ON THE VOLTAGE AMPLIFICATION BY ELECTRIC WAVE-FILTERS TERMINATED IN NEGATIVE RESISTANCES.

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1. *Introduction.*

A wave-filter is a device which transmits freely a certain band of frequencies and attenuates infinitely all other frequencies. An ideal filter of purely reactive elements and non-reactive characteristic impedance terminated in its characteristic impedance would produce no attenuation in the transmission band and infinite attenuation at all other frequencies. Commercial filters, however, fall short of this ideal and the frequencies in the transmission band also undergo a finite attenuation. The attenuation in the filter network is due to the fact that the filter elements cannot be made purely reactive in actual practice and there is an appreciable loss in the condensers at high frequencies. In addition, if the filter is inserted between two impedances different from the filter characteristic impedance, there is a further loss due to reflections at the input and output ends. The effect of a filter inserted between two impedances is therefore the combination of attenuation and reflection effects. The network attenuation can be reduced by using iron or permalloy core inductances and condensers having low loss at high frequencies. The attenuation due to reflection can be reduced by matching both the input and output impedances with filter characteristic impedance. Still the attenuation in the transmission band cannot be reduced below a certain value. In a multiplex communication system, several filters each consisting of two or more sections may have to be introduced in a short section of 200 miles as shown in Fig. 1. On an open-wire line section between the carrier terminal and a carrier repeater station, a voice-frequency channel is simultaneously worked with the carrier channel so that between the points x and y in the circuit the carrier channel frequencies have to pass through five filters, namely, A, B, C, D and E. Assuming an attenuation of three decibels in the transmission band of each filter, the total filter attenuation of about 15 decibels, to which if the line attenuation of about 15 decibels and equaliser attenuation of 4 decibels be added, the total loss will amount to about 34 decibels.

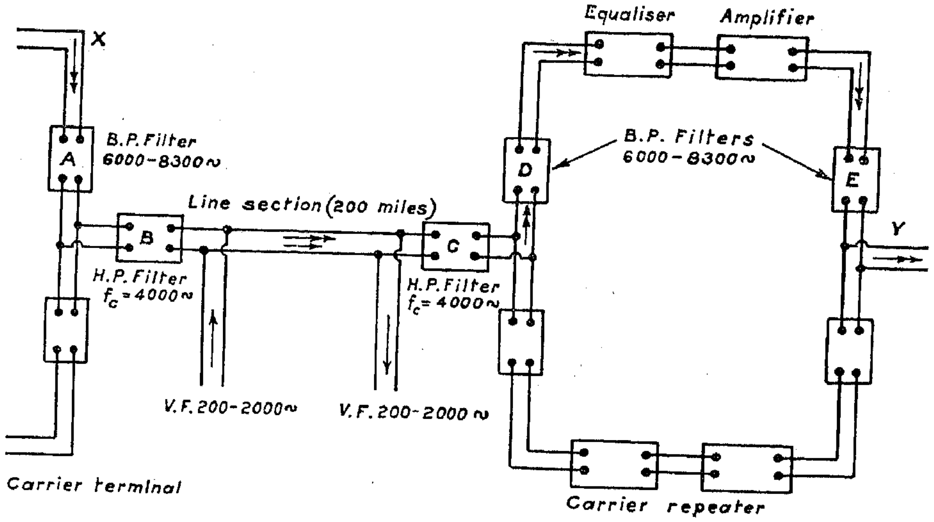

FIG. 1. Simultaneous working of Voice-Frequency and Carrier Channels.
Carrier Channel——→→
V.F. Channel——→

The repeater must therefore be of the high gain type having a gain of at least 30 decibels. If, on the other hand, the filter loss of 15 decibels could somehow or other be reduced to zero, low gain repeaters or high gain repeaters spaced at greater distances apart could be used. The reduction of attenuation in the transmission band of a wave-filter is also desirable in other fields of communication.

The author has found out that if a wave-filter be terminated in the negative resistance $-Z_0$ where Z_0 is a pure resistance of the same value as the filter characteristic impedance, a large gain can be obtained instead of attenuation for frequencies in the transmission band due to the negative reflection loss or reflection gain. The result is of great value to the communication engineers as the same device can be used to filter out a particular band of frequencies and amplify it at the same time. The amplitude distortion is within the limits of tolerance as laid down by the Comité Consultatif International Des Communications Telephoniques a Grande distance. The cut-off is much sharper than that obtained with the positive resistance termination Z_0. A filter network having zero attenuation in the transmission band can be manufactured by joining several sections with a negative resistance element between any two sections.

2. *The Negative Resistance.*

When a source of power is applied to the terminals of an ordinary positive resistance a current flows in at the terminal connected to the positive pole of the source and out at the other terminal. This direction of current flow is positive and the value of the resistance R is given by the relation R=E/I when E is the applied voltage and I is the current in amperes. Similarly, a definite current I may be passed through the resistance and a potential difference E=RI will appear across its terminals. With positive resistances it makes no difference whether we apply an e.m.f. or pass a current. The resistance absorbs energy from the circuit at a rate W given by $W=I^2R$ watts.

It is possible to construct a device which has the property of keeping the ratio of the voltage across a pair of terminals to the current at the terminals constant but with the relative direction of the voltage and current opposite to that which a positive resistance would give. In such devices the resistance is negative and the device contributes power to the circuit with which it is connected. Each such device necessarily includes a source of energy such as a battery and some means such as a vacuum tube for controlling the delivery of this energy to the circuit. There are two varieties of such devices, namely, (1) in which the internal arrangement of the device is such that if a definite voltage is applied to the terminals, a current flows in a direction opposite to the applied e.m.f., and (2) in which if a definite current is passed through the system, the potential difference across the terminals will be opposite in direction to that caused by a positive resistance. These two arrangements are entirely different and cannot be used interchangeably in a given circuit. The voltage-current characteristics of a positive and a negative resistance are shown in Fig. 2 (*a*) and 2 (*b*) respectively. There are many forms of negative resistances, namely, the electric arcs, the imperfect metallic contacts, the ordinary triodes and screen grid tubes used as dynatrons and a one-way amplifier in which the input and output terminals are interconnected. The primary

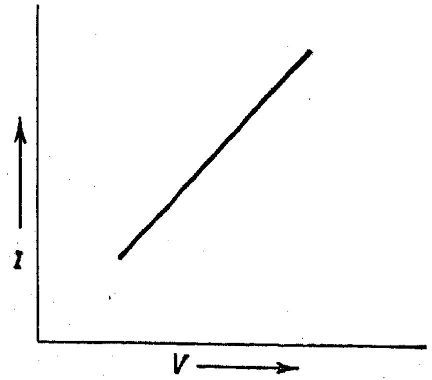

Fig. 2(*a*). Characteristic of a Positive Resistance.

