Pramāṇa, Vol. 21, No. 1, July 1983, pp. 11–27. © Printed in India.

Spin structure of nucleon

R RAMACHANDRAN
Department of Physics, Indian Institute of Technology, Kanpur 208 016, India

MS received 11 March 1983

Abstract. The quark spin content of the nucleons is subjected to constraints implied by sum rules due to global approximate chiral symmetries and perturbative QCD effects. The model, so obtained, has a large polarisation residing in the flavour singlet constituents of hadron. Predictions for the expected longitudinal and transverse spin asymmetries in deep inelastic lepton-nucleon scattering are made on the basis of the standard form of the electromagnetic and charged weak currents.

Keywords. Perturbative QCD; spin structure; partons; sea and gluon polarisation; longitudinal and transverse spin asymmetries.

1. Introduction

The main experimental support for quantum chromodynamics comes from the study of nucleon structure function, revealed in deep inelastic lepton-nucleon scattering. (For a review see Buras 1980). The Q^2-dependances of $F_2^{\text{em}}(x, Q^2)$, $F_2^{\nu}(x, Q^2)$ and $F_3^{\nu}(x, Q^2)$ have confirmed the leading order predictions. The next to leading order, however, depends on the details of the renormalisation prescription and for every process there is an optimum scheme for which the prediction is unique and agrees with the experimental values. This state of affairs encourages one to expect that the spin dependant structure functions of nucleon $G_1(x, Q^2)$ and $G_2(x, Q^2)$, studied by observing the polarisation asymmetries, will satisfy the expected scaling properties and appropriate scaling violations, further confirming QCD.

The quark distribution and the internal spin structure of the nucleons are, *per se*, non-perturbative aspects of QCD and, as of now, there exists no procedure for calculating them from first principles. Nevertheless, we should expect that they are governed by the constraints imposed by the flavour symmetries (such as isospin, SU (3) etc., the origins of which can be traced to the small symmetry breaking quark masses, compared to the intrinsic scale Λ of QCD), current algebra (such as Björken sum rule, which is a consequence of the approximate chiral symmetry (Björken 1970)) and such other dynamical restrictions as implied by general principles like Regge behaviour etc. In this paper, we use the various constraints and clues to arrive at the form of the spin structure of the nucleon and translate the same into expected asymmetries in the lepton-nucleon scattering. In particular we will emphasize the constraints implicit in the leading order (Q^2) evolution of the structure functions (Bajpai and Ramachandran 1980) and the experimental signals in which the effects may become observable.

2. Basic model

The spin-dependant part of the electroproduction ($e + p \rightarrow e + X$) may be expressed in terms of the antisymmetric tensor

$$W_{\mu\nu}^{(\text{spindep.})} = (1/2\pi) \int d^4 \zeta \exp(iq.\zeta) \langle p, s \mid [J_\mu(\zeta), J_\nu(0)] \mid p, s\rangle$$

$$= \epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda}{(p.q)} \{s^\sigma G_1(x, Q^2) + [1/(p.q)] \, s^\sigma (p.q)$$

$$- (q.s) \, p^\sigma] \, G_2(x, Q^2)\} \tag{1}$$

where x is the scaling variable $Q^2/2(q.p)$ and $Q^2 = -q^2$. Since in longitudinally polarised spin-dependent experiments s is parallel to p, $G_2(x, Q^2)$ does not contribute. Thus $G_2(x, Q^2)$ will arise only in the transverse polarisation measurements. In quark-parton model, we expect scaling and the scaled function ($M^2 \nu G_1(x, Q^2) \xrightarrow[Q^2 \to \infty]{} g_1(x)$) measures the spin structure of the quarks that make up the hadron. It is possible to identify $g_1(x)$ with spin structure through

$$2 g_1(x) = \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i) \tag{2}$$

where the subscript refers to the various flavours of quarks and $\Delta q_i \equiv q_i^+(-q_i^-)$, $q_i^\pm(x)$ is the probability of finding a quark with flavour i of $\pm$ helicity in the positive helicity hadron (here proton) carrying a longitudinal momentum fraction x of the hadron, when measured in an infinite momentum frame. The Q^2 -dependance of $g_1(x, Q^2)$ (and hence that of $\Delta q_i(x, Q^2)$) is determined by the use of an operator product expansion and the renormalisation of the relevant operators introduces appropriate $\ln Q^2/Q_0^2$ dependance determined by the anomalous dimensions of the operators thereof (Politzer 1974). More intuitively Altarelli-Parisi equations (Altarelli and Parisi 1977) can be set up to give a set of coupled integro-differential equations. The role of anomalous dimensions is played here by the convolution of the probability $p(z)$ of finding a parton (quark or gluon) within another carrying a definite momentum fraction (z) of the former per unit interval of t ($\equiv \ln Q^2/Q_0^2$). These coefficients can be read off from the basic vertices of QCD in the leading order and in principle to any order.

It is convenient to split $\Delta q_i(x, Q^2)$ into a valence part and a sea part and work with the moments, defined through

$$\Delta q_i^{v, n} \equiv \int_0^1 dx \, x^{n-1} [\Delta q_i(x, Q^2) - \Delta \bar{q}_i(x, Q^2)], \tag{3}$$

$$\Delta q_i = \Delta q_i^v + \Delta q_{i0}. \tag{4}$$

The moments of the valence part satisfies a first order equation in t:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta\, q_i^{v,n} = (\alpha_s(t)/2\pi)\, \tilde{A}_{qq}^n\, \Delta\, q_i^{v,n};\quad \alpha_s(t) = \alpha_s(0)/(1+b\, \alpha_s(0)\, t) \tag{5}$$

which can be solved to give

$$\Delta\, q_i^{v,n}(t) = \Delta\, q_i^{v,n}(0)\; [\alpha_s(0)/\alpha_s(t)]^{\tilde{A}_{qq}^n/2\pi b} \tag{6}$$

The $n = 1$ moment, which measures the net difference in the number of quarks with $+$ and $-$ helicities in a positive helicity hadron, is t-independent, since $\tilde{A}_{qq}^1 = 0$ in the leading order. In proton, $\Delta\, q_u^{v,1}$ and $\Delta\, q_d^{v,1}$ will be some fixed number. To determine this value, we appeal to various aspects of flavour symmetry. If the proton is described by an SU(6) wave function for its constituent valence quarks (Kuti and Weisskopf 1971) then we will have

$$\Delta\, q_u^{v,1} = 4/3,\quad \Delta\, q_d^{v,1} = -\,1/3. \tag{7}$$

together with the unwanted consequence of $(G_A/G_V)_{p\to n} = 5/3$ Sehgal (1974) uses, instead, Björken sum rule for G_A/G_V:

$$\Delta\, q_u^{v,1} - \Delta\, q_d^{v,1} = (G_A/G_V)_{p\to n}. \tag{8}$$

and the experimental value for G_A/G_V ($\simeq$ 1·25), together with a similar relation for ($\Xi^- \to \Xi^0$) β decay transition connected by the flavour SU(3) symmetry to give

$$\Delta\, q_u^{v,1} = 0{\cdot}97,\quad \Delta\, q_d^{v,1} = -\,0{\cdot}28. \tag{9}$$

In contrast, Carlitz and Kaur (1977) propose that the valence quark spin is governed by Feynman's leading quark hypothesis and most of the momentum and helicity is carried by the 'leading quark' in the hadron (Kaur 1977). Accordingly

$$\Delta\, q_u^v(x) = \cos 2\,\Theta(x)\; [q_u^v(x) - 2/3\; q_d^v(x)], \tag{10}$$

$$\Delta\, q_d^v(x) = -\,(1/3)\cos 2\,\Theta(x)\, q_d^v(x), \tag{11}$$

with
$$\cos 2\,\Theta(x) = [1 + 0{\cdot}052\,(1-x^2)/\sqrt{x}]^{-1} \xrightarrow[x\to 1]{} 1. \tag{12}$$

This parametrisation implies, since $\cos 2\,\Theta(x)$ rapidly approaches unity (as $x\to 1$) and $q_u^v(x)$ dominates over $q_d^v(x)$ at large x, that the valence u-quarks in this region of x, have their spins aligned almost fully along the proton spin. The first moment—the net number of valence spin—is, however, not very different from the values in (7) or (9). Typically they are $\Delta\, q_u^{v,1} = 1{\cdot}01$ and $\Delta\,{}_d^{v,1} = -\,0{\cdot}25$. Since this model appears to be in general agreement with the phenomenology at large x, we will henceforth treat the valence spin to be given by (10)–(12) in our analysis.

