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# Influence of a third body on the luminosity of a contact binary system

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**Abstract.** Based on Roche equipotential surface, a theoretical model of contact binary system has been constructed where the system is disturbed by the presence of a third body. Using this model, ratio of the luminosity in the line of sight to the total luminosity has been calculated over the entire cycle of the system as a function of the parameters like mass-ratio q' between masses of the third body and the primary, distance A of the third body from common centre of gravity and angle I between orbital planes of the third body and the binary system. These calculations show a number of anomalous behaviour of the luminosity even at a large distance A = 3 and a small mass ratio q' = 0.1 where phase of the cycle however play a very important role. At higher mass-ratio q' > 0.8, anomaly is still higher. Thus it is suggested that perturbation by a third body can produce anomalous light curve in some contact binary systems.

*Key words* : eclipses–binaries: close–stars: peculiar

## 1. Introduction

A contact binary system (extreme case of a close binary system) can be classified into an A-type (e.g. star V566 Oph and RR Cen) and W-type (e.g. stars W UMa and AB And) (Rucinski 1974; Binnendijk 1977). Such objects are a challenge to the theory of stellar structure. Most of the W-type binaries show anomalous behaviour in mass-luminosity relations, temperature excess of the secondary component and anomalies in their time periods (e.g. see Niarchos & Duerbeck 1991; Van't Veer 1991; Maceroni & Van't Veer 1993 and references therein). For understanding the anomalous light curves of these binary systems, Lucy (1968 a, b, 1976) presented a model based on common convective envelope. This model with some modifications has been used by a number of people e.g. Mochnacki & Doughty (1972 a, b) expressed potentials in cylindrical co-ordinates; Wilson & Devinney (1973) introduced reflection effect; Rucinski (1973, 1974, 1976) introduced another parameter; and Berthier (1975) studied linear limb-darkening effect. On the other hand, Anderson & Shu (1977) proposed a common radiative and convective envelope model and Binnendijk (1977) introduced a star-spot model to explain the anomalous behaviour of the light curves of W-type contact binaries. All these models are however inadequate to explain the observed light curves of W-type contact binary systems (For details see Kähler 1989; Mochnacki 1985; Rucinski 1985 and references therein). There are probably some complex relations in the mass and energy transfer between the components.

To understand the above mentioned anomalous behaviour of contact binary systems, it would be interesting to study the perturbation of a third body on the luminosity of a contact binary system, since we have evidence that $\lambda$ - Tauri system (Ebbighausen & Struve 1959) and Algol system ($\beta$ - Persei) (Kopal 1959) have their third companion. In this paper, a model of contact binary system has been constructed where the system is perturbed by a third body. The system is in radiative and hydrostatic equilibrium and it has a common radiative envelope. From the conditions of Roche equipotential surface, ratio of luminosity in the line of sight to the total luminosity, $l_1$, has been calculated and thus it has been shown how these things are dependent on the parameters - (i) the ratio between the masses of third body and primary component, (ii) the distance between the third body and common centre of gravity, (iii) the angle between two planes containing the binary components and the third body and (iv) on the longitudes of the secondary component and the third body. The next section describes the details of the model used while the method of calculations and the results have been discussed in the other sections of the paper.

## 2. Theoretical model

Let the masses $m_1$ and $m_2$ of the primary and secondary components respectively of a binary system move around their common centre of gravity and the system is perturbed by the presence of a third body of mass $m_3$ (cf. Fig. 1a). We assume for simplicity that the third body is moving around the common centre of gravity of the binary system without affecting it significantly. The origin of the co-ordinate system is taken as the centre of gravity of the primary component $m_1$; the x-axis is the line joining the centres of gravities of $m_1$ and $m_2$; the axis of rotation of the primary component is the z-axis. It is taken perpendicular to the orbital plane of the binary system which is also the equatorial plane of the primary component (see Fig. 1a).

