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Reflection of P-waves in a prestressed dissipative layered crust

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Abstract. The paper deals with overall reflection and transmission response of seismic P-waves in a multilayered medium where the whole medium is assumed to be dissipative and under uniform compressive initial stress. The layers are assumed to be homogeneous, each having different material properties. Using Biot's theory of incremental deformation, analytical solutions are obtained by matrix method. Numerical results for a stack of four layers – modelling earth's upper layers, show a decreasing trend in both the Reflection Coefficients R_D^{PP} and R_D^{PS} of the reflected P and S-waves.

Keywords. Reflection; P-wave; S-wave; dissipative; homogeneous layers; Biot's theory; matrix method; reflection coefficients.

1. Introduction

The study of reflection and transmission of seismic body waves through multilayered media is an important part of seismic sounding techniques. It is recognized that these studies provide a very convenient method of investigating the earth's interior. Although other approximations are possible, the simplest representation of the system of rocks beneath the earth's surface might be supposed to consist of a series of plane, parallel layers, each having its own characteristic – but constant within the layer – parameters of velocity and density [12]. Observation of propagation of stress waves in solids (or fluids) show that dissipation of strain energy occurs even when the waves have small amplitude. This dissipation results from imperfection in elasticity, loss by radiation, by geometrical spreading and scattering [5, 7–11, 13, 14]. A convenient measure of attenuation in waves is the dimensionless loss factor (or specific dissipation constant) Q^{-1}. It is related to the rate at which the mechanical energy of vibration is converted irreversibly into heat energy and does not depend on the detailed mechanism by which energy is dissipated. For P-waves Q_α^{-1} is given by [12].

$$Q_\alpha^{-1} = \frac{2v}{V}$$

where V and v are the real and imaginary parts of the complex P-wave velocity. It is also known that, surprisingly, Q^{-1} is independent of frequency, pressure and temperature [5].

In the focal region, prior to an earthquake, considerable tectonic thrust builds up as a uniaxial stress system. It is of some interest to investigate reflection characteristics, through a theoretical model of a stack of layers under uniaxial compressive prestress. Biot [2] has provided a detailed theory of incremental deformation of a medium in a state of prestress brought about by even arbitrary finite deformation. Later, Dahlen [3], in a limited context of *initial elastic deformation* arrives at identical set of equations, excepting the constitutive equations for the incremental stresses. If restricted to two-dimensions, Dahlen's equations fail to reduce to the equations for incompressible medium derived elaborately by Biot. Secondly, the elastic moduli in the transverse direction also change due to the uniaxial prestress. Consequently, we adhere completely to Biot's theory.

For treatment of the equations for a stack of layers, we adopt a simple matrix method based on Kennett [6]. In this paper we restrict to two-dimensional propagation.

2. Formulation of the problem

Consider an initially stressed, dissipative medium consisting of 'n' parallel homogeneous layers overlying a half-space. The interfaces are ordered as $Z_1, Z_2, \ldots, Z_n$ where the origin $Z = 0 < Z_1$ is on a hypothetical free surface from which P-wave originate and travel downwards, ultimately as plane waves. The reflected waves are received at the same surface. To keep the analysis simple in the first instance, as is often done, we disregard stress-free condition on $Z = 0$, that is to say, regard the top layer $Z < Z_1$ as semi-infinite. The topmost layer is layer number 1 and the bottom layer $n + 1$ and thicknesses of the intermediate layers are designated as $H_2, H_3, \ldots, H_n$ (figure 1). The physical quantities associated with layer number 'm' will be denoted by symbols with suffix m.

In general, if we have an isotropic elastic solid under uniform initial horizontal compression $-S_{11}$ (tensile $S_{11} < 0$) parallel to x-axis, which undergoes additional infinitesimal deformation, then according to Biot [2], the incremental stresses consist of two parts: one part due to additional deformation and the other due to infinitesimal rotation ω_2 acting to rotate the initial stress system:

$$\bar{\sigma}_{11} = S_{11} + s_{11}, \quad \bar{\sigma}_{33} = s_{33}, \quad \bar{\sigma}_{13} = s_{13} + S_{11}\omega_2 \tag{1}$$

