Probing MeV Dark Matter at Low–Energy $e^+e^−$ Colliders

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It has been suggested that the pair annihilation of Dark Matter particles $\chi$ with mass between 0.5 and 20 MeV into $e^+e^−$ pairs could be responsible for the excess flux (detected by the INTEGRAL satellite) of 511 keV photons coming from the central region of our galaxy. The simplest way to achieve the required cross section while respecting existing constraints is to introduce a new vector boson $U$ with mass $M_U$ below a few hundred MeV. We point out that over most of the allowed parameter space, the process $e^+e^− \rightarrow U\gamma$, followed by the decay of $U$ into either an $e^+e^−$ pair or an invisible ($\nu\bar{\nu}$ or $\chi\bar{\chi}$) channel, should lead to signals detectable by current $B$–factory experiments. A smaller, but still substantial, region of parameter space can also be probed at the $\Phi$ factory DAΦNE.

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Within the context of Einsteinian gravity, evidence for the existence of Dark Matter (DM) is overwhelming $^1$. Analyses of the cosmic microwave background anisotropy, and other data on the large scale structure of the Universe, have determined many cosmological parameters with unprecedented precision $^2$. Along with analyses of Big Bang nucleosynthesis (BBN) $^3$, this data also show the DM must be largely non–baryonic. Since neutrinos can contribute only a small fraction $^4$, this strongly points towards the existence of an exotic, neutral, stable particle $\chi$, with relic density $\Omega_\chi$ satisfying $^2$

$$\Omega_\chi h^2 = 0.113 \pm 0.0085 \quad \text{(at 1σ)},$$

where the scaled Hubble constant $h \simeq 0.7$ $^2$.

Dark Matter particles should clump due to gravitational attraction. At the galactic center, their density might be so high that their annihilation into lighter, known particles could lead to visible signals. Final states containing hard photons play a special role in this, since photons travel in straight lines and are easy to detect.

Looking for an excess of hard photons from the center of our galaxy, the INTEGRAL satellite indeed observed a large flux of photons with energy of 511 keV $^4$. This sharp line can only come from the annihilation of non–relativistic $e^+e^−$ pairs. However, most estimates of positron production by astrophysical sources fall short of the required flux giving rise to the speculation $^5$ that the annihilation of light DM particles $\chi$ into $e^+e^−$ final states could be responsible for this signal. While the mass $m_\chi$ can exceed $m_e$ substantially (since the positrons produced in $\chi$ annihilation quickly lose energy through scattering on neutral atoms), it has been pointed out $^6$ that annihilation into $e^+e^−\gamma$ final states, in spite of being a higher-order process, would over–produce MeV photons if $m_\chi > 20$ MeV, leading to $m_e \leq m_\chi \leq 20$ MeV. $^2$

Recently, it has been argued $^7$ that there exists evidence for a non–astrophysical source of MeV photons, in which case the upper end of the range $^2$ would be favored.

In order to produce the required flux of positrons, one needs a crossing section $\sigma_{\text{ann}}(\chi\bar{\chi} \rightarrow e^+e^-)$

$$10^{-3} \text{ fb} \leq v \sigma_{\text{ann}} \cdot (m_\chi/1 \text{ MeV})^{-2} \cdot \kappa \leq 1 \text{ fb}. \quad (3)$$

Here $v$ is the relative velocity of the two $\chi$ particles in their c.m.s frame, and $\sigma$ is to be computed at $v \sim 10^{-3}c$. Note that models of the galaxy fix the DM mass density; the number density, which enters quadratically in the calculation of the positron flux, therefore scales as $m_\chi^{-1}$. Finally, $\kappa = 1$ if $\chi$ is self–conjugate (i.e. a Majorana particle), whereas $\kappa = 2$ if $\chi \neq \bar{\chi}$, since then only half of all encounters of DM particles can lead to annihilation events. To be on the safe side, in $^3$ we have expanded the range given in the original publication $^2$ by an order of magnitude in either direction. This may be overly conservative $^2$, since one here only needs the DM density averaged over a significant volume, which is thought to be better known than that right at the galactic center.