The potential $\psi$ at a point $P$ of the contact binary system can be written as (Mochnacki & Doughty 1972a, Kopal 1959):-

$$\psi = \frac{Gm_1}{r_1} + \frac{Gm^2}{r^2} + \frac{\varpi^2 p^2}{2} + \frac{Gm_3}{r_3^3} \{R^2 P_2(\sigma) + \frac{m_1 - m_2}{m_1 + m_2} \frac{R^3}{r_3} P_3(\sigma)\} \quad (1)$$

where $r_1$ and $r_2$ are the distances of the point $P$ from the centres of gravities of the stars $m_1$ and $m_2$ respectively (see Fig. 1b). $p$ is the perpendicular distance of the point $P$ from the axis

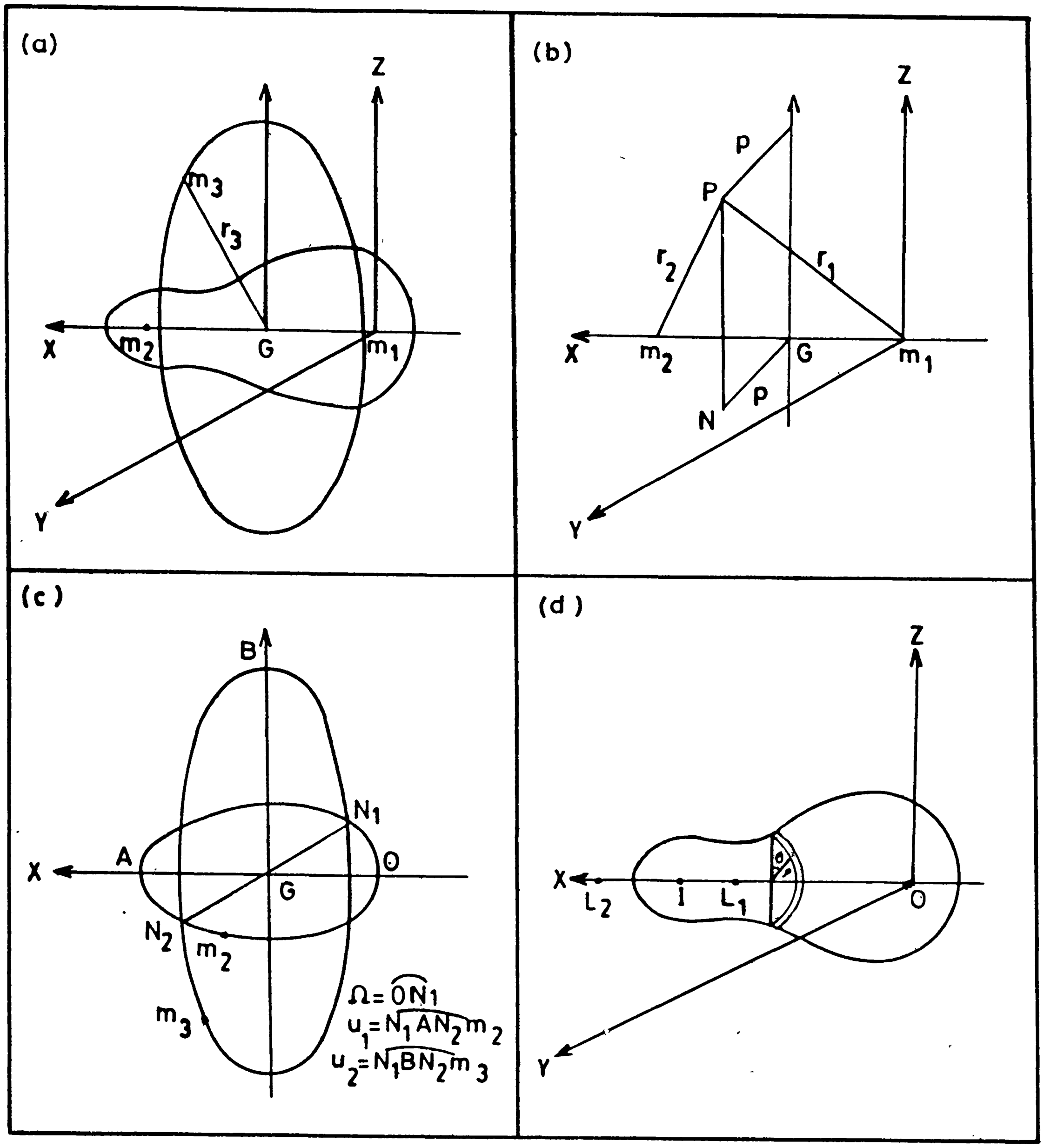


**Figure 1**. (a) The (x, y, z) co-ordinate system. The x-axis ($m_1Gm_2$) is the axis of revoution of the third body while the axis parallel to z axis but passing from G is the axis of revolution of the binar system. (b) Graphical representation of the distance parameters. (c) Graphical representation of the angular parameters. (d) The cylindrical co-ordinate system ($x$, $\rho$, $\theta$). The centres of gavities of masses $m_1$ and $m_2$ are located at 0 and 1 respectively. $L_1$ and $L_2$ are the Langrangian points.