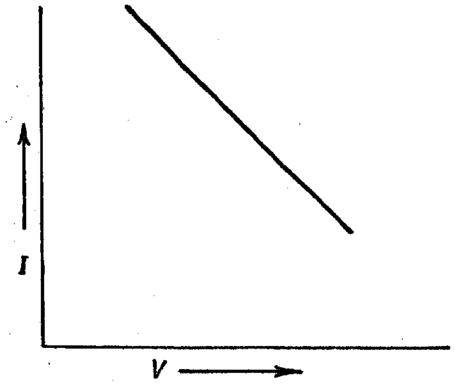

Fig. 2(*b*). Characteristic of a Negative Resistance.

difficulty is the instability of the devices. The thermionic vacuum tubes are no doubt more stable than the electric arcs but even the negative resistance obtained with commercial tubes does not stay constant over long periods of time. In this paper, the screen grid tube used as a dynatron has been employed to give the negative resistance and all precautions have been taken to make it stable over the period of observation. Another difficulty is that it is not easy to vary the negative resistance value of a given vacuum tube system beyond certain limits. External circuits, however, enable one to accomplish variations beyond these limits. Fig. 3 (*a*) shows how the absolute value of the given negative resistance $-r'$ can be reduced and Fig. 3 (*b*) shows how $-r'$ can be increased.

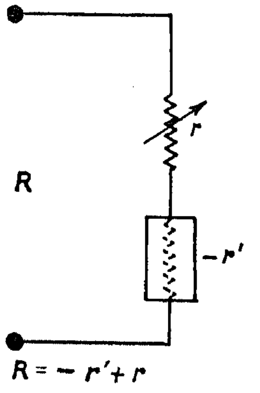

FIG. 3(*a*).

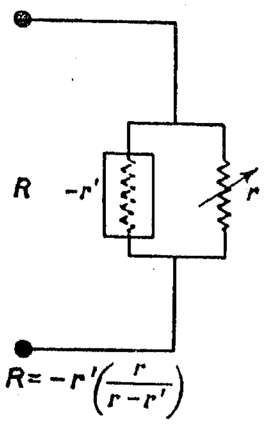

FIG. 3(*b*).

3. *Termination of Wave-Filters.*

The characteristic impedance of a single filter section is the impedance of the infinite number of such sections linked together. It follows that if a network of finite number of sections be terminated in the output by the characteristic impedance and the impedance measured across the input terminals, it shall be equal to the characteristic impedance. If Z_0 be the characteristic impedance and Z_1 and Z_2 be the series and shunt arm impedances respectively, the value of the characteristic impedance calculated from the above definition will be given by $Z_0 = \sqrt{(Z_1Z_2 + \frac{1}{4}Z_1^2)}$. If Z_1 and Z_2 are assumed to be pure reactances, it is evident that Z_0 will be a pure resistance at some frequencies and a pure reactance at others. Now if the characteristic impedance of a finite number of sections is a pure resistance and each section is made up of pure reactances, all power will be delivered to the non-reactive termination and there will be no attenuation. If the characteristic impedance of a finite number of sections is a pure reactance, no power will be delivered to the termination and there will be infinite attenuation. Since the characteristic impedance changes from a pure resistance to a pure reactance as one passes from the transmission to the attenuation band, there will be no attenuation in the transmission band and infinite attenuation outside this band. In commercial filters, however, as the elements are not purely reactive and the condenser losses appreciable, there is always a small attenuation in the transmission band. Now suppose a wave-filter, having a characteristic impedance in the transmission band of Z_o which is a pure resistance, is fed from a source of impedance Z_o and also terminated by a pure resistance Z_o as shown in Fig. 4. Then in the transmission band, there will be either no attenuation or a finite attenuation. After the cut-off frequency is passed the impedance looking into 1—2 terminals Z_{12} is neither a pure resistance nor equal to Z_o in value. Z_{12} and Z_{34} will be given by

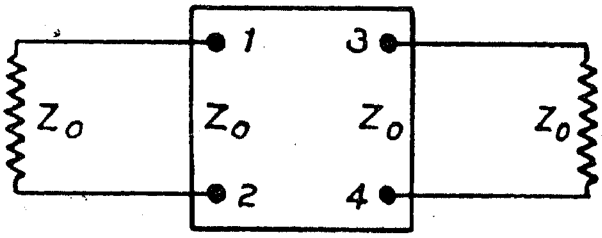

FIG. 4. A Wave-Filter fed from Z_0 and terminated in Z_0.

$$Z_{12} = Z_{34} = Z_o \left(\frac{Z_o + Z' \tanh \gamma}{Z' + Z_o \tanh \gamma}\right) \tag{1}$$

where Z' is the characteristic impedance in the attenuating band and γ = propagation constant. Hence there will be reflection losses at the input and output ends due to the mismatching of the impedances Z_o and Z_{12} and Z_o and Z_{34} respectively resulting in attenuation. The effect of a filter inserted between two impedances is a combination of attenuation and reflection effects.

In general, the total insertion loss is made up of two portions, namely, (1) the network attenuation, and (2) the reflection losses due to mismatching of terminations. The network attenuation is given by the relation

$$D_1 = 20 \log_{10} \left(1 + \frac{Z_1}{2Z_2} + \frac{Z_o}{Z_2}\right) \text{ decibels} \tag{2}$$

The input end reflection loss is given by

$$D_r = 20 \log_{10} \frac{Z + Z_o}{\sqrt{4ZZ_o}} \text{ decibels} \tag{3}$$

where Z = input impedance.

The output end reflection loss is given by

$$D'_r = 20 \log_{10} \frac{Z' + Z_o}{\sqrt{4Z_oZ'}} \text{ decibels} \tag{4}$$

where Z' = output impedance. The total insertion loss of the network inserted between impedances Z and Z' is then in decibels given by

$$D_T = 20 \log_{10}\left(1+\frac{Z_1}{2Z_2}+\frac{Z_0}{Z_2}\right)+20\log_{10}\frac{Z+Z_0}{\sqrt{4ZZ_0}}$$
$$+20\log_{10}\frac{Z'+Z_0}{\sqrt{4Z_0Z'}}-20\log_{10}\frac{Z+Z'}{\sqrt{4ZZ'}} \qquad (5)$$

A reflection loss $20 \log_{10} \frac{Z+Z'}{\sqrt{4ZZ'}}$ has been subtracted because the reference condition is one of impedance mismatching.

