\theta_c \frac{(ig)}{\sqrt{2}} \frac{1}{2} \int \frac{d^4 l}{(2\pi)^4} \Big\{ \bar{p}_i \gamma^\sigma (1-\gamma_s) \frac{i}{\boldsymbol{p}-\boldsymbol{l}} (\lambda_+)_{ij} \not{\epsilon}$$

$$\times \frac{i}{\boldsymbol{p}-\boldsymbol{l}} \gamma^\lambda (\lambda_-)_{ji}\, n_i$$

$$\left[\frac{(-i)\,(g_{\sigma\rho} - (l_\sigma l_\rho / m_W^2))\,(l^\rho l_\lambda - m_g^2 g_\lambda^\rho)\,(-i)\,(-m_g^2)}{(l^2 - m_W^2)\,(l^2 - m_g^2)\,(-m_g^2)}\right]\Big\}$$

$$= -ig^3 \frac{\cos\theta_c}{4\sqrt{2}} (\lambda_+ \lambda_-)_{ii} \int \frac{d^4 l}{(2\pi)^4} \Big\{\bar{p}_i\, [-m_g^2 \gamma^\sigma (\boldsymbol{p}-\boldsymbol{l}) \not{\epsilon} (\boldsymbol{p}-\boldsymbol{l}) \gamma_\sigma$$

$$+ D\boldsymbol{l}(\boldsymbol{p}-\boldsymbol{l}) \not{\epsilon} (\boldsymbol{p}-\boldsymbol{l})\boldsymbol{l}]\,(1-\gamma_s) n_i \frac{1}{(l^2 - m_g^2)(l^2 - m_W^2)(p-l)^4}\Big\}$$

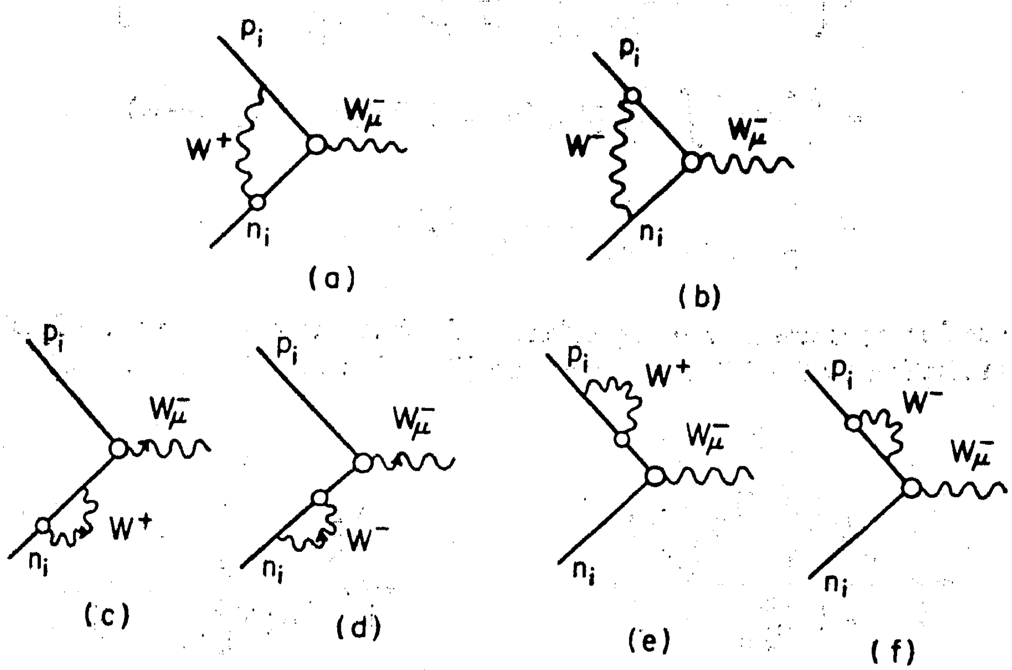


**Figure 4.** Diagrams with colour octet main vertex.

where $D=(1+m_g^2/m_W^2-l^2/m_W^2)$, $\theta_c$ is the Cabibbo angle and $\epsilon_\mu$ is the polarisation vector. (In the limit $m\rightarrow 0$, all terms proportional to $p$ and odd powers of $l$ can be put equal to zero)

$$M_{\rm II}^{(a)} = -\frac{ig^3}{4\sqrt{2}}\cos\theta_c\,(\lambda_+\lambda_-)_{ll}\int\frac{d^4l}{(2\pi)^4}$$

$$\frac{\bar{p}_l\,[-m_g^2\,l^2\,\not\epsilon+Dl^4\not\epsilon]\,(1-\gamma_5)n_l}{(l^2-m_g^2)\,(l^2-m_W^2)\,(p-l)^4}. \tag{5}$$