of revolution which is also perpendicular to the binary orbital plane and passes through the common centre of gravity $G$. $\varpi$ $\left(= \sqrt{\frac{G(m_1 + m_2)}{R}}\right.$, according to Kepler's third law) is the angular velocity of both the components in the relative orbit. $R$ denotes the radius vector of $m_2$ referred to the centre of gravity of $m_1$ while $r_3$ denotes the radius vector of $m_3$ referred to the centre of gravity of the binary system. $P_2$ and $P_3$ are the second and third order Legendre polynomials of $\sigma$, which is given by the relation (Kopal 1959):-

$$\sigma = \cos(u_1 - \Omega)\cos(u_2 - \Omega) + \sin(u_1 - \Omega)\sin(u_2 - \Omega)\cos I \tag{2}$$

where $\Omega$ denotes the longitude of the node, I is the angle of inclination of the orbital plane of third body from that of binary system and $u_1$ and $u_2$ are the longitudes of $m_2$ and $m_3$ respectively reckoned from the node in the planes of their respective orbits (see Fig. 1c).

In the relation (1), the first two terms are the potentials due to masses $m_1$ and $m_2$ respectively, the third term comes from the motion around the axis of rotation and the last term accounts for the disturbance created by the third body $m_3$.

The normalized total potential $C$ at a point $r$ on the surface of the boundary system derived from the relation (1) for a relative circular orbit of the binary (where $R$ can be taken as 1) can be written as :-

$$C = \frac{2B_1}{r_1} + \frac{2B_2}{r_2} + p^2 + \frac{2B_3}{r_3^3}\left\{P_2(\sigma) + (1-q)\frac{B_1}{r_3}P_3(\sigma)\right\} \tag{3}$$

where

$$q = \frac{m_2}{m_1};\ q' = \frac{m_3}{m_1};\ B_1 = \frac{1}{1+q};\ B_2 = qB_1;\ B_3 = q'B_1;\ C = \frac{2B_1\psi}{Gm_1}.$$

Following Mochnacki & Doughty (1972a), we express the potential $C$ in cylindrical co-ordinates (x, $\rho$, $\theta$) (vide Fig. 1d) assuming that the third body moves in a circular orbit i.e. $r_3$ = A, as:-

$$C = \frac{2B_1}{\sqrt{x^2+\rho^2}} + \frac{2B_2}{\sqrt{(x-1)^2+\rho^2}} + \{(x-B_2)^2 + \rho^2\sin^2\theta\} + \frac{2B_3}{A^3}\left\{P_2(\sigma) + \frac{(1-q)B_1}{A}P_3(\sigma)\right\} \tag{4}$$

If $C_1$ and $C_2$ be the initial potentials corresponding to the Lagrangian points $L_1$ and $L_2$, then the photospheric potential $C_p$ lying between $C_1$ and $C_2$ is conveniently represented in terms of the fill-out ratio, F, as :-

$$C_p = C_1 + (C_1 - C_2)(1 - F) \tag{5}$$

where $C_1 \geq C_p \geq C_2$ and $1 \leq F \leq 2$.

The local surface gravity, $g$ for an element at the surface is given by

$$g = \sqrt{(f_x^2 + f_y^2 + f_z^2)} \tag{6}$$

where the derivatives $f_x, f_y, f_z$ of a function $f(x, \rho, \theta) = C - C_p$ are given in Appendix 1; and the surface element of area $ds$ is given by

$$ds = -\frac{g}{f_y \sin\theta + f_z \cos\theta} d\theta dx. \tag{7}$$

If $\bar{l}$ is a unit vector in the line of sight, then we can write

$$\bar{l} = l_0 \bar{i} + \mathrm{m}_0 \bar{j} + \mathrm{n}_0 \bar{k} \tag{8}$$

where $\bar{i}$, $\bar{j}$ and $\bar{k}$ are the unit vectors along the X, Y and Z axes respectively and $l_0 = \cos\psi \sin i$, $m_0 = \sin\psi \sin i$ and $n_0 = \cos i$; $\psi$ is the phase angle with respect to the passage of the secondary component in front of the primary (heavier) and $i$ is the inclination of the orbital plane.