Coming now to the problem under consideration, it can be shown that if a network having impedance Z_1 and Z_2 ohms in the series and shunt arms respectively be terminated in the output by a *pure negative resistance* $-Z_0$ where $Z_0 = \sqrt{(Z_1Z_2+\frac{1}{4}Z_1^2)}$, the impedance measured across the input terminals of the network will be $-Z_0$ (Fig. 5). The impedance Z_{AB} is given by $Z_{AB} = \frac{1}{2}Z_1+Z_p$ where Z_p = parallel impedance. The parallel impedance in this case is given by $\frac{1}{Z_p}=\frac{1}{Z_2}+\frac{1}{\frac{1}{2}Z_1-Z_0}$, so that

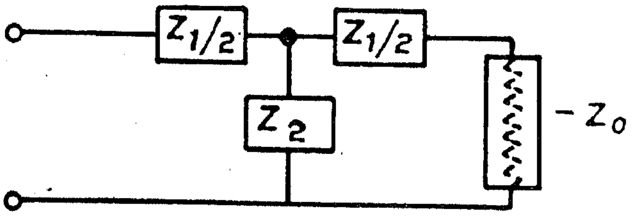

FIG. 5. A filter section terminated in the negative resistance $-Z_0$.

$$Z_p=\frac{Z_2(\frac{1}{2}Z_1-Z_0)}{\frac{1}{2}Z_1-Z_0+Z_2} \quad \text{and} \quad Z_{AB}=\frac{1}{2}Z_1+\frac{Z_2(\frac{1}{2}Z_1-Z_0)}{\frac{1}{2}Z_1-Z_0+Z_2}.$$

If $Z_{AB} = -Z_0$, then $Z_0^2 = \frac{1}{4}Z_1^2+Z_1Z_2$, so that $Z_0 = \sqrt{(Z_1Z_2+\frac{1}{4}Z_1^2)}$. Now if two impedances Z_0 and $-Z_0$ be connected across the network in the input and output respectively as shown in Fig. 6, then at the section XX′,

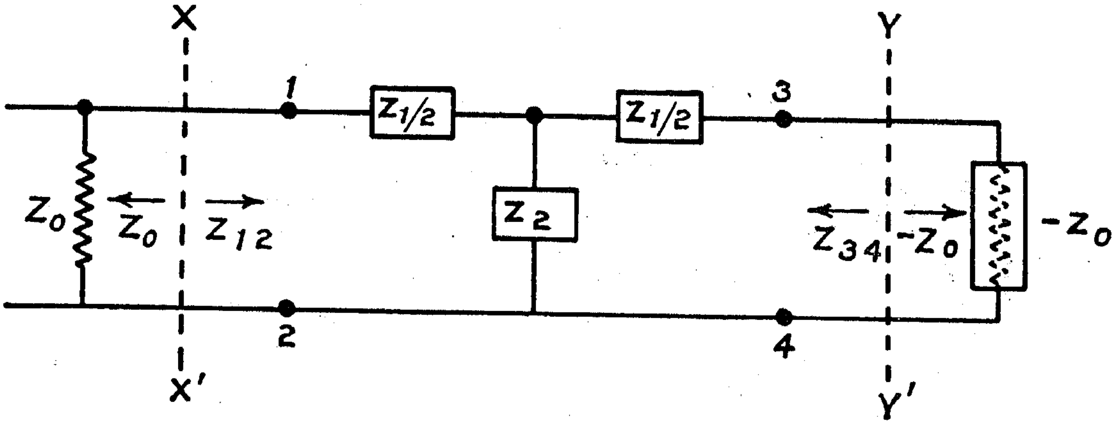

FIG. 6. A filter section fed from Z_0 and terminated in $-Z_0$.

the impedance looking on the left will be Z_o and that looking on the right will be $-Z_o$ and at the section YY′ the impedance looking on the right will be $-Z_o$ and that looking on the left will be Z_o, so that $Z_{12}=-Z_o$ and $Z_{34}=Z_o$. Since, in actual case, the magnitude of the characteristic impedance in the transmission band also undergoes variation, the total reflection loss at any frequency can be obtained from a knowledge of the magnitude of the characteristic impedance at that frequency and that of the negative resistance termination.

If V_s = the sending end voltage, V_R = the receiving end voltage (*i.e.*, across the non-reactive termination Z_o), I_s = the sending end current, I_R = the receiving end current, V_P = the voltage across the parallel impedance and I_P = the current in the shunt arm, then the vector diagram of currents and voltages for normal positive impedance termination may be drawn as shown in Fig. 4(*a*). If V'_s, V'_R, V'_P, I'_s, I'_R and I'_P be the corresponding quantities in case of the section in Fig. 6 with negative impedance termination, then the vector diagram of currents and voltages may be drawn as shown in Fig. 6(*a*).

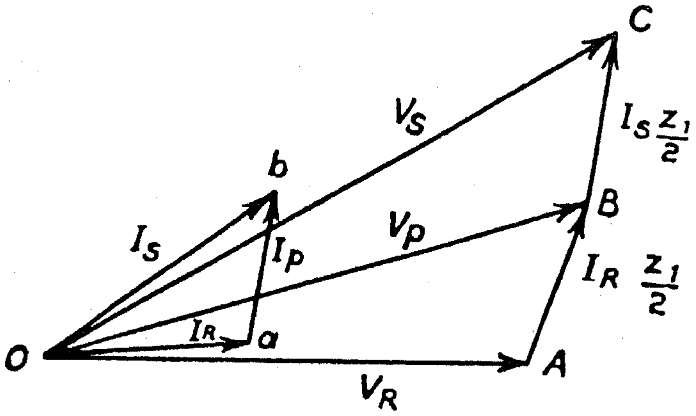

FIG. 4(*a*). Vector diagram of voltages and currents for positive impedance termination.

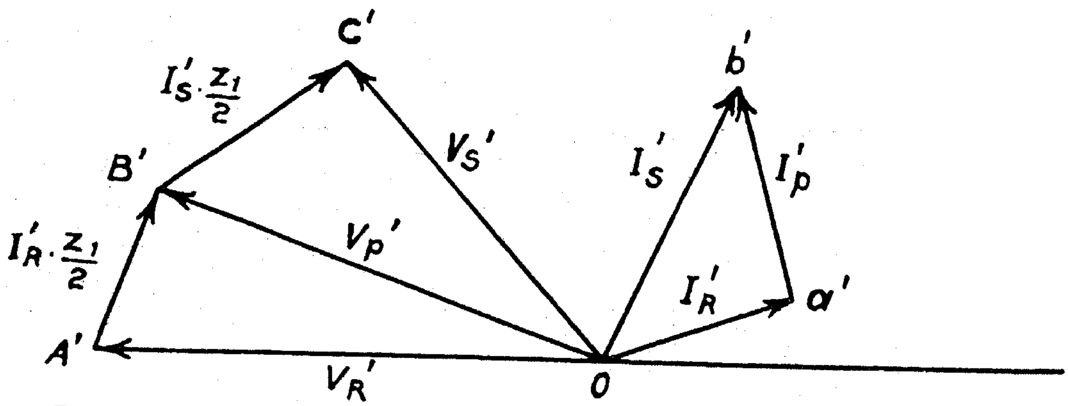

FIG. 6(*a*). Vector diagram of voltages and currents for negative impedance termination.