We now turn to the analysis of the flavour singlet parts, which includes $\sum_{i=1}^{2f} \Delta q_i$, the combination that sums over the valence and the sea quarks of all flavours and $\Delta G \equiv G^+(x, Q^2) - G^-(x, Q^2)$ where $G^\pm(x, Q^2)$ denote the gluon distribution in the proton. The QCD evolution equations for them are given by:

$$\frac{\mathrm{d}}{\mathrm{d}t} \sum_i \Delta q_i^n(t) = (\alpha_s(t)/2\pi) \left[\tilde{A}_{qq}^n \sum_i \Delta q_i^n(t) + 2f \tilde{A}_{qG}^n \Delta G^n(t) \right] \tag{13}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \Delta G^n(t) = (\alpha_s(t)/2\pi) \left[\tilde{A}_{Gq}^n \sum_i \Delta q_i^n(t) + \tilde{A}_{GG}^n \Delta G^n(t) \right] \tag{14}$$

Since the anomalous dimensions ($\tilde{A}^n/2\pi b$) are known constants in QCD to any desired order, the above equations are solved easily to give the Q^2 dependance of the moments and hence that of the structure functions. For $n = 1$, since $\tilde{A}_{qq}^n = 0$ and $\tilde{A}_{qG}^n = 0$ in the leading order, this suggests Δq_i^1 for each i is constant independent of Q^2. This then means that the net difference of quarks with either helicity for each flavour is constant. When this equation is taken together with the conservation of angular momentum (*i.e.* the proton spin $\frac{1}{2}$ is made up of the quark and gluon spin, together with the negligible orbital angular momentum), we are led to the equation (Bajpai and Ramachandran 1980)

$$\sum_i \Delta q_i^1 + \Delta G^1 = \tfrac{1}{2}, \tag{15}$$

the Q^2 independence of which leads to, using (14)

$$\tilde{A}_{Gq}^1 \sum_i \Delta q_i^1 + \tilde{A}_{GG}^1 \Delta G^1 = 0. \tag{16}$$

Solving these equations, we obtain (with $\tilde{A}_{Gq}^1 = 2$ and $\tilde{A}_{GG}^1 = (33 - 2f)/6$ where f is the number of flavours)

$$\sum_i \Delta q_i^1 = (33 - 2f)/(9 - 2f) \text{ and } \Delta G^1 = -12/(9 - 2f) \tag{17}$$

We notice two features of these equations: (i) They are rather large ($f = 3$, 4 or 5 for example) compared to the valence spin, obtained in the various models (ii) As one crosses the flavour thresholds, there is an accompanying large jump in the value of net quark spin (which resides mostly among the sea quarks) and gluon spin. These conclusions are not altered even if we consider some moderate contributions of orbital angular momentum*. We should expect, therefore, observable experimental signals, related to these effects.

3. Longitudinal asymmetries

The longitudinal asymmetries in $\vec{e}\,\vec{p}$ scattering (the arrow on top of e and p imply polarisation of the same), measuring

$$A = \frac{d\sigma(\uparrow\downarrow) - d\sigma(\uparrow\uparrow)}{d\sigma(\uparrow\downarrow) + d\sigma(\uparrow\uparrow)}$$

is related to the function $g_1(x, Q^2)$ through

$$A(x, Q^2) = \frac{2x g_1(x, Q^2)}{F_2(x, Q^2)} A_{eq}, \tag{18}$$

where A_{eq} is some known kinematical factor. $A(x, Q^2)/A_{eq}$ is referred to as the longitudinal asymmetry function, $A_{LL}(x, Q^2)$ in the literature. Thus, explicitly

$$A_{LL}(x, Q^2) = \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i) \Big/ \sum_i e_i^2 (q_i + \bar{q}_i) \tag{19}$$

For $\vec{e}\;\vec{p}$ longitudinal asymmetry

$$A_{LL}^p(x, Q^2) = \frac{\frac{4}{9}\Delta q_u^v + \frac{1}{9}\Delta q_d^v + \sum e_i^2 \Delta q_{i0}}{\frac{4}{9} q_u^v + \frac{1}{9} q_d^v + \sum e_i^2 q_{i0}} \tag{20}$$

The parametrisations used to obtain the plot for A_{LL}^p given in figure 1 are summarised in table 1. We use the CDHS analysis of the unpolarised deep inelastic muon scattering data (de Groot *et al* 1979; Falciano 1981). For the spin structure, the valence spin is given by Carlitz-Kaur model (equations (10) to (12)) and the

*If the orbital angular momentum is considered, the essential conclusions of large singlet polarisation persists with a modulating factor

$$\left[1 - 2\langle L_z\rangle + \frac{24\pi}{\alpha_s}\frac{1}{(33-2f)}\langle \dot{L}_z\rangle\right]$$

for $\sum_i \Delta q_i^1$ and a factor

$$[1 - 2\langle L_z\rangle + (\pi/\alpha_s)\langle \dot{L}_z\rangle]$$

for ΔG^1. It is not possible to determine these factors from first principles. However we may estimate it as follows: We may expect $\langle L_z\rangle \sim \langle \vec{r} \times \vec{p}\rangle_z$ to be of order $\langle p_T\rangle$, primordial average transverse momentum and from the leading 2nd order QCD $\langle p_T\rangle \sim (\alpha_s Q^2)^{1/2} = \sqrt{\alpha_s}\, Q_0\, e^{t/2}$ and thus $\langle dL_z/dt\rangle \sim \frac{1}{2}\langle L_z\rangle$. It is observed that $\langle dL_z/dt\rangle$ and $\langle L_z\rangle$ are of the same sign and hence tend to neutralise each other in the modulating factors. Even for a substantial magnitude of $\langle L_z\rangle \sim 0{\cdot}1$ (which is 20% the angular momentum of proton) the modulating factor is about 1·5, implying an enhancement of our conservative values. In summary, it is unlikely that these factor will so conspire as to nullify the large polarisation structure implied here.

