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Probing proton spin structure through hadronic reactions

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Abstract. Inclusive and semi-inclusive photon producing polarized proton reactions have been employed to probe the spin structure of the proton. Combinations of cross-sections are suggested which may measure valence quarks polarization and gluon polarization in the proton separately. The general formalism is used to predict numerical results using a model of spin structure based on Altarelli-Parisi equations.

Keywords. Spin structure; valence quark polarization; gluon polarization; polarization models; spin correlation; Altarelli-Parisi equations.

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1. Introduction

Perturbative quantum chromodynamics has by now been established as a strong candidate theory for the physics of strong interactions (see *e.g.* Ellis 1979) in particular, the high P_T data are fairly satisfactorily explained by this theory (Gluck *et al* 1978; Field 1978; Feynman *et al* 1978). The next logical thing is therefore to calculate spin effects in the high P_T reactions using perturbative QCD. Several attempts have been made in literature to apply QCD in this direction (Babcock *et al* 1979). The general formalism of application of perturbative QCD requires the knowledge of the structure functions of the various constituents of a hadron and the perturbative calculation can proceed only when this 'non perturbative' piece of information is available apart from the knowledge of fragmentation functions (Buras 1980; Dokshitser *et al* 1980). Various models have been proposed for the spin-dependent structure functions of the various constituents of a proton (to take a particular hadron) (Babcock *et al* 1979). These models can be employed along with the usual perturbative QCD procedure to calculate spin asymmetries in polarized proton reactions. The results are obviously model-dependent. Moreover, in general, the various polarizations of the different constituents of the proton (*viz.* valence quarks, sea quarks and gluons) all contribute to the spin asymmetry being calculated and it is not possible to separate out the effects of the various constituent polarizations.

It is desirable to have polarized proton reactions where the observable spin asymmetry or spin correlation depends only on the polarization of one of the constituents of the proton. This would immediately serve two purposes; it would be possible to confront the various models for the polarization of a particular constituent of the proton, against experimental information unambiguously, without at the same time having to commit oneself to some model for the polarization of the other constituents of the proton as well. Secondly it may be possible to 'work back' from experimental information and parametrise the spin-dependent structure functions.

The purpose of this paper is to suggest combinations of cross-sections which isolate the effects of valence quarks' polarization from gluon polarization in a proton. We will show that for the reactions being considered, sea quarks' polarization plays a negligible role. We write down the required combinations of cross-sections, and employ a particular model for the proton spin structure to predict numerical results. We motivate this model as we go along. However, this needs to be emphasized that the isolation of valence polarization from gluon polarization would be a *general feature* of the combination of cross-sections being suggested by us; and would in no way depend on the spin structure model being used for numerical calculation. The model would be used for illustrative purposes as well as for the intrinsic interest in the model itself.

2. Polarized photon-producing proton reactions

Since gluons are one variety of the constituents of the proton, whose polarization we wish to isolate, it is particularly useful to begin by considering the photon-producing reactions. This can be seen by noting that the basic processes which produce a photon are quark gluon compton scatterings and quark antiquark annihilation diagrams

$$qg \to q\gamma,$$

and $$q\bar{q} \to g\gamma. \tag{1}$$

Since antiquarks are found only in the 'sea' of the proton, the compton scattering is expected to dominate in the inclusive reaction

$$pp \to \gamma X. \tag{2}$$

This reaction may therefore be expected to give information on the gluon polarization in the proton. For this purpose, we consider the polarized proton scattering

$$pp\uparrow \to \gamma\uparrow\downarrow X, \tag{3}$$

where one of the protons is longitudinally polarized and the (circular) polarization of the emerging photon is being measured. As usual, we define the spin correlation function

$$A = \frac{E_\gamma \dfrac{d\sigma(pp\uparrow \to \gamma\uparrow X)}{dp_\gamma^3} - E_\gamma \dfrac{d\sigma(pp\uparrow \to \gamma\downarrow X)}{dp_\gamma^3}}{E_\gamma \dfrac{d\sigma(pp\uparrow \to \gamma\uparrow X)}{dp_\gamma^3} + E_\gamma \dfrac{d\sigma(pp\uparrow \to \gamma\downarrow X)}{dp_\gamma^3}}. \tag{4}$$

To write the expression for $E_\gamma (d\sigma/dp_\gamma^3)$, we employ the kinematics

$$s = (p_A + p_B)^2, \quad t = (p_B - p_\gamma)^2, \quad u = (p_A - p_\gamma)^2, \tag{5}$$

where p_A, p_B, p_γ denote respectively the 4 momenta of the unpolarized proton, the polarized proton and the final photon. The basic processes like (1) *viz*

$$ab \to c\gamma, \tag{6}$$

are similarly described by

$$\hat{s} = (p_a + p_b)^2, \quad \hat{t} = (p_b - p_\gamma)^2, \quad \hat{u} = (p_a - p_\gamma)^2. \tag{7}$$

It then follows that

$$\hat{s} = x_a x_b s, \quad \hat{t} = x_b t \quad \text{and} \quad \hat{u} = x_a u, \tag{8}$$

where x_a, x_b denote the fractions of momentum carried by partons a and b respectively

$$x_a = p_a/p_A, \quad x_b = p_b/p_B. \tag{9}$$

The condition $\hat{s} + \hat{t} + \hat{u} = 0$ (10)

(*i.e.* quarks and gluons are massless) then implies

$$\frac{x_2}{x_a} + \frac{x_1}{x_b} = 1, \tag{11}$$

where

$$x_1 = -\frac{u}{s} \text{ and } x_2 = -\frac{t}{s}. \tag{12}$$

It is also convenient to express x_1, x_2 as

$$x_1 = \tau \cot\frac{\theta}{2}, \quad x_2 = \tau \tan\frac{\theta}{2}, \tag{13}$$

$$\tau = P_T/(s)^{1/2},$$

where P_T is the transverse momentum of the photon produced and θ is the angle it makes with the direction of collision, in the centre of mass frame.

The master equation which gives the cross-section for the inclusive production of a photon in pp scattering is

$$E_\gamma \frac{d\sigma}{dp_\gamma^3} = \frac{1}{\pi} \int_H dx \sum_{a,b,c} P_{a/A}(x_a, Q^2)\, P_{b/B}(x_b, Q^2)\, \frac{d\sigma^{ab \to c_\gamma}}{d\hat{t}}. \tag{14}$$

(see for comparison Gluck and Reya 1977)

where the integral is a line integral over the hyperbola (11) (denoted by H) in the $x_a - x_b$ plane. $P_{a/A}(x_a, Q^2)$ is the density of the parton a carrying a fraction x_a of the momentum of the proton A. $d\sigma^{ab \to c_\gamma}/d\hat{t}$ are the elementary cross-sections (table 1) calculated using the familiar Feynman rules (Politzer 1974; Marciano and Pagels 1978). As a technical point, we note that the element of integration dx in (14) is the differential of the arc length of the hyperbola (11) thus

$$\frac{dx}{dx_a} = \left[1 + (dx_b/dx_a)^2\right]^{1/2}. \tag{15}$$

The line integral (14) can therefore be written as an integral over x_a as

$$E_\gamma \frac{d\sigma}{dp_\gamma^3} = \frac{1}{\pi} \int_{x_{a\min}}^{1} dx_a \left[\sum_{a,b,c} P_{a/A}(x_a, Q^2) P_{b/B}(x_b, Q^2) \frac{d\sigma^{ab \to c_\gamma}}{d\hat{t}}\right] \frac{dx}{dx_a} \tag{16}$$

Table 1. Cross-sections of polarized parton scattering with emission of a single polarized real photon.