Since the direction cosines of the normal $\bar{n}$ to a surface element are $-f_x/g$, $-f_y/g$, $-f_z/g$ along the X, Y and Z directions, we can write

$$\mathrm{g}\,(\bar{n}\,.\,\bar{l}) = g\,(-\frac{f_x}{\mathrm{g}}.\bar{i} - \frac{f_y}{\mathrm{g}}.\bar{j} - \frac{f_z}{\mathrm{g}}.\bar{k}).(l_0\,.\,\bar{i} + m_0.\bar{j} + n_0.\bar{k}) \tag{9}$$

$$= -(f_x l_0 + f_y m_0 + f_z n_0) \tag{10}$$

where $(\bar{n}\,.\,\bar{l})$ is the cosine of the foreshortening angle for the observer. Now we can write $l_1$, the ratio of the luminosity in the line of sight to the total luminosity using von Zeipel's theorem (1924) as

$$l_1 = \int g(\bar{n}\,.\,\bar{l})ds \,/\, \int g ds \tag{11}$$

where the integrals are taken over the uneclipsed parts of the visible discs of the two components.

The value of $l_1$ can now be calculated from equation (11) for different values of $m_3/m_1$, $A$, $I$, $u_1$, $u_2$ and $\Omega$. Since the values of $u_1$ and $u_2$ change continuously, we have calculated the value of $l_1$ over the entire cycle of the system (see Appendix 2).

## 3. Method of calculations

Equations (4) to (11) are employed in the following manner to calculate the value of $l_1$; a few appropriate values of $q'$, $A$, $I$, $u_1$, $u_2$ are selected. The Langrangian points $L_1$ and $L_2$ are then evaluated by the Newton - Raphson method by integrating the equation.

$$x_{k+1} = x_k - \frac{f_x(x_k, 0, 0)}{f_{xx}(x_k, 0, 0)}$$

where $x_k$ and $x_{k+1}$ are the $k^{th}$ and $(k+1)^{th}$ iteration of the x – values (see also Appendix 1). Corresponding to these Langrangian points, initial potentials $C_1$ and $C_2$ are calculated from equation (4) which in turn gives the photospheric potential $C_p$ for the value of fill-out ratio $F$ (vide equation (5)). For given $x$ and $\theta$, the cylindrical radius p is evaluated by iterating the equation :

$$\rho_{k+1} = \rho_k - \frac{f(x, \rho_k, \theta)}{f(x, \rho_k, \theta)}$$

while for any point along the x-axis, the following equation should be iterated :

$$x_{k+1} = x_k - \frac{f(x_k, 0, 0)}{f(x_k, 0, 0)}$$

which in turn gives the total surface of the binary system. Employing equations (6), (7), (10) and (11), we can calculate the value of $l_1$. Results of all these calculations are presented in Figures 2 and 3.

To ascertain whether an element is eclipsed or not, following procedure is adopted; the line of sight is extended out from the centre of this element, and the potential values are sampled at points along this line; if any of these values are greater than the potential specifying the eclipsing surface, the element is rendered invisible and hence is rejected.

## 4. Results and discussions

Here the distortions of the Roche surface arises simply from a combination of mass-point components, centrifugal forces, tidal forces and the disturbing forces. Since it has been assumed that the common centre of gravity of the binary system is negligibly affected by the third body, we can therefore assume that the orbit of the system remains the same. During our calculation of the value of $l_1$, we have considered the mass-ratio $m_2/m_1 = 0.54$, the fill out ratio $F = 1.13$ which are for W-UMa binary system (Whelan, Worden & Mochnacki 1973). For this system, the values of Lagrangian points $L_1$ and $L_2$ are 0.5630070 and 1.5952926 respectively. The corresponding values for potentials $C_1$ and $C_2$ are 3.966216 and 3.550863 respectively; sizes of the primary components are $a(\theta = 0°, \rho = 0) = -0.472447$; $b(\theta = 90°, x = 0) = 0.4412252$; $c(\theta = 0°, x = 0) = 0.4146707$. We have always considered $\Omega = 0°$, $\varphi = 0°$ in our calculations. The values of other parameters used in the calculations are marked in the Figures 2 and 3. To find out integrations in equation (11), we have taken sixty points over the whole surface of the binary system, remembering that each point satisfy conditions $g\,(\bar{n} . \bar{l}) > 0$ and $g - g\,(\tilde{n} . \tilde{l}) > 0$