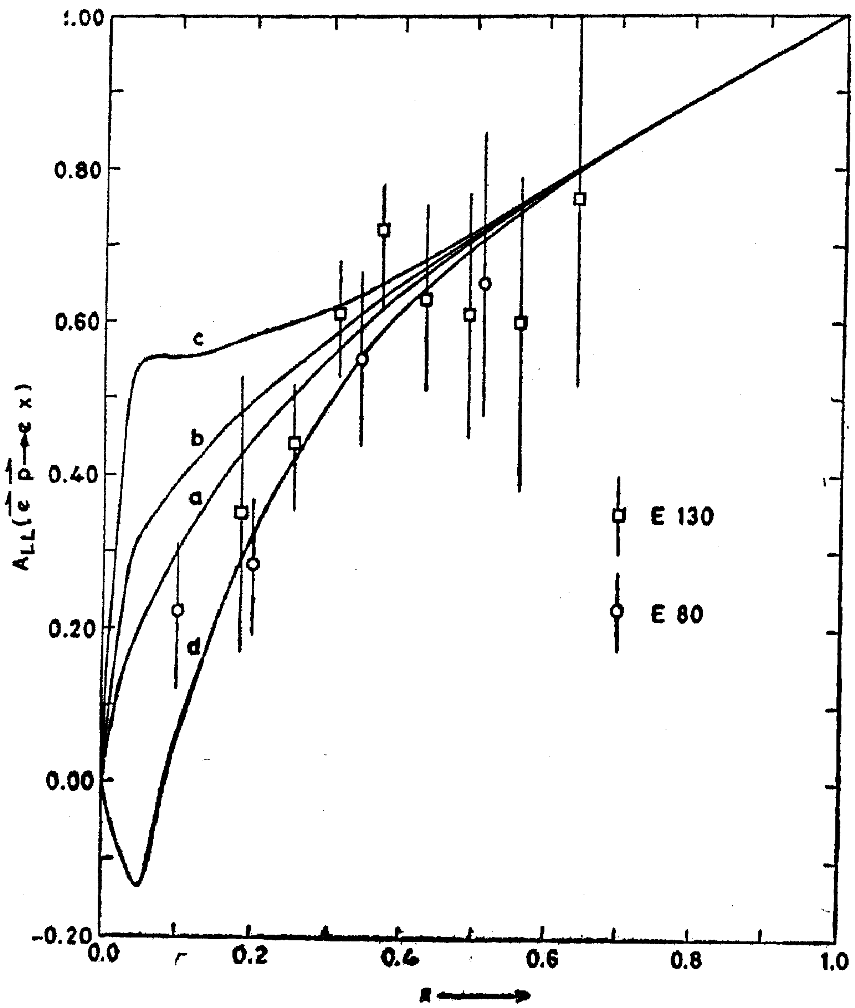

Figure 1. Longitudinal asymmetry $A_{LL}(\vec{e} + \vec{p} \to e + X)$ vs $x(= Q^2/2M\nu)$ the scaling variable. Theoretical predictions give the curve marked a when sea polarisation is negligible, and the curves b d when 3, 4 and 5 flavours are excitedrespectively. Parametrisation used are as given in table 1 for the unpolarised structure functions and as in equations (10), (11) and (22) with ϵ chosen as 0·05 in all computations. Data refer to the only available results from SLAC-Yale experiments (Alguard *et al* 1976, 1978 (E-80) and Baum *et* al 1982 (E-130).

Table 1. CDHS parametrisation of unpolarised structure functions.

Structure functions	CDHS parametrisation
$q_u^v(x, Q_0^2)$	$2{\cdot}13\,(1 - x)^{2.8}/x^{1/2}$
$q_d^v(x, Q_0^2)$	$1{\cdot}21\,(1 - x)^{3.8}/x^{1/2}$
$\sum_i q_{0i}(x, Q_0^2)$	$0{\cdot}27\,(1 - x)^{8}/x$

flavour singlet sea quarks must satisfy constraints implied by (17). If the sea polarisation derives equal contribution from u, d and s flavours, then we have

$$\Delta q_0^1 = [(33 - 2f) / (9 - 2f - 0{\cdot}76)] / 6. \tag{21}$$

For the x-dependance, while the sea quarks distribution $q_0(x)$ has the form $(1-x)^8/x$ indicating the presence of wee partons (peaking for small x), we may expect the spin structure to be given by

$$\Delta q_0(x, Q^2) = c\, x^{\epsilon}\, q_0(x);\ \epsilon > 0, \tag{22}$$

where for a given ϵ, c is obtained by the constraint (21). Further, since for all values of x, $|\Delta q_0(x)|/q_0(x) \leqslant 1$ and hence all moments satisfy $|\Delta q_0^n|/q_0^n \leqslant 1$ we find that $\epsilon <$ (0·14, 0·07 and 0·07) for 3, 4 and 5 flavours respectively for the parameters employed*.

Shown in figure 1 are the predicted curves for longitudinal asymmetry A_{LL}, demonstrating the effect of sea polarisation at small x region, when 3, 4 or 5 flavours are excited. The magnitude and the sign of the deviation from the 'no sea' parametrisation may be observed as also the comparison with results from E80 and E130 Yale-SLAC polarised $\vec{e}\ \vec{p}$ scattering experiments (Alguard *et al* 1976, 1978; Baum *et al* 1980; Oppenheim 1982). Our predictions may also be compared with those of other existing model (not shown in the figure), such as (i) SU(6) prediction (Kuti and Weisskopf 1971) (ii) various different parametrisation of valence quark spin structure (Close 1974; Look and Fishbach 1977; Sehgal 1974; Carlitz and Kaur 1977; Jaffe 1975) and (iii) other unorthodox models like source theory (Schwinger 1977) and Fire strings (Preparata 1981). For a recent summary see Bjorken 1982.

3.1 A_{LL}^N, *longitudinal asymmetry in* $\vec{e}\ \vec{N} \to e\ X$

There are, as of now, no measurements of asymmetry for scattering off neutron. However, such information is needed in order to verify Björken sum rule, which has been an ingredient in our parametrisation. To determine the neutron spin structure, it will be appropriate to measure the asymmetry parameters off an isoscalar target. There is currently a proposal (E = 138, SLAC) which aims to achieve this. The asymmetry, expected is

$$A_{LL}^N(x, Q^2) = \frac{(5/18)\,(\Delta q_u^v + \Delta q_d^v) + \sum_i e_i^2\, \Delta q_{0i}}{(5/18)\,(q_u^v + q_d^v) + \Sigma\, e_i^2\, q_{0i}}. \tag{24}$$

Figure 2 gives the plot for $A_{LL}^N(x)$, using the same set of parameters, as employed to predict A_{LL}^p. From this it is straightforward to extract the structure functions $g_1^p(x)$

*Alternatively the spin structure may behave similar to the valence quark structure near $x = 0$ as dictated by the usual Regge behaviour of the spin flip amplitudes. (Recall as $x \to 0$, $\nu \to \infty$ implies finite Q^2 Regge kinematical region; $1/\sqrt{x}$ form is related to $\nu^{\alpha(0)}$, with Regge intercept $\alpha(0) = \frac{1}{2}$. We then, may make an ansatz

$$\Delta q_0 \simeq c'\,(1-x)^p/\sqrt{x} \tag{23}$$

In order that (21) is satisfied and at the same time $|\Delta q_0^n|/q_0^n \leqslant 1$ is maintained, we find $p > 66$ for $f = 4$. This implies a sharply falling function for Δq_0 and because of the kinematical zero present in the asymmetry function at $x = 0$, experimentally this parametrisation is indistinguishable from $\Delta q_0 = 0$. (*i.e.* no polarisation in the sea).