$ab\uparrow \to c\gamma\uparrow\downarrow$	$\Sigma^{ab\uparrow\to c\gamma\uparrow\downarrow}$
$qg\uparrow\to q\gamma\uparrow$ $\bar{q}g\uparrow\to\bar{q}\gamma\uparrow$	$\alpha_e\alpha_s\left[-\frac{1}{3}\left(\frac{\hat{s}}{\hat{u}}+\frac{\hat{u}}{\hat{s}}\right)\right]e_q^2$
$qg\uparrow\to q\gamma\downarrow$ $\bar{q}g\uparrow\to\bar{q}\gamma\downarrow$	0
$gq\uparrow\to q\gamma\uparrow$ $g\bar{q}\uparrow\to\bar{q}\gamma\uparrow$	$\alpha_e\alpha_s\left[-\frac{1}{3}\frac{\hat{s}}{\hat{t}}\right]e_q^2$
$gq\uparrow\to q\gamma\downarrow$ $g\bar{q}\uparrow\to\bar{q}\gamma\downarrow$	$\alpha_e\alpha_s\left[-\frac{1}{3}\frac{\hat{t}}{\hat{s}}\right]e_q^2$
$q\bar{q}\uparrow$, $\bar{q}q\uparrow$ $\to g\gamma\uparrow$	$\alpha_e\alpha_s\left[\frac{8}{9}\frac{\hat{u}}{\hat{t}}\right]e_q^2$
$q\bar{q}\uparrow$, $\bar{q}q\uparrow$ $\to g\gamma\downarrow$	$\alpha_e\alpha_s\left[\frac{8}{9}\frac{\hat{t}}{\hat{u}}\right]e_q^2$

With each entry $\Sigma^{ab\uparrow\to c\gamma\uparrow\downarrow}$, a factor of $\pi/\hat{s}^2$ is to be multiplied to get $\frac{d\sigma^{ab\uparrow\to c\gamma\uparrow\downarrow}}{d\hat{t}}$.

where

$$x_{a_{\min}} = \frac{x_2}{1-x_1} \quad \text{from (11).} \tag{17}$$

To calculate the spin correlation (4), we first write the cross-section for the production of a photon of definite helicity (± 1) in the scattering of a polarized proton of helicity $+1$. Using (16) we write

$$E_\gamma \frac{d\sigma^{(pp\uparrow\to\gamma\uparrow\downarrow X)}}{dp_\gamma^3} = \frac{1}{\pi}\int_{x_{a_{\min}}}^{1} dX_a \sum_{a,b,c}\left[P_{a/A}(x_a,Q^2)P_{b\uparrow/B\uparrow}(x_b,Q^2)\frac{d\sigma^{ab\uparrow\to c\gamma\uparrow\downarrow}}{d\hat{t}} + P_{a/A}(x_a,Q^2)P_{b\downarrow/B\uparrow}(x_b,Q^2)\frac{d\sigma^{ab\downarrow\to c\gamma\uparrow\downarrow}}{d\hat{t}}\right]\frac{dx}{dx_a}, \tag{18}$$

where $P_{b\uparrow/B\uparrow}$ is the density of the partons of variety b aligned along the spin of the polarized proton B and $P_{b\downarrow/B\uparrow}$ denotes the density of those aligned opposite.

The spin correlation A (4) is therefore given by:

$$A=\frac{\int_{x_{a_{\min}}}^{1}\mathrm{d}x_a\sum_{a,b,c}\left[P_{a/A}(x_a,Q^2)\Delta P_{b/B}(x_b,Q^2)\Delta\frac{\mathrm{d}\sigma^{ab\to c\gamma}}{\mathrm{d}\hat{t}}\right]\frac{\mathrm{d}x}{\mathrm{d}x_a}}{\int_{x_{a_{\min}}}^{1}\mathrm{d}x_a\sum_{a,b,c}\left[P_{a/A}(x_a,Q^2)P_{b/B}(x_b,Q^2)\frac{\mathrm{d}\sigma^{ab\to c\gamma}}{\mathrm{d}\hat{t}}\right]\frac{\mathrm{d}x}{\mathrm{d}x_a}}. \tag{19}$$

where

$$\Delta P_{b/B}=P_{b\uparrow/B\uparrow}-P_{b\downarrow/B\uparrow}$$

and

$$\Delta\frac{\mathrm{d}\sigma^{ab\to c\gamma}}{\mathrm{d}\hat{t}}=\frac{\mathrm{d}\sigma^{ab\uparrow\to c\gamma\uparrow}}{\mathrm{d}\hat{t}}-\frac{\mathrm{d}\sigma^{ab\uparrow\to c\gamma\downarrow}}{\mathrm{d}\hat{t}}. \tag{20}$$

In arriving at (19), the parity invariance of cross-sections has been used

$$\frac{\mathrm{d}\sigma^{ab\uparrow\to c\gamma\uparrow}}{\mathrm{d}\hat{t}}=\frac{\mathrm{d}\sigma^{ab\downarrow\to c\gamma\downarrow}}{\mathrm{d}\hat{t}}$$

and

$$\frac{\mathrm{d}\sigma^{ab\uparrow\to c\gamma\downarrow}}{\mathrm{d}\hat{t}}=\frac{\mathrm{d}\sigma^{ab\downarrow\to c\gamma\uparrow}}{\mathrm{d}\hat{t}}. \tag{21}$$

Since the partons b in the polarized proton B can either be valence quarks, sea quarks and antiquarks or gluons, the correlation A would get contribution from each of $\Delta q_{\text{valence}}$, Δq_{sea}, $\Delta\bar{q}_{\text{sea}}$ and Δg. Obviously the photon-inclusive reactions are not able to disentangle the various polarizations *viz* Δq_v, Δq_{sea} and Δ_g, from one another. It is however still useful to consider them, since as we shall see, it is possible to locate kinematic regions where only one of these polarizations is dominant. Sections 3 and 4 are therefore devoted to the calculation of spin correlation A. We will first outline the model of proton spin structure that we will employ and then discuss the numerical results. When we consider semi-inclusive reactions, it will be possible to make a 'clean' separation between various constituent polarizations.

3. Altarelli-Parisi equations' induced model for the proton spin structure

This model has already been reported (Bajpai and Ramachandran 1980; Bajpai *et al* 1981) and some consequences of it explored. We therefore confine ourselves to an outline. We write the parton distribution functions as the sum of partons aligned along the spin of the proton and those aligned opposite

$$q_i(x)=q_i\uparrow(x)+q_i\downarrow(x),$$
$$g(x)=g\uparrow(x)+g\downarrow(x) \tag{22}$$

where i runs over quark flavours (antiquarks included) and x is the fractional momentum of a parton. Q^2 dependence is understood to be present for each parton distribution.

The distribution of net quark and gluon polarization is similarly written as

$$\Delta q_i(x)=q_i\uparrow(x)-q_i\downarrow(x),$$
$$\Delta g(x)=g\uparrow(x)-g\downarrow(x). \tag{23}$$

The net polarization must add up to the spin of the proton, thus

$$\int\left[\frac{1}{2}\sum_{i=1}^{2f}\Delta q_i(x)+\Delta g(x)\right]\mathrm{d}x=\frac{1}{2}-L_z, \tag{24}$$

where f is the number of flavours. The sum runs up to $2f$ as antiquarks are included; L_z is the orbital angular momentum.