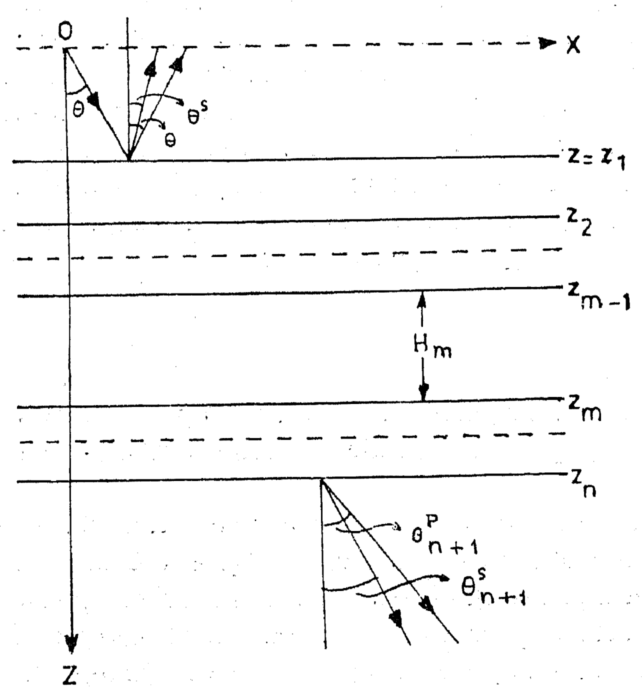


Figure 1. Geometry and schematic of the problem.

where s_{ij} are incremental stresses referred to axes which rotate with the medium (Biot [2], eq. (4.13)) and

$$\omega_2 = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right). \tag{2}$$

For infinitesimal incremental strain e_{ij}, the incremental stress s_{ij} will be linear functions of e_{ij}. Assuming these to be orthotropic in nature we can write

$$\begin{aligned} s_{11} &= B_{11}e_{xx} + B_{13}e_{zz}, \quad e_{xx} = \frac{\partial u}{\partial x} \\ s_{33} &= B_{31}e_{xx} + B_{33}e_{zz}, \quad e_{zz} = \frac{\partial w}{\partial z} \\ s_{31} &= 2Qe_{zx}, \quad e_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right). \end{aligned} \tag{3}$$

Also, after careful consideration of existence of strain-energy,

$$B_{31} - B_{13} = S_{11} \tag{4}$$

(Biot [2], eq. (6.2)). The elastic constants $B_{11}, \ldots, Q$ in general may depend on the initial stress S_{11}. Biot ([2], eq. (8.31e)) after analysis of an incompressible medium, selects for an original isotropic compressible medium (Lamé constants λ, μ), relations equivalent to

$$\begin{aligned} B_{11} &= \lambda + 2\mu - S_{11}, \quad B_{13} = \lambda - S_{11} \\ B_{31} &= \lambda, \quad B_{33} = \lambda + 2\mu, \quad Q = \mu. \end{aligned} \tag{5}$$

A salient feature of these relations is that the moduli in the x-direction (the direction of initial stress) increases due to the initial compressive stress while those in the transverse z-direction remain unchanged. To account for dissipation in the medium λ and μ are to be regarded complex: $\lambda = \lambda_r + i\lambda_i$, $\mu = \mu_r + i\mu_i$.

The two-dimensional dynamical equations of motion as obtained by Biot [2] are

$$\begin{aligned} \frac{\partial s_{11}}{\partial x} + \frac{\partial s_{13}}{\partial z} - S_{11}\frac{\partial \omega_2}{\partial z} &= \rho\frac{\partial^2 u}{\partial t^2} \\ \frac{\partial s_{31}}{\partial x} + \frac{\partial s_{33}}{\partial z} - S_{11}\frac{\partial \omega_2}{\partial x} &= \rho\frac{\partial^2 w}{\partial t^2}. \end{aligned} \tag{6}$$

For time-harmonic plane wave propagation of frequency $f = \omega/2\pi$, we may assume a factor $\exp[i(\omega t - kx)]$. Insertion of (3) with (5) in (6) results in two O.D.E's for u and w, the displacement components. However, for developing a matrix method we introduce stresses

$$\begin{aligned} \tau_{11} &= s_{11}, \quad \tau_{33} = s_{33} \\ \tau_{13} &= s_{13} - S_{11}\omega_2 \end{aligned} \tag{7}$$

and the quantities [6]

$$W = iw, \quad U = u, \quad T = i\tau_{33}, \quad S = \tau_{13}. \tag{8}$$

Constructing the stress-displacement vector

$$\mathbf{b} = [W, U, T, S]^T \tag{9}$$

eq. (6), with the aid of (7) and (3) can be written as a first order system. For subsequent computational purpose we nondimensionalize all quantities: the displacements by z_1 (thickness traversed by the waves in the top layer) and stresses by μ_{1r}, the real part of shear modulus of the top layer. Denoting the respective nondimensional quantities by superscript *, the first order system can be written as