4. *Experimental.*

Formation of negative resistances.—A screen-grid tube (Marconi S 21) under secondary emission condition in series with a variable positive non-reactive resistance R was employed to obtain "negative resistances" of different values. The circuit arrangement is shown in Fig. 7. The magnitude of this negative resistance and the range of voltage over which it can be used depends upon (*a*) the anode voltage, (*b*) the temperature of the filament, and (*c*) the shape and the material of the electrodes. The filament current (I_f), the anode current (I_a), the screen-grid voltage (V_s) and the anode voltage (V_a) upon which depends the stability of the resistance were maintained constant throughout the experiment. Iron in hydrogen resistance inserted in the filament circuit kept I_f constant.

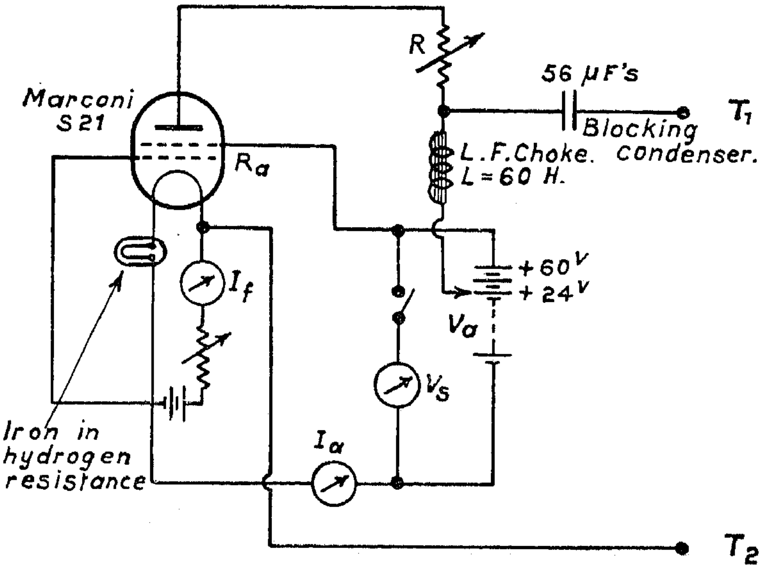

FIG. 7. Negative Resistance Circuit.

The falling portion PQ of the V_a-I_a characteristic shown in Fig. 8 was used for obtaining the negative resistance condition. The magnitude of the negative resistance was given by $\delta V_a/\delta I_a$ (*i.e.*, $-48{,}000\,\Omega$) which was constant whichever point on PQ was selected as the working point. C is a blocking condenser to prevent the direct current from the high tension battery from circulating in the main circuit. The L.F. choke (L) prevents the high frequency currents from circulating in the parallel battery path. The impedances of C and L calculated at various frequencies are shown below in Table I.

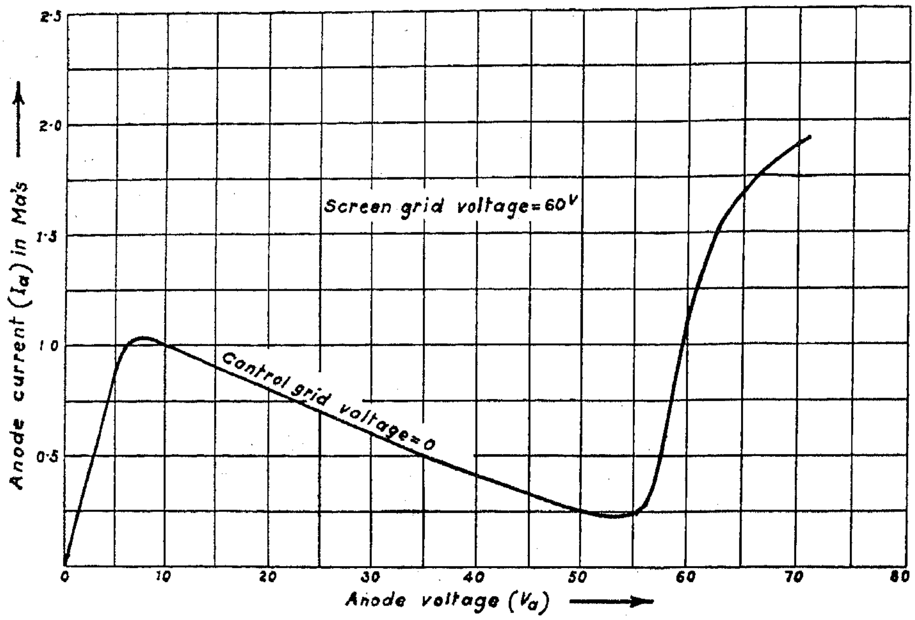

FIG. 8. Va—Ia characteristic of the screen-grid tube (Marconi S 21).

TABLE I.

	Impedance at Different Frequencies		
	100 c.p.s.	3000 c.p.s.	8000 c.p.s.
C (56 μ F's)	28·2 Ω	0·94 Ω	0·2 Ω
L (60 H)	37,600 Ω	11,28,000 Ω	30,08,000 Ω

The resultant negative impedance obtained across the terminals T_1 and T_2 was taken very approximately equal to be a pure resistance of value $-R_a+R$, *i.e.*, $-R'$ ohms. The maximum inclination of the resultant vector obtained with $-R'$ ohms impedance (*i.e.*, -120 ohms) and series condenser impedance of 28·2 ohms at 100 c.p.s. worked out to be about 13°, and at higher frequencies, the angle would be very small.

Measurement of total insertion loss and network attenuation.—For this experiment, a low-pass filter (cut-off frequency $f_c=2700\sim$ and characteristic impedance $Z_0=120\Omega$) and a band-pass filter (cut-off frequencies f_1 and f_2 of 6000 and 8300 cycles per second respectively, frequency of infinite attenuation $f_\infty=8500$ c.p.s. and characteristic impedance$=600\,\Omega$) designed and constructed by the author were used. For measuring the network attenuation, the input and output voltages V_s and V_R respectively were measured by a valve voltmeter with 120 ohms and 600 ohms positive non-reactive resistances in the output of the low-pass and band-pass filters respectively. The attenuation in decibels at any frequency is given by the relation $\alpha = 20 \log_{10} \frac{V_s}{V_R}$. The circuit arrangement is shown in Fig. 9. For measuring the total insertion loss of the filter, the input and output impedances were 120 and -120 ohms respectively in case of the low-pass filter and were 600 and -600 ohms respectively in case of the band-pass filter. The input and output voltages V'_s and V'_R respectively were measured by a valve voltmeter and attenuation at any frequency similarly calculated in decibels. The circuit arrangement for measurement is shown in Fig. 10. Tables II and III show the total insertion loss and the network attenuation at various frequencies for the low-pass and the band-pass filters respectively.