The nth moments of the distributions Δq_i and Δg, defined as

$$\Delta q_i^n(Q^2)=\int_0^1 \mathrm{d}x\, x^{n-1}\,\Delta q_i(x,Q^2),$$

$$\Delta g^n(Q^2)=\int_0^1 \mathrm{d}x\, x^{n-1}\,\Delta g(x,Q^2), \tag{25}$$

have a simple Q^2 dependence (Altarelli and Parisi 1977) given by the Altarelli-Parisi equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta q_i^n(t)=\frac{\alpha_s(t)}{2\pi}[\tilde{A}_{qq}^n\Delta q_i^n(t)+\tilde{A}_{qG}^n\Delta g^n(t)],$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta g^n(t)=\frac{\alpha_s(t)}{2\pi}\left[\tilde{A}_{Gq}^n\sum_{i=1}^{2f}\Delta q_i^n(t)+\tilde{A}_{GG}^n\Delta g^n(t)\right], \tag{26}$$

where $t=\ln\frac{Q^2}{\Lambda^2}$; Λ is QCD scale parameter and α_s is the strong coupling. The coefficients $\tilde{A}_{qq}^n$, $\tilde{A}_{Gq}^n$, $\tilde{A}_{qG}^n$, $\tilde{A}_{GG}^n$ are calculable; (Altarelli and Parisi 1977) in particular for $n=1$ the values are

$$\tilde{A}_{qq}^1=0,\quad \tilde{A}_{qG}^1=0$$

$$\tilde{A}_{Gq}^1=2,\quad \tilde{A}_{GG}^1=\frac{33-2f}{6}\quad \text{(for the SU(3) group).} \tag{27}$$

It follows from (27) that

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta q_i^1(t)=0. \tag{28}$$

Differentiating (24) with respect to t we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}\sum_{i=1}^{2f}\Delta q_i^1(t)+\Delta g^1(t)\right]=-\frac{\mathrm{d}L_z}{\mathrm{d}t}. \tag{29}$$

Combining (29) with (28) we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta g^1(t)=-\frac{\mathrm{d}L_z}{\mathrm{d}t}. \tag{30}$$

The second of equations (26) therefore becomes

$$\tilde{A}_{Gq}^1\sum_{i=1}^{2f}\Delta q_i^1(t)+\tilde{A}_{GG}^1\Delta g^1(t)=-\frac{2\pi}{\alpha_s}\frac{\mathrm{d}L_z}{\mathrm{d}t}. \tag{31}$$

Solving (24) and (31) for $\sum_{i=1}^{2f}\Delta q_i^1(t)$ and $\Delta g^1(t)$, we get

$$\sum_{i=1}^{2f} \Delta q_i^1 = \frac{33-2f}{9-2f}\left[1-2L_z+\frac{24\pi}{\alpha_s}\frac{1}{33-2f}\frac{\mathrm{d}L_z}{\mathrm{d}t}\right],$$

$$\Delta g^1 = -\frac{12}{9-2f}\left[1-2L_z+\frac{\pi}{\alpha_s}\frac{\mathrm{d}L_z}{\mathrm{d}t}\right]. \tag{32}$$

It is easy to see that $\mathrm{d}L_z/\mathrm{d}t$ is a constant (to this order of Altarelli-Parisi equations), for on solving (32) for $\mathrm{d}L_z/\mathrm{d}t$, we have

$$\frac{\mathrm{d}L_z}{\mathrm{d}t} = \frac{-\alpha_s(t)}{\pi}\left[\sum_{i=1}^{2f}\Delta q_i^1 + \frac{33-2f}{12}\Delta g^1\right], \tag{33}$$

which can be solved to give

$$L_z = L_z(0)\frac{t}{t_0} - \frac{1}{2}\left[1+\frac{2f-9}{33-2f}\sum_{i=1}^{2f}\Delta q_i^1\right]\left(\frac{t}{t_0}-1\right). \tag{34}$$

(using the Q^2 dependence of α_s *i.e.* $\alpha_s = 12\pi/(33-2f)t$) where $L_z(0)$ is the value of L_z at $t = t_0$.

Most conventional models take L_z to be small (Babcock *et al* 1979). It would then be appropriate to assume that $\Sigma\Delta q_i^1$ and Δg^1, to a good approximation, are given by

$$\sum_{i=1}^{2f}\Delta q_i^1 = \frac{33-2f}{9-2f}, \tag{35a}$$

$$\Delta g^1 = -\frac{12}{9-2f}. \tag{35b}$$

We would use (35) as our basic constraint for determining spin-dependent distributions.

Ramachandran and Bajpai (1981) use the above constraint and after breaking up the quark distribution into valence and sea parts

$$\Delta q_i = \Delta q_i^V + \Delta q_i^s, \tag{36}$$

they use the Bjorken sum rule (Bjorken 1966) and SU (3) symmetry for the valence part to get for the first moments

$$\Delta q_u^{1V} = 0{\cdot}97 \text{ and } \Delta q_d^{1V} = -0{\cdot}28, \tag{37}$$

for u and d quark polarization in the proton respectively.

We parametrise Δq_u^V and Δq_d^V by

$$\Delta q_u^V(x) = \alpha_u q_u^V(x),$$

and
$$\Delta q_d^V(x) = \alpha_d q_d^V(x), \tag{38}$$

and use (38) to fix α_u and α_d.

Using (35) we can now write for the sea quark polarization

$$\sum_{\substack{\text{sum over sea quarks}\\ \text{and antiquarks}}} \Delta q_i^1 = \frac{33-2f}{9-2f} - 0{\cdot}69, \tag{39}$$

since $\Delta q_u^{1V} + \Delta q_d^{1V} = 0{\cdot}69$ from (37).

We parametrise the sea polarization by

$$\Delta q_0 = C q_0 x^{\beta}, \tag{40}$$

where q_0 is the (total) density of sea quarks (and antiquarks) in the proton. We choose the sea to be SU(3) symmetric and suppress the higher flavours by the ansatz

$$\sum_{i=1}^{2f} q_{0i}(x) = \left(6 + \sum_{j=7}^{2f} e^{-m_j/\Lambda}\right) q_0(x), \tag{41}$$

where m_j is the mass parameter of the jth quark. (q_{0i} denotes density of the quark of ith flavour.) C and β in (40) are fixed by (39) and by the additional condition

$$\Delta q_0^2 / q_0^2 = \varepsilon, \tag{42}$$

where ε is the net momentum asymmetry associated with the polarized sea quarks. ε may be taken from $\frac{1}{4}$ to $\frac{1}{2}$.

Alternatively we can use the Carlitz-Kaur model for valence quarks (Carlitz and Kaur 1977; Kaur 1977). In this model, the valence quarks lose their 'memory' of the spin direction of the proton at small x. The model gives

$$\Delta q_u^V = \cos[2\theta(x)]\,[u^V(x) - \tfrac{2}{3} d^V(x)],$$

$$\Delta q_d^V = -\frac{1}{3}\cos[2\theta(x)]\,[d^V(x)], \tag{43}$$

where

$$\cos[2\theta(x)] = \left[1 + \frac{H_0(1-x^2)}{x}\right]^{-1}, \tag{44}$$

and H_0 is fixed at 0·052 by the Bjorken sum rule.

For the gluon polarization, we similarly use the ansatz

$$\Delta g(x) = d g(x) x^{\gamma},$$

and $$\Delta g^2 / g^2 = \sigma, \tag{45}$$

where σ is the net momentum asymmetry associated with gluon polarization. The constraint (35) is used to determine d and γ.

Clearly these models for polarization of valence quarks, sea quarks and gluons can be used for different parametrizations for total densities of these constituents. We use three different parametrizations for our calculation.

3.1 *Pakvasa, Parashar, Tuan* (PPT) *parametrization*

This parametrization for unpolarized quark and gluon distributions is given by Pakvasa *et al* (1974) and is used by Peierls *et al* (1977) for their fits to the Drell-Yan processes. At some specific value of Q^2 (assumed to be 3 GeV^2), the valence quark distribution is given by

$$q_u^V(x) = 1{\cdot}79(1-x)^3(1+2{\cdot}3x)/\sqrt{x},$$

$$q_d^V(x) = 1{\cdot}07(1-x)^{3{\cdot}1}/\sqrt{x}, \tag{46}$$

and the sea quark distribution by

$$\sum_{i=1}^{2f} q_{0i}(x) = 0{\cdot}90\frac{(1-x)^7}{x}. \tag{47}$$

For Q^2 development, we use the form used by Gluck and Reya (1977).