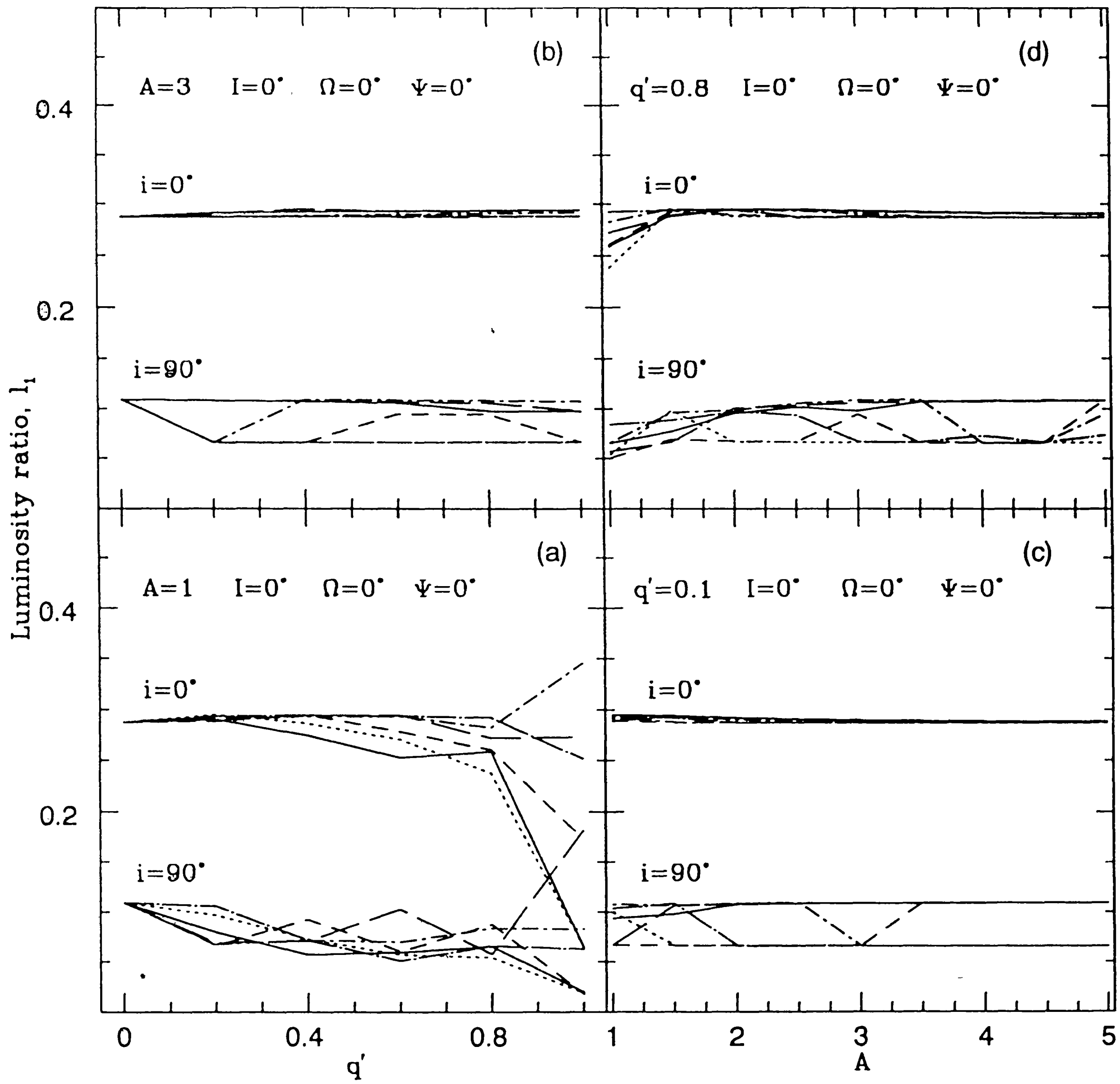


**Figure 2.** For a given set of parameters marked in the figures, variation of luminosity ratio $l_1$ with mass ratio q' $(\equiv \frac{m_3}{m_1})$ is shown in (a) and (b) while the same with the radial distance of the third body, A is shown in (c) and (d). Solid, dot, short dash, long dash, dot-short dash and dot-long dash curves represent the variations for $u_1$ = 0°, 90°, 135°, 225°, 270° and 315° respectively.