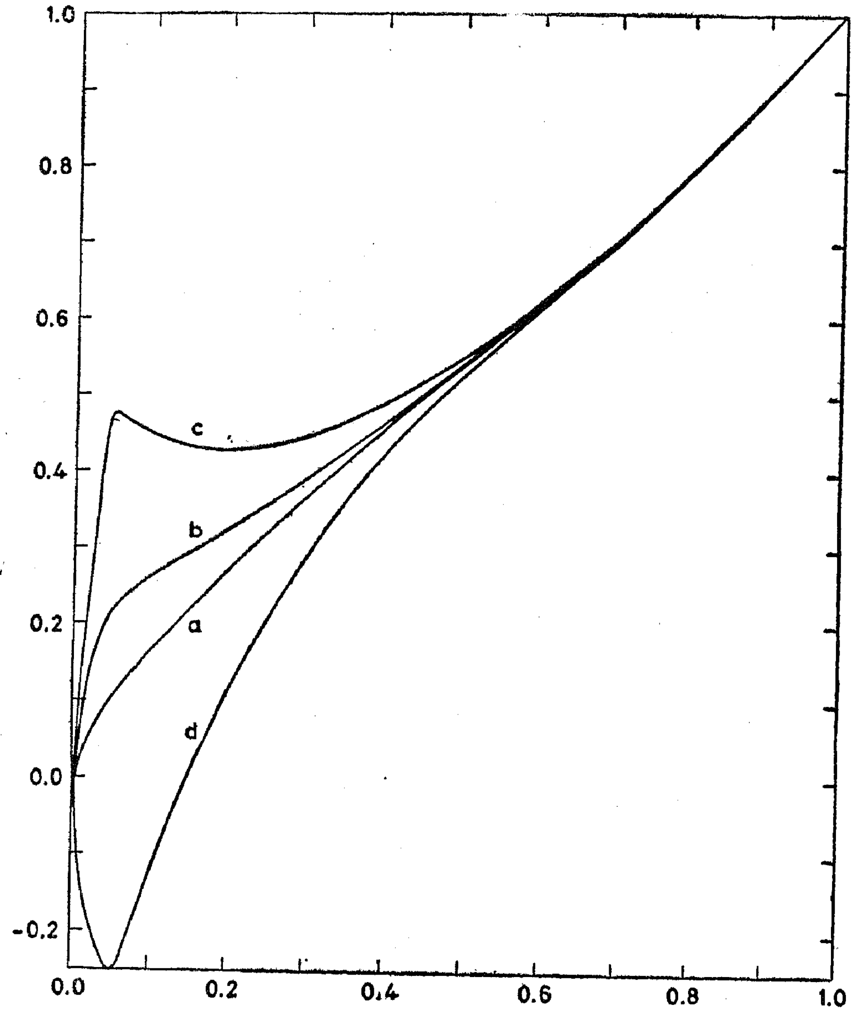

Figure 2. $A_{LL}\,(\vec{e} + \vec{N} \to e + X)$ vs x. **a-d.** refer to the same set of parameters as in figure 1.

and $g_1^n(x)$ and verify the Björken sum rule, which is a consequence of chiral symmetry, implicit in QCD, to the extent we may ignore quark masses:

$$\int_0^1 [g_1^p(x) - g_1^n(x)] = \frac{1}{6}(\Delta\, q_u^{v,1} - \Delta\, q_d^{v,1}) = \frac{1}{6}(G_A/G_V)_{p \to n} \tag{25}$$

Carlitz-Kaur parametrisation for the valence spin structure ensures that the sum rule is satisfied and thus any deviation of the experimental evaluation of the sum rule is attributable to the QCD non-leading correction, for which there is now an estimate. It is, however, more significant to test the sum rule of $g_1^p(x)$ and $g_1^n(x)$ separately. In our model, we expect

$$\int_0^1 g_1^p(x)\,\mathrm{d}x = \tfrac{1}{2}\,[4/9\,\Delta\, q_u^{v,1} + 1/9\,\Delta\, q_d^{v,1} + \Sigma\, e_i^2\,\Delta\, q_{i0}^1]$$

$$= 0{\cdot}21 + \begin{Bmatrix} 1{\cdot}26 & \text{3 flavours} \\ 3{\cdot}36 & \text{4 flavours} \end{Bmatrix}, \tag{26}$$

and $$\int_0^1 g_1^n(x)\,\mathrm{d}x = \tfrac{1}{2}\,[4/9\,\Delta\, q_d^{v,1} + 1/9\,\Delta\, q_u^{v,1} + \Sigma\, e_i^2\,\Delta\, q_{i0}^1]$$

$$= 0{\cdot}001 + \begin{Bmatrix} 1{\cdot}26 & 3 \text{ flavours} \\ 3{\cdot}36 & 4 \text{ flavours} \end{Bmatrix}. \tag{27}$$

The terms in the curly brackets arise from the large sea polarisation, coming mostly from the experimentally insensitive small x-region. While a direct experimental confirmation of the presence of large values of the integrals should be the main evidence for large sea polarisation effects, it should be recognized that this is concentrated in a kinematical region, hard to reach and the structure function extracted is expected to have large uncertainties in this region.

4. Transverse spin in nucleon

Transverse polarisation of the electron and proton in $\vec{e}\,\vec{p} \to e + X$ measures in addition to $g_1(x)$ the function $g_2(x)$ $(= \mathrm{Lt}_{Q^2\to\infty}\ M\nu^2\, G_2\,(x, Q^2))$. In parton model the combination $g_1(x) + g_2(x)$ is equal to $k_+(x) - k_-(x)$, where $k_\pm\,(x)$ is the probability (weighted by the (charge)^2 of the quark) of finding a quark of spin up (down), with a momentum function x in the infinite momentum frame in a proton or neutron spinning up (perpendicular to the infinite momentum). When the quark is moving rapidly ($x \gg 0$), then we expect it to be spinning along the direction of momentum, which implies that $k_+(x) \simeq k_-(x)$. Thus

$$g_1(x) + g_2(x) = 0,\ x \gg 0. \tag{28}$$

The longitudinal asymmetry, we have already seen, is governed by the function $g_1(x)$ and its magnitude and the ln Q^2 dependance are governed by the relevant twist-2 operator of the symmetry type (bilinear in quark) $O^{\lambda,\,\mu_1,\,\mu_2,\ldots,\,\mu_{n-1}}$ (traceless symmetric in all indices). It can be shown that the operator of the symmetry type $O^{[\lambda,\,\sigma],\,\mu_1,\,\mu_2,\ldots,\,\mu_{n-1}}$ (symmetric in μ's and antisymmetric in λ and σ) dictates the dynamics of the moments:

$$\int_0^1 \mathrm{d}x\, x^{n-1} \left\{\frac{n-1}{n}\, g_1\,(x, Q^2) + g_2\,(x, Q^2)\right\} = M_n\,(Q^2) \tag{29}$$

In contrast to the similar moments of $g_1(x, Q^2)$ which were discussed in §2, the numerical magnitudes of the moments in (29) have been argued to be very small, that we may assume $M_n(Q^2){=}0$. (See Wandzura and Wilczek 1977 for the relevant arguments). For large n this is simply the reaffirmation of (28). Further, notice that

$$g_1(x) + g_2(x) = \int_x^1 \frac{\mathrm{d}x'}{x'}\, g_1(x'), \tag{30}$$

solves (29) and thus in terms of the quarks spin distributions

$$\Delta q_i^T (x, Q^2) = \int_x^1 \frac{dx'}{x'} \Delta q_i^L (x) (Q^2). \tag{31}$$

The transverse asymmetries are thus given by

$$A_{NN} (x) = \frac{\int_x^1 dx'/x' \, \Sigma \, e_i^2 \, \Delta q_i^L (x')}{\int \sum_i e_i^2 \, q_i (x)}. \tag{32}$$

It may be observed that $A_{NN}(x) \to 0$ as $x \to 1$, consistent with the expectation that the transverse polarisation of the hard quarks inside a nucleon should be vanishing. The transverse asymmetries are much smaller than the longitudinal asymmetries and should show sizable enhancements due to large sea polarisation effects at small values of x. Figures 3 and 4 indicate the expected values for $\vec{e}\,\vec{p} \to eX$ and $\vec{e}\,\vec{N} \to eX$ respectively. (For earlier analysis see Hidaka *et al* 1979). These measurements are expected to be made in (E-138) SLAC experiment during 1983.