$$\frac{d\mathbf{b}^*}{dz^*} = A^*\mathbf{b}^* \tag{10}$$

where

$$A^* = \begin{bmatrix} 0 & -\dfrac{B_{31}^*}{B_{33}^*}kz_1 & \dfrac{1}{B_{33}^*} & 0 \\ \dfrac{Q^*+0{\cdot}5S_{11}^*}{Q^*-0{\cdot}5S_{11}^*}kz_1 & 0 & 0 & \dfrac{1}{Q^*-0{\cdot}5S_{11}^*} \\ -\dfrac{(kz_1)^2}{(p\beta_1)^2}\dfrac{\rho}{\rho_1} & 0 & 0 & -kz_1 \\ 0 & \left[B_{11}^* - \dfrac{B_{13}^*B_{31}^*}{B_{33}^*} - \dfrac{\rho/\rho_1}{(p\beta_1)^2}\right]k^2z_1^2 & \dfrac{B_{13}^*}{B_{33}^*}kz_1 & 0 \end{bmatrix} \tag{11}$$

is the coefficient matrix. $p = k/\omega$ is the wave slowness (reciprocal of phase velocity of propagation in the x-direction) and $\beta_1 = (\mu_{1r}/\rho_1)^{1/2}$ is the shear wave velocity in the topmost layer. For reflection and transmission of body waves, p remains constant in all the layers. Finally, $S_{11}^* = S_{11}/\mu_{1r}$.

The incremental boundary forces have also been carefully examined by Biot ([2], eq. 17.56)). In our case, where the boundaries are $z = \text{const.}$, the components turn out to be τ_{13} and τ_{33}, so that at an interface $z = z_m \mathbf{b}^*$ is a continuous vector when perfect bonding is assumed.

3. Propagation in the stack

In an *intermediate* mth layer, the solution of (10) is

$$\mathbf{b}^* = e^{A_m^*(z^*-z_{m-1}^*)}\mathbf{b}_{m-1}^* \tag{12}$$

where $\mathbf{b}_{m-1}^*$ is the stress-displacement vector at the interface $z^* = z_{m-1}^*$. Hence at $z^* = z_m^*$

$$\mathbf{b}_m^* = e^{A_m^*H_m^*}\mathbf{b}_{m-1}^*. \tag{13}$$

where $H_m^* = z_m^* - z_{m-1}^*$ is the nondimensional thickness of the mth layer. Hence, recursively

$$\mathbf{b}_n^* = e^{A_n^*H_n^*}e^{A_{n-1}^*H_{n-1}^*},\ldots,e^{A_2^*H_2^*}\mathbf{b}_1^* \equiv E\mathbf{b}_1^*. \tag{14}$$

All the exponentials involved above are 4×4 matrix exponentials.

For $\mathbf{b}_1^*$ we note that it consists of the down going incident P type wave and reflected up going P and S type waves (figure 1). We construct the contributions from each of these separately and superpose. Suppressing the time harmonic term, we can write for the down going incident wave

$$u_1^* = A_1 e^{-iqz}e^{-ikx}, \quad w_1^* = e^{-iqz}e^{-ikx} \tag{15}$$

where the predominant z-component of the amplitude has been taken to be unity. Inserting in the equations of motion (6) with (3) and (5) and assuming

$$k = K \sin\theta, \quad q = K\cos\theta \tag{16}$$

so that θ is the angle of incidence, we get

$$A_1 = -\frac{(\lambda_1^* + 2\mu_1^*)\cos^2\theta + \{\mu_1^* + 0{\cdot}5S_{11}^* - 1/(p\beta_1)^2\}\sin^2\theta}{(\lambda_1^* + \mu_1^* - 0{\cdot}5S_{11}^*)\sin\theta\cos\theta} \tag{17}$$

and the velocity of propagation ω/k is given by a quadratic equation whose roots are

$$\rho_1\frac{\omega^2}{K^2} = \frac{1}{2}(R_1 \pm \sqrt{S_1}) \tag{18}$$

where

$$R_1 = \lambda_1 + 3\mu_1 - 0{\cdot}5S_{11} + \frac{S_{11}}{\rho}p^2(\lambda_1 + \mu_1 - 0{\cdot}5S_{11})$$

$$S_1 = R_1^2 - 4(\lambda_1 + 2\mu_1)(\mu_1 - 0{\cdot}5S_{11}). \tag{19}$$

If S_{11} is neglected, the positive sign in (18) yields P-waves and the negative sign, S-waves. In the presence of S_{11}, the velocities are p, that is, direction dependent and the waves are not pure, in the sense that P-waves are accompanied by some transverse component and S-waves by some longitudinal component [4]. Stresses corresponding to (15) can be readily calculated from (3) and (5). We thus obtain