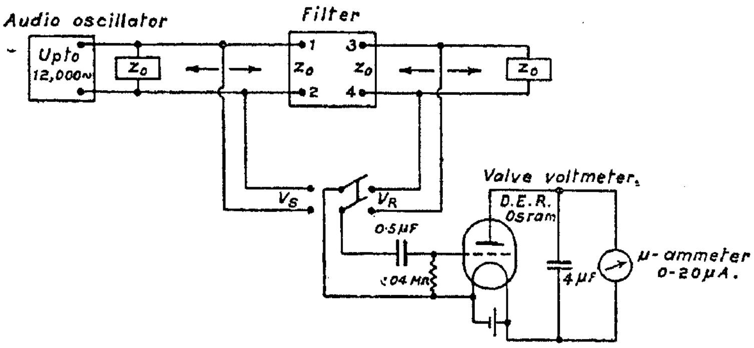

FIG. 9. Measurement of network attenuation.

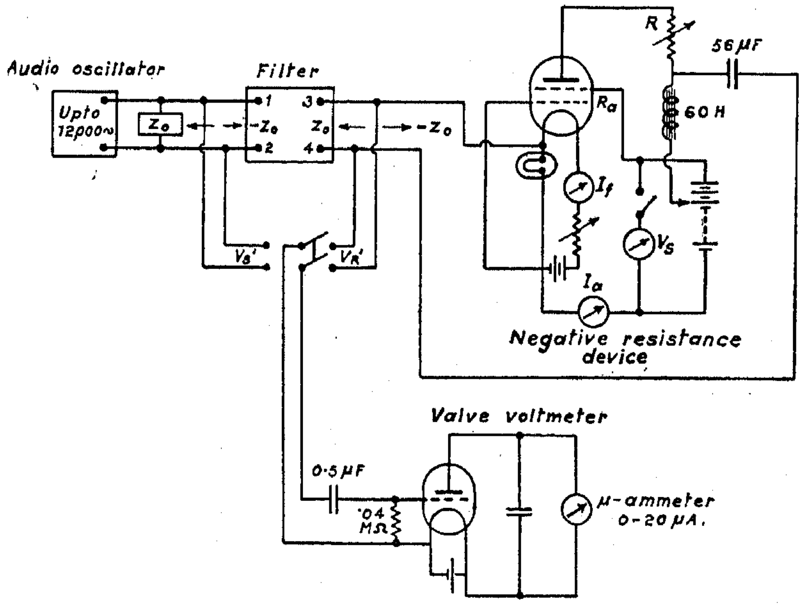

FIG. 10. Measurement of total insertion loss.

TABLE II.

Low-Pass Filter.

Frequency c.p.s.	Termination = 120 ohms		Termination = −120 ohms		Total reflection loss (decibels)
	V_s/V_R	Network attenuation (decibels)	V'_s/V'_R	Total insertion loss (decibels)	
200	1·17	0·68	0·93	− 0·63	− 1·31
470	1·21	0·83	0·74	− 2·61	− 3·44
1010	1·25	0·97	0·29	−10·75	−11·72
1370	1·25	0·97	0·29	−10·75	−11·57
1500	1·26	0·97	0·30	−10·50	−11·47
2000	1·25	0·97	0·31	−10·20	−11·17
2500	1·25	0·97	0·33	− 9·60	−10·55
3000	1·28	1·07	1·03	+ 0·13	− 0·94
3500	1·33	1·24	1·11	+ 0·45	− 0·79
4000	4·40	6·10	1·20	+ 0·82	− 5·28

TABLE III.

Band-Pass Filter.

Frequency c.p.s.	Termination = 600 ohms		Termination = −600 ohms		Total reflection loss (decibels)
	V_s/V_R	Network attenuation (decibels)	V'_s/V'_R	Total insertion loss (decibels)	
5000	1·36	1·34	1·07	+0·59	−0·75
5500	1·31	1·20	0·715	−2·90	−4·07
6000	1·19	0·76	0·86	−1·30	−2·06
6500	1·11	0·45	0·97	−0·30	−0·75
7000	1·13	0·50	0·97	−0·30	−0·80
7500	1·19	0·75	0·97	−0·30	−1·05
8000	1·27	1·04	0·91	−1·00	−2·04
8500	1·46	1·64	1·13	+1·06	−0·58
9000	1·57	1·90	1·18	+1·44	−0·46

Fig. 11 (*a*) shows that, in the case of the low-pass filter, the average attenuation in the transmission band is about 1 decibel, with +120 ohms non-reactive termination, while the average gain is from 6 to 7 decibels with −120 ohms non-reactive termination. The gain of 6 to 7 decibels corresponds to a voltage amplification of 2·0 to 2·24. Further the cut-off is much sharper in the latter case and the amplitude distortion is within the