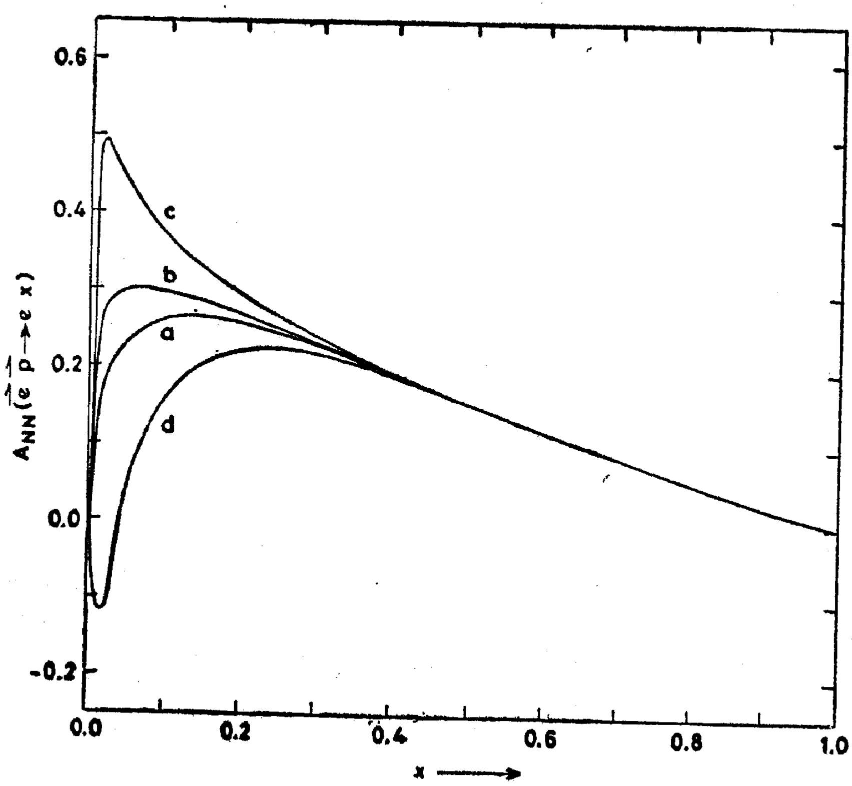

Figure 3. Transverse asymmetry in proton $A_{NN}\,(\vec{e} + \vec{p} \to e + X)$ vs x.

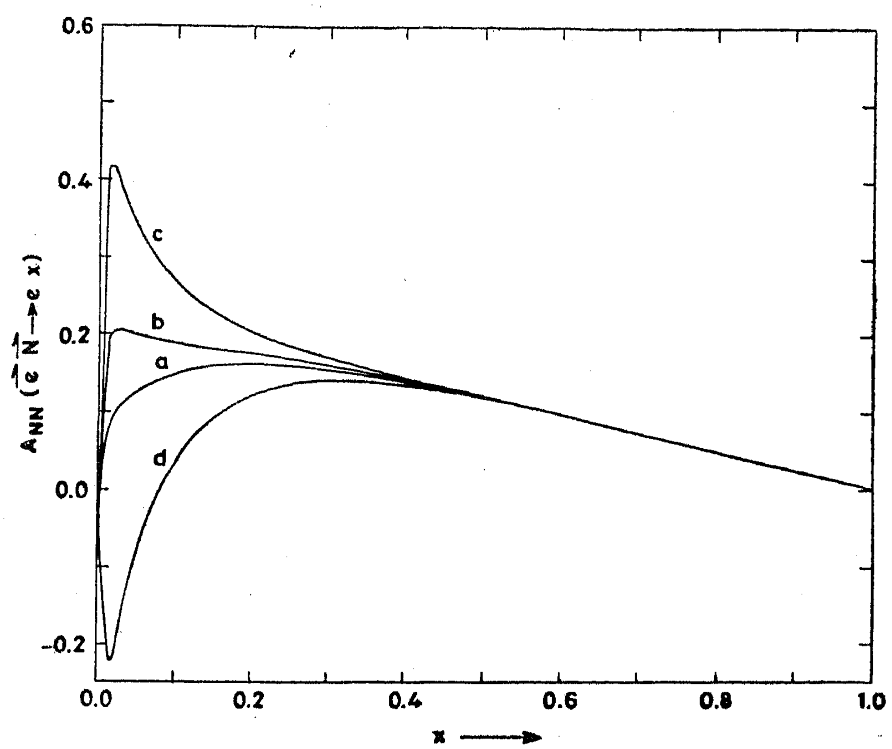

Figure 4. $A_{NN}(\vec{e} + \vec{N} \to e + X)$ vs x.

5. Neutrino induced asymmetries

In the charged current $\nu(\bar{\nu})N$ interactions, the unpolarised inelastic scattering is given by the functions $F_2(x, Q^2)$ and $F_3(x, Q^2)$. The longitudinal spin asymmetry (with target nucleon spin aligned parallel or antiparallel to the $\nu(\bar{\nu})$ spin) is once again given proportional to $g_1(x, Q^2)$. It is easily observed that the asymmetries A_{LL} are given by (with $y = (E' - E)/E$, where E and E' are the energies of $\nu(\bar{\nu})$ and charged lepton respectively):

$$A^{\nu}_{LL}(x, y) = \frac{-\Delta q_d \cos^2\theta - \Delta q_s \sin^2\theta + (1-y)^2 \Delta\bar{q}_u}{q_d \cos^2\theta + q_s \sin^2\theta + (1-y)^2 \bar{q}_u}. \tag{33}$$

and

$$A^{\bar{\nu}}_{LL}(x, y) = \frac{-\Delta q_u (1-y)^2 + \Delta\bar{q}_d \cos^2\theta + \Delta\bar{q}_s \sin^2\theta}{q_u (1-y)^2 + \bar{q}_d \cos^2\theta + \bar{q}_s \sin^2\theta}, \tag{34}$$

where $\cos\theta$ is the Cabibbo angle. $\nu(\bar{\nu})$ acts as a selective probe that measures separately d and $\bar{u}$ (u and $\bar{d}$) content of the hadrons, differentiated by the $(1-y)^2$ dependence of the antiquark (quark) portion. Integrating over the y (range y=0 to 1; the weak y dependance implicit in the ln Q^2 dependence of $q(x, Q^2)$ and $\Delta q(x, Q^2)$ may be ignored), we have for the proton and isoscalar nucleon targets.

$$A^{\nu p}_{LL} = \frac{-\Delta q^v_d \cos^2\theta - 2/3\,\Delta q_0}{q^v_d \cos^2\theta + 4/3\, q_0}, \tag{35a}$$

$$A_{LL}^{\nu N} = \frac{-1/2\,(\Delta q_d^v + \Delta q_u^v) - 2/3\,\Delta q_0}{1/2\,(q_u^v + q_d^v) + 4/3\,q_0} \tag{35b}$$

$$A_{LL}^{\bar{\nu} p} = \frac{-1/3\,\Delta q_u^v + 2/3\,\Delta q_0}{1/3\,q_u^v + 4/3\,q_0} \tag{36a}$$

$$A_{LL}^{\bar{\nu} N} = \frac{-1/6\,(\Delta q_u^v + \Delta q_d^v) + 2/3\,\Delta q_0}{1/6\,(q_u^v + q_d^v) + 4/3\,q_0} \tag{36b}$$

We have assumed here that the sea distribution is SU(3) flavour-independant and contains no charm or heavy flavours. ($q_{0u}=\bar{q}_{0u}=q_{0d}=\bar{q}_{0d}=q_{0s}=\bar{q}_{0s}$). The asymmetries are mostly dominated by the valence quark spin structures and are as shown in figures 5 to 8. The effect of the spin structure in the sea is confined to a small x region and depends on the number of flavours excited. Also shown in the diagram are the curves that correspond to the absence of any polarization structure in the sea.