3.2 *Gluck and Reya parametrization*

Gluck and Reya (1977) obtain their valence and sea distributions from fits to data at $Q_0^2 = 3\,\text{GeV}^2$ and conform to Q^2 dependences as dictated by QCD. The valence distributions are

$$\begin{aligned} q_u^V(x) &= [5{\cdot}707(1-x)^3 - 6{\cdot}219(1-x)^5 + 4{\cdot}570(1-x)^7 \\ &\quad - 2{\cdot}868(1-x)^9]/\sqrt{x}; \\ q_d^V(x) &= [2{\cdot}994(1-x)^4 - 0{\cdot}767(1-x)^5 + 1{\cdot}890(1-x)^7 \\ &\quad - 3{\cdot}026(1-x)^9]/\sqrt{x}, \end{aligned} \tag{48}$$

and sea and gluon distributions are

$$\sum_{i=1}^{2f} q_{0i}(x) = [0{\cdot}019(1-x)^{5\cdot5} + 0{\cdot}007(1-x)^9 + 0{\cdot}091(1-x)^{13}]/x$$

and $$g(x) = [0{\cdot}557(1-x)^{4\cdot5} + 0{\cdot}559(1-x)^9 + 2{\cdot}037(1-x)^{13}]/x. \tag{49}$$

The Q^2 dependences are given by

$$P_{a/p}(x, Q^2) = P_{a/p}(x, Q_0^2)\left[\frac{\ln(Q^2/\alpha_a)}{\ln(Q_0^2/\alpha_a)}\right]^{\beta_a - x\ln Q^2/\delta_a}, \tag{50}$$

with

$$\begin{aligned} &\alpha_u = 3\times10^{-6}\,\text{GeV}^2, \quad \beta_u = 0{\cdot}951, \quad \delta_u = 0{\cdot}012\,\text{GeV}^2, \\ &\alpha_d = 7\times10^{-7}\,\text{GeV}^2, \quad \beta_d = 0{\cdot}791, \quad \delta_d = 0{\cdot}0015\,\text{GeV}^2, \\ &\alpha_g = 4\times10^{-5}\,\text{GeV}^2, \quad \beta_g = 0{\cdot}288, \quad \delta_g = 0{\cdot}031\,\text{GeV}^2, \end{aligned}$$

and $$\underset{\substack{q,\bar q\\ \text{sea}}}{\alpha} = 1\times10^{-3}\,\text{GeV}^2, \quad \underset{\substack{q,\bar q\\ \text{sea}}}{\beta} = 0{\cdot}67, \quad \underset{\substack{q,\bar q\\ \text{sea}}}{\delta} = 0{\cdot}583\,\text{GeV}^2$$

3.3 *Barger-Phillips*(BP) *parametrization*

This parametrization assumes, rather unrealistically, that the quark and gluon distributions are Q^2 independent (Barger and Phillips 1974). However, we use it for purposes of comparison. The distributions are:

$$\begin{aligned} q_u^V(x) &= [0{\cdot}549(1-x^2)^3 + 0{\cdot}460(1-x^2)^5 + 0{\cdot}621(1-x^2)^7]/\sqrt{x}, \\ q_d^V(x) &= [0{\cdot}072(1-x^2)^3 + 0{\cdot}206(1-x^2)^5 + 0{\cdot}621(1-x^2)^7]/\sqrt{x}, \\ \sum_{i=1}^{2f} q_0^i &= 0{\cdot}145\frac{(1-x)^9}{x}, \end{aligned} \tag{51}$$

For both first and third set of parametrizations, the gluon distribution is taken to be

$$g(x) = 3(1-x)^5/x \tag{52}$$

thus gluons carry $\frac{1}{2}$ the momentum, as customarily assumed.

4. Results of spin correlation calculation

As is clear from §3, the gluon and sea quark polarizations are expected to change as the number of participating flavours increases. Whether or not a particular flavour threshold is crossed would depend on the Q^2 of the basic reactions. We take Q^2 to be

$$Q^2 = \frac{2\hat{s}\,\hat{t}\,\hat{u}}{\hat{s}^2+\hat{t}^2+\hat{u}^2}, \tag{53}$$

where the kinematic variables $\hat{s}$, $\hat{t}$, $\hat{u}$ are defined by (7) and (8).

Since each 'process' $pp \to \gamma X$ involves a convolution over subprocesses, the Q^2 would vary over a range, for a given process. For $\theta = 90°$, (in centre of mass frame) Q^2 varies between $\frac{4}{3}\tau^2 s$ and $(\tau^2/1+\tau^2-\tau)s$ for fixed s and τ. We would choose s and τ suitably to separate out regions where different number of flavours are expected to participate. Clearly the region of three flavours is characterized by $Q^2 < (2m_c)^2$ or $Q^2 < 9{\cdot}61\,\text{GeV}^2$. The region of five flavours is given by $Q^2 > (2m_b)^2$ or $Q^2 > 100\,\text{GeV}^2$.

We start by using the PPT parametrization for total parton densities. Figure 1 shows the contributions to A from valence gluon and gluon valence scatterings for 4 and 5

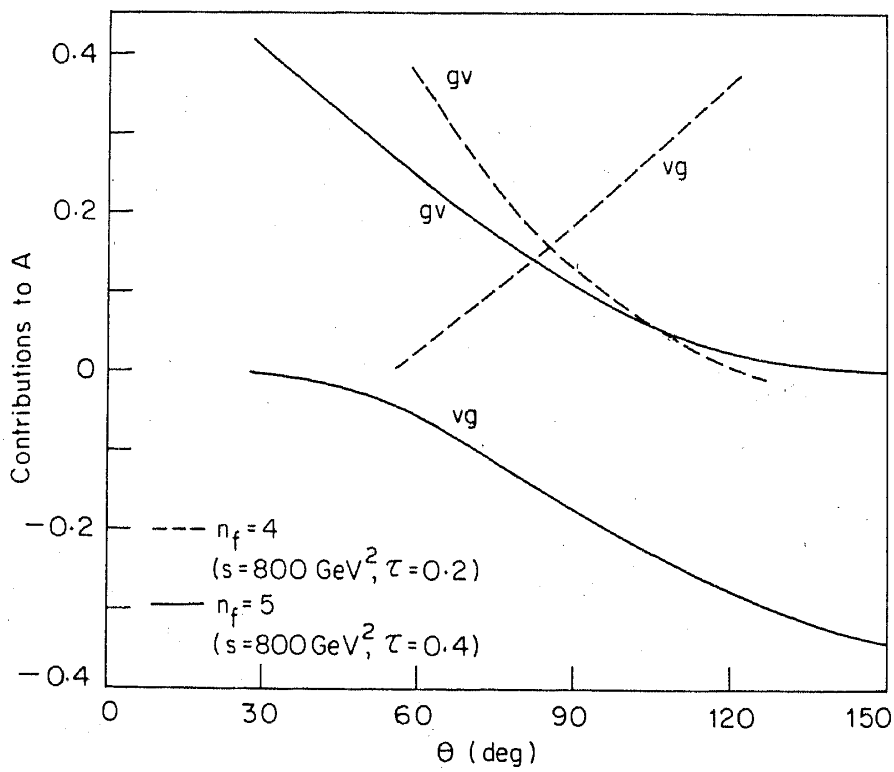

Figure 1. Variation of the contributions from Vg and gV scatterings to A, with θ, for 4 and 5 flavours.

flavours. (The terminology 'valence gluon scattering' means that the valence quark belongs to the unpolarized proton while the gluon belongs to the polarized proton participating in the reaction $pp\uparrow \rightarrow \gamma\uparrow\downarrow X$.) Other contributions to A are from sea-sea, sea gluon, sea valence, valence sea and gluon sea scatterings but our calculations show all of them to be negligible. Figure 2 shows that the contributions for $n_f = 3$ and 4 are virtually indistinguishable. We observe the following:

(i) The two contributions Vg and gV dominate in different regions. While the gluon valence scattering dominates at $\theta < 90°$, the valence gluon is dominant at $\theta > 90°$. At $\theta = 90°$, the two are comparable.