$$\mathbf{b}_{1D}^{*P} = e^{-iKz_1\cos\theta}\begin{bmatrix} i \\ A_1 \\ Kz_1\{A_1\lambda_1^*\sin\theta + (\lambda_1^* + 2\mu_1^*)\cos\theta\} \\ -iKz_1\{A_1(\mu_1^* - 0{\cdot}5S_{11}^*)\cos\theta + (\mu_1^* + 0{\cdot}5S_{11}^*)\sin\theta \end{bmatrix} \tag{20}$$

p – the constant for all the layers – in (17), can be computed from the equation

$$\sin\theta = (p\beta_1)\left(\frac{R_1^* + \sqrt{S_1^*}}{2}\right)^{1/2} \tag{21}$$

which is arrived at from (16) and (18). Here θ is given so $(p\beta_1)$ is to be obtained by solving the above nonlinear equation.

For up going reflected P type wave, we have to use the representations

$$u_1^* = A_2 e^{iq(z-z_1)}e^{-ikx}, \quad w_1^* = B_2 e^{iq(z-z_1)}e^{-ikx}. \tag{22}$$

Analysis similar to the above leads to

$$\mathbf{b}_{1U}^{*P} = B_2\begin{bmatrix} i \\ -A_2/B_2 \\ -Kz_1\left\{\frac{A_2}{B_2}\lambda_1^*\sin\theta + (\lambda_1^* + 2\mu_1^*)\cos\theta\right\} \\ -iKz_1\left\{\frac{A_2}{B_2}(\mu_1^* - 0{\cdot}5S_{11}^*)\cos\theta + (\mu_1^* + 0{\cdot}5S_{11}^*)\sin\theta\right\} \end{bmatrix} \tag{23}$$

where $A_2/B_2 = -A_1$ is obtained from (17). For up going reflected S type wave we again use representation of the type (22) with amplitudes A_3, B_3 instead of A_2, B_2. We thus obtain $\mathbf{b}^{*S}_{1U}$ similar to (23) with A_2/B_2 replaced by $A_3/B_3 = -A_1$ and θ replaced by θ^s given by

$$\sin\theta^s = \mathrm{Re}\left\{(p\beta_1)\left(\frac{R_1^* - \sqrt{S_1^*}}{2}\right)^{1/2}\right\} \tag{24}$$

appropriate for S type waves. Here Re means real part of. The total stress-displacement vector in the top layer is thus

$$\mathbf{b}_1^* = \mathbf{b}^{*P}_{1D} + \mathbf{b}^{*P}_{1U} + \mathbf{b}^{*S}_{1U}. \tag{25}$$

Finally, for the bottom most $(n+1)$th layer, only down going P and S type waves are sustained. For the former we take

$$u^*_{n+1} = A_4 e^{-iq(z-z_n)} e^{-ikx}, \quad w^*_{n+1} = B_4 e^{-iq(z-z_n)} e^{-ikx}. \tag{26}$$

As in the case of $\mathbf{b}^{*P}_{1D}$ we obtain

$$\mathbf{b}^{*P}_{n+1,D} = B_4 \begin{bmatrix} i \\ A_4/B_4 \\ Kz_1\left\{\frac{A_4}{B_4}\lambda^*_{n+1}\sin\theta^P_{n+1} + (\lambda^*_{n+1} + 2\mu^*_{n+1})\cos\theta^P_{n+1}\right\} \\ -iKz_1\left\{\frac{A_4}{B_4}(\mu^*_{n+1} - 0{\cdot}5S^*_{11})\cos\theta^P_{n+1} + (\mu^*_{n+1} + 0{\cdot}5S^*_{11})\sin\theta^P_{n+1}\right. \end{bmatrix} \tag{27}$$

where

$$\sin\theta^P_{n+1} = \mathrm{Re}\left[(p\beta_1)\left(\frac{R^*_{n+1} + \sqrt{S^*_{n+1}}}{2}\right)\right] \tag{28}$$