A cross section in the range $^3$ implies that $e^+e^- \leftrightarrow \chi\bar{\chi}$ reactions were in equilibrium down to temperatures well below $m_\chi$. Since successful BBN requires a starting temperature $T \geq 0.7$ MeV $^2$, it is safe to assume that $\chi$ particles indeed were in thermal equilibrium. Their relic density then turns out to be $^10$ inversely proportional to the thermal average of their total annihilation cross section into all final states containing only SM particles:

$$\Omega_{DM} h^2 \propto \langle v \sigma_{\text{ann}} \rangle^{-1}. \quad (4)$$

Given the mass constraint $^2$, to the leading order, annihilation is possible only into $e^+e^-$ and $\nu\bar{\nu}$ final states; the first channel must exist, since $\chi$ particles are supposed to annihilate into $e^+e^-$ pairs even today. Assuming $\chi$ forms (nearly) all DM, the relic density resulting from eq. $^1$ must fall in the range $^11$. This constraint, interpreted using eq. $^4$, and $^5$ are compatible only if the present $\sigma_{\text{ann}}$ is strongly suppressed compared to that at decoupling. This is most easily achieved $^5, 11, 12$ if $\chi\bar{\chi}$ annihilation proceeds only from a $P$–wave initial state, in which case $\sigma_{\text{ann}} \propto v^2$; note that $v^2 \sim 0.1$ when $\chi$ particles decoupled, while $v^2 \sim 10^{-6}$ today.
In a renormalizable theory, some particle must mediate $\chi \rightarrow e^+e^-$ annihilation. The simplest possibility is to introduce a light spin–1 boson $U$ coupling to both $e^+e^-$ and $\chi\bar{\chi}$ states. If $\chi$ is a Majorana spin–1/2 fermion or complex scalar, $\sigma_{\text{ann}}(\chi \rightarrow f \bar{f})$ is given by

$$
\sigma_{\text{ann}} = \frac{\beta_f g_f^2}{12\pi s} \left\{ \left( s - 4m_f^2 \right) \left[ s\Sigma_f + m_f^2 \left( 6\Pi_f - \Sigma_f \right) \right] \left( s - M_U^2 \right)^2 + \frac{\xi}{M_U^2} \left( \frac{m_f^2 m_{\chi}^2}{M_U^2} \right)^2 \left( 3\Sigma_f - 6\Pi_f \right) \right\},
$$

where $\xi = 1$ (0) for spinor (scalar), $\beta_f = \sqrt{1 - 4m_f^2/s}$, $\Pi_f = g_{fL}g_{fR}$ and $\Sigma_f = g_{fL}^2 + g_{fR}^2$ with $g_{fL}$ and $g_{fR}$ being the left– and right–handed $U$ couplings. The $U\chi\bar{\chi}$ coupling $g_f$ is purely axial vector for a Majorana $\chi$.

The first line in eq. (5) is a pure $P$–wave contribution. We discount, henceforth, a Dirac $\chi$ as it would, in general, have a large $S$–wave contribution to $\sigma$ thereby making it difficult to reconcile the constraints $[12]$ and $[13]$.

Since $\Gamma_U \ll M_U$ for realistic couplings, the usual $[10]$ non–relativistic expansion of the cross section in powers of $v$ breaks down if $2m_\chi \sim M_U$, and the thermal averaging has to be done numerically $[13]$. The strong velocity dependence of the cross section $(s - 4m_\chi^2 \sim v^2 m_\chi^2)$ for $v^2 \ll 1$ in eq. (5) also has to be treated properly. For simplicity, we assume a thermal velocity distribution, with a rather high temperature $T = 10^{-4}m_\chi$: smaller temperatures, which are probably more realistic, would require larger couplings, which would be easier to test at $e^+e^-$ colliders.

**Fig. 1:** Parameter space for a Majorana $\chi$ with $g_{eL} = g_{eR} = 0$ and $g_\chi = 10g_{eR}$. In between the solid curves, $\chi$ has the correct relic density, and the correct cross section to explain the flux of 511 keV photons emerging from the galactic center; the red (black) curves correspond to $2m_\chi < (>)M_U$. The dotted (blue) line indicates the upper bound on $g_{eR}$ from $(g_\chi - 2)$ measurements. The dashed curves show the maximal sensitivity of DAΦNE to $e^+e^- \rightarrow U\gamma$ production, for $U \rightarrow e^+e^-$ (upper, dark green, curve) and $U$ decaying invisibly (lower, magenta curve). The DM constraints are essentially independent of the ratio $g_\chi/g_{eR}$, whereas the $g_\chi - 2$ constraint as well as the sensitivity limits are independent of $g_{eR}$. Results for $g_{eR} = g_{eL}$ are similar to those for scalar $\chi$ (Eq. 2).