We may observe that charm production occurs through the Cabibbo favoured processes $\nu + s \to \mu^- + c$ and $\bar{\nu} + \bar{s} \to \mu^+ + \bar{c}$. Since $(s, \bar{s})$ strange quarks are part of the flavour singlet sea, neutrino induced charm production and the longitudinal asymmetry therein could serve to measure the sea content and the polarisation thereof. They will have flat y distributions (because there is no admixture of $\nu\bar{q}$ or $\bar{\nu}q$

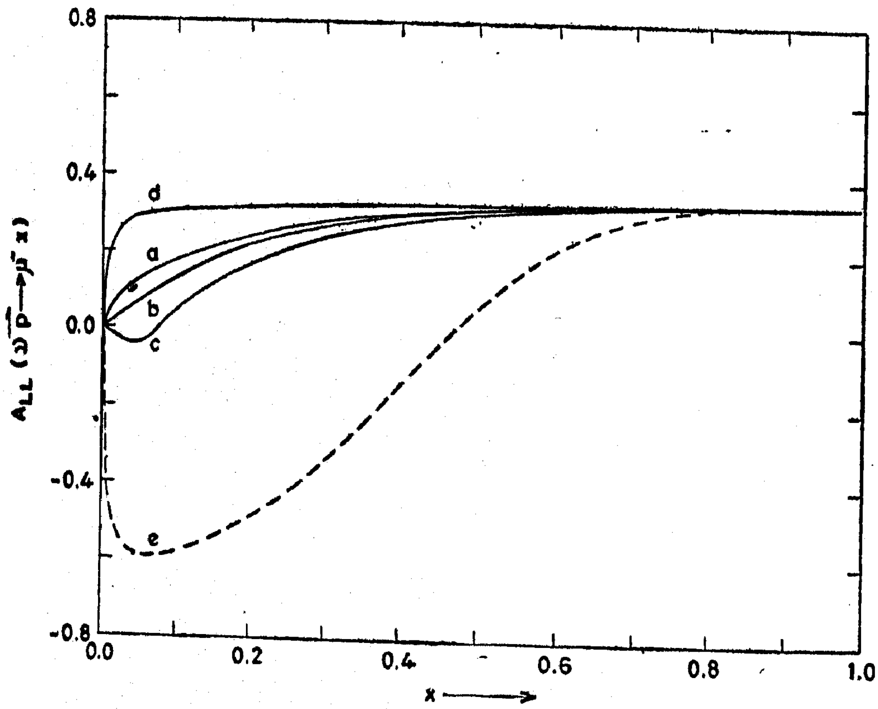

Figure 5. ν-induced longitudinal asymmetry. $A_{LL}\,(\nu + \vec{p} \to \mu^- + X)$ vs x. **a-d.** refer to the same parametrisation as in figure 1. **e.** (..) is the prediction when charmed particle is produced in the final state ($A_{LL}\,(\nu + \vec{p} \to \mu^- + C + X)$ vs x where C is any charmed hadron.

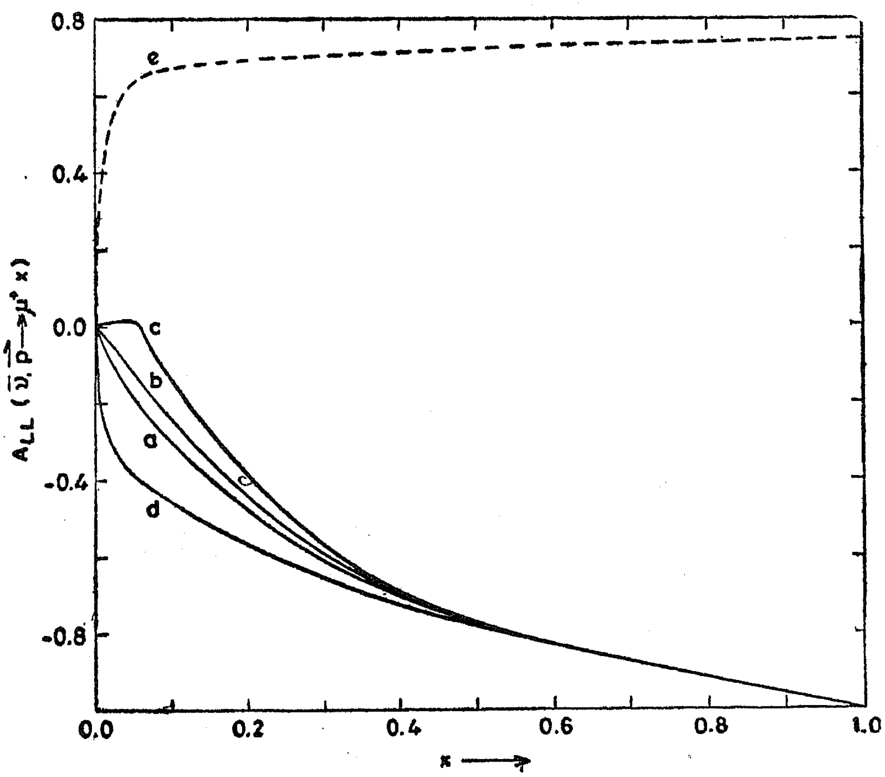

Figure 6. $A_{LL}(\bar{\nu} + \vec{p} \to \mu^- + X)$ vs x together with the curve for related charm production.

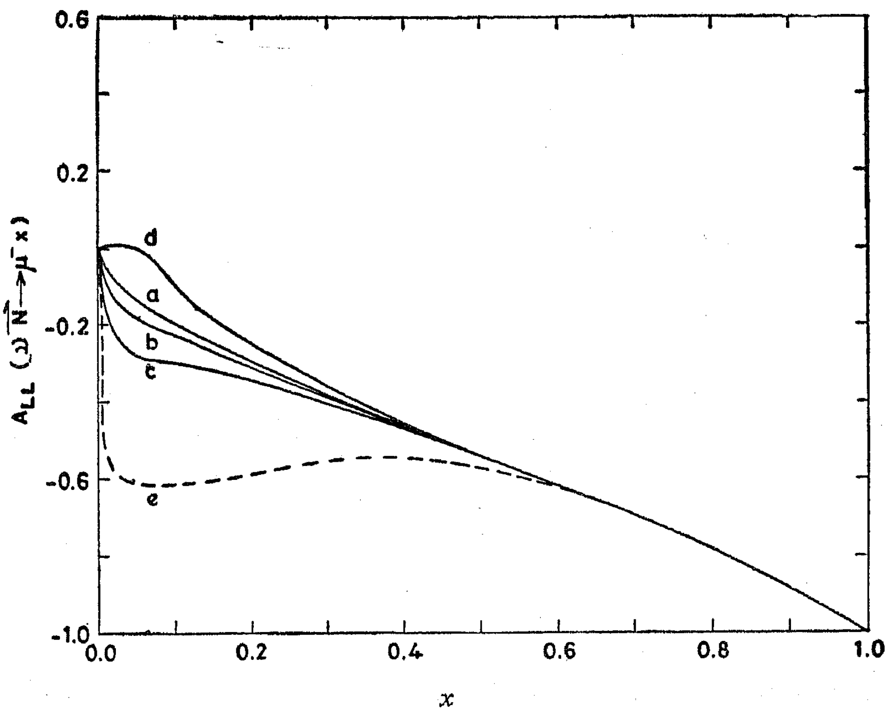

Figure 7. $A_{LL}(\nu + \vec{N} \to \mu^- + X)$ vs x together with the curve for related charm production.