The *angular distribution* thus furnishes a method for distinguishing between the effects of valence polarization and gluon polarization. The two show up in different kinematic regions. The reason for this can be understood by noting that at smaller θ values, the integrals defining A (equation 19) get contribution from low x region (of the unpolarized proton) (Expression (17) for $x_{a_{\min}}$ immediately shows this). Since the gluon dominates in the region of small x, gV scattering is expected to dominate at small angles. Similarly, at large angles Vg scattering dominates.

(ii) The contribution from gluon valence scattering retains its sign as the beauty threshold is crossed. This is as expected since the valence quark polarization does not change sign as the number of flavours increases, in the model taken by us.

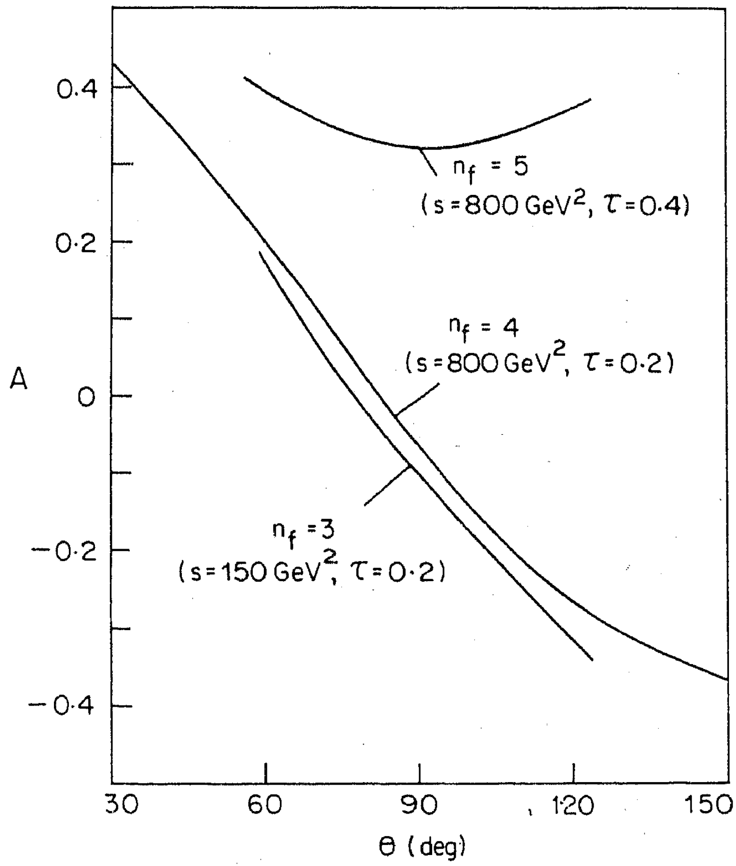

Figure 2. Variation of A with θ for 3, 4 and 5 flavours.

(iii) The contribution from the valence gluon scattering changes sign from negative to positive as the beauty threshold is crossed. This reflects the reversal of gluon polarization as the number of flavours increases from 4 to 5.

4.1 *Comparison with other parametrizations*

Besides the PPT parametrization for the unpolarised parton distributions, we have also calculated A with Barger-Phillips parametrization and Gluck-Reya parametrization. The results are identical with the PPT parametrization. This illustrates an important characteristic of the function A. The *function A depends only on the polarisation model* and is independent of the total unpolarised distribution. To check this, we show in figure 3, the behaviour of A with the Carlitz-Kaur model. Figure 3 clearly shows that the inclusive photon producing scattering can distinguish between various polarization models. We see that the predictions of the Carlitz-Kaur model (equation (43)) are strikingly different from that of the Bajpai and Ramachandran model (given by equation (38)). We have thus found that

(i) A depends only on the polarization models and is independent of total densities' parametrizations. (ii) Its magnitude is substantial and should be easily observable. (iii) A depends essentially on valence and gluon distributions only and sea quarks (and antiquarks) play a negligible role. (iv) The gluon valence and the valence gluon scatterings dominate in different regions; thus $\theta < 90°$ region *gives information on valence polarisation* while $\theta > 90°$ region gives information on gluon polarization.

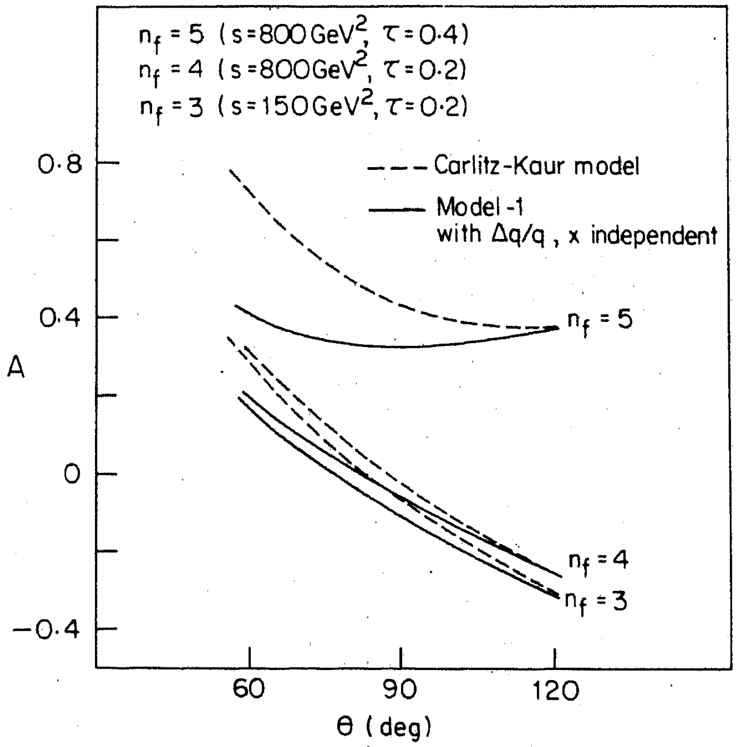

Figure 3. Behaviour of A for Barger-Phillips parametrization.

5. Clean separation of valence and gluon polarisation

We have seen in §4 that the valence and gluon polarizations can be investigated in different kinematical regions. This, however, is not completely satisfactory as, a clean separation is not achieved and angular measurements are necessary. In this section, we obtain a combination of cross-sections which achieves this clean separation. We consider the semi-inclusive process

$$\bar{p}p \rightarrow \gamma K X, \tag{54}$$

where $\bar{p}$ is an antiproton and K stands for K^+, K^-, K^0 or $\bar{K}^0$ meson.

The cross-section for this process is obtained from (14) by multiplying by a factor whose origin is explained below:

$$E_\gamma \frac{\mathrm{d}\sigma^{\bar{p}p\rightarrow\gamma KX}}{\mathrm{d}p_\gamma^3} = \left[\frac{1}{\pi}\int_H \mathrm{d}x \sum_{a,b,c} P_{a/\bar{p}}(x_a, Q^2) P_{b/p} \frac{\mathrm{d}\sigma^{ab\rightarrow c\gamma}}{\mathrm{d}\hat{t}}\right] \times \left[\int_{z_{\min}}^{1} \mathrm{d}z\, D_c^K(z)\right], \tag{55}$$

where the integral is, as before a line integral over the hyperbola H defined by (11).