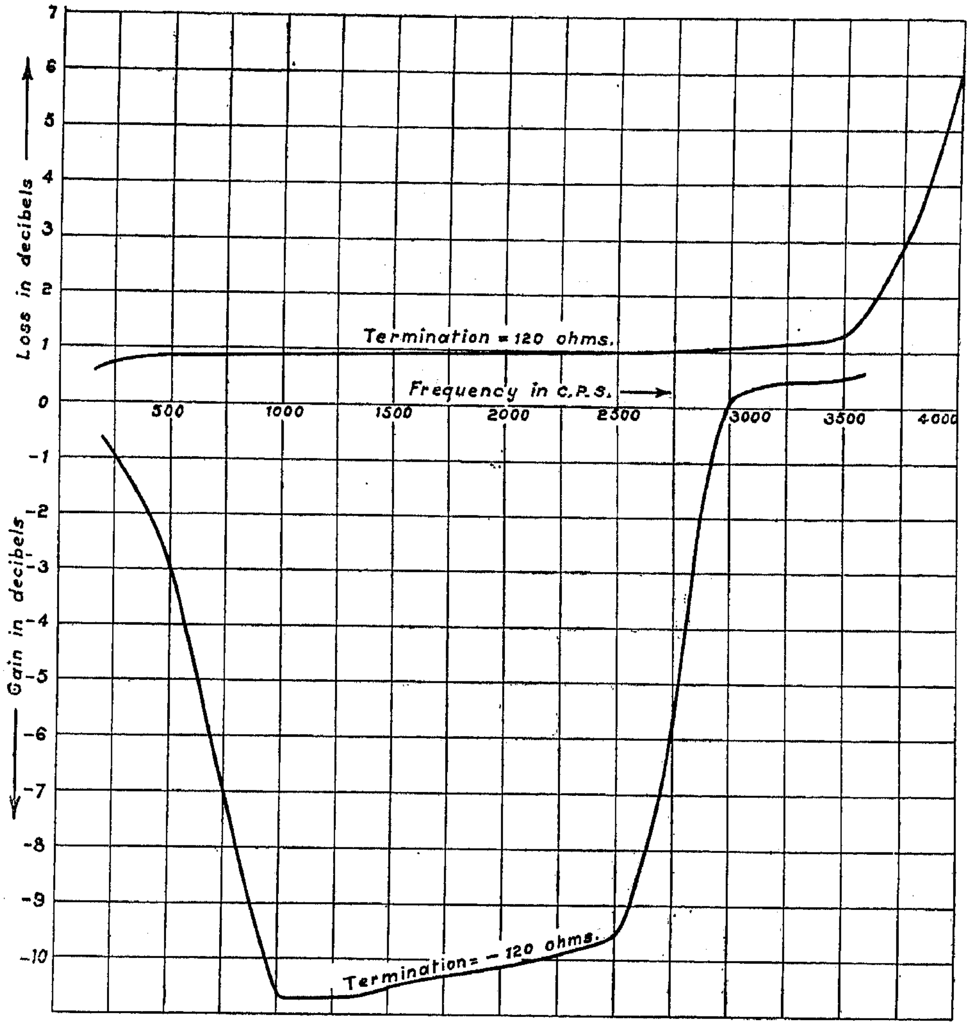

FIG. 11(*a*). Low-Pass Filter Characteristics with Positive and Negative Resistance Terminations.

limits of tolerance as the response at 200~ is 7·3 decibels less and that at 2500 ~ about 1·6 decibels higher than the response at 800 ~ (the mean speech frequency). Fig. 11 (*b*) gives similar conclusions about the band-pass filter. The average gain in case of the band-pass filter is about 1·5 decibels corresponding to a voltage amplification of about 1·2. The cut-off is much sharper and the amplitude distortion is well within the limits of tolerance.

The reflection losses at various frequencies have been obtained by subtracting the network attenuation from the total insertion loss and shown in Figs. 12 (*a*) and 12 (*b*). It is evident from the negative signs that the reflection losses are "reflection gains" in this case. Relative figures for reflection gains at various frequencies calculated from the characteristic impedance and negative impedance values and plotted on the same sheet

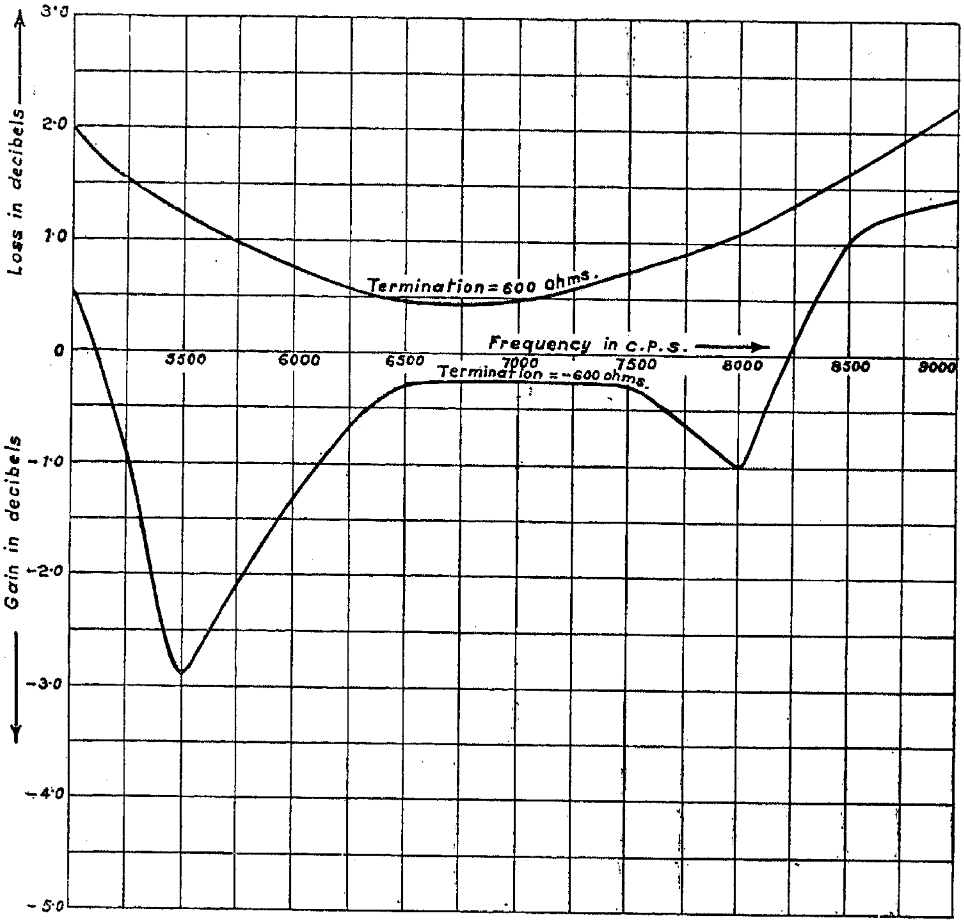

FIG. 11(*b*). Band-Pass Filter Characteristics with Positive and Negative Resistance Terminations.

show an agreement between the theoretical and experimental curves, thereby indicating that reflection of energy has taken place at the junction of positive and negative resistances such that an increase in amplitude has thereby resulted instead of the usual decrease.

5. *Conclusion.*

The following conclusions have been arrived at by the author:—

1. An electrical wave-filter, fed from a source having a non-reactive impedance of Z_o and terminated by the negative resistance $-Z_o$ (where $|Z_o| = \sqrt{Z_1 Z_2 + \frac{1}{4} Z_1^2}$), gives a voltage gain for frequencies in the transmission band instead of the voltage attenuation obtained by the positive resistance termination (Z_o).

2. The rise of the characteristic near the cut-off frequency of the network is much steeper in case of the negative resistance termination and the cut-off is thereby improved.