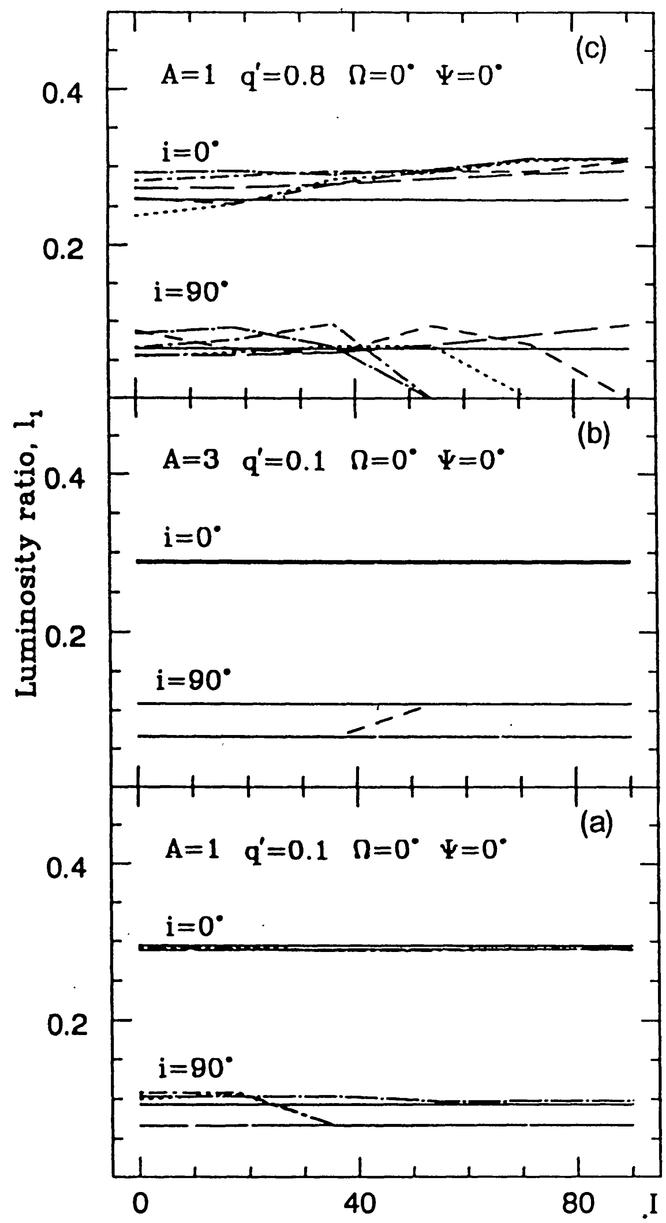


**Figure 3.** Variation of $l_1$ with the angle $l$. Other symbols are the same as Figure 2.

Inspection of Figures 2 and 3 indicates that :-

1. In case of $i = 0°$ the mean values of $l_1$ are always larger (close to ~ 0.3) in comparison to the case $i = 90°$ where the value is close to ~ 0.1. But the variations in the values of $l_1$ for different values of $u_1$ are generally larger for $i = 90°$ case in comparison to the case for $i = 0°$ except in the case of Fig. 2(a).

2. In Fig. 2(a), one can see that for inclination of the orbital plane, $i = 0°$, the value of $l_1$ changes significantly with $q'$ when the mass of the third body is close to the mass of primary. Generally it increases to some extent, thereafter it begins to fall slowly and the fall is remarkably large for some $u_1$ when $q' \rightarrow 1$. The fall is large when longitude $u_1 = 0°$ to 90°. However this nature is not so prominent for all values of $u_1$. For $u_1 = 270°$, the value of $l_2$ however increases for $q' \geq 0.8$ in contrast to its decreasing nature for other values of $u_1$.

3. For distance $A = 3$, there are no appreciable changes in the values of $l_1$ for different values of $q'$ and $u_1$ in the case of inclination $i = 0°$ (see Fig. 2(b)). However the changes are noticiable for $i = 90°$. For example, for $u_1 = 90°$ the value of $l_1$ decreases from 0.109 to 0.066 when the value of $q'$ increases from 0.0 to 0.2 and thereafter it remains almost constant with the increasing value of $q'$.

4. A comparison of Fig. 2(a) with Fig. 2(b) indicates that for a given set of parameters, variation in the value of $l_1$ with $q'$ is more for A = 1 case than A=3 case.