Amplitude $M_{\rm II}^{(b)}$ is the same as $M_{\rm II}^{(a)}$ but for the order of $\lambda$-matrices because

$$\gamma^\sigma(1-\gamma_5)\,(\mathbf{p}-\mathbf{l})\gamma^\mu\,(\mathbf{p}-\mathbf{l})\gamma^\lambda = \gamma^\sigma\,(\mathbf{p}-\mathbf{l})\gamma^\mu\,(\mathbf{p}-\mathbf{l})\gamma^\lambda\,(1-\gamma_5).$$

The order of $\lambda$-matrices now is $\lambda_-\lambda_+$ and the amplitude is

$$M_{\rm II}^{(b)} = \frac{-ig^3\cos\theta_c}{4\sqrt{2}}\,(\lambda_-\lambda_+)_{ll}\int\frac{d^4l}{(2\pi)^4}\,\frac{\bar{p}_l\,[-m_g^2l^2\,\not\epsilon+Dl^4\not\epsilon]\,(1-\gamma_5)n_l}{(l^2-m_g^2)\,(l^2-m_W^2)\,(p-l)^4}. \tag{6}$$

For the amplitudes $M_{\rm II}^{(c,\,d,\,e,\,f,)}$ we have to evaluate the corresponding self energies and the $O(\alpha)$ contributions are just the external leg-wave function renormalisation coming from these self energies (figure 5).

$$-i\sum\nolimits_{ij}^{\pm}(\mathbf{p}) = (-i)\frac{(ig)^2}{4}\cos\theta_c\int\frac{d^4l}{(2\pi)^4}\,\frac{\gamma^\sigma(\mathbf{p}-\mathbf{l})\gamma^\lambda(1-\gamma_5)}{(p-l)^2}(\lambda_\pm)_{ij}$$

$$\times\left[\frac{(-i)\,(g_{\sigma\rho}-l_\sigma l_\rho/m_W^2)\,(-m_g^2\,g_\lambda^\rho+l^\rho l_\lambda)\,(-i)\,(-m_g^2)}{(l^2-m_W^2)\,(l^2-m_g^2)\,(-m_g^2)}\right],$$

$$=\frac{ig^2}{4}\cos\theta_c\int\frac{d^4l}{(2\pi^4)}[2m_g\,(\mathbf{p}-\mathbf{l})+D\,(\mathbf{l}\mathbf{p}\mathbf{l}-l^2\mathbf{l})]\,(1-\gamma_5)$$

$$\times\;\frac{1}{(p-l)^2\,(l^2-m_g^2)\,(l^2-m_W^2)}\,(\lambda_\pm)_{ij} \tag{7}$$

These self-energies are now $\gamma_5$-dependent. We shall carry out the wave function renormalisation by the following procedure (Bollini *et al* 1973; Hiida 1963).

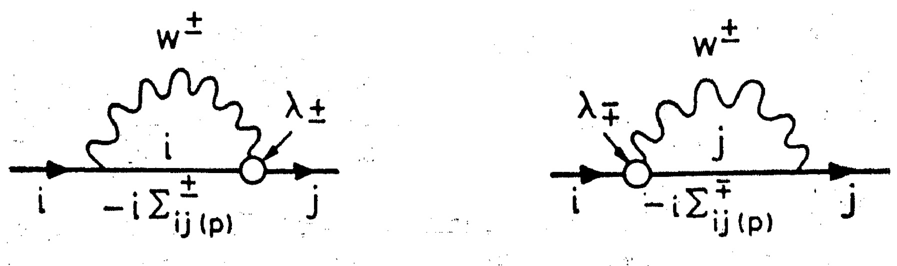


**Figure 5.** The colour non-diagonal quark self-energies.

The self-energies are of the form,

$$\sum(\mathbf{p})-\delta m=B\mathbf{p}+C\mathbf{p}^2+E\mathbf{p}^3-A\mathbf{p}\gamma_5-F\mathbf{p}\gamma_5 p^2+\Sigma_f.$$