3. The amplitude distortion introduced in case of negative resistance

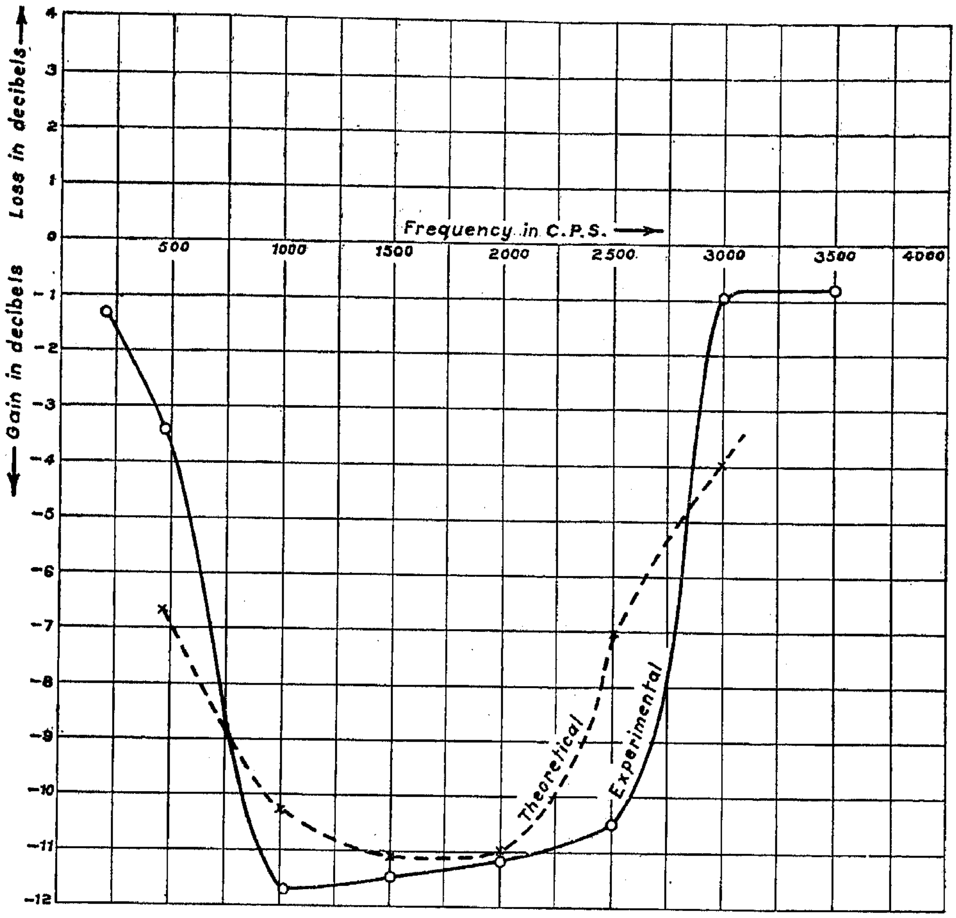

FIG. 12(*a*). Variation of the reflection gain with frequency in case of L.-P. Filter.

terminations is well within the limits of tolerance. Table IV compares the response figures arrived at by the experiment and the figures laid down by the Comité Consultatif International Des Communications Telephoniques a Grande distance (C.C.I.).

TABLE IV.

	Response		
	at 800 ~	at 200 ~	at 2500 ~
* Experimental result (L. P. filter) .. .	0	$-7\cdot3$ decibels	$+1\cdot6$ decibels
C. C. I. limits (Message telephony and Picture telegraphy)	0	$\pm8\cdot7$,,	$\pm8\cdot7$,,

* Figures obtained with reference to the response at 800~ which is taken as zero.

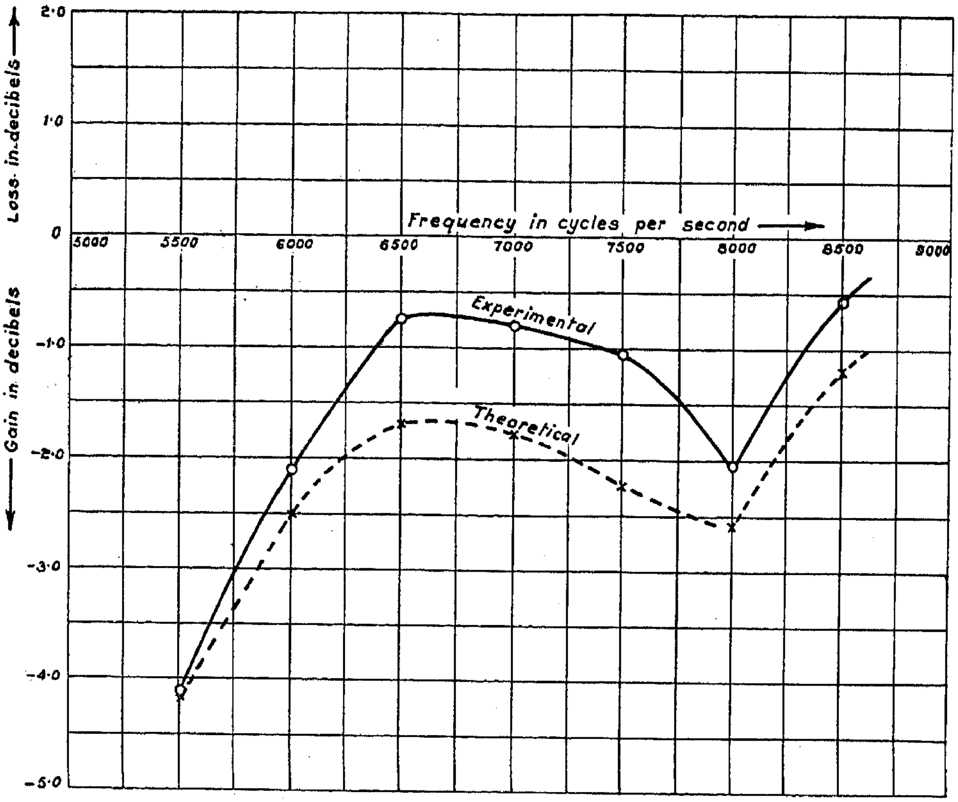

FIG. 12(*b*). Variation of the reflection gain with frequency in case of B.-P. Filter.

6. *Summary.*

Ideal wave-filters should have zero attenuation in the transmission band whereas in commercial filters consisting of several sections this attenuation cannot be reduced below 3 or 4 decibels. In multiplex communication systems, the filter attenuation becomes very large as the frequency bands of a channel have to pass through several filters and a high gain repeater has therefore to be inserted to overcome the total filter attenuation and the circuit loss.

The paper relates to the author's experiments on wave-filters terminated in negative resistances formed by screen grid thermionic tubes under secondary emission condition. He has found out that if a wave-filter be terminated in a negative resistance $-Z_0$ where Z_0 is a pure resistance of the value of the characteristic impedance in the transmission band, a voltage gain of several decibels can be obtained for frequencies in the transmission band, the cut-off improves and the amplitude distortion is well within the limits of tolerance laid down by the C.C.I.