R^*_{n+1} and S^*_{n+1} are quantities identical to R^*_1 and S^*_1 (cf. eq. (19)), save that λ_1 and μ_1 are to be replaced λ_{n+1} and μ_{n+1}. Similarly A_4/B_4 is given by an expression like that of A_1 (eq. 17)) save for λ_1, μ_1, θ we have to write $\lambda_{n+1}, \mu_{n+1}, \theta^P_{n+1}$. For the down going S type waves we get in a similar manner $\mathbf{b}^{*S}_{n+1,D}$ with a form similar to (27) except that A_4, B_4 are to be replaced by similar amplitudes A_5, B_5 and θ^P_{n+1} replaced by θ^S_{n+1} given by

$$\sin\theta^S_{n+1} = \mathrm{Re}\left[(p\beta_1)\left(\frac{R^*_{n+1} - \sqrt{S^*_{n+1}}}{2}\right)\right] \tag{29}$$

and A_5/B_5 given by right hand side of (17) with λ_1, μ_1, θ replaced by $\lambda_{n+1}, \mu_{n+1}, \theta^S_{n+1}$. Thus,

$$\mathbf{b}^*_{n+1} = \mathbf{b}^{*P}_{n+1,D} + \mathbf{b}^{*S}_{n+1,D}. \tag{30}$$

The expressions for $\mathbf{b}^*_1$ and $\mathbf{b}^*_{n+1}$ from (25) and (30) can now be inserted in (14). If we denote the successive vectors [ ] in the expressions for $\mathbf{b}^{*P}_{1D}, \mathbf{b}^{*P}_{1U}, \mathbf{b}^{*S}_{1U}, \mathbf{b}^{*P}_{n+1,D}, \mathbf{b}^{*S}_{n+1,D}$ by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ and $\mathbf{v}_5$, we get the system of equations

$$[-\mathbf{v}_2, -\mathbf{v}_3, E^{-1}\mathbf{v}_4, E^{-1}\mathbf{v}_5][B_2, B_3, B_4, B_5]^T = e^{-iKz_1\cos\theta}\mathbf{v}_1. \tag{31}$$

Solving these equations we get "reflection coefficients", $R_D^{PP} = B_2$, $R_D^{PS} = B_3$ and "transmission coefficients", $T_D^{PP} = B_4$, $T_D^{PS} = B_5$.

4. Numerical calculations for model crust

In general the earth's continental crust consists of three layers: granitic, basaltic and a thin sedimentary layer at the top. For computations of reflection (and transmission) coefficients we consider the earth's crust beneath the Indo-Gangetic plain, which lies between the Himalayas and the Peninsula. Surface wave dispersion across this region has been investigated by several investigators [1]. Inversion of these data gives the crustal and upper mantle structure of the region. Such a model of crust is given by Bhattacharya [1] and is given below:

Region	Thickness of layer (km)	P-wave velocity (km/sec)	S-wave velocity (km/sec)	Density (gm/cm³)
1. Sedimentary	3·5	3·40	2·00	2·00
2. Granitic	16·5	6·15	3·55	2·60
3. Basaltic	23·0	6·58	3·80	3·00
4. Upper Mantle	∞	8·19	4·603	3·30