The extra factor is the probability that the parton c in the subprocess

$$ab \rightarrow c\gamma, \tag{56}$$

fragments into the kaon K. The integration over z in (55) signifies that the kaon energy is not being measured.

We assume that the quark fragmentation functions are related by parity invariance and isospin invariance *i.e.*

$$D_u^{K^+} = D_{\bar{u}}^{K^-} = D_d^{K^0} = D_{\bar{d}}^{\bar{K}^0}, \tag{57a}$$

$$D_{\bar{u}}^{K^+} = D_u^{K^-} = D_{\bar{d}}^{K^0} = D_d^{\bar{K}^0}, \tag{57b}$$

$$D_{\bar{s}}^{K^+} = D_s^{K^-} = D_{\bar{s}}^{K^0} = D_s^{\bar{K}^0}, \tag{57c}$$

$$D_s^{K^+} = D_{\bar{s}}^{K^-} = D_s^{K^0} = D_{\bar{s}}^{\bar{K}^0}, \tag{57d}$$

$$D_d^{K^+} = D_{\bar{d}}^{K^-} = D_u^{K^0} = D_{\bar{u}}^{\bar{K}^0}, \tag{57e}$$

$$D_{\bar{d}}^{K^+} = D_d^{K^-} = D_{\bar{u}}^{K^0} = D_u^{\bar{K}^0}, \tag{57f}$$

We further assume that the 'sea' fragmentation functions (*i.e.* the fragmentation of a particular quark into a kaon when that quark is not one of the valence quarks of the kaon) are all equal. By noting that the valence quarks in the four kaons are:

$$K^+: (u, \bar{s}),\ K^0: (d, \bar{s}),\ K^-: (\bar{u}, s),\ \bar{K}^0: (\bar{d}, s), \tag{58}$$

this requirement means that the fragmentation functions in (57b) and (57d-f) are all equal. We are thus left with three independent fragmentation functions; we would respectively denote by D_1, D_2, D_3 the fragmentation functions in (57c), (57a) and (57b).

Photon-producing quark gluon compton scatterings and quark antiquark annihilation both contribute to the cross-section of the process (54). Using the fragmentation functions D_1, D_2, D_3, we can write down the cross-sections for the production of different kaons. We use the following differences of cross-sections:

$$E_\gamma \frac{d\sigma^{\bar{p}p\to\gamma K^+X}}{dp_\gamma^3} - E_\gamma \frac{d\sigma^{\bar{p}p\to\gamma K^0X}}{dp_\gamma^3}$$

$$= \frac{1}{\pi}\int_H dx \left[\left(\frac{4}{9}p_{u/\bar{p}} - \frac{1}{9}P_{d/\bar{p}}\right)p_{g/p}\frac{d\sigma^{qg\to q\gamma}}{d\hat{t}} + p_{g/\bar{p}}\left(\frac{4}{9}P_{u/p} - \frac{1}{9}P_{d/p}\right)\frac{d\sigma^{gq\to q\gamma}}{d\hat{t}}\right](F_2 - F_3) \tag{59a}$$

and

$$E_\gamma \frac{d\sigma^{\bar{p}p\to\gamma K^-X}}{dp_\gamma^3} - E_\gamma \frac{d\sigma^{\bar{p}p\to\gamma \bar{K}^0X}}{dp_\gamma^3}$$

$$= \frac{1}{\pi}\int_H dx \left[\left(\frac{4}{9}P_{\bar{u}/\bar{p}} - \frac{1}{9}P_{\bar{d}/\bar{p}}\right)P_{g/p}\frac{d\sigma^{qg\to q\gamma}}{d\hat{t}} + P_{g/\bar{p}}\left(\frac{4}{9}P_{\bar{u}/p} - \frac{1}{9}P_{\bar{d}/p}\right)\frac{d\sigma^{gq\to q\gamma}}{d\hat{t}}\right](F_2 - F_3) \tag{59b}$$

(Contribution from all other subprocesses cancels out)

where

$$F_2 = \int_{z_{\min}}^{1} dz\, D_2(z) \tag{60}$$

and similarly for F_3.

In (59a), the second term is clearly dominant since the first term is proportional to the sea quark densities in the antiproton. Similarly in (59b), the first term is dominant.

We now consider the polarized proton scattering where the polarization of the final photon is being detected in the semi inclusive process

$$\bar{p}p\uparrow \to \gamma\uparrow\downarrow KX \tag{61}$$

and consider the following combinations of cross-sections:

$$B = \frac{[\xi(\bar{p}p\uparrow\to\gamma\uparrow K^+X) - \xi(\bar{p}p\uparrow\to\gamma\uparrow K^0X)] - [\xi(\bar{p}p\uparrow\to\gamma\downarrow K^+X) - \xi(\bar{p}p\uparrow\to\gamma\downarrow K^0X)]}{[\xi(\bar{p}p\uparrow\to\gamma\uparrow K^+X) - \xi(\bar{p}p\uparrow\to\gamma\uparrow K^0X)] + [\xi(\bar{p}p\uparrow\to\gamma\downarrow K^+X) - \xi(\bar{p}p\uparrow\to\gamma\downarrow K^0X)]} \tag{62a}$$

$$C = \frac{[\xi(\bar{p}p\uparrow\to\gamma\uparrow K^-X) - \xi(\bar{p}p\uparrow\to\gamma\uparrow \bar{K}^0X)] - [\xi(\bar{p}p\uparrow\to\gamma\downarrow K^-X) - \xi(\bar{p}p\downarrow\to\gamma\downarrow \bar{K}^0X)]}{[\xi(\bar{p}p\uparrow\to\gamma\uparrow K^-X) - \xi(\bar{p}p\uparrow\to\gamma\uparrow \bar{K}^0X)] + [\xi(\bar{p}p\uparrow\to\gamma\downarrow K^-X) - \xi(\bar{p}p\uparrow\to\gamma\downarrow \bar{K}^0X)]} \tag{62b}$$

where ξ stands for $E_\gamma \dfrac{d\sigma}{dp_\gamma^3}$.

Experimentally, the process (61) would be carried out by scattering a $\bar{p}$ beam off a quasistationary polarized proton target. Calculation is however done at the centre of the mass frame.

Proceeding similar to the steps used in arriving at (19) and using (59) we have from (62)

$$B = \frac{\int_H dx\left[\frac{1}{3}P_{\mathrm{sea}/\bar{p}}\Delta P_{g/p}\Delta\frac{d\sigma^{qg\to q\gamma}}{d\hat{t}} + P_{g/\bar{p}}\left(\frac{4}{9}\Delta P_{u/p} - \frac{1}{9}\Delta P_{d/p}\right)\Delta\frac{d\sigma^{gq\to q\gamma}}{d\hat{t}}\right]}{\int_H dx\left[\frac{1}{3}P_{\mathrm{sea}/\bar{p}}P_{g/p}\frac{d\sigma^{qg\to q\gamma}}{d\hat{t}} + P_{g/\bar{p}}\left(\frac{4}{9}P_{u/p} - \frac{1}{9}P_{d/p}\right)\frac{d\sigma^{gq\to q\gamma}}{d\hat{t}}\right]} \tag{63a}$$