5. In Fig. 2(c) and (d), one can see that for $i = 0°$ the value of $l_1$ increases a little for higher values of $q'$ and for values of A = 1 to 1.5. On the other hand, for $i = 90°$ curves show anomalous behaviour for different values of $A$ and $u_1$ which is larger for $q' = 0.8$ than that for $q' = 0.1$.

6. In Fig. 3(a) and (b), the value of $l_1$ changes with I for some values $u_1$ in the case of $q' = 0.1$, $A = 1.3$ and $i = 90°$. But Fig. 3(c) shows anomalous behaviour in the variation of $l_1$ value with $I$ and $u_1$ for both $i = 0°$ and 90°. From these figures, it is clear that for generating an anomalous variation in the luminosity of a close triple system, distance A is less important in comparison to mass ratio $q'$.

## 5. Conclusions

From the above discussions, it is clear that during the entire cycle of a contact binary system anomalous variation in the luminosity of the system may occur, even at long distance A = 3 and small mass-ratio $q' = 0.1$. For some combination of $u_1$ and $l_2$, this anomaly is very large indicating that they play a very important role in generating anomalous light-curve of such systems. The anomaly in the light-curve is generally larger for higher mass-ratios. But it is maximum for different values $I$ and $u_1$ for the case A = 1, $q' = 0.8$ (see Fig. 2c).

It can, therefore, be concluded that the presence of a third body in a compact binary system may produce a peculiar light curve and hence can offer an explanation for the presence of an anomalous behaviour with mass-ratio $m_2/m_1$ in the light-curve of the W-type contact binaries like W UMa.

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## Appendix I

The equipotential surface representing the photosphere can be specified by the function $f(x, \rho, \theta)$ as :

$$f(x, \rho, \theta) = C - C_p = 0$$

where $C$ and $C_p$ are given by equations (3) and (5) of the text respectively. The derivatives $f_x$, $f_y$, $f_z$, $f_\rho$ and $f_{xx}$ of $f(x, \rho, \theta)$ are given by the expressions :

$$f_x = E_1 x + E_2(x - 1) + 2(x - B_2);$$

$$f_y = (E_1 + E_2 + 2)\rho \sin\theta ;$$

$$f_z = (E_1 + E_2)\rho \cos\theta ;$$

$$f_\rho = (E_1 + E_2)\rho + 2\rho \sin^2\theta ;$$

$$f_{xx} = E_1 + E_2 + D_1 x^2 + D_2(x - 1)^2 + 2$$

where

$$B_1 = \frac{1}{1 + q}, \; B_2 = qB_1,$$

$$E_1 = -\frac{2B_1}{r_1^3}, \; E_2 = -\frac{2B_2}{r_2^3}$$

$$D_1 = -\frac{3E_1}{r_1^2}, \; D_2 = -\frac{3E_2}{r_2^2}$$

## Appendix II

In order to find variation in $u_2$ corresponding to a given variation in $u_1$, we used Kepler's 2nd and 3rd law of planetary motion along with the polar equation of an orbit. Suppose we take N values of $u_1$ for completing the entire cycle then $\Delta u_1 = \frac{360°}{N}$. We used N = 8 in our calculations. The time $\Delta t$ taken to change angle $\Delta u_1$ can be written as

$$\Delta t = \frac{r_1^2 P_1 \Delta u_1}{2\pi a_1^2 (1 - e_1^2)^{1/2}}$$

where

$$P_1 = \sqrt{\frac{4\pi^2 a_1^3}{G(m_2 + m_1)}};$$

$$r_1 = \frac{a_1(1 - e_1^2)}{1 + e_1 \cos(u_1 - \omega)};$$

and other symbols are the orbital elements of $m_2$.

During time $\Delta t$, the change $\Delta u_2$ in angle $u_2$ is given as

$$\Delta u_2 = \frac{2\pi a_2^2(1 - e_2^2)^{1/2}}{P_2 r_2^2} \times \Delta t$$

where

$$P_2 = \sqrt{\frac{4\pi^2 a_2^3}{G\left(m_3 + \frac{m_2 m_1}{m_2 + m_1}\right)}};$$

$$r_2 = \frac{a_2(1 - e_2^2)}{1 + e_2 \cos(u_2 - \omega)};$$

and $a_2$, $e_2$ etc. are the orbital elements of $m_3$. At each step, new values of $u_1$ and $u_2$ and $\Delta t$ can be calculated.

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