Figs. 1 (2) show the parameter space spanned by $M_U$ and the product of couplings $g_\chi g_{eR}$ for the case that $\chi$ is a Majorana fermion (complex scalar). The regions allowed by the constraints $[12]$ are enclosed by the solid curves, with distinct allowed regions for $2m_\chi < M_U$ ($2m_\chi > M_U$). We consider only $g_{eL} = 0$ since this implies $g_{eL} = 0$ if $U$ is a gauge boson of a gauge group $G_U$ that simply multiplies the gauge group of the Standard Model (SM). Note that we scattering data imposes the very strong constraint $[12]$ $g_{eL} + g_{eR}^2 < M_U^2 G_F$. For $g_{eL} = g_{eR}$ and $g_{eR} = 0$, this would exclude the entire DM–allowed range (which is invariant under $g_{eR} \leftrightarrow g_{eL}$) of Fig. 1, and most of Fig. 2. Finally, for $g_{eL} = 0$, DM constraints apply essentially to the product $g_{eR}g_\chi$. The allowed values matter only if $2m_\chi \sim M_U$, in which case the value of the decay width $\Gamma_U$ is irrelevant.

Only a narrow range of couplings is allowed in Fig. 1 on account of an $S$–wave contribution surviving for our choice of $g_{eL} = 0$. Even though proportional to $m_\chi^2$, for small $m_\chi$ it would lead to a present annihilation cross section above the range $[13]$, unless $2m_\chi$ is quite close to $M_U$. In the latter case, the kinetic energy of the DM particles in the early universe was sufficient to allow efficient annihilation through the exchange of on–shell $U$–bosons, whereas those at the galactic center are so slow that they can only annihilate through the exchange of off–shell $U$–s. This also gives the required large enhancement of the cross section at decoupling relative to that at the galactic center, even for a non–vanishing $S$–wave contribution. However, if $2m_\chi$ is too close to (but still below) $M_U$, the relic density constraint $[14]$ will require very small couplings, too small to satisfy the constraint $[15]$. For small $M_U$, the allowed range of $m_\chi$ values is, therefore, very narrow, e.g. 0.72 MeV $\leq m_\chi \leq 0.76$ MeV for $M_U = 2$ MeV. For larger $M_U$, and correspondingly larger $m_\chi$, the $S$–wave contribution $\propto m_\chi^2$ becomes less important, and a wider range of values of $m_\chi$ is allowed, e.g. $m_\chi > 11$ MeV for $M_U = 60$ MeV; this lower bound on $m_\chi$ increases only slowly for even larger values of $M_U$. Recall that values of $m_\chi$ close to the upper bound of 20 MeV are preferred $[16]$ since they allow to describe an excess of MeV photons from the galactic center.

With the $S$–wave contribution vanishing for a scalar $\chi$, the allowed parameter space is much wider (Fig. 2). The DM constraints are now compatible with the entire range of $m_\chi$ except for values very close to $M_U/2$ where today’s $\sigma_{\text{ann}}$ comes out too small (for $2m_\chi$ just below $M_U$) or too large (for $2m_\chi$ just above $M_U$) $[23]$ if the couplings are chosen to satisfy the constraint $[14]$. For example, for $M_U = 2$ MeV, the range 0.91 MeV $\leq m_\chi \leq 1.04$ MeV is excluded. Note that in both Figures, smaller couplings correspond to larger (smaller) $m_\chi$ if $2m_\chi < (>)M_U$.

Since it is natural to assume $g_{eL} = 0$ for our choice of $g_{eR}$, the only relevant model–independent laboratory constraints come from processes involving only electrons, the most sensitive being the anomalous magnetic mo-
In the opposite limit, namely together exclude scenarios with $g_2$ and $g_1 = 1$. Notation is as in Fig. 1, except that the indicated sensitivities are now those that can be achieved at the $B$–factories.

moment of the electron $\chi$ at 95% C.L. The resulting upper limit on the product $g_e g_X$ is shown as dotted (blue) line in Figs. 1 and 2. While the data weakly favor a small positive contribution from $U$–boson loops can account for it only if $g_{eL} \approx g_{eR}$. This would either imply a sizable $U \bar{v}$ coupling, or a model where the $U$ gauge group is embedded non–trivially into the electroweak gauge group, rendering the construction of a complete, renormalizable model more complicated. As long as $g_{eL}$ is zero, $(g_e - 2)$ imposes a rather severe constraint (Figs.1&2) entirely independent of $g_X$. With DM–constraints operating essentially on the product $g_X g_{eR}$, the parameter space that also satisfies the $(g_e - 2)$ constraint becomes larger for larger $g_X$. Together, the constraints exclude scenarios with $g_X < g_{eR}$. In the opposite limit, namely $g_X = 1$ ($g_{eR}$), they together exclude scenarios with $M_U$ much above 200 MeV, as shown in Fig. 2. Other constraints on the $U$–boson have been discussed in the literature in the literature; however, they are more model–dependent in that they involve couplings that are not required from DM phenomenology. Hence we ignore these constraints here, and instead turn to a discussion how this model can be tested at $e^+e^-$ colliders. Scenarios with $g_{eL} = 0$ but $g_e \neq 0$ may also be favored by BBN.