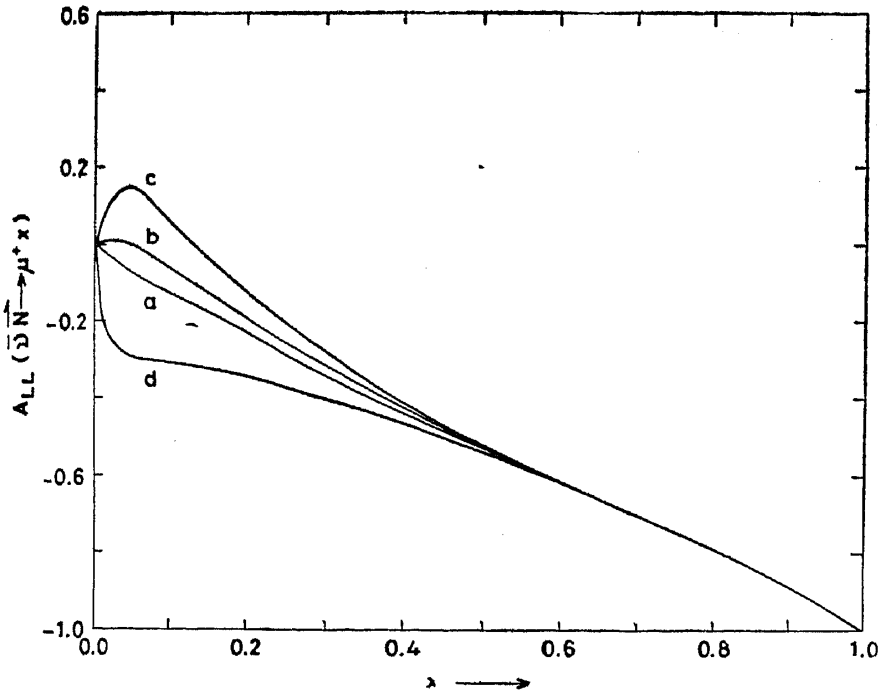

Figure 8. $A_{LL}{}'(\bar{\nu} + \vec{N} \to \mu^+ + X)$ vs x together with the curve for related charm production.

interactions, when we have a charmed quark as a final state) and the predicted asymmetries are:

$$A_{LL}^{\nu p \to \mu^- c X} = \frac{-\Delta q_d^v \sin^2\theta - \Delta q_0 \cos^2\theta}{q_d^v \sin^2\theta + q_0 \cos^2\theta} \tag{37}$$

$$A_{LL}^{\bar{\nu} p \to \mu^+ \bar{c} X} = A_{LL}^{\bar{\nu} N \to \mu^+ \bar{c} X} = \frac{\Delta q_0}{q_0} \tag{38}$$

$$A_{LL}^{\nu N \to \mu^- c X} = \frac{-1/2(\Delta q_d^v + \Delta q_u^v)\sin^2\theta - \Delta q_0 \cos^2\theta}{1/2(q_d^v + q_u^v)\sin^2\theta + q_0 \cos^2\theta} \tag{39}$$

These imply larger asymmetries, characteristic of large sea polarisations and could provide a clean test for the structures. For easy comparison, they are plotted in the same graphs that show the asymmetries of non-charm inclusive νp and νN charged current processes.

5.1 *y distributions of asymmetry*

We may make a more definitive prediction for the asymmetries by integrating over the x variable and plotting the various asymmetries as a function of the y-variable. In view of the fact that the first moment of the spin structures are given by the constraints on the flavour singlet piece and further are more or less model-independent for the valence spin, our predictions for y-distributions (which involve the second moments only) should be free from any serious model-dependent ambiguity. We find

$$A_{LL}^{\nu p}(y) = \frac{-\Delta q_d^{v,2} \cos^2\theta + \Delta q_0^2 [-1 + (1-y)^2]}{q_d^{v,2} \cos^2\theta + q_0^2 [1 + (1-y)^2]} \tag{40}$$

$$A_{LL}^{\bar{\nu} p}(y) = \frac{-(1-y)^2 \Delta q_2^{v,2} + \Delta q_0^2 [1 - (1-y)^2]}{(1-y)^2 q_u^{v,2} + q_0^2 [1 + (1-y)^2]}. \tag{41}$$

Similar expressions for $A_{LL}^{\nu N}$ and $A_{LL}^{\bar{\nu} N}$ can be written by substituting for $\Delta q_u^{v,2}$, $\Delta q_d^{v,2}$; $q_u^{v,2}$ and $q_d^{v,2}$ the average values $1/2(\Delta q_u^{v,2} + \Delta q_d^{v,2})$ and $1/2(q_u^{v,2} + q_d^{v,2})$ respectively. For the parametrisation we have presented, the various second moments are as follows: $q_u^{v,2} = 0{\cdot}233$, $q_d^{v,2} = 0{\cdot}095$, $q_0^2 = 0{\cdot}03$, $\Delta q_u^{v,2} = 0{\cdot}158$, $\Delta q_d^{v,2} = -0{\cdot}029$ and $\Delta q_0^2 = 0{\cdot}007$ (3 flavours), 0·020 (4 flavours) and $-$ 0·019 (5 flavours). The graphs showing the y-dependances of A_{LL} form figures 9 and 10.

For the corresponding charmed particle production by ν and $\bar{\nu}$ beam, there will be flat y-distribution with a value $\mp 0{\cdot}67$ (being $\mp \Delta q_0^2/q_0^2$) if we may ignore the small Cabibbo suppressed contributions.

The neutrino asymmetries, requiring polarised targets are not easily determined with the presently available experimental facilities.

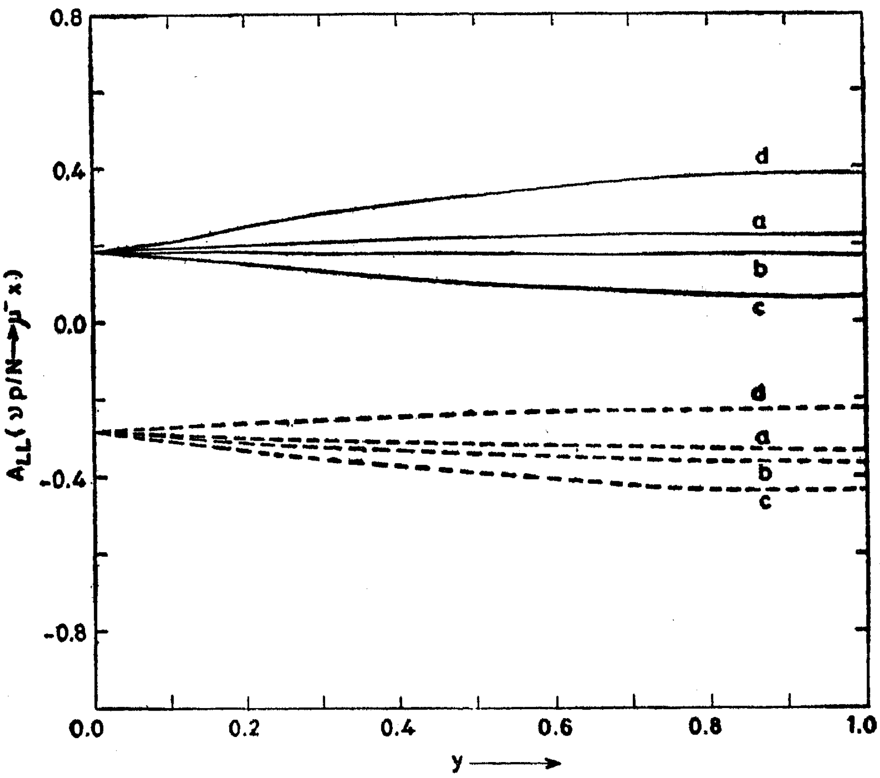

Figure 9. The y-distributions of longitudinal ν-asymmetries: $A_{LL}(\nu + \vec{p} \to \mu^- + X)$ are given by solid lines and $A_{LL}(\nu + \vec{N} \to \mu^- + X)$ by dashed lines. $y = E'/E$, where E and E' are energies of ν and μ respectively.