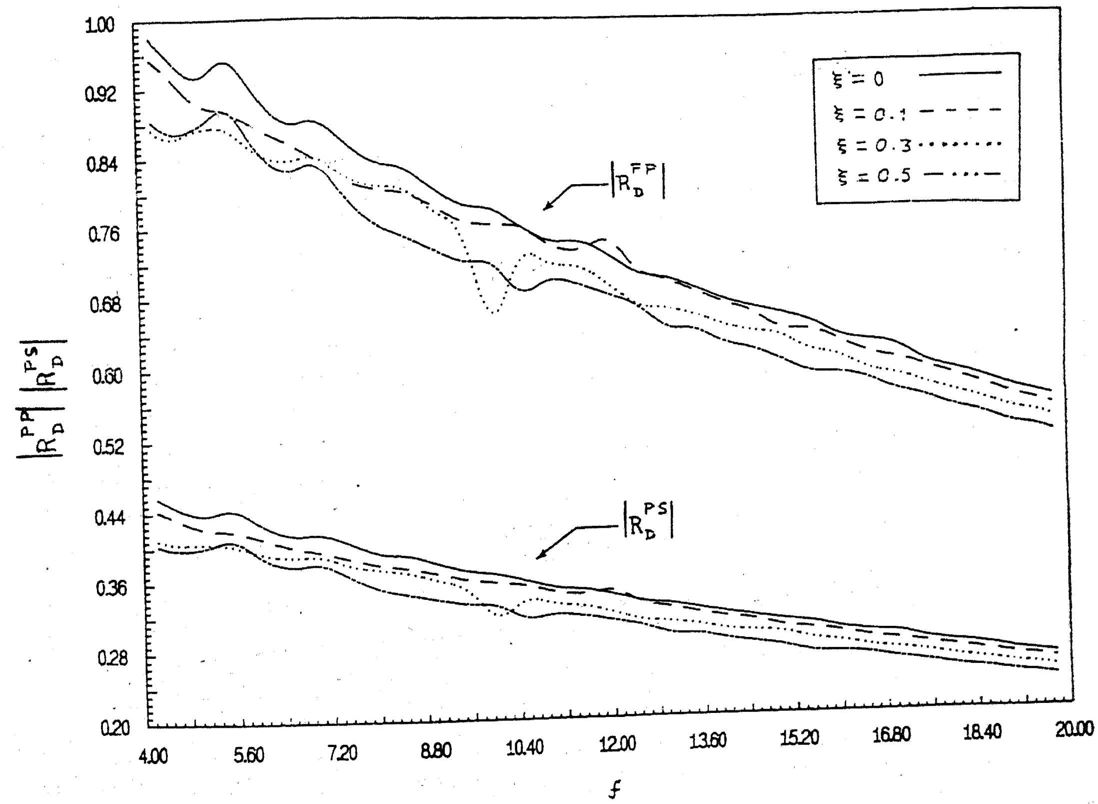
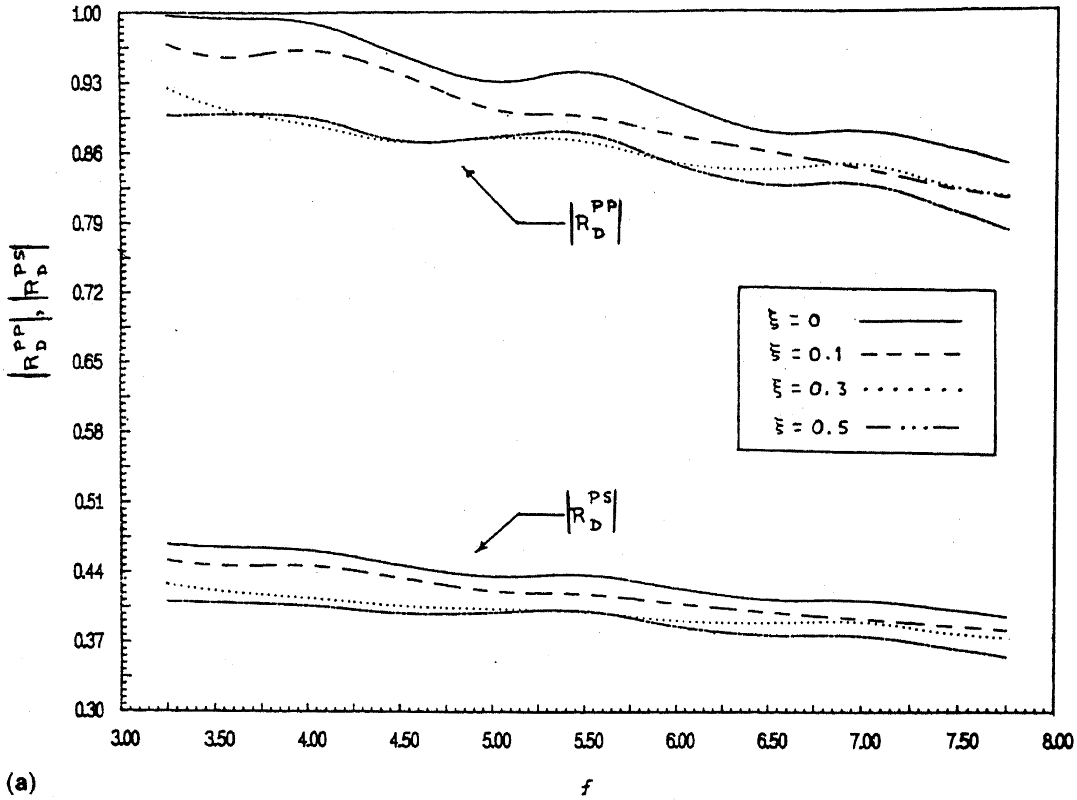
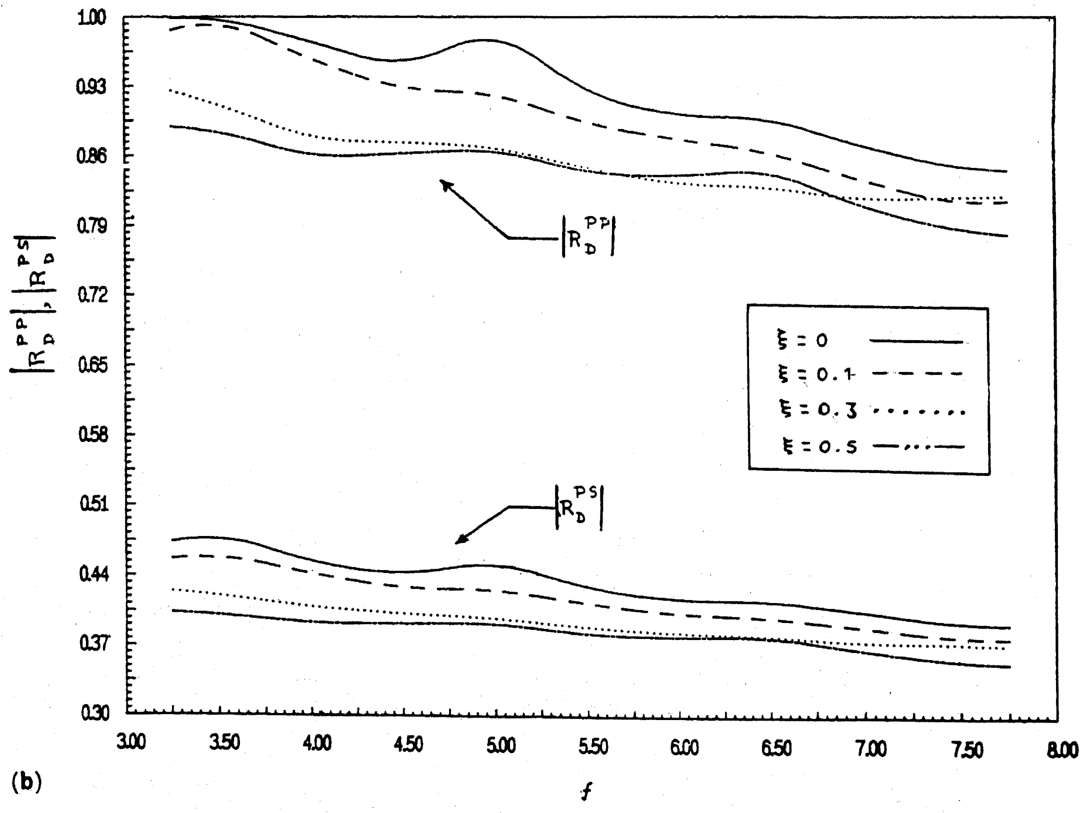