As noted above, the $U$ boson must couple to $e^+e^-$ pairs in order to explain the excess flux of 511 keV photons. The process $e^+e^- \rightarrow U\gamma$ will therefore have a non–vanishing cross section if the cms energy $\sqrt{s} > M_U$:

$$\frac{d\sigma}{d\cos \theta} = \frac{\alpha}{2\pi} \frac{g_{eL}^2 + g_{eR}^2}{2s^3 (s - M^2_U)} \left[ \frac{s^2 + M^4 + (s - M^2_U)^2}{\sin^2 \theta} - \frac{(s - M^2_U)^2}{2} \right],$$

where $\alpha$ is the fine structure constant and $\theta \equiv \theta_0$ is the emerging angle of the photon. Since existing constraints imply $M_U \leq 0.2$ GeV, the produced $U$ bosons can decay only into $e^+e^-$ pairs, of DM particles $\chi$, or perhaps neutrinos. This gives rise to two possible collider signatures, with $e^+e^-\gamma$ and $\gamma\rightarrow \text{“nothing”}$ final states and we explore the detectability of both. We focus on low–energy colliders, since the signal cross section will drop $\propto 1/s$ for $s \gg M^2_U$. Specifically, we analyze the two signatures at the $\Phi$ factory DAΦNE, which operates at $\sqrt{s} = m_\phi = 1.02$ GeV, and at the $B$ factories, which operate at $\sqrt{s} = 10.6$ GeV; the reduced cross section at the latter is over–compensated by the higher accumulated luminosity ($\sim 500$ fb$^{-1}$ for both $B$–factories combined, as compared to $\sim 500$ pb$^{-1}$ at the $\Phi$ factory).

The $e^+e^-\gamma$ final state receives a large contribution from $\mathcal{O}(\alpha^3)$ QED processes. But whereas the signal events have invariant mass of the outgoing $e^+e^-$ pair $M_{ee}$ very close to $M_U$, the background distribution has peaks at $M_{ee} \approx \sqrt{s}$ (from $t$–channel diagrams with soft $\gamma$ emission) and at a few $m_e$ (from $s$–channel diagrams). We therefore require that (i) the produced particles must not be too close to the beam pipe, $|\cos \theta| < 0.9$ for $i = e\gamma, \gamma$; (ii) $M_{ee} \in [M_U - 1 \text{ MeV}, M_U + 1 \text{ MeV}]$, where the spread is given by the mass resolution of the KLOE detector; we assumed that the BaBar and BELLE detectors at the $B$ factories have similar resolution. The second cut implies that the photon is quite energetic, since $E_{\gamma} = (s - M^2_{ee})/(2\sqrt{s})$. The signal is considered detectable if $N_{\text{signal}} > 5\sqrt{N_{\text{backgd}}}$.

The resulting sensitivity limits are indicated by the (upper) dashed (dark green) curves in Figs. 1 and 2. Signal and background have been calculated with the CompHEP package, augmented to include $U$ bosons. Note that these are maximal sensitivities in that we assume a branching ratio $B(U \rightarrow e^+e^-) = 1$. This may well be realistic for $2m_{\chi} > M_U$, but for the assumptions made in the Figures is not realistic otherwise; since we assumed $\Phi \gg g_{eR}, g_{eL}$ in these plots, the invisible $U \rightarrow \chi\bar{\chi}$ decay mode will dominate if it is open. However, since the $e^+e^-\gamma$ final state is background dominated, the sensitivity limit on $g_{eR}$ only scales like $B(U \rightarrow e^+e^-)\alpha^{-0.25}$.