$$C=\frac{\int_H \mathrm{d}x\left[\left(\frac{4}{9}P_{\bar{u}/\bar{p}}-\frac{1}{9}P_{\bar{d}/\bar{p}}\right)\Delta P_{g/p}\Delta\frac{\mathrm{d}\sigma^{qg\to q\gamma}}{\mathrm{d}\hat{t}}+P_{g/\bar{p}}\frac{1}{3}\Delta P_{\text{sea}/p}\Delta\frac{\mathrm{d}\sigma^{gq\to q\gamma}}{\mathrm{d}\hat{t}}\right]}{\int_H \mathrm{d}x\left[\left(\frac{4}{9}P_{\bar{u}/\bar{p}}-\frac{1}{9}P_{\bar{d}/\bar{p}}\right)P_{g/p}\frac{\mathrm{d}\sigma^{qg\to q\gamma}}{\mathrm{d}\hat{t}}+P_{g/\bar{p}}\frac{1}{3}P_{\text{sea}/p}\frac{\mathrm{d}\sigma^{gq\to q\gamma}}{\mathrm{d}\hat{t}}\right]} \tag{63b}$$

where $\Delta P_{u/p}$ etc and $\Delta\mathrm{d}\sigma/\mathrm{d}\hat{t}$ are defined in (20). In arriving at (63) we have used parity invariance (21) and the fact that

$$P_{\text{sea}/p}\equiv P_{\bar{u}/p}=P_{\bar{d}/p}=P_{u/\bar{p}}=P_{d/\bar{p}}\equiv P_{\text{sea}/\bar{p}}. \tag{64}$$

We note that the combinations B and C are determined by Compton scatterings alone and the annihilation diagrams do not contribute. This is due to our assumption that the gluon fragmentation for the four kaons is identical.

$$D_g^{K^+}=D_g^{k^-}=D_g^{K^0}=D_g^{\bar{K}^0}. \tag{65}$$

The clean separation between gluon polarization and valence polarization, that we aimed at, has now been achieved. Except for a negligible term, B is determined by valence polarization only, while C is determined by gluon polarization.

Experimentally, a photon tagged with a kaon is detected. The circular polarization of the photon is measured; but no measurements are required for the kaon.

6. Results and discussion

As before, we begin by using the PPT parametrization for parton distributions. Figure 4 shows the contributions to B of the two terms in the numerator of the expression defining B (equation 63). We plot the magnitudes of these two contributions, without regard to sign. The first term is proportional to the sea content of antiproton and the gluon polarization in the proton (marked *sg* in the figure)..The contribution from the second term (marked gV) is proportional to the gluon content in the antiproton and the valence polarization in the proton. We find that the contribution from the first term (*sg*) is about two orders of magnitude lower than that of the second term, for 3 flavours, and is about 4 orders lower for 5 flavours. It is therefore clear *that B depends only on valence polarization.* Figure 5 shows the behaviour of B with θ. B remains positive throughout and is almost independent of the number of flavours. This reflects the fact that in our model, the valence polarization is independent of the number of flavours.

6.1 *Other parametrizations*

We have calculated B using the other two sets of parametrizations for the unpolarised parton densities as well, namely the BP and GR parametrizations. B changes very little when PPT parametrization is replaced by these. B is thus independent of the parametrization used for total parton densities and depends only on the polarisation model. We plot in figure 6 the behaviour of B for the Carlitz-Kaur model. As compared with figure 5 the B value has almost doubled at lower angles. This reflects the fact that at lower angles (*i.e.* high x), the Carlitz-Kaur model gives polarization approaching 100% (for u quark).

The observable B can thus distinguish between the various models for valence

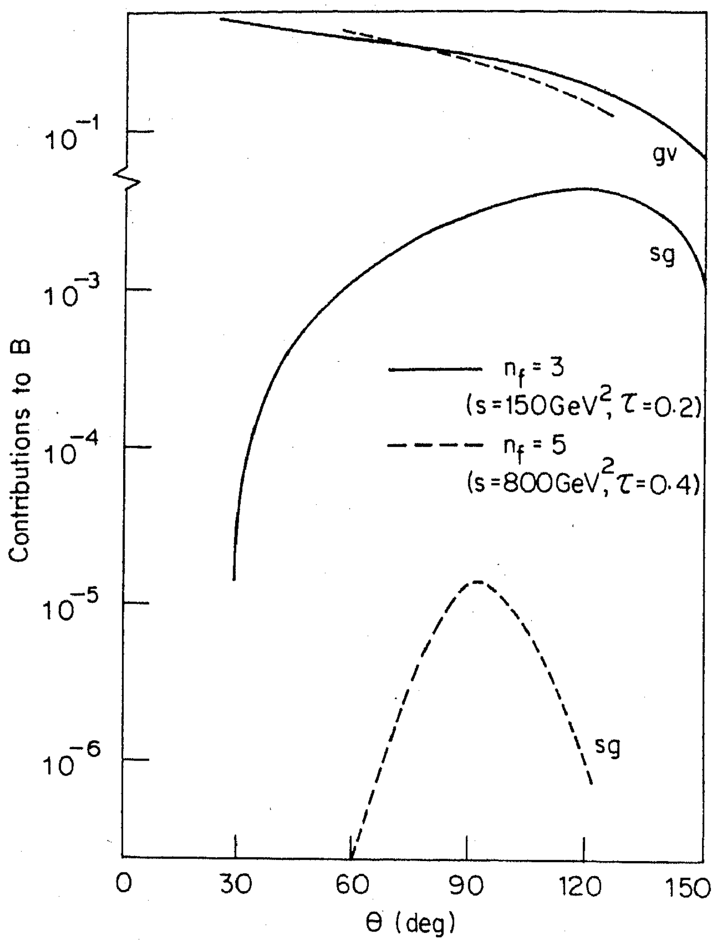

Figure 4. Relative magnitudes of contributions to B from sea (q, $\bar{q}$) gluon and gluon valence scatterings for 3 and 5 flavours.

polarization but is insensitive to the choice of parametrization for unpolarized (*i.e.* total) valence distribution.

Figure 7 shows the relative magnitudes of the two terms contributing to the numerator of the expression defining C (equation 63b). We plot the magnitudes of the two terms without regard to sign. The first is proportional to the gluon polarization in the proton and the valence content in the antiproton (marked Vg in the figure). The second is proportional to the gluon content in the antiproton and quark sea polarization in the proton (marked gs). Clearly the first term dominates, the second always being smaller by at least two orders of magnitude. It is therefore clear that the observable C *is determined essentially by gluon polarization* alone.

Figure 8 shows the behaviour of C with θ for 3, 4 and 5 flavours. As the beauty threshold is crossed, C changes its sign, reflecting the change in sign of the gluon polarization. When PPT parametrization is replaced by GR parametrization for gluon distribution or by a steeper distribution namely

$$g(x) = \frac{9}{2}\frac{(1-x)^8}{x}$$

C changes very little. On the other hand, when the momentum asymmetry associated with gluon polarization (σ defined in (45)) is decreased from $\frac{1}{3}$ to $\frac{1}{4}$, the magnitude of C

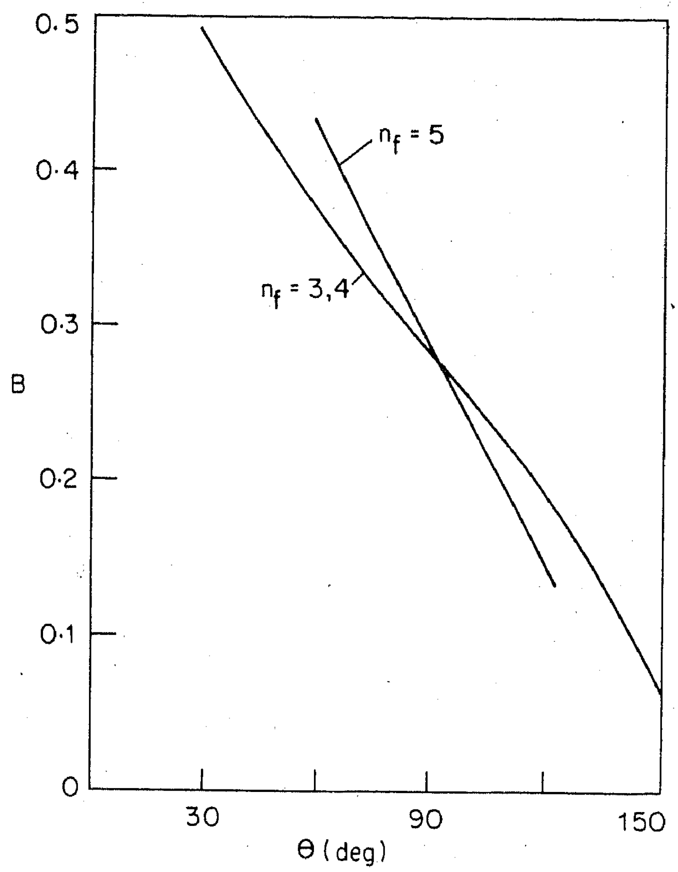

Figure 5. Variation of B with θ for 3, 4 and 5 flavours.