Figure 2. Amplitudes of reflection coefficients $|R_D^{PP}|$ and $|R_D^{PS}|$ for near vertical propagation: $\theta = 1°$.

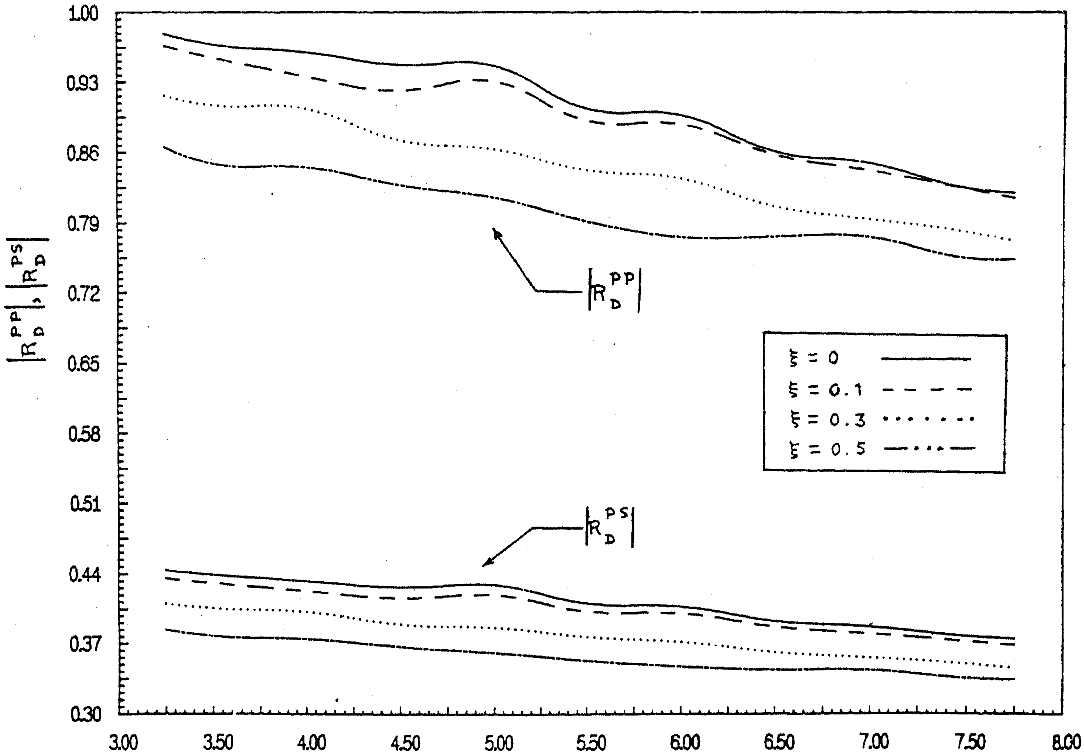


(a)



(b)

Figure 3a, b.