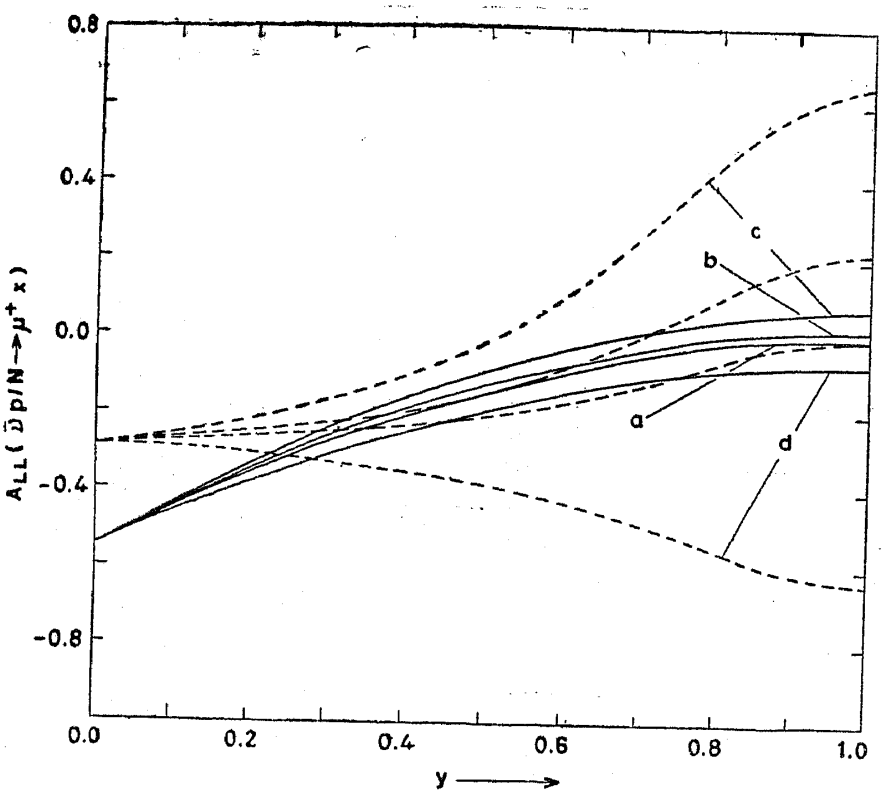

Figure 10. The y distributions for $\bar{\nu}$-asymmetries. $A_{LL}(\bar{\nu} + \vec{p} \to \mu^{+} + X)$ is given by solid line and $A_{LL}(\bar{\nu} + \vec{N} \to \mu^{+} + X)$ by dashed lines.

6. Conclusions

We have compiled the predictions for the polarisation asymmetries in deep inelastic lepton-nucleon scattering. The asymmetries measure the proton and neutron spin structures. While, it is customary to regard that most of the spin resides in the valence quark, the possibility that the sea or the flavour singlet quarks may have a substantial non-trivial spin structure, as suggested by the leading order QCD effects is also considered. However, when these spin ingredients are translated into consequent experimentally measurable asymmetries, it is discovered that they are not easy to confirm, in view of the small momentum carried by the sea quarks. Their effects are often masked by the valence quark induced asymmetries. We have considered in detail the ν and $\bar{\nu}$ induced asymmetries, since in νN interaction, it is possible to separate the contribution due to νq and $\bar{\nu}\bar{q}$ interaction by looking at the y-distribution. Again the effects are confined to low x-region and kinematic suppression is difficult to avoid except in the ν-induced charmed particle production.

Alternate methods for probing sea polarisations have been given elsewhere. Asymmetry in massive lepton pair production (Drell-Yan processes) in pp collision probes directly the product of the valence quark and the sea quark structure.* (Baldrachchini

*The longitudinal asymmetry for large angle $\mu^{+}\mu^{-}$ pair with $x_F = p_{||}/\sqrt{s}$ and $\tau = m_{\mu\mu}/\sqrt{s}$ (where $p_{||}$ is the longitudinal momentum of $\mu^{+}\mu^{-}$ pair and $m_{\mu\mu}$ is the invariant mass of $\mu^{+}\mu^{-}$ pair) is given by

$$A_{LL}(\tau, x_F) = \frac{\Delta V(x_a, Q^2)\,\Delta q_0(x_b, Q^2) + (x_a \longleftrightarrow x_b)}{V(x_a, Q^2)\,q_0(x_b, Q^2) + (x_a \longleftrightarrow x_b)}$$

where $V(x_a, Q^2) = 4/9\, q_u^v + 1/9\, q_d^v$, $\tau^2 = x_a x_b$ and $x_F = x_a - x_b$

et al 1980). At large momentum transfers, the asymmetries in similar process measure, in addition, gluon spin structure (Bajpai *et al* 1981) and they appear to be quite sensitive to the large polarisation structures in the flavour singlet sector of the nucleon. It is also possible to study the initial state spin structure by looking for the transmitted asymmetries in processes such as $p + p \to \Lambda + X$. However, since the relevant hard processes involve u and d quarks (in addition to the s-quarks) and further since their fragmentation into Λ_s is not very much less when compared with the fragmentation of s-quark (Ramachandran and Bajpai 1982) the resultant asymmetries are found dominated by the valence spin structure and no reliable information regarding flavour singlet spin structure emerges.

Acknowledgement

The author acknowledges useful discussions with R P Bajpai, M Rafat and H S Mani. He also thanks K S Sateesh for computations.

References

Alguard M J *et al* 1976 *Phys. Rev. Lett.* **37** 1258, 1261
Alguard M J *et al* 1978 *Phys. Rev. Lett.* **41** 70
Altarelli G and Parisi G 1977 *Nucl. Phys.* **B126** 298
Bajpai R P and Ramachandran R 1980 *Phys. Lett.* **B97** 125
Bajpai R P, Noman M and Ramachandran R 1981 *Phys. Rev.* **D24** 1832
Baldrachchini F, Craigie N S, Roberto V and Socolowsky M 1980 *A survey of polarisation asymmetries predicted by QCD.* ICTP preprint IC/80/148
Baum G *et al* 1980 *Proc. 20th Int. Conf. on High Energy Physics at Madison, Wisconsin* July 1980
Björken J D 1970 *Phys. Rev.* **D1** 1376
Björken J D 1982 *Fermilab-conf*-82/65 Thy (to be published in the Proc. 5th Int. Symposium on High Energy Spin Physics at BNL September 1982)
Buras A 1980 *Rev. Mod. Phys.* **52** 199
Carlitz R and Kaur J 1977 *Phys. Rev. Lett.* **38** 673, 1102
Close F 1974 *Nucl. Phys.* **B80** 269
de Groot J G H *et al* 1979 *Phys. Lett* **B82** 456
Falciano S 1981 *Phys. Lett* **B104** 416
Hidaka K, Monsay E and Sivers D 1979 *Phys. Rev* **D19** 1503
Jaffe R 1975 *Phys. Rev.* **D11** 1953
Kaur J 1977 *Nucl. Phys.* **B128** 219
Kuti J and Weisskopf V F 1971 *Phys. Rev.* **D4** 3418
Look G and Fishbach E 1977 *Phys. Rev.* **D16** 211
Oppenheim R 1982 *Proc. 5th International Symposium on High Energy Spin Physics at BNL* September 1982
Politzer H D 1974 *Phys. Rep.* **C14** 1290
Preparata G 1981 *Proc. 4th International Symposium on High Energy Spin Physics at Lausanne* September 1982
Ramachandran R and Bajpai R P 1982 *Phys. Lett.* **B115** 313
Schwinger J 1977 *Nucl. Phys.* **B123** 223
Sehgal L M 1974 *Phys. Rev.* **D10** 1663
Wandzura S and Wilczek F 1977 *Phys. Lett.* **B72** 195