Thus, DAΦNE can probe couplings $g_{eR}$ down to about $10^{-3}$, whereas the $B$ factories would be sensitive to couplings as small as $3 \cdot 10^{-4}$. In both cases the sensitivity gets somewhat worse at small $M_U$ as the background peaks at small $M_{ee}$. Note that these sensitivity limits are completely independent of $g_X$, and of the nature of the DM particle (scalar or Majorana fermion). In particular, for the scenario of Fig. 1, the $B$ factories should be able to probe the entire parameter space with $2m_{\chi} > M_U$ in this channel. The scenario in Fig. 2 is the worst case scenario for collider experiments. With the DM constraints essentially fixing the product of couplings $g_{eR}g_X$ (for $g_{eL} = 0$), a large $g_X$, as in Fig. 2, therefore leads to small $g_{eR}$, and hence small production cross sections.
As already noted, for \( g_\chi \gg g_{eR} \) we expect a large invisible branching ratio for the \( U \) boson if \( 2m_\chi < M_U \). The signal, then, consists of a single monochromatic photon with \( E_\gamma = (s - M_U^2)/(2\sqrt{s}) \). Unfortunately, the experimental resolution on \( E_\gamma \) is considerably worse than that for \( M_\chi \). On the other hand, the physics (SM) background now comes from \( \nu\gamma \) final states, and is thus \( O(\alpha G_F^2 s) \). After simple acceptance cuts, \( E_\gamma > 100 \) MeV, \(|\cos\theta| < 0.9\), the background is already completely negligible at the \( \Phi \) factory. Even at the \( B \) factories we expect \( \ll 1 \) background event once we require \(|\cos\theta| < 0.9\), \( E_\gamma > 0.5\sqrt{s} - 200 \) MeV \[22\]. In other words, the photon plus “nothing” signal is rate, rather than background, limited. We neglect instrumental backgrounds here as these are very specific to the experiment.

The corresponding sensitivity limits are shown by the (lower) dashed (maroon) lines in Figs. 1 and 2. We again show the maximal sensitivity, i.e. here we assumed 100% branching ratio for invisible \( U \) decays. We see that DAΦNE can probe a coupling \( g_{eR} \gtrsim 8 \cdot 10^{-5} \) in this channel, while the \( B \) factories would be sensitive to \( g_{eR} \gtrsim 3 \cdot 10^{-5} \), independent of \( g_\chi \). In particular, the \( B \) factories would probe the entire parameter space of Fig. 1 with \( M_U \geq 4 \) MeV in this channel. Even in the worst case scenario of Fig. 2, much of the DM-allowed parameter space would lead to a detectable signal.

A dominant invisible decay mode for the \( U \) is not realistic if \( 2m_\chi > M_U \), unless \( U \) also has significant couplings to neutrinos, which however is disfavored. We therefore also investigated \( \chi\chi\gamma \) production through off-shell \( U \) exchange. The signal cross section is now proportional to the product \( g_{eR} g_\chi \) (for \( g_{eR} = 0 \), just like the DM annihilation cross section \[6\]). Using a modified CompHEP, we find a detectable signal at the \( B \)-factories (\( \geq 10 \) events with \(|\cos\theta| < 0.9\), \( E_\gamma > 0.5\sqrt{s} - 200 \) MeV in 500 fb\(^{-1} \)) if \( g_{eR} g_\chi \geq 2.4 \cdot 10^{-4} \). This limit depends only weakly on \( M_U \) and \( m_\chi \), so long as \( m_\chi \) is not very close to \( M_U/2 \). This would be sufficient to probe at least the upper end of the DM allowed region with \( 2m_\chi > M_U \) in Fig. 2.

In summary, we carefully delineated the allowed parameter space of models with MeV Dark Matter whose annihilation is mediated by the exchange of a spin-1 \( U \) boson. Model parameters must be chosen such that the correct thermal relic density and the correct present annihilation cross section are reproduced; the latter is motivated by the signal of 511 keV photons from the center of our galaxy. The parameter space is further constrained by the anomalous magnetic moment of the electron. We found that these models can be tested decisively at existing low-energy \( e^+e^- \) colliders if the \( U \) boson has similar (or greater) coupling strength to electrons as to DM particles (Fig. 1). Such models would be relatively easy to construct by introducing an additional gauge group with small coupling constant. Models where the coupling of the \( U \) boson to DM particle is much stronger than that to electrons are much more difficult to probe at colliders (Fig. 2); such a pattern could emerge if \( U \) couples to electrons only through mixing with SM gauge bosons, but has direct coupling to DM particles. The single photon plus “nothing” channel allows to probe much of the parameter space compatible with the DM constraints even in this unfavorable situation. We therefore come to the rather surprising conclusion that the solution of the Dark Matter puzzle might be found at the existing \( \Phi \) and \( B \) factories.

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[19] See \url{http://www.lnf.infn.it/kloe/kloedef.html}.
[21] There is a tiny range of \( m_\chi \) values very, very close to \( M_U/2 \) in 500 fb\(^{-1} \) at the Φ factory. Even at the \( B \) factories.
[22] For a less restrictive cut on \( E_\gamma \) we do expect a significant number of background events at the \( B \) factories. In fact, BELLE and BaBar should be able to determine the number of neutrinos to a few % using this final state.