(c)

Figure 3 (*Continued*). Amplitudes of reflection coefficients $|R_D^{PP}|$ and $|R_D^{PS}|$ for wide angle propagation: (a) $\theta = 2°$ (b) $\theta = 5°$ (c) $\theta = 10°$.

The above yield the real part of Lamé constants of each layer. For the imaginary parts, the loss factors Q_α^{-1} of P-waves as given in Waters [12]

$$Q_\alpha(\text{granite}) = 311, \quad Q_\alpha(\text{basalt}) = 561$$

Q_α for sedimentary rocks is highly disperse, so, as an example we take old red sandstone for which $Q_\alpha = 93$ – a figure nearing the mean of dispersal of the values. Since the role of dissipation is small, the computed values are not expected to change very much on account of actual deviation. For the upper mantle we take $Q_\alpha = 849$ from data discussed in Ewing *et al* ([5], p. 278). Further data on imaginary part of shear modulus are provided by loss factor Q_β^{-1} of S-waves:

$$Q_\beta = \frac{4}{3}\left(\frac{\beta^2}{\alpha^2}\right) Q_\alpha$$

which is obtained from the often used assumption of zero dilatational viscosity [12, 5].

For initial stress-free basalt rock, strength $\leqslant 11{,}000$ atmospheres and if we consider hydrostatic pressure at a depth of 40 km to be present, the approximate range of the compressive initial stress $\xi = -S_{11}^*$ could be (0, 0·3). We therefore consider the parametric values $\xi = 0, 0{\cdot}1, 0{\cdot}3$ and $0{\cdot}5$, over a slightly enhanced range.

For selecting suitable frequency range, we consider the cases of seismic prospecting method of weight-dropping devices in which near vertical propagation takes place and explosion seismology technique where it is wide angle propagation. In the former case, f is taken within the range of 4–20 Hz [12] with $\theta = 1^\circ$. In the second case the range chosen is 3–8 Hz ([5], p. 202) with θ ranging from 2° to 10°.

In the numerical treatment of (31) we use Gauss's method for matrix inversion. The computation of the matrix exponentials in E (eq. (14)) is performed using the Cayley–Hamilton theorem. The latter requires the eigenvalues of matrices like A^* (eq. (11)), which is a simple task, because of the fact that the characteristic equation for the eigenvalues Λ of A^* reduces to a quadratic in Λ^2. The solution of (21) is performed by Muller's method.

We restrict presentation of the results to R_D^{PP} and R_D^{PS} only. In figures 2 and 3, we present the variation of the amplitudes of these quantities with frequency f, for different values of initial stress parameter ξ. In figure 2, the results for near vertical propagation are presented. There is a general trend of *diminution* in the reflection coefficients for increasing ξ, which becomes significant towards the higher frequencies in the band. The results for wide angle propagation for $\theta = 2^\circ, 5^\circ$ and 10° are presented in figures 3(a), (b) and (c) respectively. Here too, is a general trend of diminution in the reflection coefficients for increasing ξ. The trend of diminution increases with increasing θ.

It may be mentioned here that when P-waves propagate *vertically* in an unbounded initially stressed *homogeneous* medium, there is no effect of initial stress on the velocity of propagation [4]. This fact can be verified from (18), (19), with $k = 0 = p = \theta$ for the case. For reflections from the stack, there are no up going S-waves, $A_1 = 0$ (verifiable by the limit $\theta \to 0$ in (17)), $A_z = A_4 = 0$ and the reflection and transmission coefficients B_z, B_4 are given by a pair of equations similar to (31).

5. Conclusion

The focal regions at plate boundaries of the earth prior to earthquakes are at considerable thrust due to tectonic movement. For understanding the reflection and transmission characteristics of body waves in such regions appropriate mathematical model studies are required. Herein, is considered, a stack of dissipative layers under uniaxial thrust to which the theory of incremental deformation given by Biot [2] is applicable. The governing equations can be compactly treated by matrix method, as in the case of initial stress free case, for the reflection and transmission of body waves. A numerical model study of a stack of four layers – sedimentary, granitic, basaltic and upper mantle, for near vertical as well as wide angle reflections, shows significant diminution in the magnitudes of both P and S waves.

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