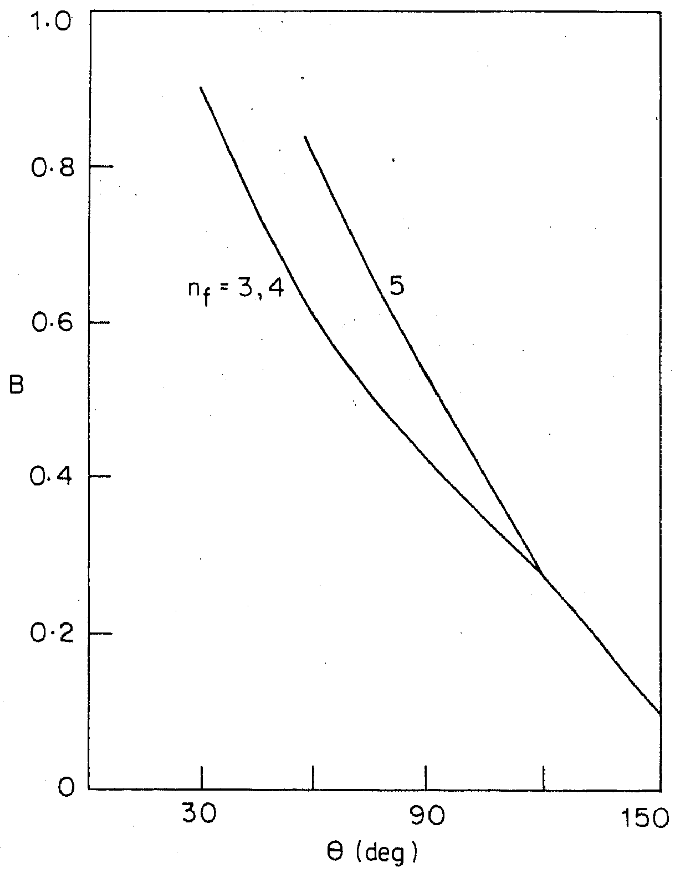

Figure 6. Behaviour of B in the Carlitz-Kaur model.

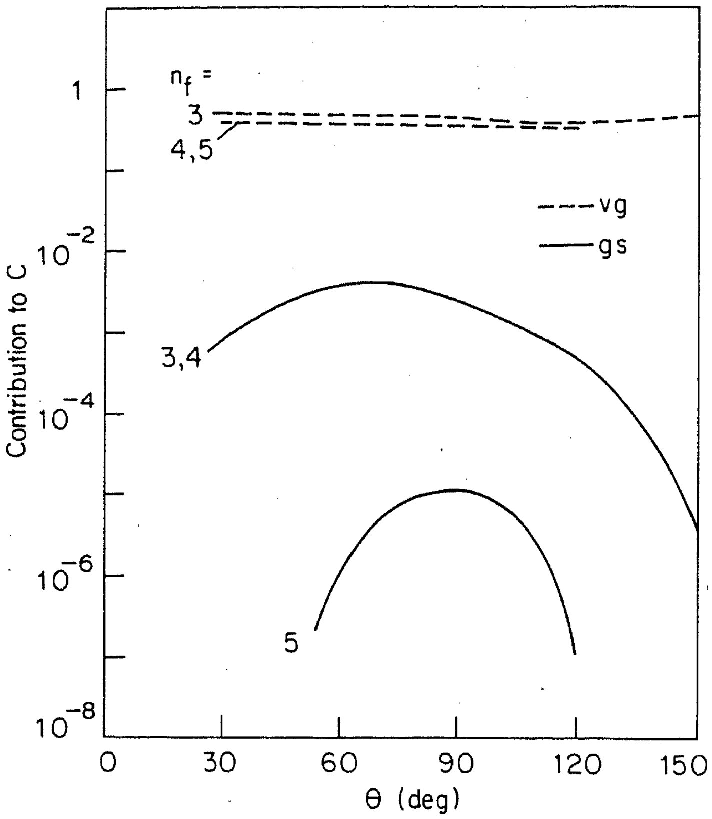

Figure 7. Relative magnitudes of the contributions to C from valence gluon and gluon sea $(q, \bar{q})$ scatterings.

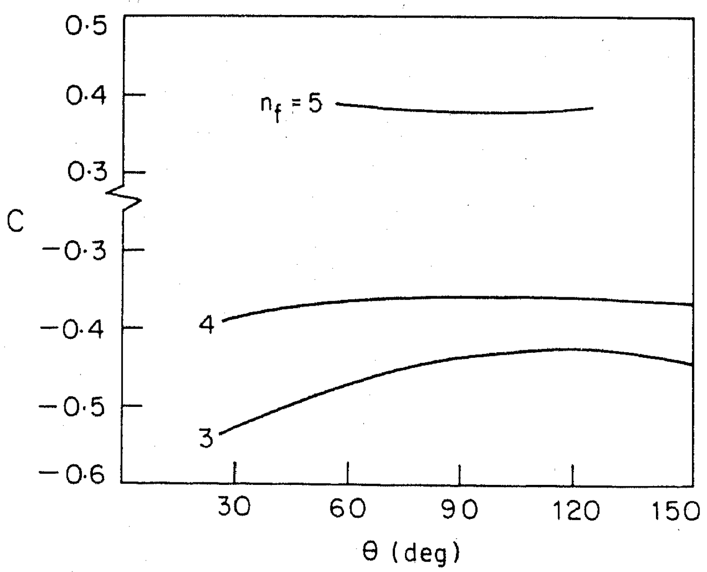

Figure 8. Behaviour of C with θ.

drops proportionately. Thus, while C *is not sensitive to the parametrisation of the total gluon density, it depends on the gluon polarization model.*

7. Conclusion

We have studied the photon inclusive and semi-inclusive scattering of a polarized proton off an unpolarized proton or antiproton. In our attempt to separate the valence polarization from the gluon polarization, we have defined combinations of cross-sections A, B and C. Employing function A, the valence polarization and the gluon polarization can be separated from each other by working in two different kinematic regions (characterized by $\theta_{\text{c.m.}} < 90°$ and $\theta_{\text{c.m.}} > 90°$). The necessity of measuring the angle is avoided by using the observables B and C. In every kinematical region, B depends only on valence polarization while C depends on gluon polarization. A clean separation is thus achieved.

We have found that the functions A, B and C only depend on the polarization models and are independent of the parametrizations of the total quark and gluon densities, being employed. This feature implies that these functions can be used to test models for spin structure of the proton, the test not depending on the specific characteristics of a particular parametrization for unpolarised densities.

To complete the investigation of spin structure, we need to pinpoint processes which would isolate the q, $\bar{q}$ sea polarization, which has played a negligible role in the processes considered here. This will be reported elsewhere.

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