Only $B$ and $A$ will contribute to the amplitude, as for on-shell quarks $\Sigma_f=0$ and other terms give zero (In our case $B=A$). To take into account of the $A$ term the fermion fields are scaled to $\psi'=\left(1-\frac{A}{2}\gamma_5\right)\psi$. This amounts to replacing the free Lagrangian by

$$\mathscr{L}_0=\bar{\psi}\,[i\gamma^\mu\partial_\mu\,(1+A\gamma_5)-m]\psi$$

and adding a compensating counter term $-\bar{\psi}\,i\gamma^\mu\partial_\mu A\gamma_5\psi$ which cancels the $-A\mathbf{p}\gamma_5$ term in $\Sigma$. The $B$-term is accounted for by the usual wave function renormalisation which yields $\left(1+\frac{B}{2}\right)M_0$ for self energy on each external leg, where $M_0$ is the bare vertex. $B(=A)$ is given by

$$B^{\pm}_{ij}=\left.\frac{\partial\Sigma^{\pm}_{ij}(\mathbf{p})}{\partial\mathbf{p}}\right|_{\mathbf{p}=0}=\frac{-g^2}{4}\cos\theta_c\,(\lambda_\pm)_{ij}\times$$

$$\int\frac{d^4l}{(2\pi)^4}\frac{[m_g^2\,l^2-Dl^4]}{(p-l)^4\,(l^2-m_g^2)\,(l^2-m_W^2)}\,. \tag{8}$$

Therefore the $O(\alpha)$ corrections for figures 4c to 4f are given by

$$M_{\text{II}}^{(c+d)}=\frac{ig}{\sqrt{2}}\bar{p}_i\gamma_\mu(\lambda_+)_{ji}\,B^-_{ji}\,(1-\gamma_5)n_i, \tag{9}$$

$$=-\,ig^3\frac{\cos\theta_c}{4\sqrt{2}}(\lambda_+\lambda_-)_{ii}\int\frac{d^4l}{(2\pi)^4}\frac{\bar{p}_i\,[m_g^2\,l^2-Dl^4]\,\gamma_\mu\,(1-\gamma_5)n_i}{(p-l)^4\,(l^2-m_g^2)\,(l^2-m_W^2)}, \tag{10}$$

$$M_{\text{II}}^{(e+f)}=\frac{ig}{\sqrt{2}}\bar{p}_iB^-_{ij}\,(1+\gamma_5)\,\gamma_\mu\,(\lambda_+)_{ji}\,n_i,$$

$$=-ig^3\,\frac{\cos\theta_c}{4\sqrt{2}}\,(\lambda_-\lambda_+)_{ii}\int\frac{d^4l}{(2\pi)^4}\frac{\bar{p}_i\,[m_g^2\,l^2-Dl^4]\,\gamma_\mu\,(1-\gamma_5)\,n_i}{(p-l)^4\,(l^2-m_g^2\,)\,(l^2-m_W^2\,)},$$

where we have dropped correction terms of $O(g^4)$. The amplitudes $M_{\text{II}}^{(c+d)}$ (eq. (9)) and $M_{\text{II}}^{(e+f)}$ (eq. (10)) cancel $M_{\text{II}}^{(a)}$ (eq. (5)) and $M_{\text{II}}^{(b)}$ (eq. (6)) respectively.

## 5. Conclusions

Thus, in conclusion, what we have shown above is that the strength of weak vertices for quarks in broken colour gauge theories remains unaltered upto $O(\alpha)$ by colour octet contributions which can manifest themselves in higher order corrections. Therefore, the results of unbroken colour gauge theories on weak decay matrix elements will still hold upto $0(\alpha)$.

It is not, however, clear whether cancellations would occur for such contributions (i.e. octet contributions to vertices diagonal in colour) in higher orders. This would call for a formal proof which would require study of renormalization of a broken colour gauge theory (a la Rajasekaran and Roy) in full detail.

## Acknowledgements

We wish to thank G Rajasekaran for discussions and P P Divakaran for a critical reading of the manuscript.

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