This research is of great value in the field of electrical communications as the same device can be used to filter out a certain band of frequencies and to amplify it at the same time. A no-loss filter can be manufactured by inserting a negative resistance element between any two sections. By employing no-loss or gain filters in the channel, very high grade circuits can be obtained and only a low gain repeater will be necessary.

7. *Appendix.*

Design of the band-pass filter (6000 c.p.s.—8300 c.p.s.)

It is required to design a band-pass filter whose lower cut-off frequency $f_1 = 6000$ c.p.s., upper cut-off frequency $f_2 = 8300$ c.p.s., frequency of infinite attenuation $f_\infty = 8500$ c.p.s. and characteristic impedance $= 600\,\Omega$.

The type of structure chosen is shown in Fig. 13.

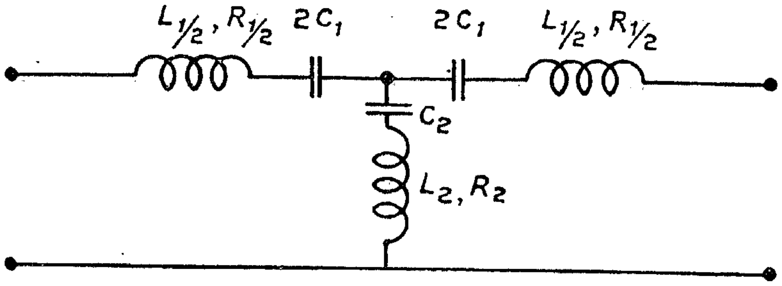

FIG. 13. Structure of the Band-Pass Filter.

The value of m is given by

$$m = \sqrt{1 - \frac{(f_2/f_1)^2 - 1}{(f_\infty/f_1)^2 - 1}} \qquad (1)$$

$$= 0{\cdot}296.$$

$$\text{Hence } L_1 = \frac{Z_o m}{\pi(f_2 - f_1)} \text{ Henries} = 0{\cdot}0248 \text{ Henries} \qquad (2)$$

Therefore $L_1/2 = 0{\cdot}0124$ Henries.

$$C_1 = \frac{f_2 - f_1}{4\pi f_1^2 Z_o m} \text{ Farads} = 0{\cdot}0288\,\mu\text{F's} \qquad (3)$$

Therefore $2C_1 = 0{\cdot}0576\,\mu$F's.

$$L_2 = \frac{Z_o}{\pi(f_2 - f_1)} \times \frac{1 - m^2}{4m} \text{ Henries} = 0{\cdot}064 \text{ Henries} \qquad (4)$$

$$C_2 = \frac{(f_2 - f_1)m}{(f_2^2 - m^2 f_1^2)\pi Z_o} \text{ Farads} = 0{\cdot}0054\,\mu\text{F's} \qquad (5)$$

The values of elements are therefore

$L_1/2 = 0{\cdot}0124$ Henries; $2C_1 = 0{\cdot}0576\,\mu$F's; $L_2 = 0{\cdot}064$ Henries.
$C_2 = 0{\cdot}0054\,\mu$F's.

In constructing the filter, the Gambrell Coils (having inductances as given above) and mica condensers were used. Only one section was employed.

Design of the low-pass filter.

It is required to design a low-pass filter whose cut-off frequency $f_c = 2650$ c.p.s. and the characteristic impedance $Z_o = 120$ ohms.

The type of structure chosen is shown in Fig. 14.

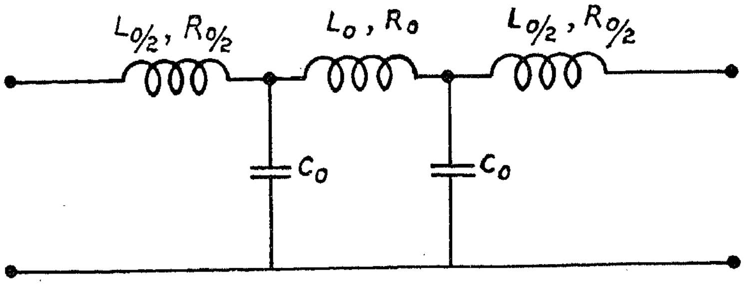

FIG. 14. Structure of the Low-Pass Filter.

Then

$$L_o = \frac{Z_o}{\pi f_c} \tag{1}$$

$$C_o = \frac{1}{\pi f_c Z_o} \tag{2}$$

The cut-off frequency is given by the equation

$$f_c = \frac{1}{\pi \sqrt{L_o C_o}} \tag{3}$$

Hence

$$L_o = \frac{120}{\pi \times 2650} = 14{\cdot}4 \text{ milli-henries, and } \frac{L_o}{2} = 7{\cdot}2 \; mH.$$

$$C_o = \frac{1}{\pi \times 2650 \times 120} = 1{\cdot}0 \; \mu F.$$

Design of coil L_o—

Here $L_o = 0{\cdot}0144$ Henries; $b = 2$ cms.; $c = 1{\cdot}5$ cms.; $a = 2{\cdot}25$ cms.; $R = 3{\cdot}0$ cms. (Fig. 15).

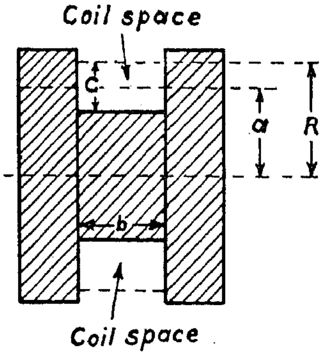

FIG. 15.

Using Brooks and Turner's formula for a multi-layer coil, we obtain,

$$L_o = \frac{4\pi^2 a^2 N^2}{b+c+R} F_1 F_2 \tag{4}$$

where

$$F_1 = \frac{10b + 12c + 2R}{10b + 10c + 1{\cdot}4R}$$

$$F_2 = \tfrac{1}{2} \log_{10} \left(100 + \frac{14R}{2b + 3c}\right)$$

whence $N = 643{\cdot}8 = 644$ turns roughly.

Using 26 S.W.G. D.S.C. wire, the number of turns per layer = 36.
Hence the number of layers = 18. The D.C. resistance $R_0 = 9 \cdot 3$ ohms.

Design of coil $\frac{L_0}{2}$ —

Here $\frac{L_0}{2} = 0 \cdot 0072$ Henries; $b = 2$ cms.; $c = 1$ cm.; $a = 2 \cdot 0$ cms.;
$R = 2 \cdot 5$ cms.

From the same formula, the number of turns N works out to be 477. The number of turns per layer being 36, the number of layers will be 13. The D.C. resistance works out to be 6 ohms.

The filter was constructed in two sections by coils wound in the laboratory and by mica condensers.